

CSI 2103: Data Structures

HW Assignment 3

Yonsei University

Spring 2022

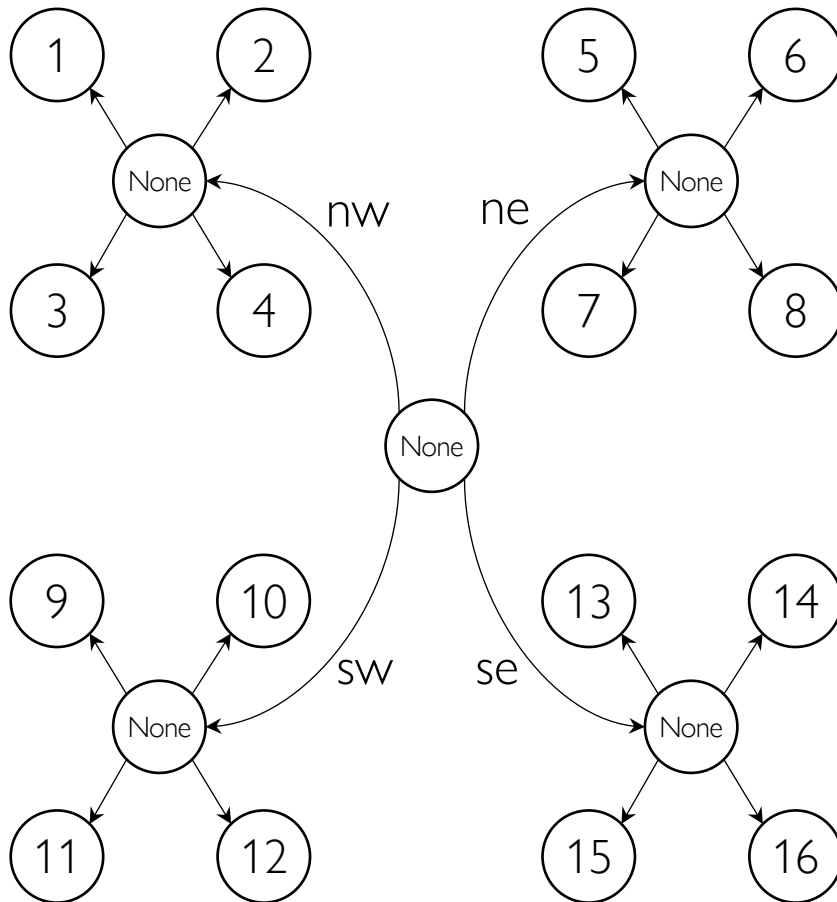
Seong Jae Hwang

Quadtree from Array for HW3



Quadtree

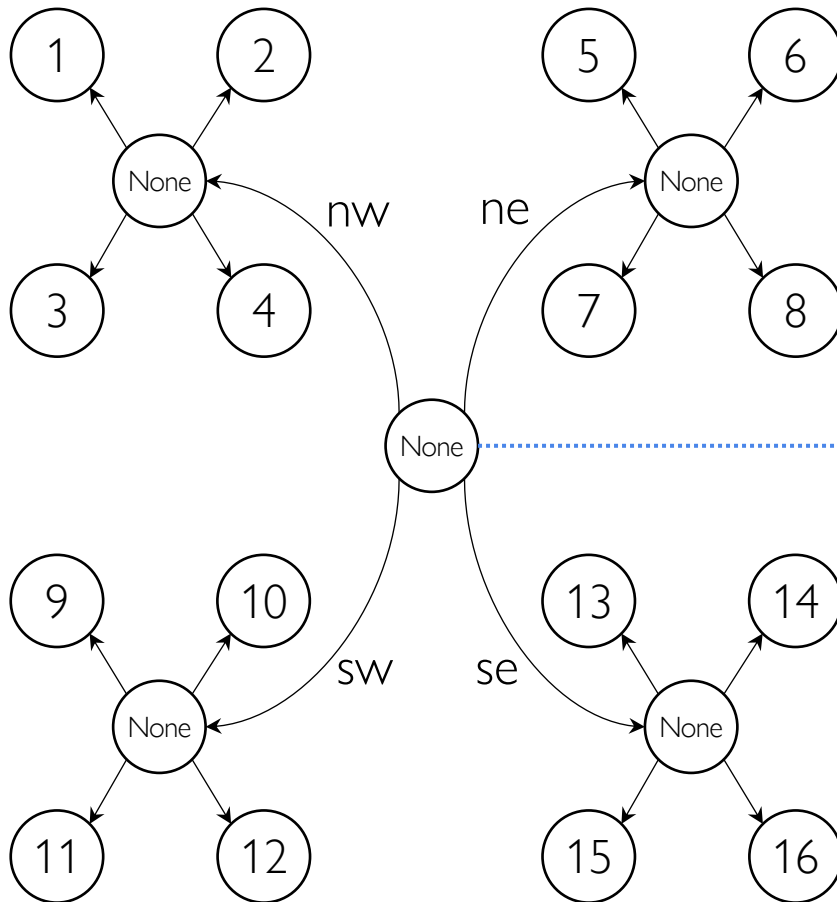
- At most 4 children
- Our quadtree is always full



	column			
	x = 0	x = 1	x = 2	x = 3
y = 0	1	2	5	6
y = 1	3	4	7	8
y = 2	9	10	13	14
y = 3	11	12	15	16

Quadtree

- At most 4 children
- Our quadtree is always full



	column			
	x = 0	x = 1	x = 2	x = 3
y = 0	1	2	5	6
y = 1	3	4	7	8
y = 2	9	10	13	14
y = 3	11	12	15	16

color: None
level: 0

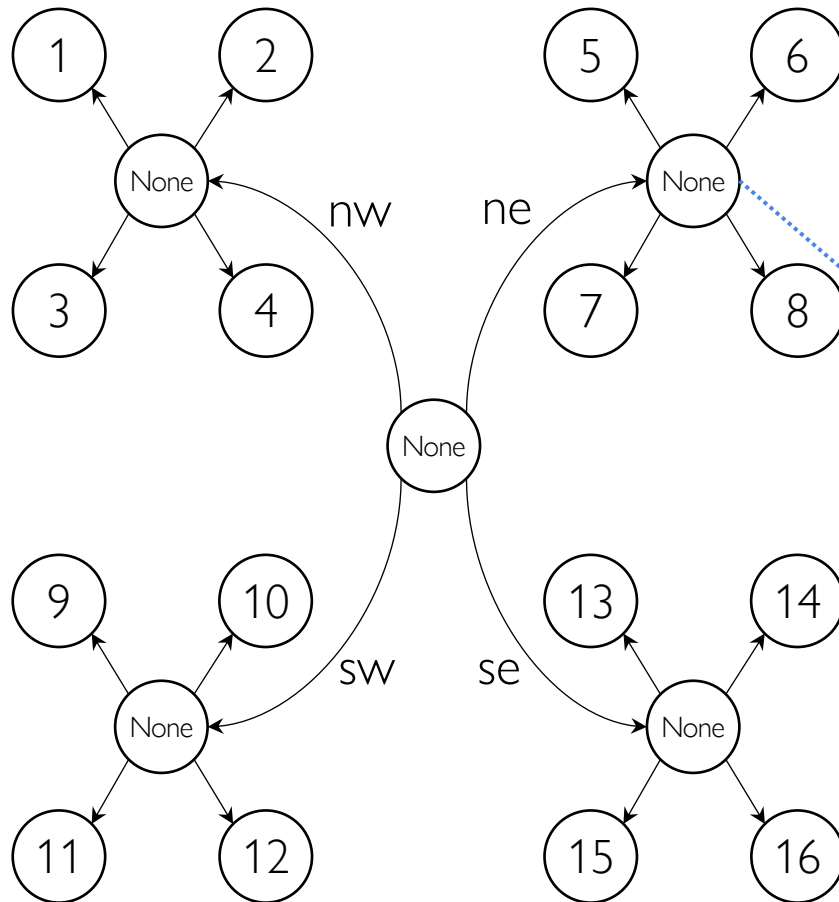
4 children pointers:
{nw, ne, sw, se}

Image area corners:
upleft_x: 0
upleft_y: 0
downright_x: 3
downright_y: 3

Subtree rooted at qt.root covers the **entire image area** defined by a box with upleft corner at (ul_x=0, ul_y=0) and downright corner at (dr_x=3, dr_y=3)

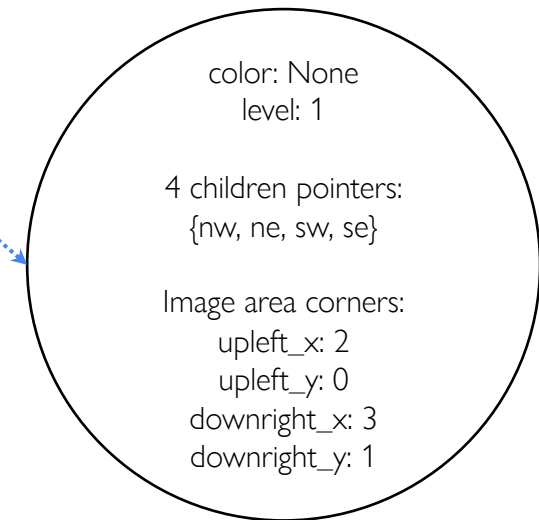
Quadtree

- Each child covers its corresponding **quadrant** with respect to the parent



row

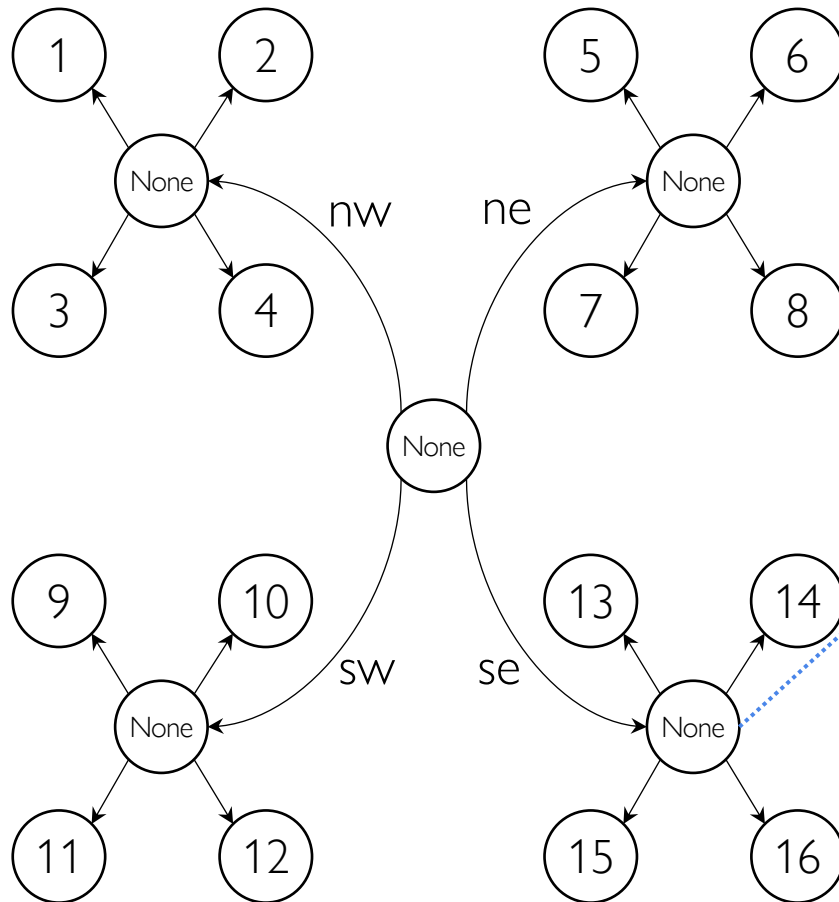
	column			
	x = 0	x = 1	x = 2	x = 3
y = 0	1	2	5	6
y = 1	3	4	7	8
y = 2	9	10	13	14
y = 3	11	12	15	16



Subtree rooted at root.ne covers the **northeast quadrant** defined by a box with upleft corner at (ul_x=2, ul_y=0) and downright corner at (dr_x=3, dr_y=1)

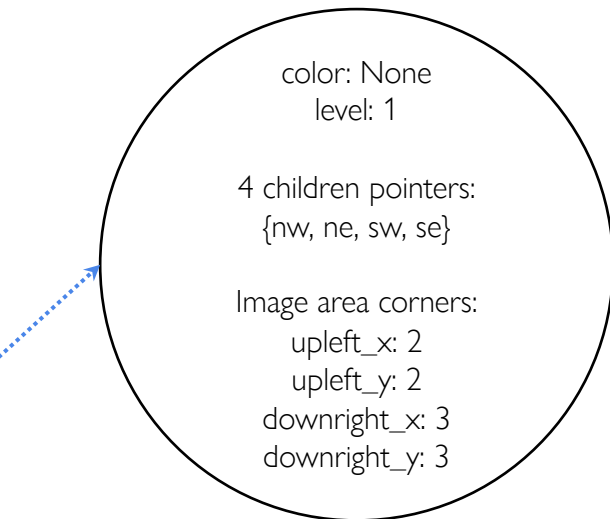
Quadtree

- Each child covers its corresponding **quadrant** with respect to the parent



row

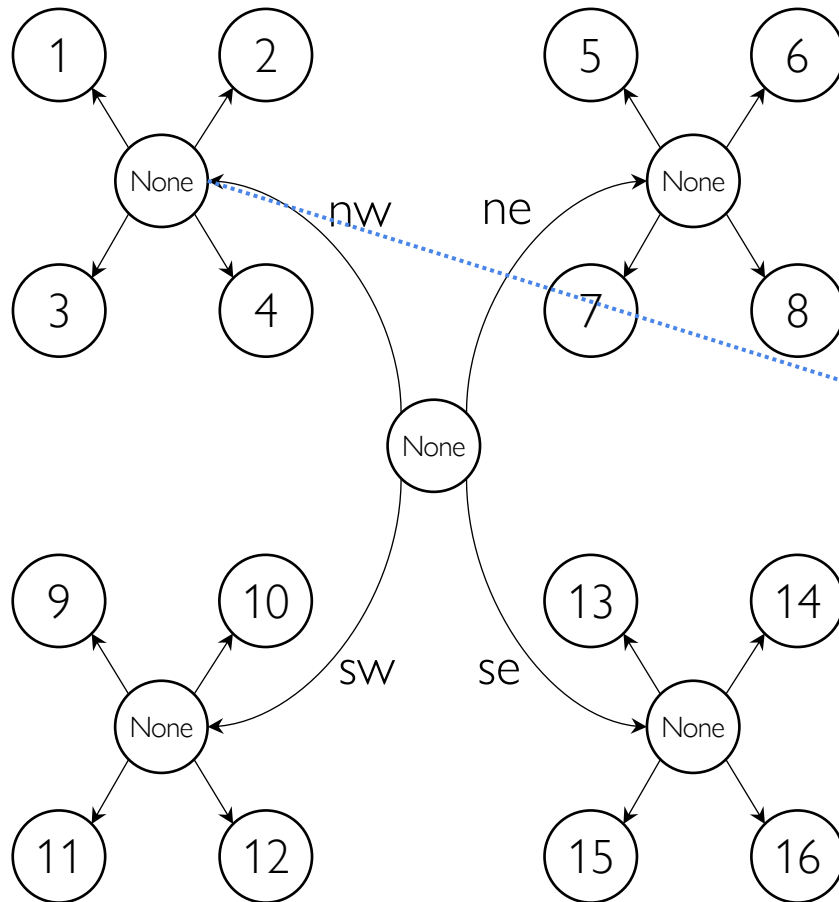
	column			
	x = 0	x = 1	x = 2	x = 3
y = 0	1	2	5	6
y = 1	3	4	7	8
y = 2	9	10	13	14
y = 3	11	12	15	16



