Resource Allocation to Maximize Fairness and Minimize Interference for Maximum Spectrum Reuse in 5G Cellular Networks

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***Abstract*—The large number of internet-connected devices will continue to drive growth in data traffic in an exponential way, forcing network operators to increase the capacity of wireless networks. To do so in the cost-effective way a paradigm shift is occurring in 5G cellular networks from high power macro base station to small cell heterogeneous networks known as microcells, picocells, and femtocells. This paradigm shift of 5G cellular networks gives many opportunities ranging from increase capacity to reuse the scarce spectrum resources to coexistence to interference minimization etc. The coexistence of heterogeneous small cells makes the resource allocation, interference management, and maximum fairness among the users more complicated. In this paper, we formulate the resource allocation for spectrum reuse maximization, interference minimization and user level fairness in heterogeneous small cells 5G cellular networks as a NP-hard problem. We design centralized and probability based heuristic for the above resource allocation problem in-order to minimize interference and to achieve maximum spectrum reuse and fairness among the users in feasible computational complexity. We show through extensive network simulations that our proposal outperforms existing centralized interfering model (INT) and distributed random access (DRA) in both low and high-density networks.**

***Index Terms*—LTE-A, 5G cellular networks, Random, Algorithm, Heterogeneous networks.**

I. Introduction

The upcoming 5G technology is expected to have heterogeneous multi-tier deployment of the huge number of femtocells, picocells, and microcells along with macrocell structure [1], [2]. The technique of Small Base Station (SBS) is being developed for efficient spectrum frequency reuse and to reduce load over Macro Base Station (MBS). Deployment of these SBSs such as Femto Access Point (FAP), Pico Access Point (PAC), Micro Access Point (MAP) are an economically better solution rather than deploying more MBSs. These access points are of different capacity and coverage (ranges from few hundred meters to few kilometers) but smaller than macrocells. Microcell is the early form of small cells which used as a complement of macrocells over a remote location, at hotspots etc. Picocells and femtocells are derived from the microcells concept. The coverage area of picocell is reduced up to 30% of microcell. Picocell improves the capacity and coverage area over a hotsopt region. To fulfill the continuous high data demand of indoor and in-vehicle, Third Generation Partnership Project (3GPP) came out with femtocells with further reduced transmit power. Femtocells are cost effective and low powered due to which this technique benefits both operators and users. The technique to deploy femtocells not only manages the traffic but also improves the capacity and coverage for indoor wireless users.

A user can connect in three modes to a small cell. These three access control mechanisms are open access, close access, and hybrid access. In open access mode, whenever a user of the same operator comes in a region of a small cell, get connected to it. In the case of closed access, only the authorized users of the small cell will get connected. The restriction is considered to improved Quality of Service (QoS) for authorized users on the cost of resource sharing. Hybrid access mode is a combination of these two where a limited resource of small cells are available for all users and rest are dedicated to authorized users.

The deployment of small cells faces many technical challenges due to different coverage area, non-uniform nature and random deployment scenario, most notably, resource allocation, interference management, user level fairness and spectrum reuse. A typical deployment scenario of small cell heterogeneous networks (HetNet) is shown in Fig. 1. A femtocell may get interference from picocell, microcell or macrocell and same may happen for any of the cells. The most important question is how to allocate the resources to the small cells and their users so that they can meet the desired performance criteria. Two types of resource allocation methods are defined such as shared spectrum [4], and split spectrum [5] for small cell networks. In the first case, all the small cells including macrocell can use the same frequency band whereas

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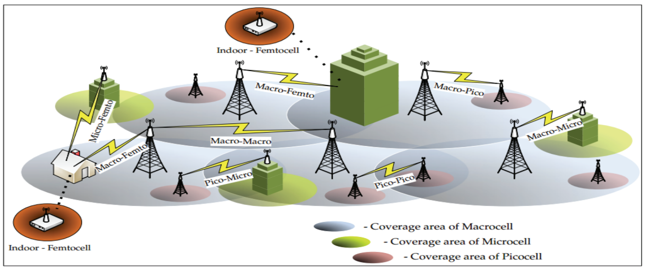


Fig. 1. Heterogeneous small cell network [3].

in second case small cells use different frequency band as the macrocell. The first scheme results in more dynamic resource allocation but the interference among different cells may result in a drastic performance decrease. The split spectrum scheme is a simpler resource allocation and interference management technique but results in less spectrum re-usability scenario and unable to adopt the dynamic nature of the 5G cellular networks.

Due to the random deployment of small cells in an area, the coverage range of neighboring cells get interfered and this will result in the degradation of 5G cellular network's efficiency. As the total available resources are very limited compared to the total number of users in the system, so it becomes very constitutive to make some comparison among the users and respective serving Small cell Access Point (SAP) before assigning actual resources to them. Thus, a priority constraint comes into the function which actually decided based on QoS level. In this regard, we categorize users into two categories i.e., high priority and low priority based on their QoS criteria. In this context, we propose new resource allocation methods that maximize the re-usability of the spectrum, minimize the interference among cells and keep user level fairness, all together in the consideration. In order to achieve the objectives, we convert the heterogeneous network into the link conflict graph $G(V, E)$ as shown in Fig. 2 and then apply the proposed algorithms. The contributions of the paper are summarized as follows.

1) Resource allocation to maximize fairness and minimize interference for maximum spectrum reuse in 5G cellular network is formulated as an optimization problem.
2) Centralized and probability based heuristics are proposed to solve the resource allocation problem.
3) The worst case time complexity of the proposed centralized algorithm is $O(\mathbb{N}^2 + N\mathbb{N}\Delta)$, where $\mathbb{N} = |V|$ is the number of nodes, N is the total number of Physical Resource Block (PRB) and Δ is the degree of $G(V, E)$.
4) Through simulation analysis, we validate the proposed algorithms.