Subtree rooted at root.se covers the **southeast quadrant** defined by a box with upleft corner at (ul_x=2, ul_y=2) and downright corner at (dr_x=3, dr_y=3)

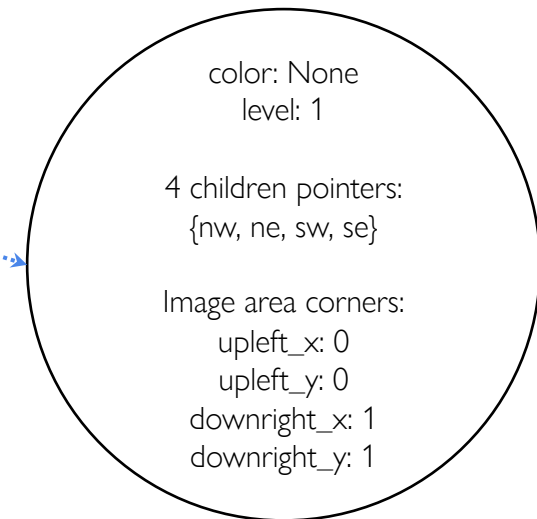
Quadtree

- Each child covers its corresponding **quadrant** with respect to the parent



column

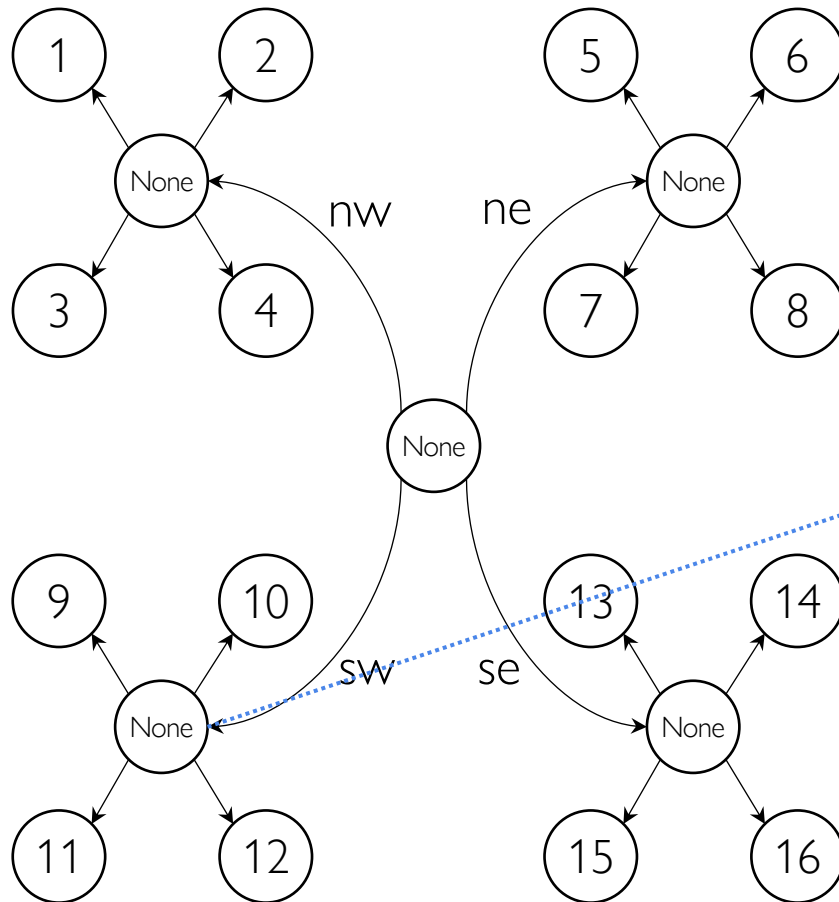
	x = 0	x = 1	x = 2	x = 3
y = 0	1	2	5	6
y = 1	3	4	7	8
y = 2	9	10	13	14
y = 3	11	12	15	16



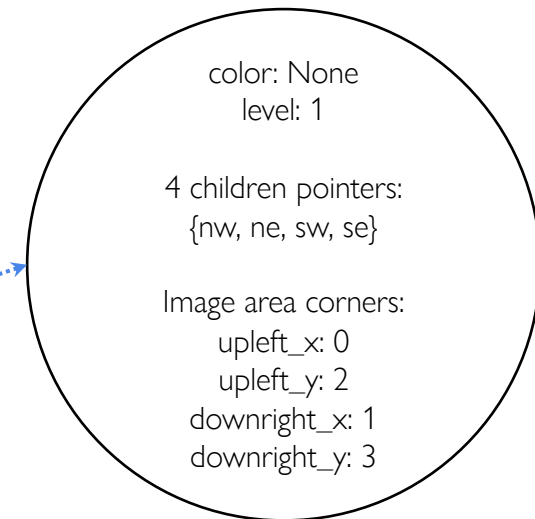
Subtree rooted at root.nw covers the **northwest quadrant** defined by a box with upleft corner at (ul_x=0, ul_y=0) and downright corner at (dr_x=1, dr_y=1)

Quadtree

- Each child covers its corresponding **quadrant** with respect to the parent



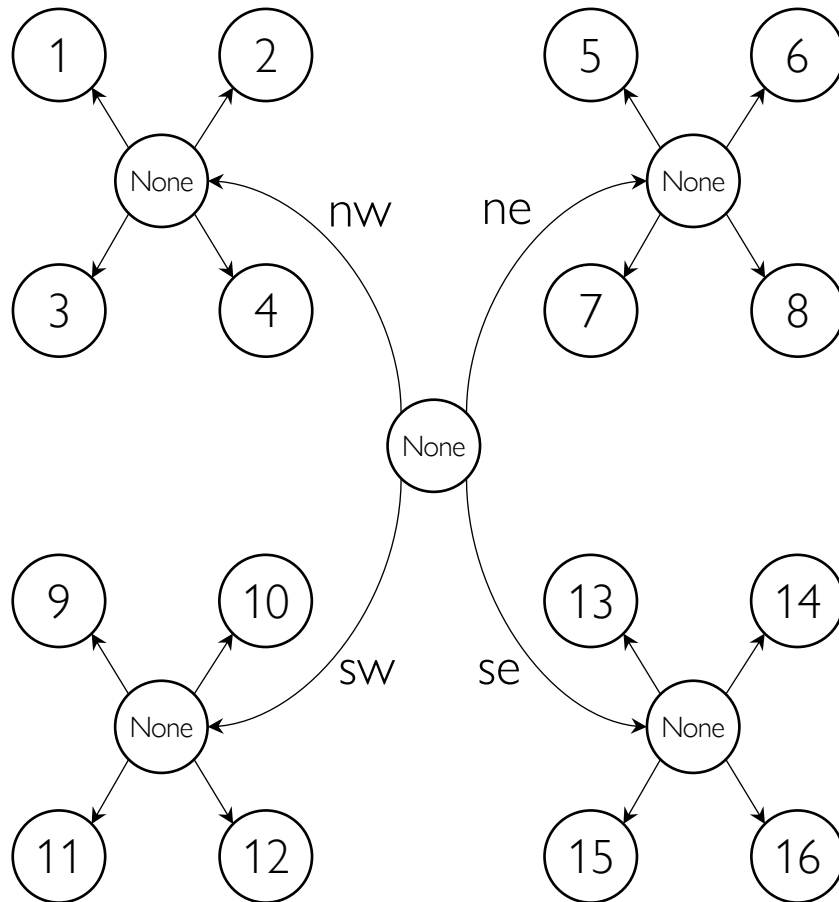
	column			
	x = 0	x = 1	x = 2	x = 3
row				
y = 0	1	2	5	6
y = 1	3	4	7	8
y = 2	9	10	13	14
y = 3	11	12	15	16



Subtree rooted at root.sw covers the **southwest quadrant** defined by a box with upleft corner at (ul_x=0, ul_y=2) and downright corner at (dr_x=1, dr_y=3)

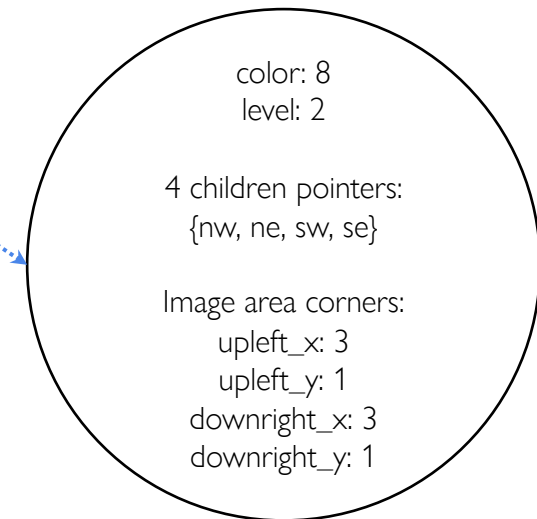
Quadtree

- A leaf is a node which covers only a pixel. A leaf always has a color.



row

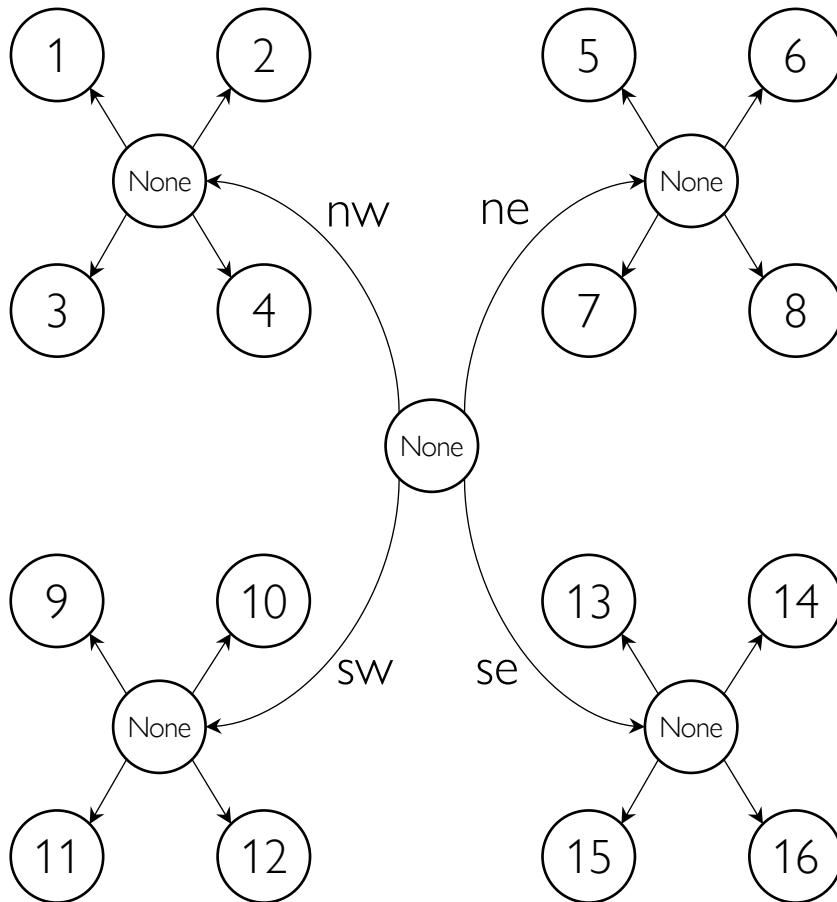
	column			
	x = 0	x = 1	x = 2	x = 3
y = 0	1	2	5	6
y = 1	3	4	7	8
y = 2	9	10	13	14
y = 3	11	12	15	16



This subtree is a leaf that covers the **single pixel with color 8** defined by a “box” with upleft corner at (ul_x=3, ul_y=1) and downright corner at (dr_x=3, dr_y=1)

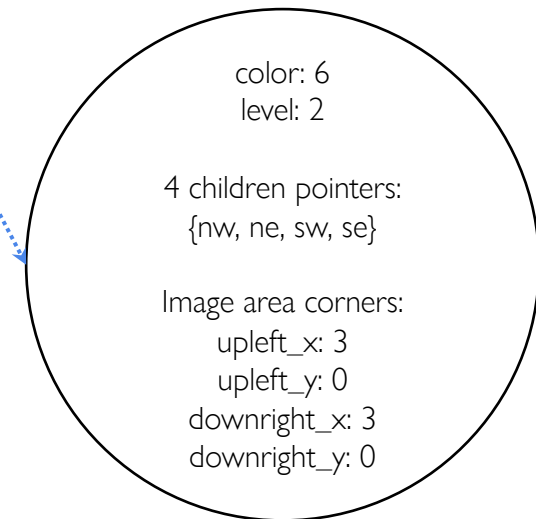
Quadtree

- A leaf is a node which covers only a pixel. A leaf always has a color.



row

	column			
	x = 0	x = 1	x = 2	x = 3
y = 0	1	2	5	6
y = 1	3	4	7	8
y = 2	9	10	13	14
y = 3	11	12	15	16



This subtree is a leaf that covers the **single pixel with color 8** defined by a “box” with upleft corner at (ul_x=3, ul_y=0) and downright corner at (dr_x=3, dr_y=0)

Quadtree Class

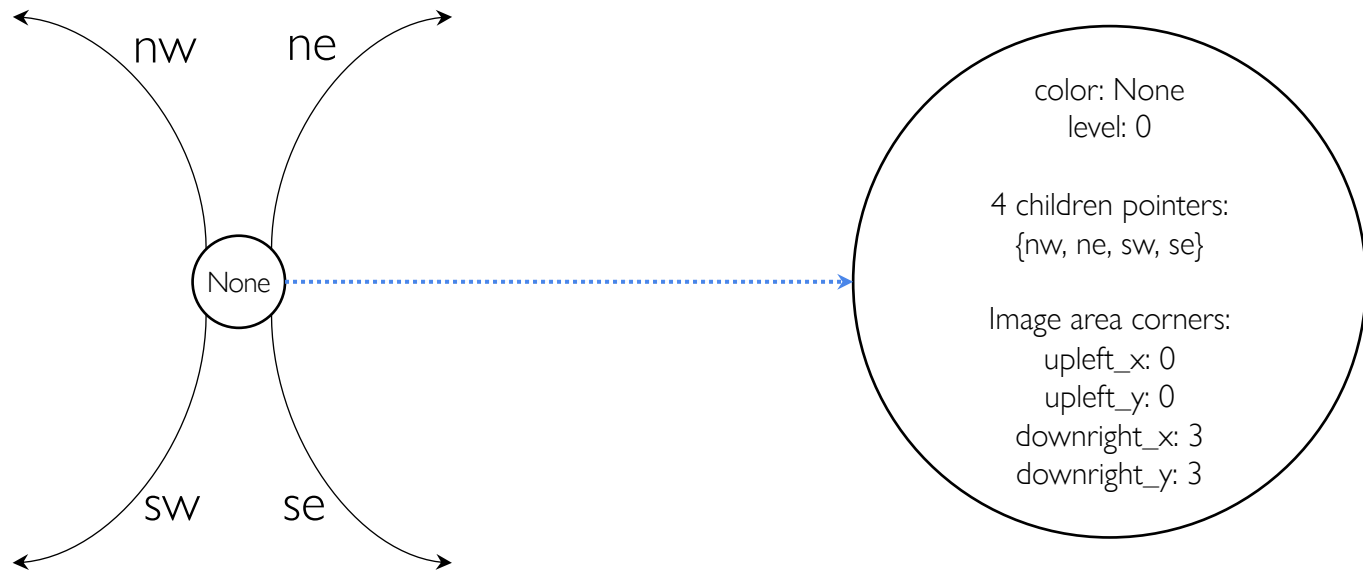
- Contains the image as `self.image` and its corresponding quadtree rooted at `self.root`
- `self.image`
 - the image array needs to be accessed throughout the recursive calls
 - our functions pass the Quadtree variable
- `self.root`
 - the actual quadtree begins from the root, so we just keep the root node

Part 1A: Build a Quadtree from Array

Build a Quadtree

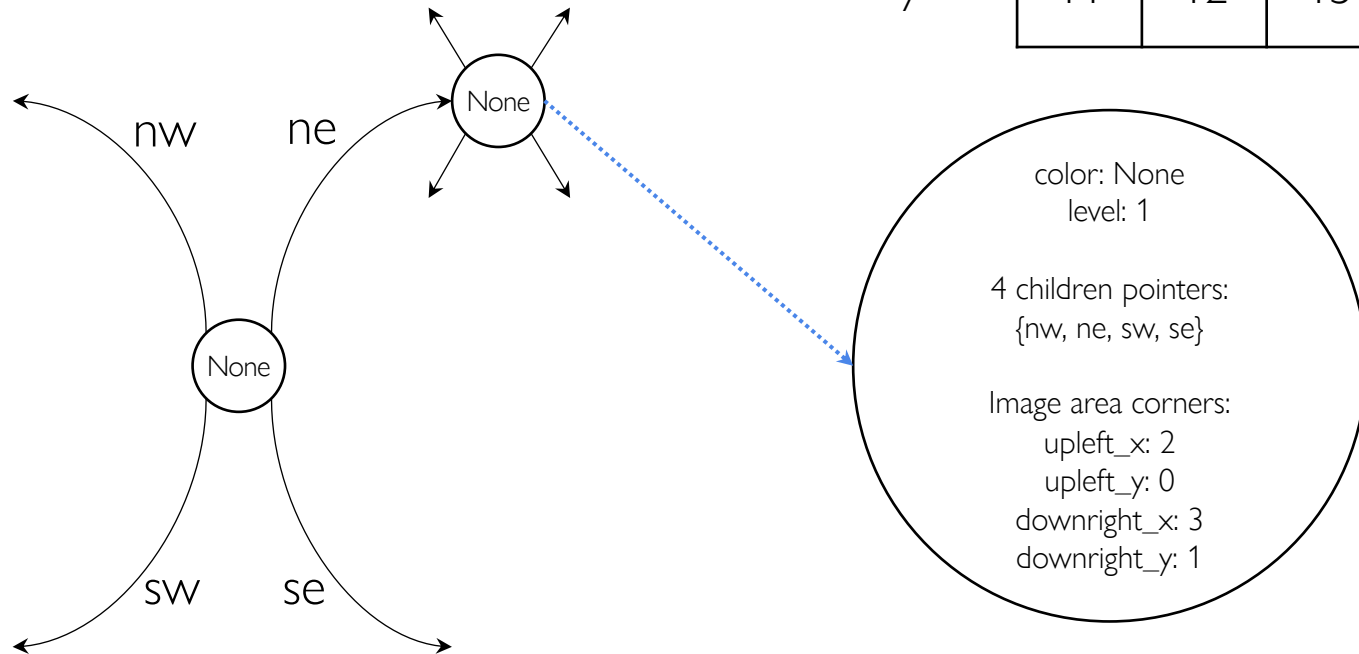
- Start from the root node with proper corners (it's a full image)

row	column			
	x = 0	x = 1	x = 2	x = 3
y = 0	1	2	5	6
y = 1	3	4	7	8
y = 2	9	10	13	14
y = 3	11	12	15	16



Build a Quadtree

- Recursively create children nodes with proper corners and levels
 - corners can be computed using `compute_corners` function

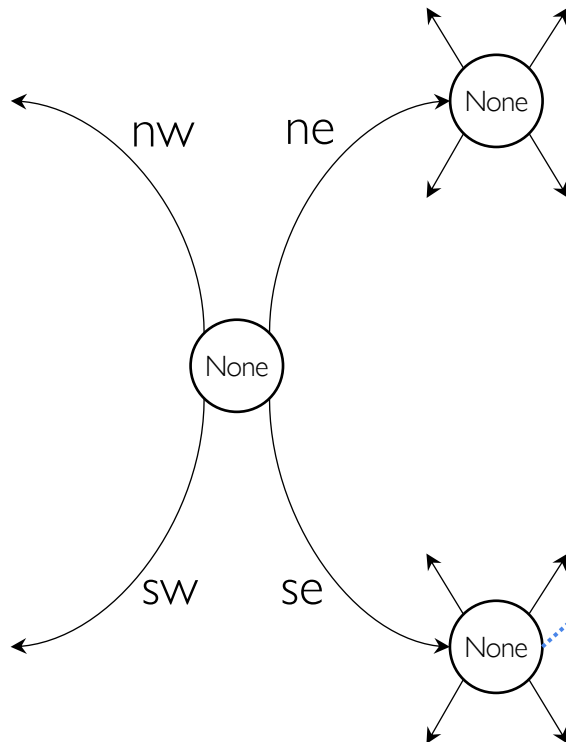


row

	column			
	x = 0	x = 1	x = 2	x = 3
y = 0	1	2	5	6
y = 1	3	4	7	8
y = 2	9	10	13	14
y = 3	11	12	15	16

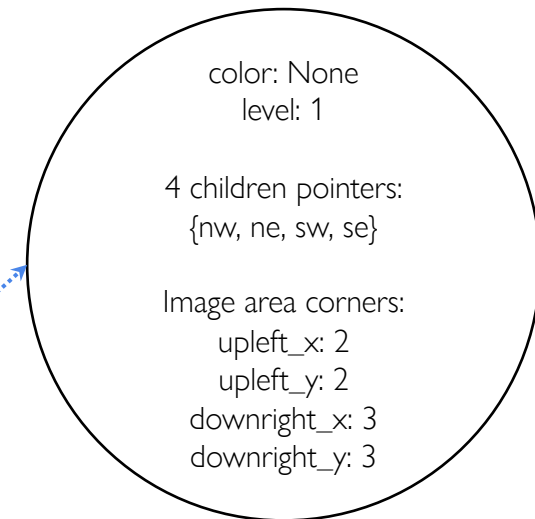
Build a Quadtree

- Recursively create children nodes with proper corners and levels
 - corners can be computed using `compute_corners` function



row

	column			
	x = 0	x = 1	x = 2	x = 3
y = 0	1	2	5	6
y = 1	3	4	7	8
y = 2	9	10	13	14
y = 3	11	12	15	16



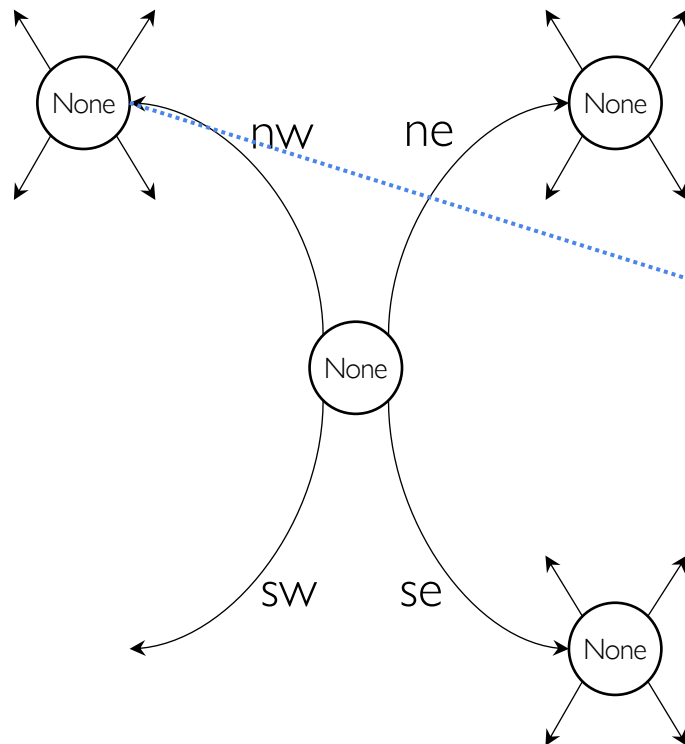
color: None
level: 1

4 children pointers:
{nw, ne, sw, se}

Image area corners:
upleft_x: 2
upleft_y: 2
downright_x: 3
downright_y: 3

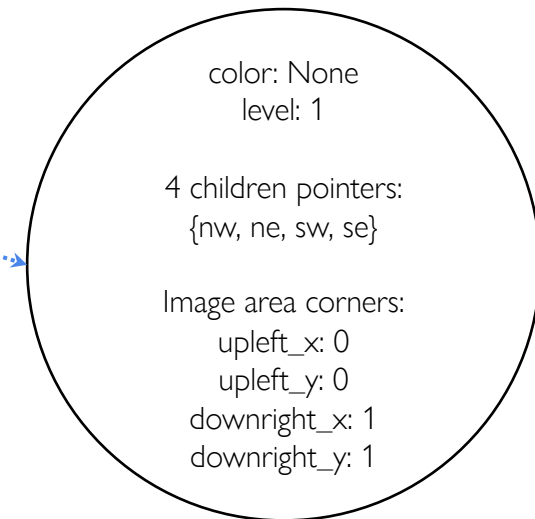
Build a Quadtree

- Recursively create children nodes with proper corners and levels
 - corners can be computed using `compute_corners` function



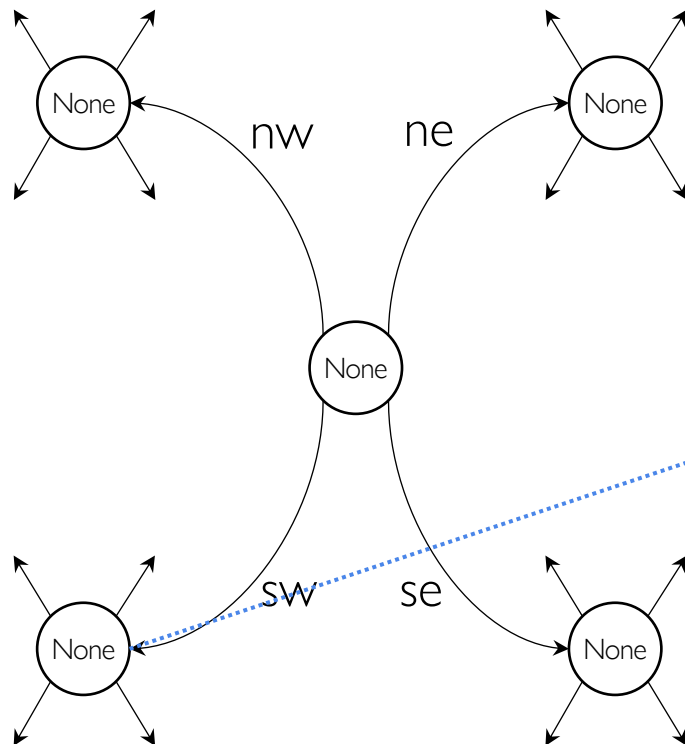
row

	column			
	x = 0	x = 1	x = 2	x = 3
y = 0	1	2	5	6
y = 1	3	4	7	8
y = 2	9	10	13	14
y = 3	11	12	15	16



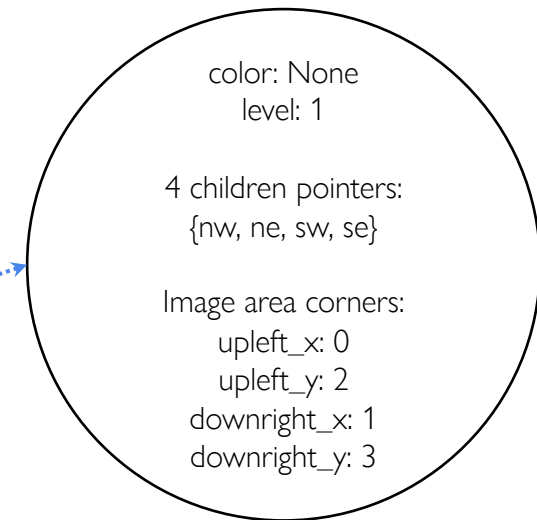
Build a Quadtree

- Recursively create children nodes with proper corners and levels
 - corners can be computed using `compute_corners` function



row

	column			
	x = 0	x = 1	x = 2	x = 3
y = 0	1	2	5	6
y = 1	3	4	7	8
y = 2	9	10	13	14
y = 3	11	12	15	16



color: None
level: 1

4 children pointers:
{nw, ne, sw, se}

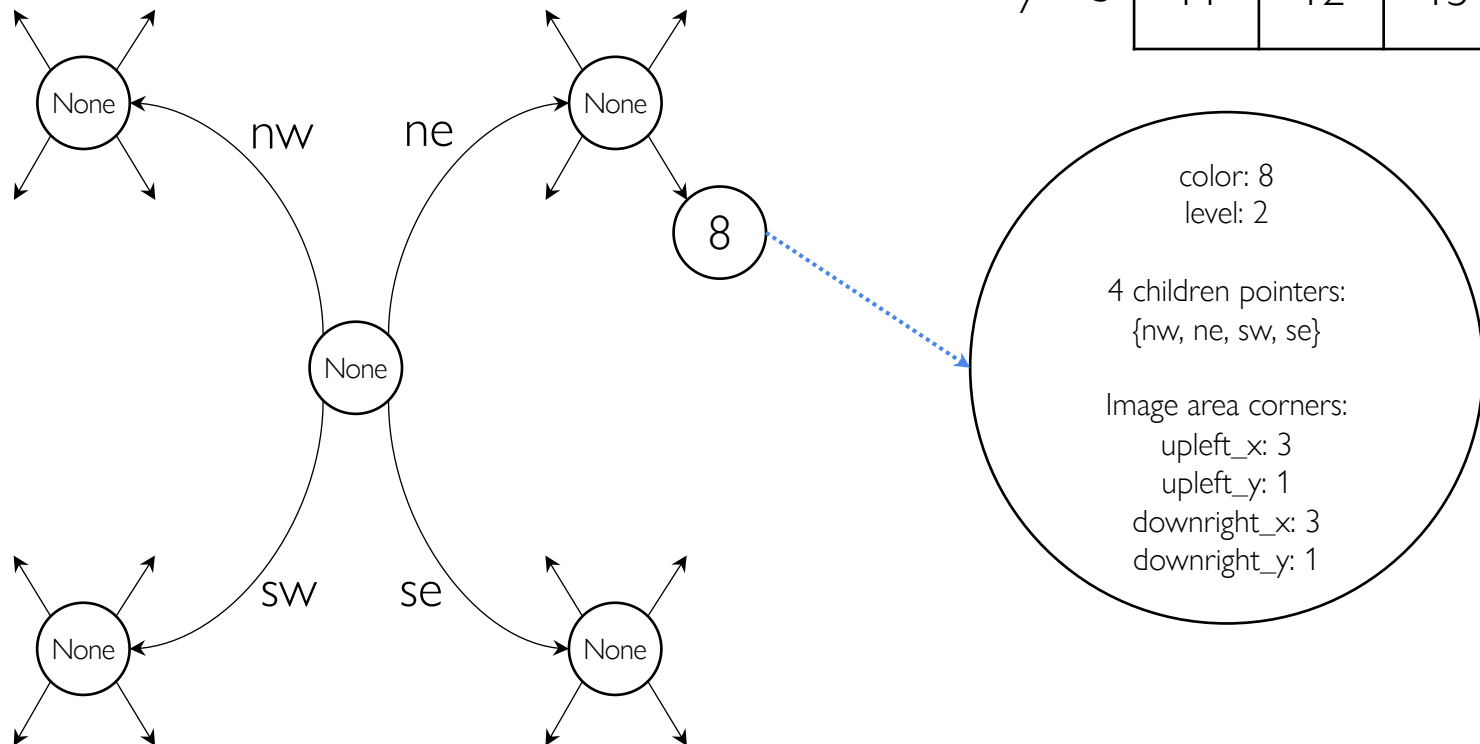
Image area corners:
upleft_x: 0
upleft_y: 2
downright_x: 1
downright_y: 3

Build a Quadtree

- If you find corners that imply it's a single pixel, assign colors and treat them as leaves