The rest of the paper is organized as follows. Section II gives an overview of related works. Section III describes system model and problem formulation. Section IV introduces our proposed centralized algorithm. Section V presents a probability based heuristic approach to solve resource allocation problem. Section VI describes simulation analysis of proposed algorithms. Section VII concludes this paper and gives the future direction of the work.

II. Related Work

In [6], a centralized algorithm for interference avoidance in OFDMA femtocell networks is proposed. This work allocates the resources to the users based on QoS criteria. In [7], authors have proposed a graph-based multi-cell scheduling framework which incorporates dynamic clustering combined with channel-aware resource allocation to provide QoS based services. In [8], authors used a clustering-based approach to develop a hybrid centralized-decentralized method for resource allocation in OFDMA based femtocell networks. Further, an extended work proposed by same authors in [9] which keeps track of QoS constraint of users and formulate the resource allocation and admission control problem in the femtocell networks as min-max optimization problem. A graph-based clustering resource allocation (GCRA) scheme is given in [10], this includes a coloring algorithm implemented in a distributed way to construct disjoint clusters in which the frequency band can be reused to improve the spectrum efficiency. An adaptive frequency reuse scheme proposed in the work [11]. In this work, authors have proposed semistatic intercell interference coordination scheme (ICIC) that sort out the inter-cell and intracell interference in the networks. In [12], a centrally controlled resource partitioning method is developed based on a graph coloring based approach to stabilizing balance between interference protection and spatial frequency reuse in randomly deployed wireless networks.

In [13], authors have proposed a heuristic based algorithm for multi-cell OFDMA downlink channel-assignment which is done in two phases, at first interference graph is mapped with MAX k-CUT problem and then channel allocation is done by instantaneous conditions. Another approach for the similar problem is proposed in [14], where a graph based branch and bound approach (dynamic graph partitioning) is proposed which is further optimized using the graph-based heuristic algorithm. In [15], a local bargaining approach has been proposed for fairness in spectrum allocation which reduces the complexity by 50% of topology-based optimization technique. In [16], a graph-based distributed method proposed to manage reuse of radio resources among femtocells, achieving a good trade-off between system throughput and user fairness. In [17] interference minimization and resource utilization problem solve using distributed graph coloring based approach. In [18], authors have proposed distributed resource-allocation algorithm (i.e., DRA), which is suitable for medium size networks. Each femtocell locally executes DRA to reserve set of resources using a randomized hashing function. Each femtocell divides the total resources into blocks proportion to the interfering neighbors.

In [19], authors have formulated the resource allocation problem as a max-min optimization problem. Authors have proposed noninterfering (NINT) model based centralized and distributed resource allocation methods in-order to achieve maximum fairness among the users. Further, an interfering

(INT) model has been proposed by the authors based on partitioned greedy graph coloring method. In this approach, in each iteration, maximal independent set pop out from the link conflict graph, get allocated with a color (tile) and these steps continue until each node get a color. Further, colors are re-allocated to each node based on max-min fairness criteria. In [20], spectrum reuse ratio in 5G cellular networks is modeled as the matrix graph-based approach. Authors have designed centralized multi-coloring based approach in-order to minimize interference and maximize the reuse ratio in the 5G cellular networks. These works are not much suitable for an adaptive behavior of heterogeneous small cell networks. It executes the whole steps once again if any change happens in the networks. These existing methods are not much suitable for the heterogeneous 5G cellular networks due to their complex computation and uniform cell structure. The main challenge arises when various small cells of heterogeneous nonuniform structured work altogether. Interference minimization, fairness and allocation of resources for maximum spectrum reuse become more challenging due to nonuniform capacity, cell radius and users association among the small cells. In order to achieve these objectives, we design the concept of user level fairness keeping interference minimum while maximizing spectrum reuse in the 5G cellular networks. Unlike the existing approaches, we have considered multi-tier HetNet (microcell, picocell, femtocell, and macrocell) availability of resources at respective heterogeneous small cells and QoS level issues at the user level. Presence of heterogeneity in capacity and coverage area of different small cells make the problem more challenging. Even in our case, we have considered the heterogeneity at the number of users getting services from different small cells and describe them on the basis of priorities, unlike other existing works.

We propose two approaches in this work, centralized and probability based heuristic, in-order to achieve the maximum efficiency of the resources in 5G cellular networks. To apply the proposed algorithms we need to gather the information of the whole network at a particular central entity. Cloud Random Access network (Cloud-RAN) is a well-accepted candidate cellular network configuration in 5G cellular network [21], [22]. Central entity is a cloud computing center with a pool of available digital processing units i.e., baseband units (BBUs), which can be used to serve the computation tasks that arrive at the cloud center.

III. System Model

In this section, we first define the network model considered in the heterogeneous network and then formulate the resource allocation problem as a multi-objective optimization problem.