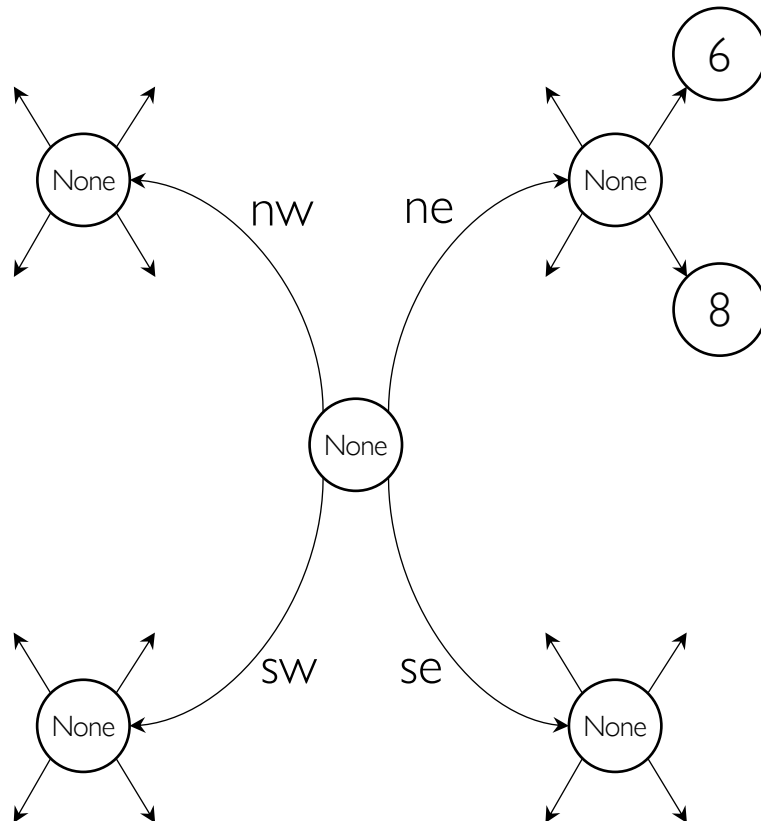
row

	column			
	x = 0	x = 1	x = 2	x = 3
y = 0	1	2	5	6
y = 1	3	4	7	8
y = 2	9	10	13	14
y = 3	11	12	15	16



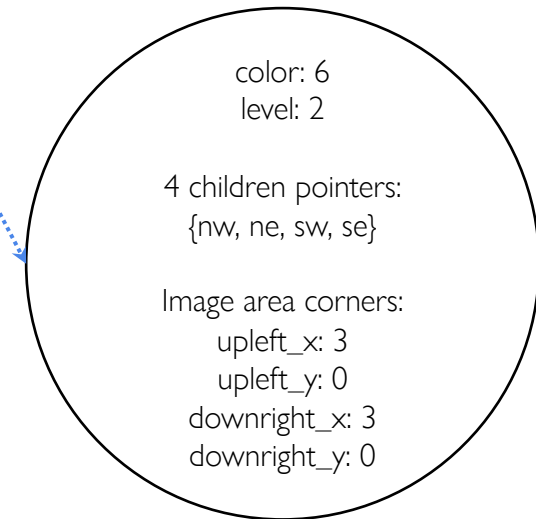
Build a Quadtree

- If you find corners that imply it's a single pixel, assign colors and treat them as leaves



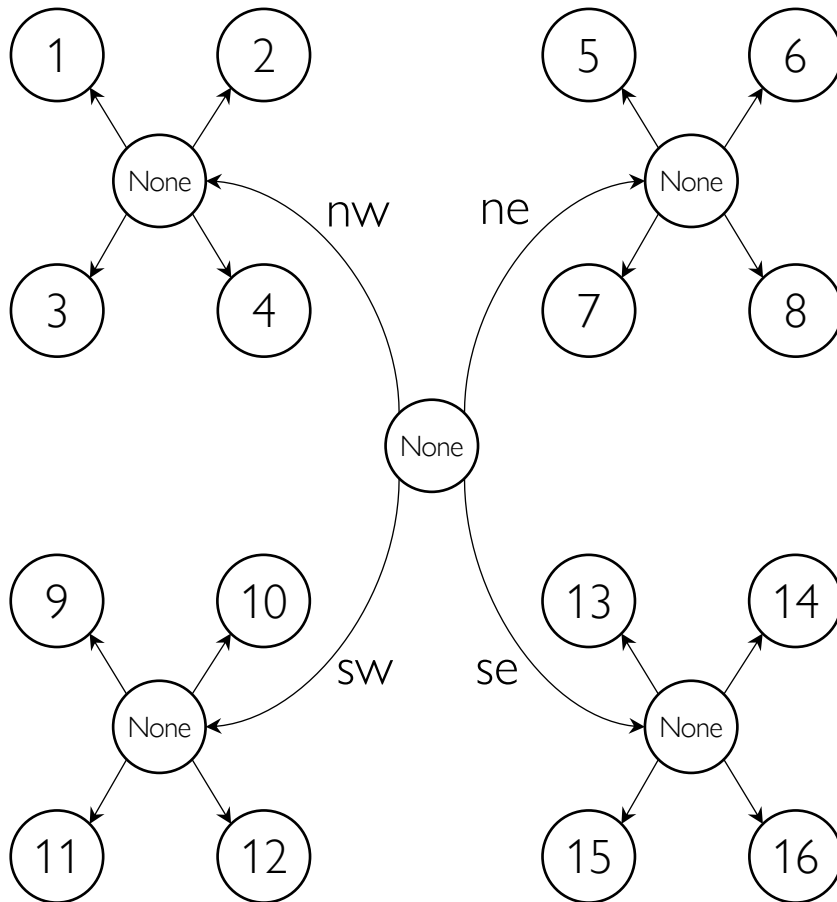
row

	column			
	x = 0	x = 1	x = 2	x = 3
y = 0	1	2	5	6
y = 1	3	4	7	8
y = 2	9	10	13	14
y = 3	11	12	15	16



Build a Quadtree

- Eventually all single pixels will be reached and the quadtree will be constructed



row	column			
	x = 0	x = 1	x = 2	x = 3
y = 0	1	2	5	6
y = 1	3	4	7	8
y = 2	9	10	13	14
y = 3	11	12	15	16

Build a Quadtree

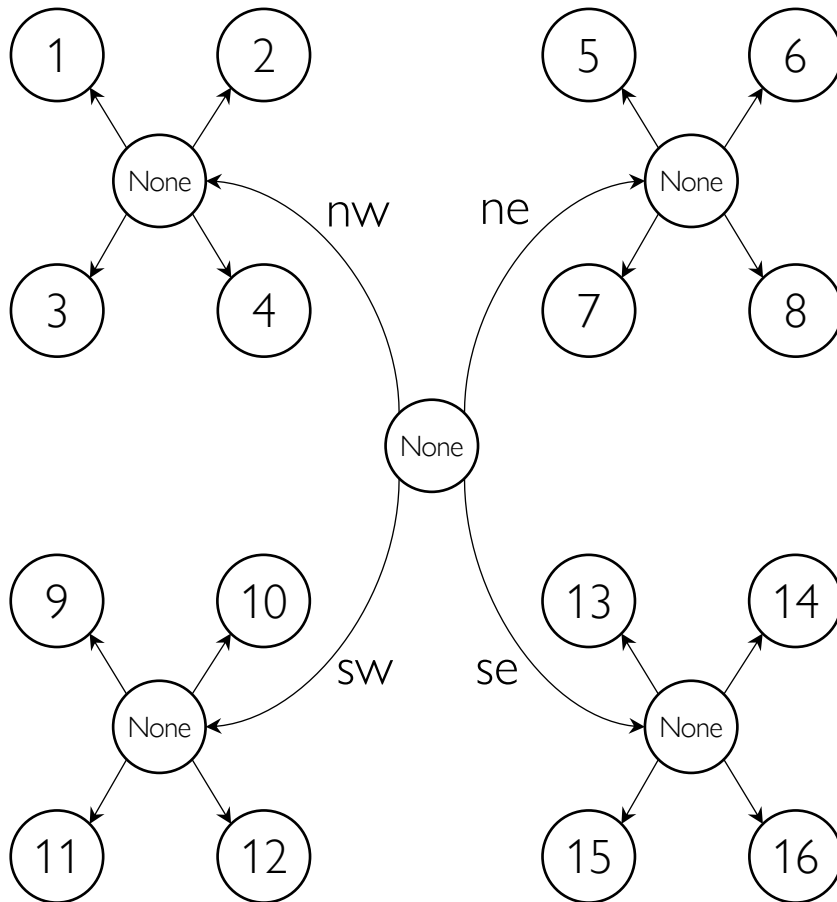
- `build_quadtree` calls `build_quadtree_recursive`
- The code itself is very short
- color is important, but the corners are also important
 - read and use `compute_image_corners`
 - ex: `Node(None, compute_image_corners(node, 'nw'), node.level+1)`
- In Part 1A, you also implement two simple helper functions:
 - `count_nodes`: recursively count the # of nodes in a quadtree
 - `max_num_nodes`: mathematically compute the maximum # of nodes you can have in a quadtree built from an N by N image

Part 1B: Reconstruct



Quadtree to Image

- Traverse the quadtree and assign color to corresponding area with upleft corner at (ul_x, ul_y) and downright corner at (dr_x, dr_y)

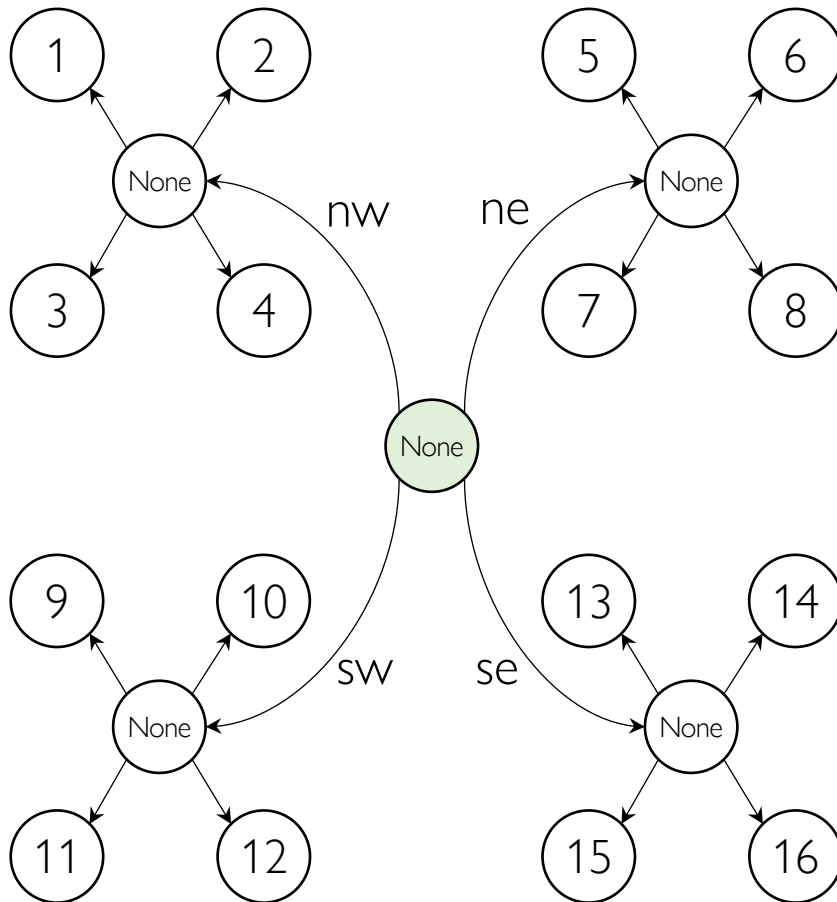


row

	column			
	x = 0	x = 1	x = 2	x = 3
y = 0				
y = 1				
y = 2				
y = 3				

Quadtree to Image

- Traverse the quadtree and assign color to corresponding area with upleft corner at (ul_x, ul_y) and downright corner at (dr_x, dr_y)

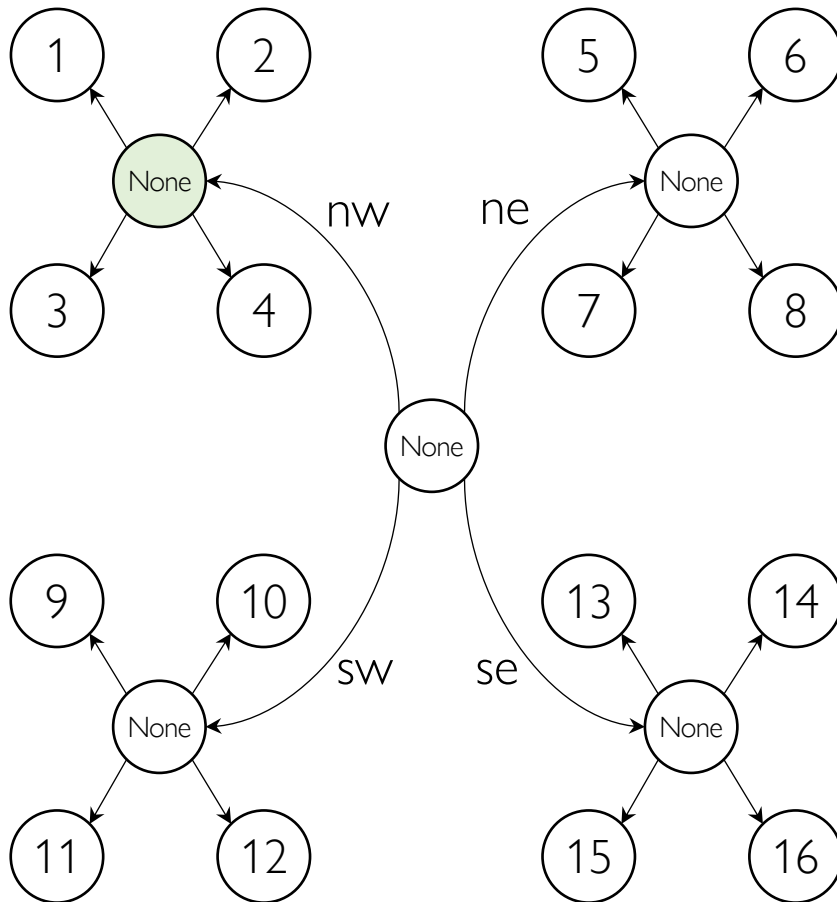


row

	column			
	x = 0	x = 1	x = 2	x = 3
y = 0				
y = 1				
y = 2				
y = 3				

Quadtree to Image

- Traverse the quadtree and assign color to corresponding area with upleft corner at (ul_x, ul_y) and downright corner at (dr_x, dr_y)