A. Network Model

We have considered an OFDMA multi-tier dense heterogeneous networks formed of dense deployment of microcells, picocells, and femtocells served by antenna MAP, PAP, and FAP, respectively. As per OFDMA, PRB is the smallest resource unit that can be assigned to a user. A PRB corresponds to 0.5 ms time and 180 kHz frequency band. A resource frame has 20-time slots derived from frame length of 10 ms [23]. In order to enable the small cell heterogeneous technology, this work targets to come out with a resource allocation strategy that copes up with interference and improves PRB efficiency keeping QoS into consideration. To achieve so, we assumed that MBS is given orthogonal frequency allocation [18], [6], and our proposed analysis consider only, users getting services from MAP, PAP, and FAP. Let the total number of microcells, picocells and femtocells as M, P and F and the total number of users associated with MAP, PAP and FAP are U_M, U_P and U_F respectively. Thus, total number of users in the system be $U = U_M + U_P + U_F$. MAP, PAP, and FAP can reuse the total available N PRBs depending on the interference criteria. The users associated with MAP, PAP and FAP are given as MUE, PUE, and FUE, respectively. All access points MAP, PAP and FAP work at fixed power level i.e., P_M, P_P and P_F, respectively.

In this work, we differentiate users based on QoS criteria. Here, we need to make clear that the differentiation among users is not based on different QoS demands instead, these users are differentiated based on the parameters they are subscribed with the operators. For example, owner of FAP or subscriber of PAP or MAP will get higher priority as compared to other users who have not subscribed. In an urban dense area, it is expected that total demand of small cells exceeds the total available N PRBs in the network. Thus, our objective is to allocate the resources in such wrangling situation which will not only take care of interference, scarcity of resources but also track QoS of users.

B. Problem Formulation

Our objective is to maximize spectrum reuse in the heterogeneous 5G cellular networks while keeping user level fairness, interference and demand of each small cell into consideration. The matrix of PRB allocation is written as follows.

$$T_{(U_M+U_P+U_F)\times N} = \begin{pmatrix} A^M_{U_M\times N} \\ B^P_{U_P\times N} \\ C^F_{U_F\times N} \end{pmatrix} \tag{1}$$

Where, $A^M_{U_M\times N} = [\alpha^m_{i,n}]$, $B^P_{U_P\times N} = [\beta^p_{j,n}]$ and $C^F_{U_F\times N} = [\gamma^f_{k,n}]$. The respective values are given as follows.

$$\alpha^m_{i,n} = \begin{cases} 1, & PRB\ n\ is\ allocated\ to\ user\ i\ associated\ with\ Microcell\ m \\ 0, & Otherwise \end{cases} \tag{2}$$

$$\beta^p_{j,n} = \begin{cases} 1, & PRB\ n\ is\ allocated\ to\ user\ j\ associated\ with\ Picocell\ p \\ 0, & Otherwise \end{cases} \tag{3}$$

$$\gamma^f_{k,n} = \begin{cases} 1, & PRB\ n\ is\ allocated\ to\ user\ k\ associated\ with\ Femtocell\ f \\ 0, & Otherwise \end{cases} \tag{4}$$

To achieve the optimal PRB assignment, we formulate the problem as PRB reuse ratio $\bar{f}$ as follows.

$$\max_{\alpha,\beta,\gamma} \bar{f} = \frac{1}{\sum_{n=1}^{N} f(n)} \left(\sum_{n=1}^{N} \left(\sum_{m=1}^{M} \sum_{k=1}^{U_M} w_{k,n}^{m} \alpha_{k,n}^{m} + \sum_{p=1}^{P} \sum_{j=1}^{U_P} w_{j,n}^{p} \beta_{j,n}^{p} + \sum_{f=1}^{F} \sum_{i=1}^{U_F} w_{i,n}^{f} \gamma_{i,n}^{f} \right) \right) \quad (5)$$

Subject to the constraints:

$$\alpha_{k,n}^{m}, \beta_{j,n}^{p}, \gamma_{i,n}^{f} \in \{0,1\} \quad (6)$$

$$\sum_{m=1}^{M} \gamma_{o,n}^{m} + \sum_{p=1}^{P} \beta_{o,n}^{p} + \sum_{f=1}^{F} \alpha_{o,n}^{f} \leq 1, \forall z : o \in I_z, z \in U \quad (7)$$

$$\sum_{n=1}^{N} \sum_{k=1}^{U_M} \gamma_{k,n}^{m} \leq D_m \quad (8)$$

$$\sum_{n=1}^{N} \sum_{j=1}^{U_P} \beta_{j,n}^{p} \leq D_p \quad (9)$$

$$\sum_{n=1}^{N} \sum_{i=1}^{U_F} \alpha_{i,n}^{f} \leq D_f \quad (10)$$

$$f(n) = \begin{cases} 1, & \left(\sum_{m=1}^{M} \sum_{k=1}^{U_M} w_{k,n}^{m} \alpha_{k,n}^{m} + \sum_{p=1}^{P} \sum_{j=1}^{U_P} w_{j,n}^{p} \beta_{j,n}^{p} + \sum_{f=1}^{F} \sum_{i=1}^{U_F} w_{i,n}^{f} \gamma_{i,n}^{f} \right) \geq 1 \\ 0, & Otherwise \end{cases} \quad (11)$$

The optimization function (5) aims at maximizing the total PRB efficiency over the heterogeneous 5G cellular networks. In this equation weight, w presents the fairness of user taking instantaneous QoS requirement into account. The network constraints are defined in Equations (6)-(10). Equation (6) is a binary variable of Equations (2)-(4). In Equation (7), I_z is the list of interfering UEs, that means the list of UEs connected to interfering small cells. This constraint shows that no two interfering users can use the same PRB. Constraints (8)-(10) confer that the maximum number of PRBs that can be allocated to the users within a small cell i is limited to respective small cells PRBs demand D_i. In the other word, as the maximum capacity of an access point is limited so, the total number of users that can get services from an access point is also limited. Equation (11) gives a function of PRB allocation. If a PRB is allocated to any of user in the heterogeneous 5G cellular network then function value is one otherwise, zero. The formulated problem is mapped to the graph coloring problem where an objective is to maximize the re-usability of colors with the additional set of constraints of QoS and availability of resources at each SAP. Hence, it is an NP-hard combinatorial problem with non-linear constraints [24]. Thus, to solve the formulated problem in feasible time complexity, we propose centralized and probability based heuristic in the following sections.

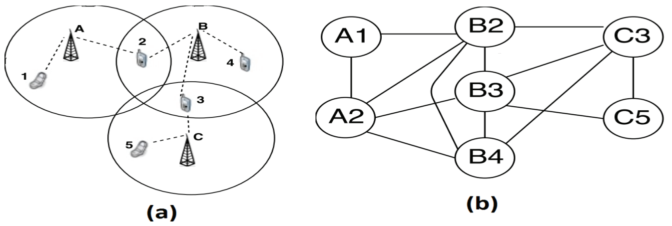


Fig. 2. (a): Heterogeneous topology, (b): Link conflict graph corresponding (a).

IV. A Centralized Algorithm

The input to the algorithm is link conflict graph $G(V,E)$. Each vertex $v \in V$ presents communication link between user $u \in U$ and its corresponding access point $a \in \{M,P,F\}$ i.e., $v = (u,a)$ and $\mathbb{N} = |V|$. If an access point can communicate with a user at more than one power level then the minimum power level allowing the communication is selected. For a user, there may be more than one links corresponding to various FAPs, PAPs, and MAPs. All communication links that cannot be activated simultaneously connected by edges $\in E$ in the link conflict graph $G(V,E)$. Two links may not be activated simultaneously due to interference or conflicting user occupancy. This scenario may occur due to inter-cell or intra-cell interference. Fig. 2a shows a heterogeneous topology and Fig. 2b presents corresponding link conflict graph. In Fig. 2a users 2 and 3 are conflicting users because both lie in the overlapping area of more than one small cells. Construction of link conflict graph is outside the scope of this paper but it can be done in very less time using the techniques given in [19], [25].

Definition 1. *(One-hop view [26]). Let i be a node and τ_i be the set of IDs of one-hop neighbors of i. We say the pair (i, τ_i) the one-hop view of node i.*

We propose a centralized resource allocation algorithm for PRB assignment to the users in the 5G cellular networks. In this algorithm, each vertex $v \in V$ corresponding to the link (u,a) is assigned a priority P_v based on instantaneous QoS requirement given as weight w in the equation (5). Let $R(a_i)$ be the maximum available PRBs at $a_i \in \{M,P,F\}$, which specifies the maximum orthogonal PRBs that can be assigned to respective SAP users.

Input to the centralized algorithm is link conflict graph $G(V,E)$, total available PRBs N, available PRBs at each node $R(a_i)$, users associated with each access point $U(a_i)$, $v.assign = 0$ for each node and priority P_v. Output of the algorithm is PRB allocated nodes. The algorithm is given in two-phase. In Phase 1, highest priority unallocated node is selected (line 2 of Algorithm 1). If two nodes are having same priority then any of them can be selected. If the selected node contains conflicting user u_c it means there are other unassigned PRB nodes those contain same user u_c. In the case of conflicting users, two or more nodes present the same user associated with different SAPs. Thus, it becomes necessary to assign PRB to one of them. Hence, a node with the highest value of $R(a_i) - |U(a_i)|$ will go for Phase 2 and rest of nodes containing conflicting user u_c are assigned with -1 (line 3). In Phase 2 of the algorithm, if total available PRBs at the respective access point is more than zero (line 5) then the smallest unassigned PRB within one-hop view is assigned to the node and respective access point update total available PRBs (lines 6-9). The process terminates when, either each node is assigned with some PRB or total available PRBs at each SAP is exhausted.

Illustrative Example. In Fig. 3, we have shown an illustrative example to execute the steps of centralized algorithm 1. We have taken the link conflict graph of Fig. 2(b). Let the non-increasing priorities of users are $u3 > u1 > u2 > u4 > u5$ (two users may have same priority based on the services they are subscribed with). Accordingly, we can write the priorities of nodes in link conflict graph such as $B3 = C3 > A1 > B2 = A2 > B4 > C5$. Node $B3$ and $C3$ are having same priority because these two nodes present the same user $u3$, likewise node $B2$ and $A2$. Let total available resources at SAPs A, B and C are $R(A) = 3$, $R(B) = 5$ and $R(C) = 2$, respectively. Based on Phase 1 of algorithm we can select the any of highest priorities nodes $B3$, or $C3$. Let us select the node $B3$ and based on line 3 of Phase 1, $C3.assign = -1$ and node $B3$ moves to next Phase 2. Based on lines 5-9 given in Phase 2, $B3.assign = 1$ and $R(B) = 4$. Thus, in iteration 1 (Fig. 3a), $C3.assign = -1$, $B3.assign = 1$, $R(A) = 3$, $R(B) = 4$ and $R(C) = 2$ (no updation took place in total available PRBs of SAPs A and C because these SAPs did not spent any of its PRB). In 2nd iteration Fig. 3b node $A1$ is selected in Phase 1 and assigned with smallest unassigned PRB in neighborhood in Phase 2 i.e., $A1.assign = 1$, $R(A) = 2$, $R(B) = 4$ and $R(C) = 2$. In 3rd iteration (Fig. 3c) node $B2$ and $A2$ are selected, $A2.assign = -1$ and node $B2.assign = 2$, $R(A) = 2$, $R(B) = 3$ and $R(C) = 2$ based on Phase 1 and Phase 2 of Algorithm 1 as described in above iteration 1. In 4th iteration (Fig. 3d) node $B4$ selected and $B4.assign = 3$, $R(A) = 2$, $R(B) = 2$ and $R(C) = 2$. Similarly, in 5th iteration (Fig. 3e) node $C5$ selected and $C5.assign = 2$, $R(A) = 2$, $R(B) = 2$ and $R(C) = 1$. This resulted PRBs assignment satisfies the set of constraints given in problem definition (5).