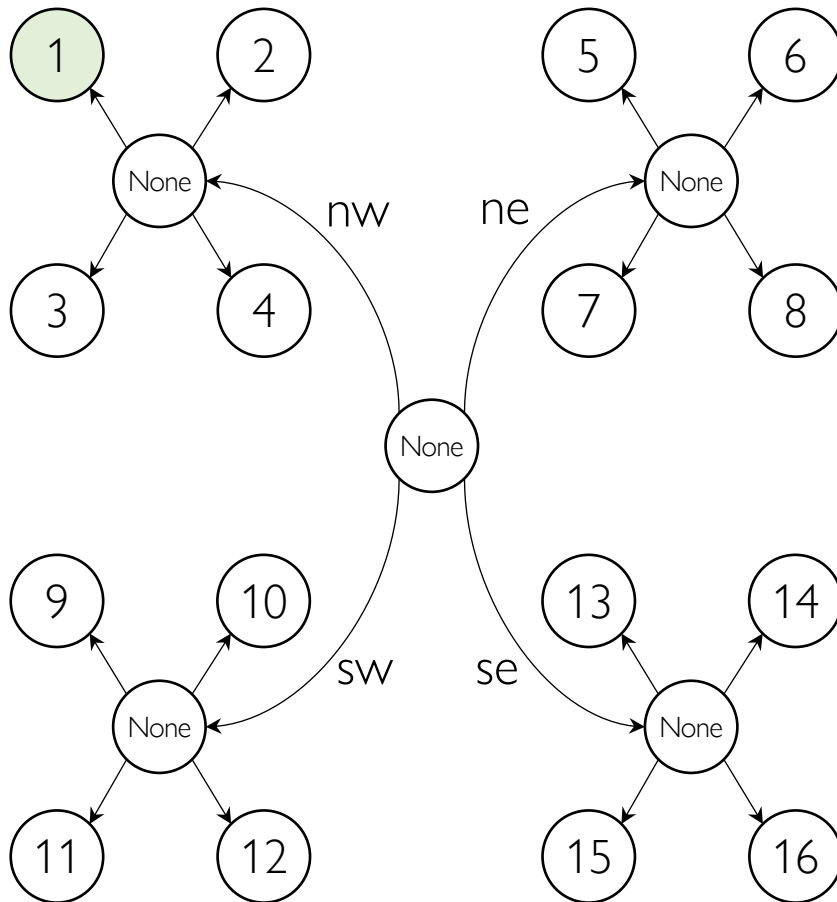


row

	column			
	x = 0	x = 1	x = 2	x = 3
y = 0				
y = 1				
y = 2				
y = 3				

Quadtree to Image

- Traverse the quadtree and assign color to corresponding area with upleft corner at (ul_x, ul_y) and downright corner at (dr_x, dr_y)

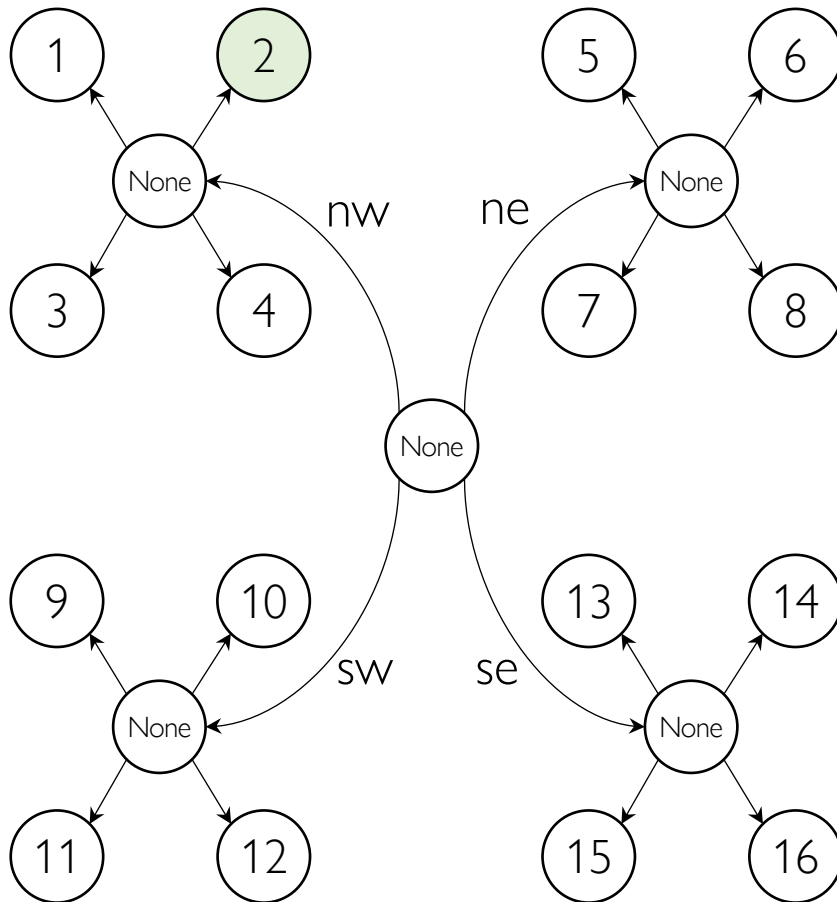


row

	column			
	x = 0	x = 1	x = 2	x = 3
y = 0	1			
y = 1				
y = 2				
y = 3				

Quadtree to Image

- Traverse the quadtree and assign color to corresponding area with upleft corner at (ul_x, ul_y) and downright corner at (dr_x, dr_y)

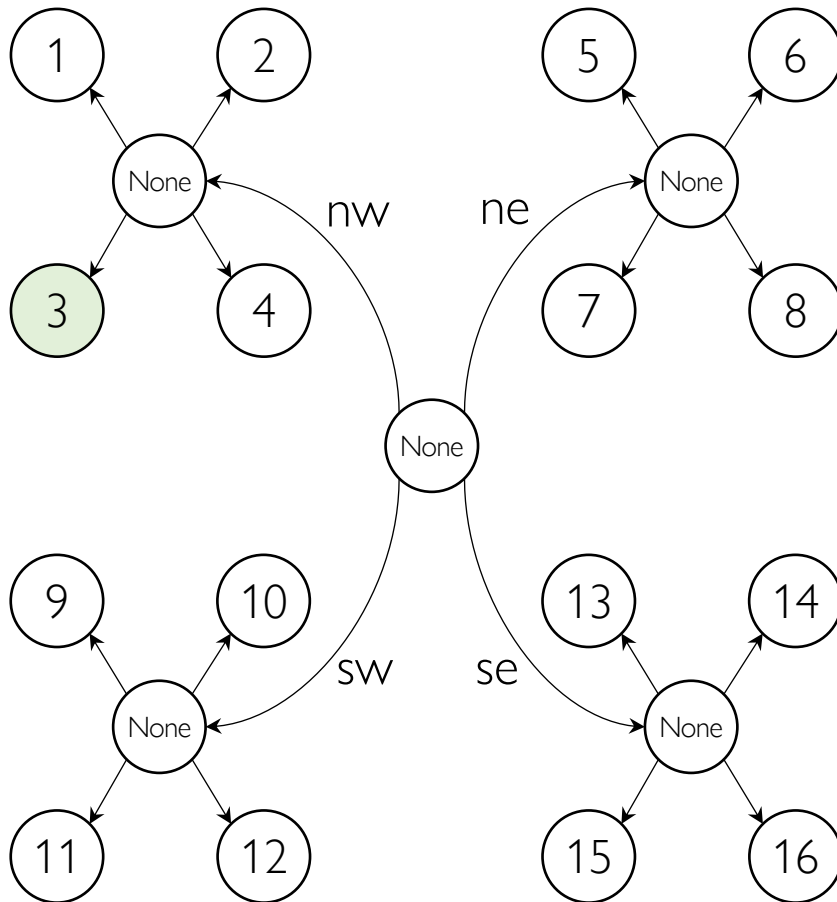


row

	column			
	x = 0	x = 1	x = 2	x = 3
y = 0	1	2		
y = 1				
y = 2				
y = 3				

Quadtree to Image

- Traverse the quadtree and assign color to corresponding area with upleft corner at (ul_x, ul_y) and downright corner at (dr_x, dr_y)

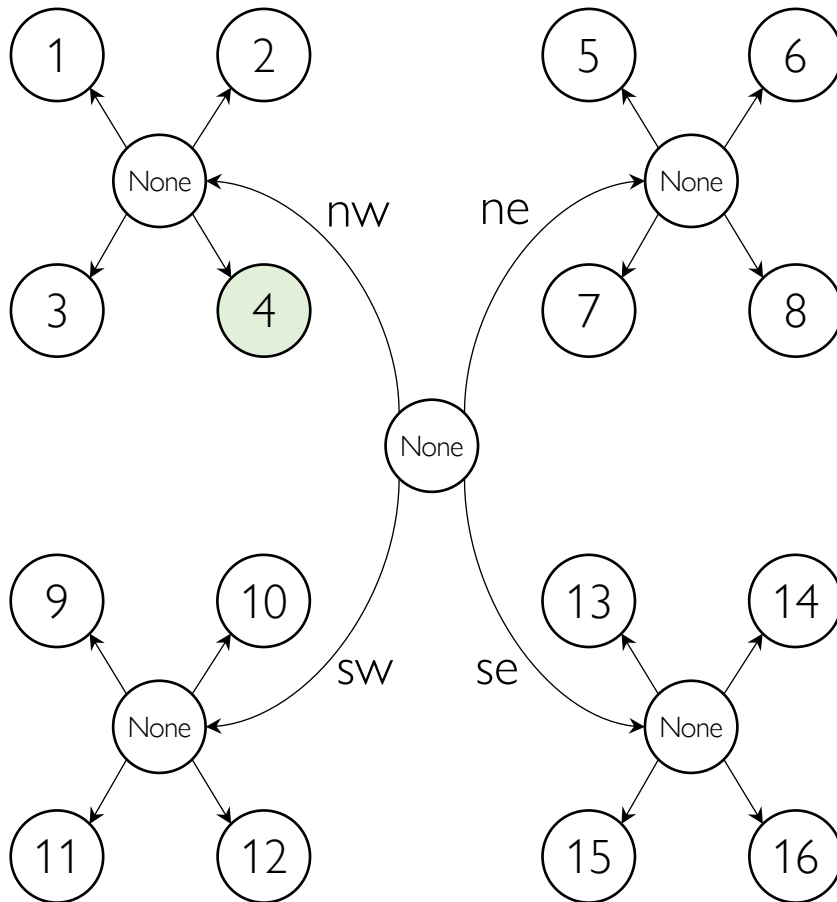


row

	column			
	x = 0	x = 1	x = 2	x = 3
y = 0	1	2		
y = 1	3			
y = 2				
y = 3				

Quadtree to Image

- Traverse the quadtree and assign color to corresponding area with upleft corner at (ul_x, ul_y) and downright corner at (dr_x, dr_y)

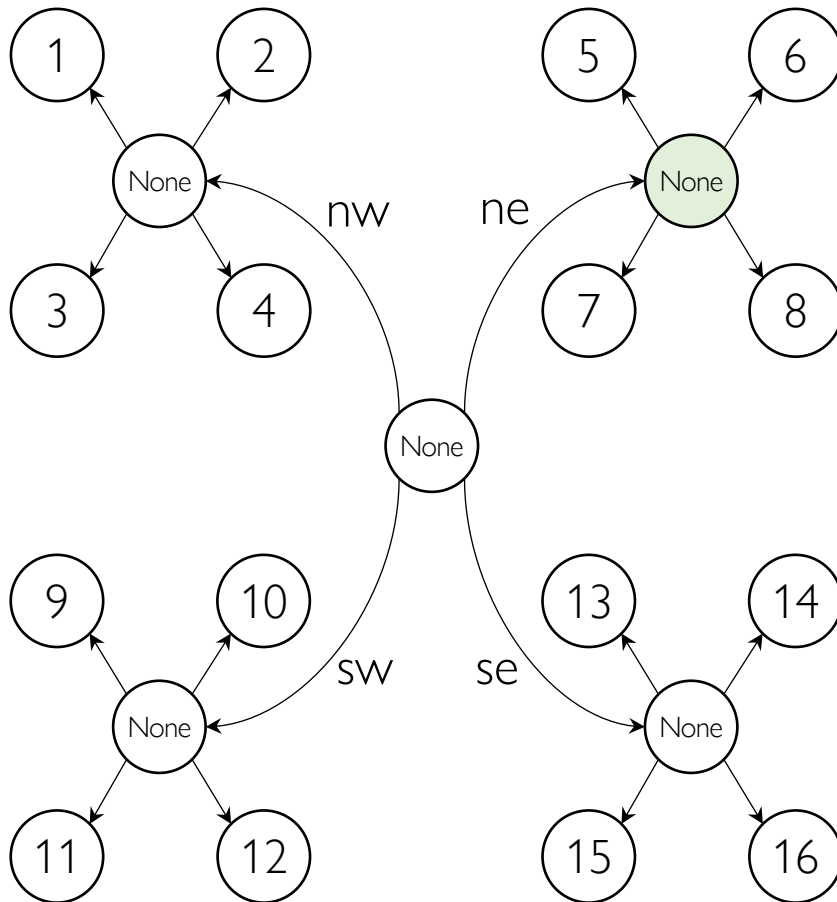


row

	column			
	x = 0	x = 1	x = 2	x = 3
y = 0	1	2		
y = 1	3	4		
y = 2				
y = 3				

Quadtree to Image

- Traverse the quadtree and assign color to corresponding area with upleft corner at (ul_x, ul_y) and downright corner at (dr_x, dr_y)

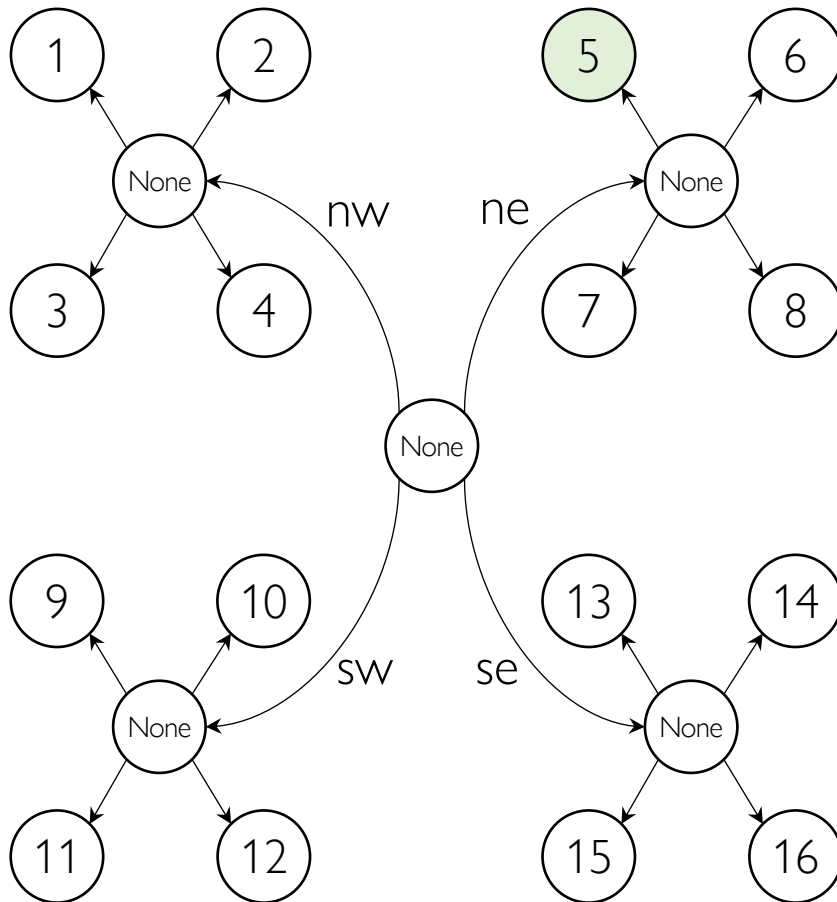


row

	column			
	x = 0	x = 1	x = 2	x = 3
y = 0	1	2		
y = 1	3	4		
y = 2				
y = 3				

Quadtree to Image

- Traverse the quadtree and assign color to corresponding area with upleft corner at (ul_x, ul_y) and downright corner at (dr_x, dr_y)