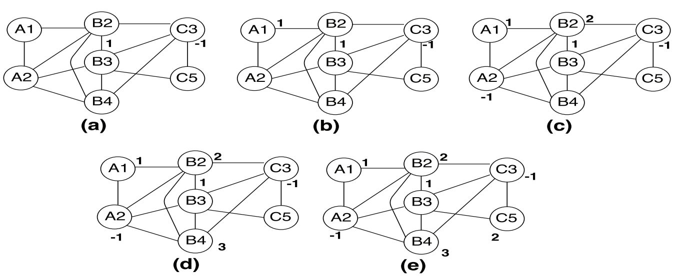


Fig. 3. Illustrative Example. (a): Iteration 1, (b): Iteration 2, (c): Iteration 3, (d): Iteration 4, (e): Iteration 5.

Algorithm 1 Centralized Resource Allocation Algorithm

Input: $Link\ Conflict\ Graph\ G(V, E)$, Set of PRBs N, $R(a_i)$, $U(a_i)$, $\forall a_i \in \{M, P, F\}$, $v.assign = 0$ and P_v $\forall v \in V$
Output: PRB allocated nodes
Repeat
1: **Phase 1: Deciding priority**
2: Select the highest priority (P_v) node $v \in V$ such that $v.assign = 0$
3: If selected highest priority node contains conflicting user u_c then among all nodes containing user u_c, the one with maximum value of ($R(a_i)$-$|U(a_i)|$) only retains and rest are assign with -1 /* $U(a_i)$ is the users associated with access point a_i and it did not receive any PRB */
4: **Phase 2: PRB Assignment**
5: If $R(a_v) > 0$ then /* Available resource corresponding to highest priority node v */
6: for n=1 to N
7: If ($\tau_v.assign \neq n$) where, τ_v is one-hop neighbors of node v
8: $v.assign = n$
9: $R(a_v) = R(a_v) - 1$
Until $\forall v \in V$ such that $v.assign \neq 0$ or $R(a_v) = 0$ /*PRBs can not be allocated any further */

A. Centralized Algorithm Analysis:

In this subsection, we analyze time complexity, message complexity and correctness proof of proposed algorithms.

Theorem 1. *The time complexity of the proposed centralized resource allocation algorithm is $O(\mathbb{N}^2 + N\mathbb{N}\Delta)$.*

Proof. The lines 1-9, given in centralized algorithm 1 executed for each vertex in the graph. The time complexity of line 2 and 3 are $O(\mathbb{N})$ and $O(\Delta)$ respectively, where Δ is the degree of link conflict graph. Line 5 takes constant time. The time complexity of lines 6-9 is $O(N\Delta)$. Phase 1 and Phase 2 execute for each node, thus total time complexity of centralized Algorithm 1 is $O(\mathbb{N})(\mathbb{N} + \Delta + N\Delta) = O(\mathbb{N}^2 + N\mathbb{N}\Delta)$. □

Theorem 2. *The proposed centralized resource allocation algorithm guarantees valid PRB assignment in the networks.*

Proof. The execution of the centralized Algorithm 1 ensures, $v.assign \neq 0$ for all $v \in V$ in $G(V, E)$. As it is given in the algorithm 1, before selecting PRB from the PRB pool set, a node scrutinize the total number of available PRBs at the respective access point and their user's priorities. A node is only allowed to select PRB $n \in N$ if $R(a_v) > 0$ and its priority is higher or equal to the other unassigned nodes. Thus, this statement validates the total PRBs availability at the respective SAP and users priorities. In order to ensure the assignment interference-free, node v assigns with the PRB n such that $v.assign = min\ n \notin \{\tau_v.assign\}$ and this validates the interference-free PRB assignment. Thus, centralized Algorithm 1 ensures valid PRB assignment in the networks. □

In the following, we show that proposed algorithm satisfies the demand of all access points a_i with the objective function given in Equation (5). The proposed algorithm only fails when the total available PRBs are exhausted but some access points with positive demand remain. From the proposed algorithms we found out the number of times a PRB is used in the whole network defined as the re-usability of a PRB n_i as follows.

$$L_i = \begin{cases} \frac{D}{N} + 1, & if\ i \leq D\%N \\ \frac{D}{N}, & if\ i > D\%N \end{cases} \quad (12)$$

Where $D = D_m + D_p + D_f$ be the total demand of PRBs in the 5G cellular network. In order to show that the proposed algorithm will satisfy the demand D, we prove following lemma. Let d_i^a and D_i represent the demand of access point a and total demand before the ith PRB has been assigned, respectively. For simplicity, we consider the value of $D\%N$ as zero.

Lemma 1. *The PRB n_i is assigned to L_i SAPs with positive demand in the i^{th} iteration. Moreover, for every SAP a, $d_i^a \leq N - i + 1$ and $L_i \leq D_i(N - i + 1)^{-1}$.*

Let us consider a dense link conflict graph (complete graph structure i.e., an edge between each pair of nodes). In this case, the centralized algorithm will take $O(N\mathbb{N}^3)$ time complexity for resource allocation. The main reason for this complex time complexity is because central entity looks priorities of rest of nodes and smallest unallocated PRB in one-hop view before assigning a PRB to any node. However, if central entity facilitates to each node to select PRB probabilistically it will lead to faster termination. Considering these scenarios we propose a probabilistic heuristic to solve the worst case condition of PRB assignment named as Probability based Heuristic (i.e., PBH) in the following Section.

V. Probability based Heuristic

In this model, nodes are assigned with PRBs in a probabilistic way. The central entity maintains the rank γ of each node in the networks. Rank is decided based on priorities of nodes, corresponding access points connection and their available resources as given in Phase 1 of centralized Algorithm 1. If the outcome of Phase 1 of Algorithm 1 gives two nodes at same priority then any of them can be ranked higher. The central entity maintains a non-increasing order of nodes based on γ value. Highest priority node gets rank 1 and next higher will get rank 2, and so on. Let rank of a node Q be γ and the smallest unassigned PRB is n such that $1 \leq n \leq N$. If $\gamma = 1$, then Q assigns with PRB n, else node Q assigns with PRB $n + \gamma - 1$ with probability $p > 0$. If the randomly assigned PRB gets interference (constraint given in Equation 7) from any one-hop view node, then central entity will reassign the new PRB $n' + c$ to this node with probability p_c, computed according to a truncated geometric distribution such that $\sum_{c=1}^{N} p_c = 1$. Here, n' presents highest PRB index assigned to any node in the graph and $c \in N$. Let $\pounds < 1$, node Q and its one-hop neighbors consists of Z nodes. $p_c = p\pounds^{|c-\gamma|}$, if $|c - \gamma| \leq Z - 1$, otherwise, $p_c = 0$. Thus, we can write the following equation.

$$\sum_{c=1}^{N} p_c = \sum_{c=1}^{N} p\pounds^{|c-\gamma|} \quad (13)$$

The above equation can be written as follows.

$$\begin{aligned} \sum_{c=1}^{N} p_c &= \sum_{c=1}^{\gamma-1} p\pounds^{c-\gamma} + p\pounds^{0} + \sum_{c=\gamma+1}^{N} p\pounds^{\gamma-c} \\ &= p(\pounds^{\gamma-1} + \pounds^{\gamma-2} + ... + \pounds^{1} + \pounds^{0} + \pounds^{1} + \pounds^{2} + \\ &\qquad ... + \pounds^{N-\gamma}) \\ &= p\left(\pounds\frac{1-\pounds^{\gamma-2+1}}{1-\pounds} + \frac{1-\pounds^{N-\gamma+1}}{1-\pounds}\right) \\ &= p\left(\frac{\pounds - \pounds^{\gamma} + 1 - \pounds^{N-\gamma+1}}{1-\pounds}\right) = 1 \end{aligned} \quad (14)$$

As p is the probability so it must lie between zero and one. To prove the validity we give a statement, as the value of $\pounds$ lies between 0 and 1. Thus, from Equation (14) we can directly say that probability $p > 0$. We can write $\pounds^{\gamma-1} + \pounds^{N-\gamma} < 2$ based on $\pounds$ value. Further, solving this en-quality we can write $0 < p < 1$ and this satisfies probability constraint.

Illustrative Example. Fig. 4 shows the execution of probability based heuristic. Fig. 4a shows the ranks assigned to nodes based on users priorities and available resources at each access point. We have considered priorities and number of available resources at each access point same as considered in the above example Fig. 3. Ordering of nodes according to the rank is given such as $B3 > A1 > B2 > B4 > C5$. Rank of $C3 = A2 = 0$ shows that these nodes present conflicting users and, with the comparison of their corresponding access points (line 3 of Algorithm 1), node $B3$ and $B2$ are assigned with higher rank. Fig. 4b shows the initial PRB assignment where higher ranked node $B3$ assigned with PRB 1, node $A2$ and $C3$ with -1. In the next iteration, each node selects their PRBs probabilistically as shown in Fig. 4c. For instance node $A1$ selects PRB $n + \gamma - 1 = 1 + 2 - 1 = 2$ with probability p. The assigned PRB at each node satisfies all the constraints mentioned in the problem definition. If somehow any node does not satisfy the interference constraint after PRB assignment, then corresponding SAP inform to the central

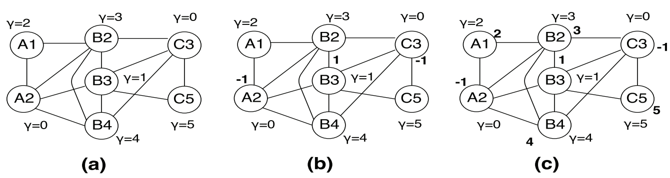


Fig. 4. Illustrative Example. (a): Rank assignment, (b): Initial PRB assignment, (c): Probability based allocation.

entity and central entity will re-assign the PRB $n' + c$ with p_c probability as described in the above heuristic.

A. Analysis of Probability based Heuristic

To maximize the total re-usability of PRBs it can be observed, nodes of each individual PRB form an independent set, finding a minimum PRB assignment can equivalently formulated as finding minimum number of independent sets such that each node is contained in at least one independent set. Following this observation, we map the PRB allocation as an antisymmetric relation [27] on the ID set $[\mathbb{N}]$. We are not considering don't care state nodes. Let probability based heuristic assigns a PRB from N to one-hop view (v, τ). For each PRB $n \in N$, there is a relation $\rhd_n$ s.t., $\forall v, x \in [\mathbb{N}]\ v \not\rhd_n x \vee x \not\rhd_n v$. A PRB n can be assigned to one-hop view (v, τ) if and only if $\forall x \in \tau : v \rhd_n x$. Let $\psi_n(v) := \{x \in [\mathbb{N}] : v \not\rhd_n x\}$ be the set of nodes that must be nonadjacent to a node v assigned with PRB n. We select v_r uniformly at random from set $[\mathbb{N}]$. Rest $\mathbb{N} - 1$ are independently included to a set τ_r with probability p. For a PRB n, consider η_n be the event that $\tau_r \cap \psi_n(v_r) \neq \phi$. In the other words ψ_n is an event that PRB n can not be assigned to a randomly chosen (v_r, τ_r).