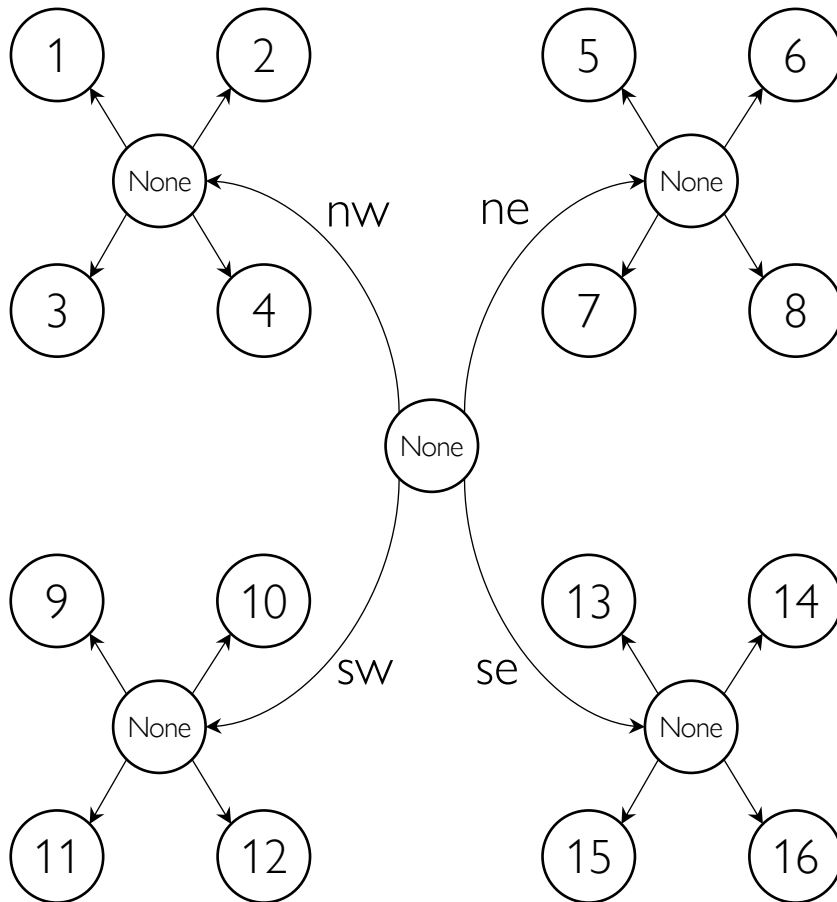


row

	column			
	x = 0	x = 1	x = 2	x = 3
y = 0	1	2	5	
y = 1	3	4		
y = 2				
y = 3				

Quadtree to Image

- Traverse the quadtree and assign color to corresponding area with upleft corner at (ul_x, ul_y) and downright corner at (dr_x, dr_y)



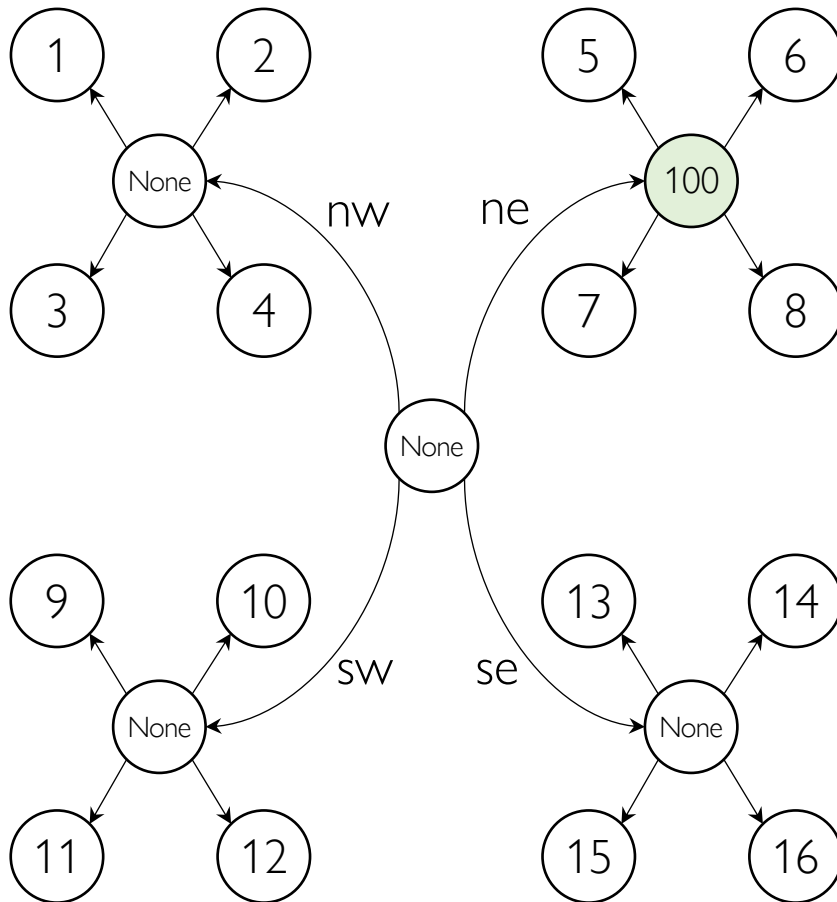
		column			
		x = 0	x = 1	x = 2	x = 3
row	y = 0	1	2	5	6
	y = 1	3	4	7	8
	y = 2	9	10	13	14
	y = 3	11	12	15	16

This is when we have colors at leaves!

What happens if we have colors at **internal nodes**?

Quadtree to Image

- Traverse the quadtree and assign color to corresponding area with upleft corner at (ul_x, ul_y) and downright corner at (dr_x, dr_y)



column

	x = 0	x = 1	x = 2	x = 3
y = 0	1	2	5	6
y = 1	3	4	7	8
y = 2	9	10	13	14
y = 3	11	12	15	16

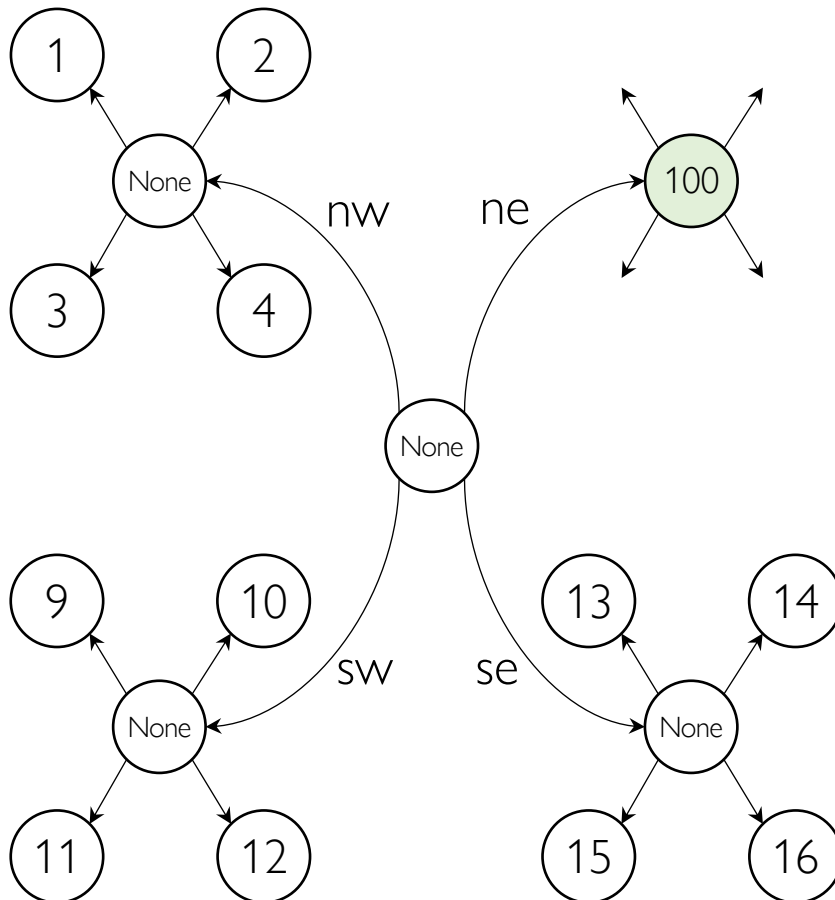
row

This is when we have colors at leaves!

What happens if we have colors at **internal nodes**?

Quadtree to Image

- Traverse the quadtree and assign color to corresponding area with upleft corner at (ul_x, ul_y) and downright corner at (dr_x, dr_y)



		column			
		x = 0	x = 1	x = 2	x = 3
row	y = 0	1	2	100	100
	y = 1	3	4	100	100
	y = 2	9	10	13	14
	y = 3	11	12	15	16

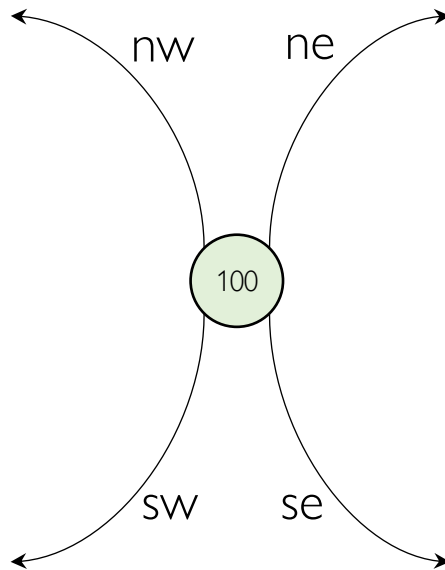
This is when we have colors at leaves!

What happens if we have colors at **internal nodes**?

- The internal node's entire area has that single value
- Its children do not exist
 - Removal happens in Part 1C
 - We do NOT need to traverse its children anyway

Quadtree to Image

- Traverse the quadtree and assign color to corresponding area with upleft corner at (ul_x, ul_y) and downright corner at (dr_x, dr_y)



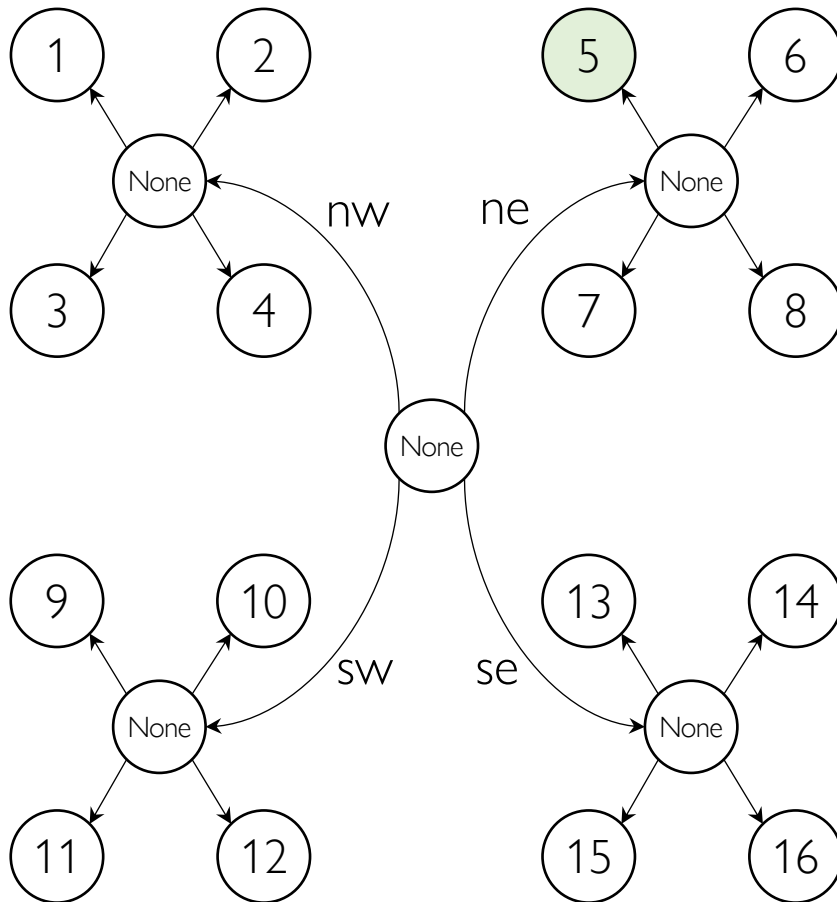
row

	column			
	x = 0	x = 1	x = 2	x = 3
y = 0	100	100	100	100
y = 1	100	100	100	100
y = 2	100	100	100	100
y = 3	100	100	100	100

One extreme case

Quadtree to Image

- Traverse the quadtree and assign color to corresponding area with upleft corner at (ul_x, ul_y) and downright corner at (dr_x, dr_y)



row

	column			
	x = 0	x = 1	x = 2	x = 3
y = 0	1	2	5	6
y = 1	3	4	7	8
y = 2	9	10	13	14
y = 3	11	12	15	16

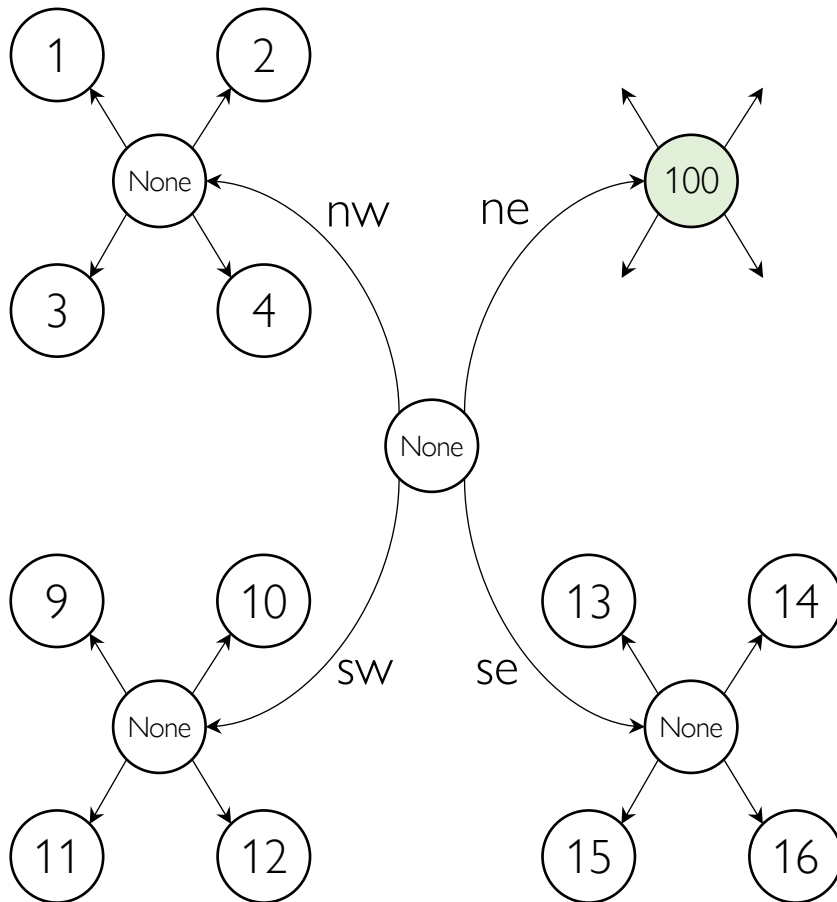
Notice that our logic does not change:

- for a leaf node, its “area” is just a single pixel
- for an internal node, its “area” is just some area larger than a pixel

The “areas” are both defined by (ul_x, ul_y) and (dr_x, dr_y)

Quadtree to Image

- Traverse the quadtree and assign color to corresponding area with upleft corner at (ul_x, ul_y) and downright corner at (dr_x, dr_y)



		column			
		x = 0	x = 1	x = 2	x = 3
row	y = 0	1	2	100	100
	y = 1	3	4	100	100
	y = 2	9	10	13	14
	y = 3	11	12	15	16

Notice that our logic does not change:

- for a leaf node, its “area” is just a single pixel
- for an internal node, its “area” is just some area larger than a pixel

The “areas” are both defined by (ul_x, ul_y) and (dr_x, dr_y)

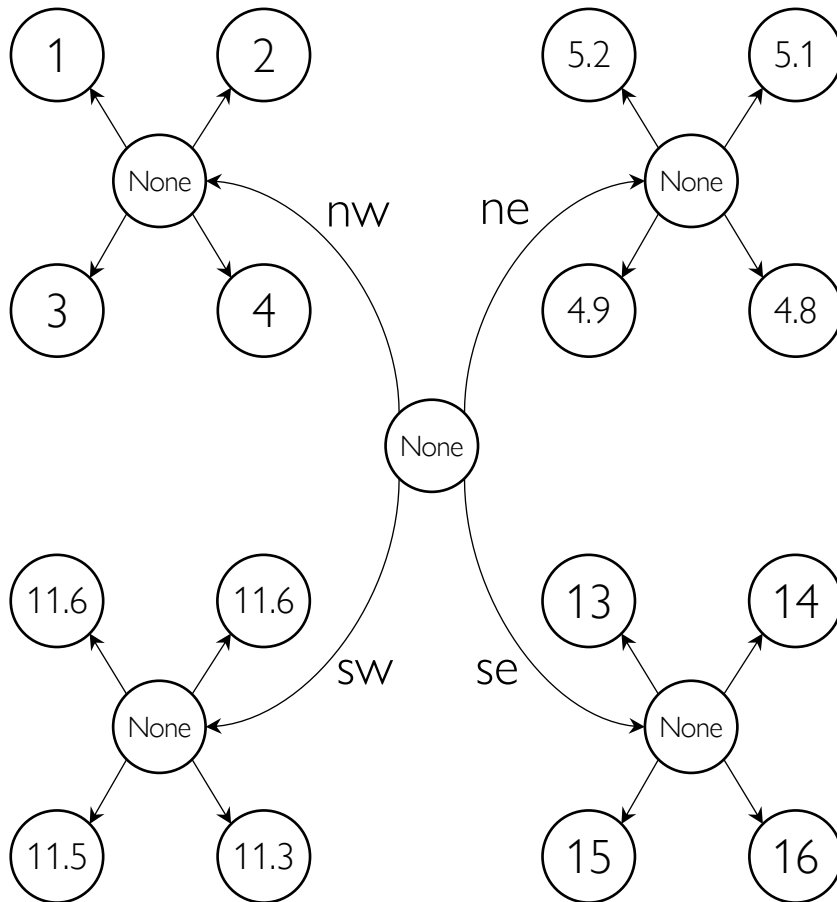
Quadtree to Image

- Implement `quadtree_to_image_recursive`
- Recursively pass down 'qt' as argument to modify its image 'qt.image' throughout the recursion
 - This is an example of a `preorder` traversal: you check the condition of the parent node before deciding to traverse its children
- Use `draw_color(qt, node, draw_box)` to directly modify qt.image
 - `draw_box` argument used for Part 2 to draw boxes on the image for visualization of "areas"
- Again, not a very long code

Part 1C: Compression: How to set colors for internal nodes?

Compression

- Suppose we have a quadtree from the image shown at the right



		column			
		x = 0	x = 1	x = 2	x = 3
row	y = 0	1	2	5.2	5.1
	y = 1	3	4	4.9	4.8
	y = 2	11.6	11.6	13	14
	y = 3	11.5	11.3	15	16

We want to “compress” the image by simplifying pixels that are

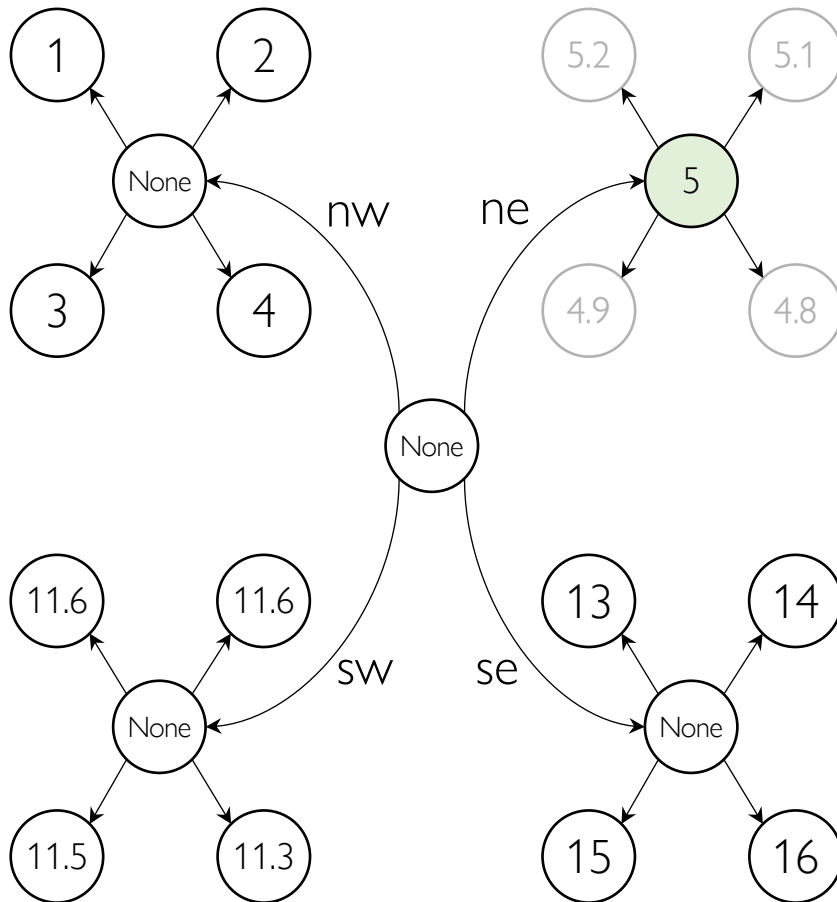
1. siblings and
2. have “similar” color values

If an internal node’s children have “similar” color values, then

1. the internal node’s color = average of the children colors
2. children nodes are removed (pruned)

Compression

- Suppose we have a quadtree from the image shown at the right



row

	column			
	x = 0	x = 1	x = 2	x = 3
y = 0	1	2	5.2	5.1
y = 1	3	4	4.9	4.8
y = 2	11.6	11.6	13	14
y = 3	11.5	11.3	15	16

We want to “compress” the image by simplifying pixels that are

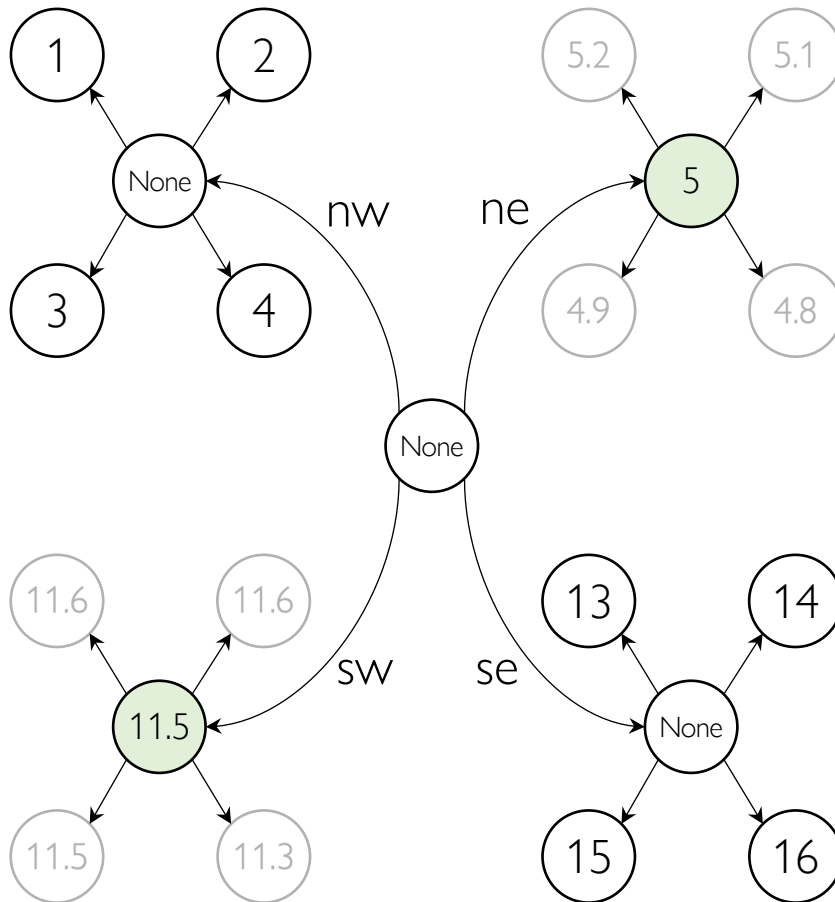
- siblings and
- have “similar” color values

If an internal node’s children have “similar” color values, then

- the internal node’s color = average of the children colors
- children nodes are removed (pruned)

Compression

- Suppose we have a quadtree from the image shown at the right



row

	column			
	x = 0	x = 1	x = 2	x = 3
y = 0	1	2	5.2	5.1
y = 1	3	4	4.9	4.8
y = 2	11.6	11.6	13	14
y = 3	11.5	11.3	15	16

We want to “compress” the image by simplifying pixels that are

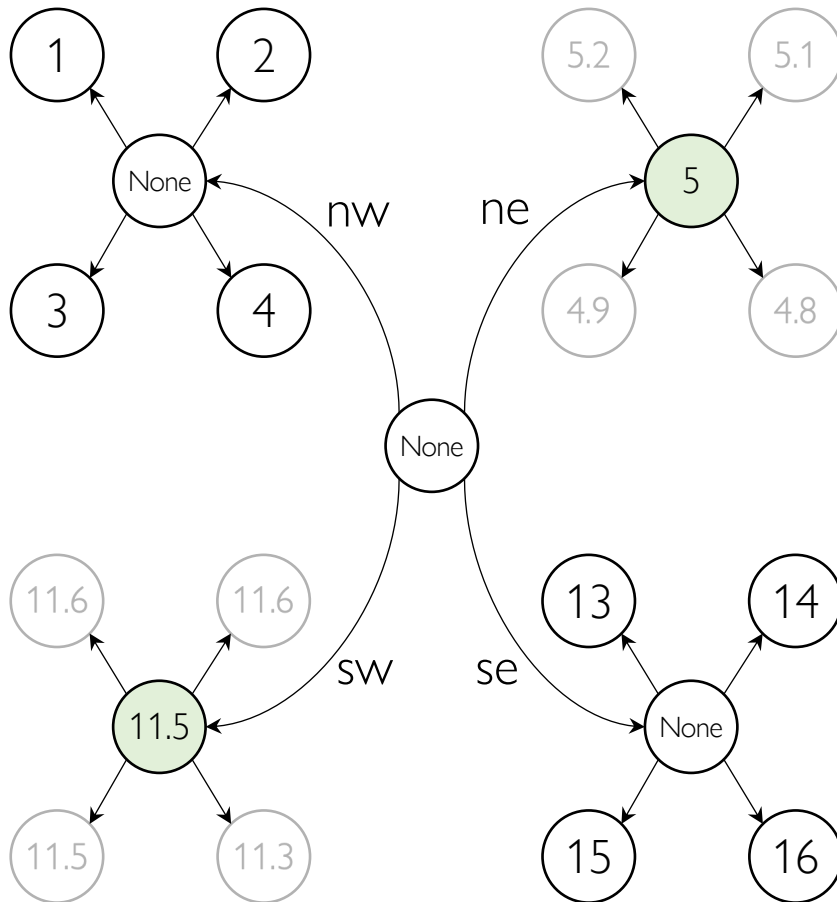
- siblings and
- have “similar” color values

If an internal node’s children have “similar” color values, then

- the internal node’s color = average of the children colors
- children nodes are removed (pruned)

Compression

- Suppose we have a quadtree from the image shown at the right



row

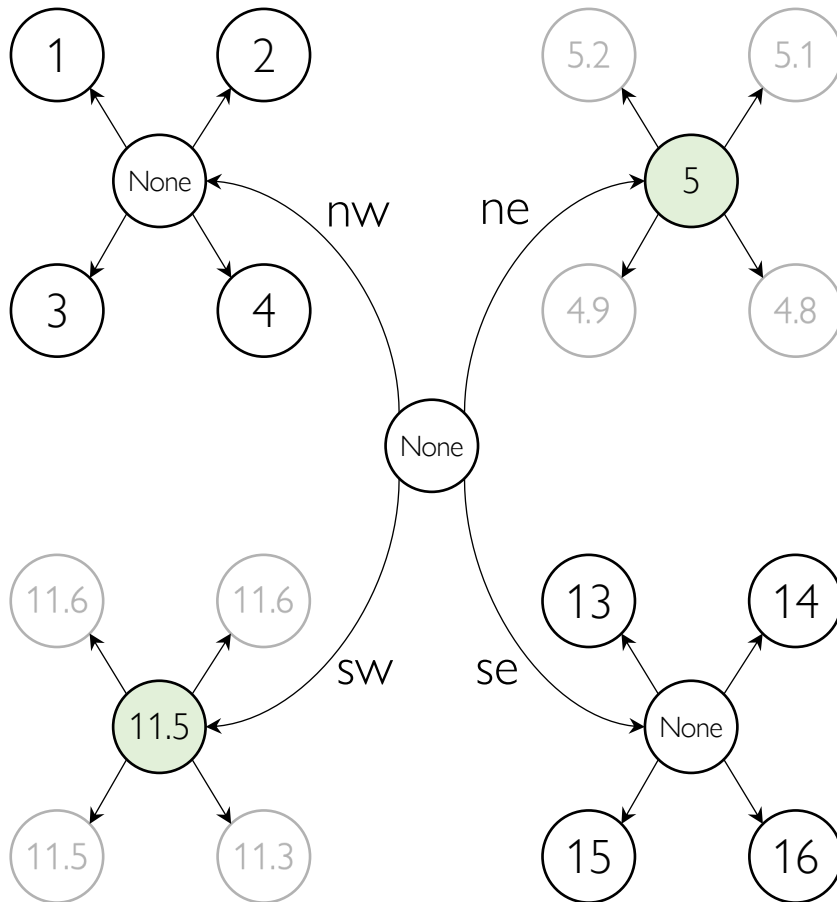
	column			
	x = 0	x = 1	x = 2	x = 3
y = 0	1	2	5	5
y = 1	3	4	5	5
y = 2	11.5	11.5	13	14
y = 3	11.5	11.5	15	16

quadtree_to_image then returns the **compressed image**:

- similar image areas are “compressed” to one (most representative) color
 - this reduces the image file size
- reduces the # of trees of quadtree

Compression

- Suppose we have a quadtree from the image shown at the right



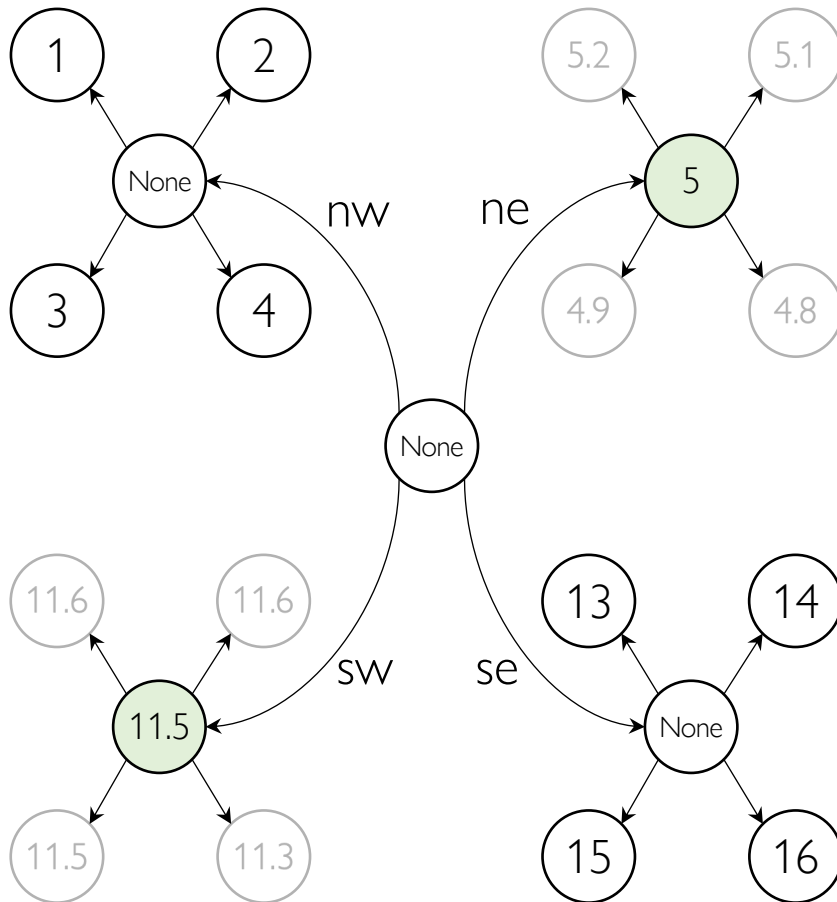
row \ column				
	x = 0	x = 1	x = 2	x = 3
y = 0	1	2	5	5
y = 1	3	4	5	5
y = 2	11.5	11.5	13	14
y = 3	11.5	11.5	15	16

use `combine_colors(node, threshold)` to update the internal node color

- It checks for similarity by computing the “difference” among the children colors
- If the “difference” is less than some “standard” we call “threshold” (difference < threshold)
 - The similarity is high, so `combine_colors` returns the average children color
- If the difference \geq threshold
 - The similarity is small, so `combine_colors` returns None