Lemma 2. *Randomly selected one-hop view nodes can not get one of n PRBs in N is bounded by* $p\left(\bigcap_{n \in N} \eta_n\right) \geqslant \prod_{i=1}^{N} \left(1 - 1/e^{|\psi_n(v_r)|p}\right)$.

Similar to the above analysis we find out the probability that a UE will get a PRB if satisfy some set of constraint. In order to do so, let $\mathbb{P}_n$ be the probability that a node can get a PRB using the probability based heuristic.

Lemma 3. *Probability $(\mathbb{P}_n)$ that each node can be assigned with PRB using probability based heuristic should satisfy* $\mathbb{P}_n = \sum_{i=1}^{N} (-1)^{i-1} \binom{N}{i} (1 - i\mathbb{P}_n/N)^{\Delta}$.

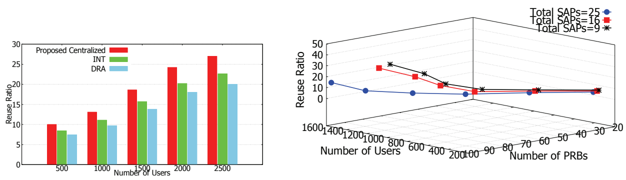
VI. Simulation and Results

In this section, we present the simulation environment and results obtained from the proposed algorithms. We have modeled the simulation environment according to 3GPP standard given in [28] and [29]. Small cells are deployed under the coverage area of the macro base station. Transmit power of MBS, MAP, PAP, and FAP are considered as 40 W, 5 W, 1 W, and 0.1 W respectively. Carrier frequency, system bandwidth, modulation scheme and number of PRBs are considered as 2 GHz, 20 MHz, 64 QAM and 100 respectively as per 3GPP standard. Transmission time interval (TTI) and radio frame length in physical channel are taken as 1 ms and 10 ms respectively. A square region of 100 m x 100 m is considered in the simulation. The macro base station is deployed in the center. FAP, PAP, and MAP are randomly deployed underlaying macrocell region. The radius of FAP, PAP, and MAP are considered as 15 m, 30 m and 50 m respectively. We have considered four priorities levels i.e., $P_1 > P_2 > P_3 > P_4$ which shows the priorities of users subscribes with femtocells, picocells, microcells and with none of any small cell, respectively. Weight w (i.e., priority P) is considered as 4, 3, 2, and 1 for users subscribed with femtocells, picocells, microcells and with none of any small cell, respectively. If a user is not subscribed with any of small cell then its priority will be considered at the lowest level P_4. We compare our proposed algorithms with the existing competitive works [18] and [19] based on our objective of re-usability (reuse ratio) formulated in Equation (5) and set of constraints given in Equations (6)-(10). We also give a comparison based on Jain fairness index [30] to the existing competitive works.

TABLE I
SIMULATION PARAMETERS

Parameters	Details
System bandwidth	20 MHz
Carrier frequency	2 GHz
Transmit power of Micro access point	5 W
Transmit power of Pico access point	1 W
Transmit power of Macro base station	40 W
Transmit power of Femto access point	0.1 W
Modulation	64 QAM
Number of users per access point	{1,2,..100}
Number of PRBs	100
TTI	1 ms
Radio frame length in physical channel	10 ms
Small cells layout	circle
Subcarrier Spacingl	15 kHz

Real Data Analysis. We have evaluated the performance of proposed algorithm using the real data set of Wireless Topology Discovery project [31]. This data set contains information of approximately 275 PDA users for a period between September 22, 2002 and December 8, 2002. Each active UE records every 20 seconds all the WiFi (APs) that are detected by its device. We consider SBSs for WiFi APs as in [32]. This approximation is reasonable for SBSs, because the radius of WiFi APs is ~ 100 meters. We have set the time slot duration of 20 seconds. In each time slot a new LCG is generated. As a user can get signal from more than one APs at a time thus, a user may represent more than one node in LCG as discussed in Fig 2. We have taken a random priorities of each user between 1 and 4. We compared the average number of required PRBs with respect to days. We found out, on October 16, 2002 the average required PRBs are more compared to any other days. This is because, it was the busiest day and most of the users activated at the same time thus, it requires more number of PRBs to be allocated.



(a) (b)

Fig. 5. (a) Comparison w.r.t. reuse ratio and number of users. (b) Comparison among number of users, PRBs and reuse ratio w.r.t. number of SAPs.

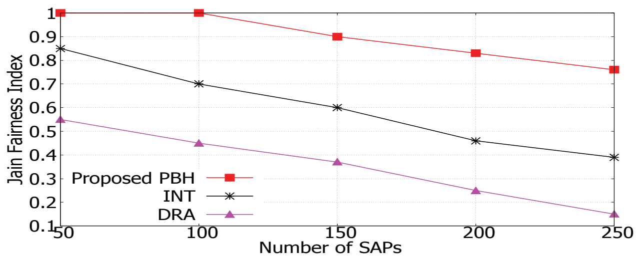


Fig. 6. Performance comparison w.r.t. Jain Fairness Index.

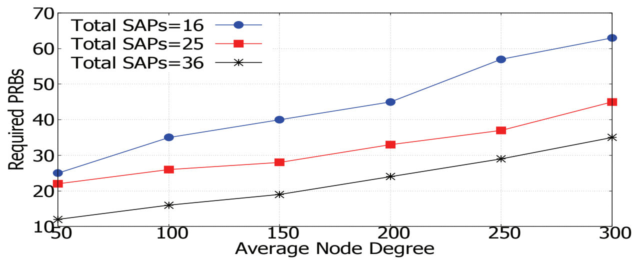


Fig. 8. Comparison between average node degree and the minimum number of required PRBs w.r.t. total access points.

Reuse Ratio Analysis. In Fig. 5a, we have shown a comparison with the existing works. The deployment probabilities of femtocells, picocells, and microcell are considered as 0.3, 0.4 and 0.3 respectively. The reuse ratio of propose algorithms is better than the existing DRA [18] and INT [19] algorithms by 26% and 16% respectively on an average. The improvement is expected because, in DRA algorithm, resource allocation is done on per femtocell basis without considering the user's density. We got better performance comparing the INT algorithm because, as in [19] users are deployed within femtocells region and when these users are deployed within the heterogeneous region the chances of interference becomes very less because of non-uniformity among the femtocells, picocells, and microcells.

We have shown a relative comparison among the total number of users, total number of access points, minimum number of required PRBs and reuse ratio in Fig. 5b. The deployment probabilities of femtocells, picocells, and microcells are considered as 0.2, 0.4 and 0.4 respectively. We can see from the result, the number of access points play a very important role in the reuse ratio of PRBs. Reuse ratio of 25 access points is less than the reuse ratio of 16 and 9 access points for the same number of users in the network. The reason behind this finding is the increase of interference among the access points. An increase of access points in the fixed region causes higher interference in the networks and this results in lower reuse ratio of PRBs.

Fairness Analysis. We obtain better Jain fairness index of proposed algorithms comparing to the existing works [18], [19]. As it can be seen from the Fig. 6, when the number of access points is less, chances of interference is less, as the number of access points is increasing the fairness decreases due to the increase in the interference. The reason for better improvement of fairness comparing to DRA is that resource allocation in DRA is done on access point level whereas in our case it is done at the user level. The reason for better fairness comparing to INT model is that, in INT model although the resource allocation is done at user level but while allocating the resources to the users it does not consider the total available resources at respective access points and their priorities. In our case, we always check the total available resources at the access point with respect to the number of users and their priorities and this results in better fairness.

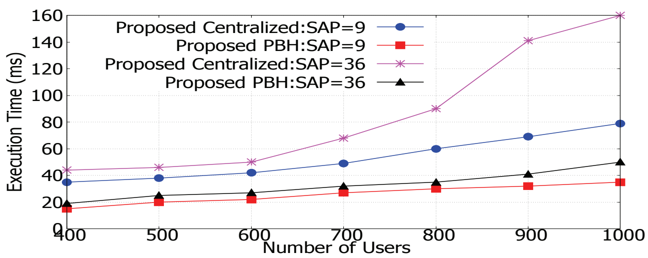


Fig. 7. Execution time comparison.

Execution Time Analysis. Fig. 7 presents a relative comparison between proposed centralized and probabilistic based algorithms against the number of users and execution time. The deployment probabilities of femtocells, picocells, and microcells are considered as 0.5, 0.2 and 0.3 respectively. An observation from the result is that with an increase of the number of access points in the network total execution time increases for the same number of users. This is because with the increase of access points the total number of nodes increases and this takes more time to assign valid PRBs in the network. The rate of increase in execution time with respect to the number of small cells and users in PBH is far less comparing to the centralized algorithm. This is because unlike centralized algorithm, in PBH each node selects PRB probabilistically once the rank assignment is done and this results in faster termination of the PBH.

PRB Analysis. Fig. 8 presents a comparison among the minimum number of required PRBs, average node degree and total access points in the network. The deployment probabilities of femtocells, picocells, and microcells are considered as 0.3, 0.3 and 0.4, respectively. As it can be observed from the result, with the increase of average node degree total number of required PRBs increases. On the other hand, when we increase the total number of access points in the network on the same average degree, the required number of PRBs also increases. The reason behind this, with the increase of access points on the same average node degree, total number of users increases in the system and degree of link conflict graph go high and this result in more number of required PRBs.

VII. Conclusion and Future Works

In this paper, we have formulated the joint resource allocation, interference minimization, user level fairness for maximum spectrum reuse in 5G heterogeneous small cell networks. We have presented centralized and probability based heuristics those assign PRBs to the users with the objective of minimizing interference, maximizing spectrum reuse and keeping the user level fairness into the account. We have proved the validity and correctness of algorithms through theoretical and simulation analysis.

The proposed algorithms depend on a central entity and this strategy may create problem during any disaster or natural calamity or it may be possible due to central point failure whole system stop working. To avoid such unavoidable scenario there is a need that each node monitors the resource allocation process locally and put very less or no dependency on the central point. This scenario will help to offload the overhead of central point and make the resource allocation, faster and robust. Inspired by such scenarios, in the future, we will extend our proposed centralized approach to the distributed algorithm. The distributed algorithm will differs from the centralized algorithm in the following facet. In the distributed approach multiple nodes will decide their resources, simultaneously. In other words, nodes which are more than one-hop away will take their decisions over the set of available PRBs, simultaneously. As the interference constraint depends on one-hop view nodes, so each node will make the decision over a particular PRB depending on one-hop view scenario. Thus, each node will look at one-hop view nodes and make the decision accordingly. This strategy will dynamize and homogenize the resource allocation process in the heterogeneous 5G networks.

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