Compression

- Suppose we have a quadtree from the image shown at the right



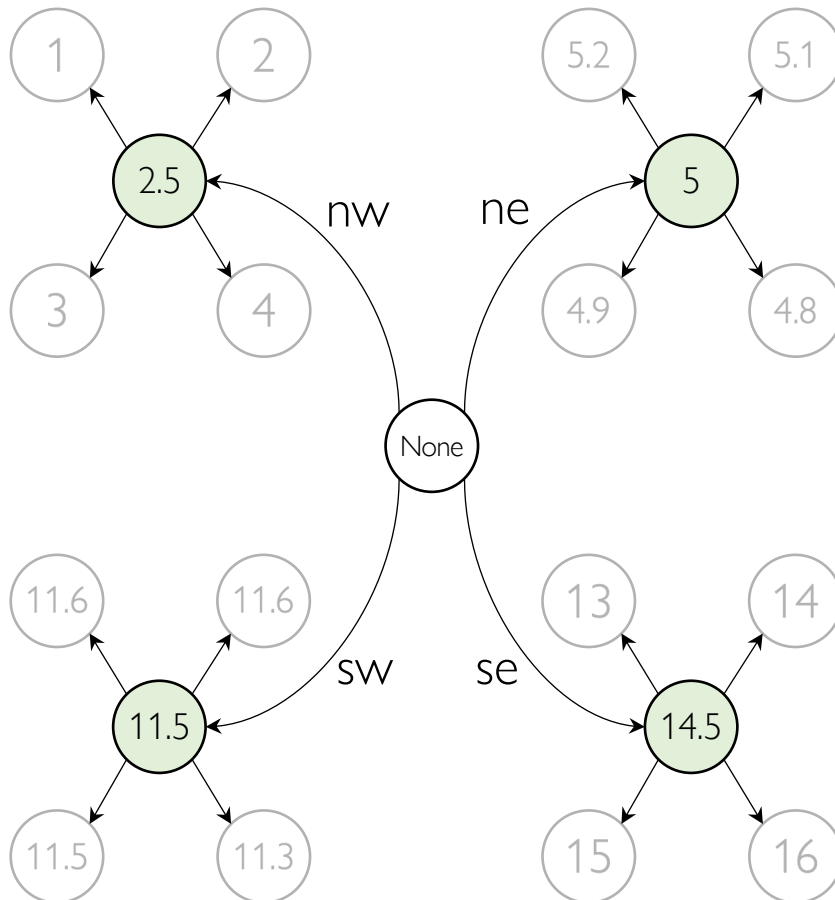
This means your choice “threshold” determines the strength of compression. This is our choice.

- Small threshold means you only combine children with very small difference
 - weak, conservative compression
- Large threshold means you are willing to combine children with larger difference
 - strong, liberal compression

		column			
		x = 0	x = 1	x = 2	x = 3
row	y = 0	1	2	5	5
	y = 1	3	4	5	5
	y = 2	11.5	11.5	13	14
	y = 3	11.5	11.5	15	16

Compression

- Suppose we have a quadtree from the image shown at the right



		column			
		x = 0	x = 1	x = 2	x = 3
row	y = 0	2.5	2.5	5	5
	y = 1	2.5	2.5	5	5
	y = 2	11.5	11.5	14.5	14.5
	y = 3	11.5	11.5	14.5	14.5

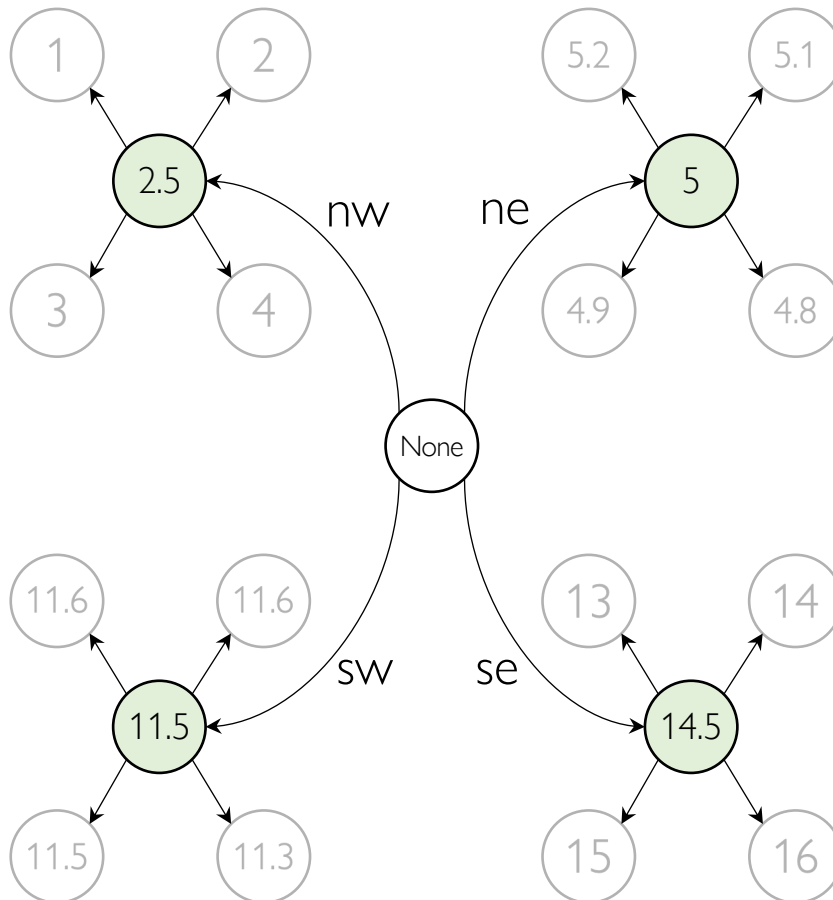
This means your choice “[threshold](#)” determines the strength of compression. This is our choice.

- Small threshold means you only combine children with very small difference
 - weak, conservative compression
- Large threshold means you are willing to combine children with larger difference
 - strong, liberal compression

Ex: very large threshold may combine all children

Compression

- Suppose we have a quadtree from the image shown at the right



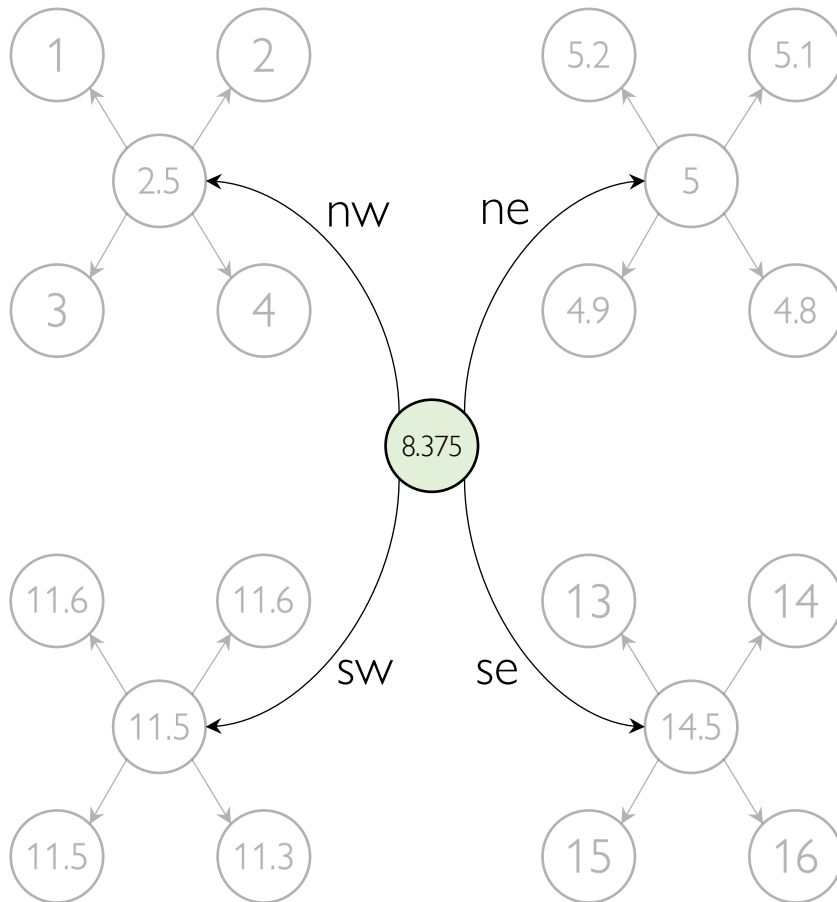
row

	column			
	x = 0	x = 1	x = 2	x = 3
y = 0	2.5	2.5	5	5
y = 1	2.5	2.5	5	5
y = 2	11.5	11.5	14.5	14.5
y = 3	11.5	11.5	14.5	14.5

Note that this can happen recursively from the bottom of quadtree!

Compression

- Suppose we have a quadtree from the image shown at the right



row

	column			
	x = 0	x = 1	x = 2	x = 3
y = 0	8.375	8.375	8.375	8.375
y = 1	8.375	8.375	8.375	8.375
y = 2	8.375	8.375	8.375	8.375
y = 3	8.375	8.375	8.375	8.375

Note that this can happen recursively from the bottom of quadtree!

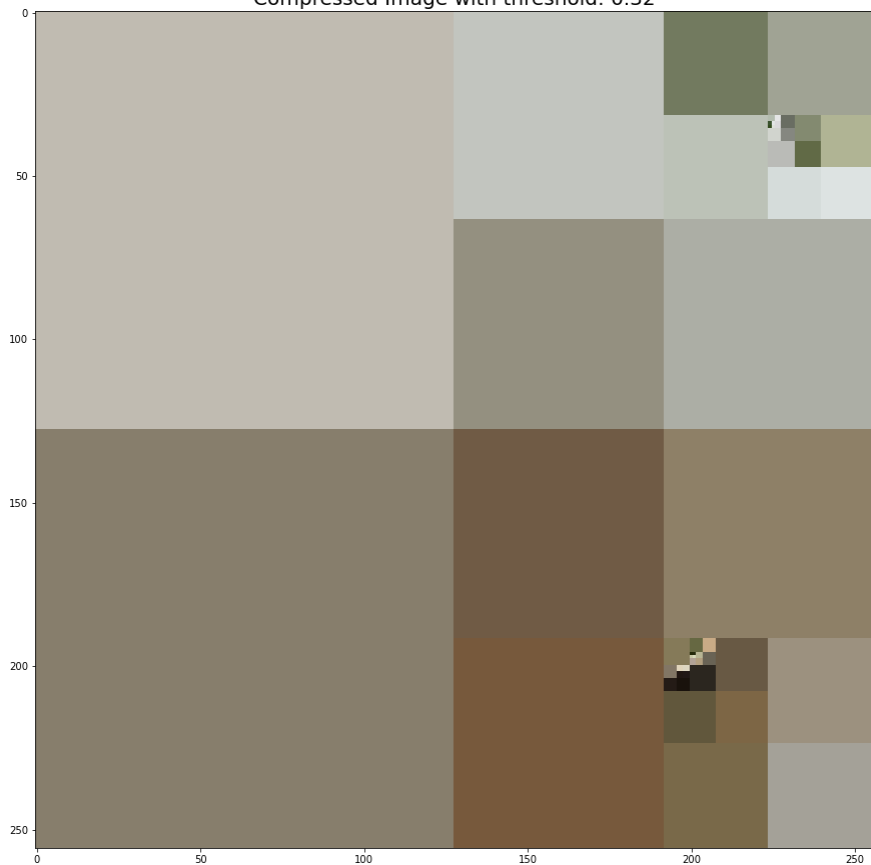
Compression

- Implement `compress_quadtree_recursive`
- Recursively pass down 'qt' as argument to update the internal node colors from the bottom of the quadtree
 - This is an example of a `postorder` traversal: you update the children colors by visiting them first
- Use `combine_colors(node, threshold)`
- Make sure to `remove children` if an internal node has a color
 - Easy to check if this happened by using `count_nodes` function you implement in Part1A
- `assignment3.ipynb` tries specific thresholds
 - I will grade with my choice of thresholds
- Again, not a very long code

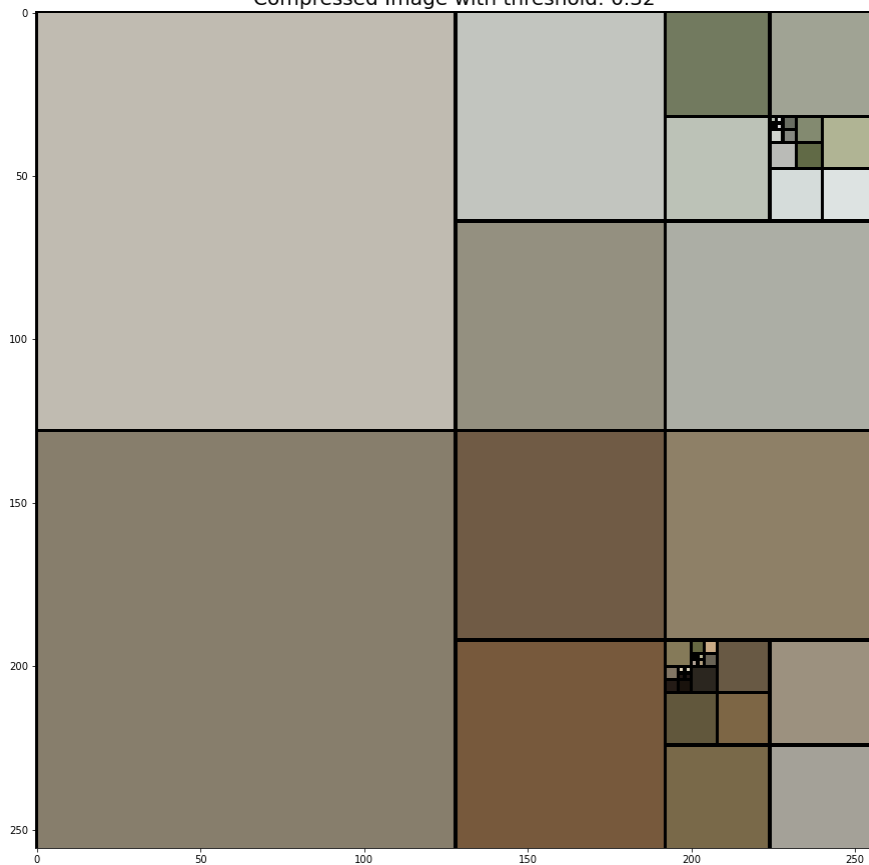
Part 2: Test on a real image



Compressed Image with threshold: 0.32



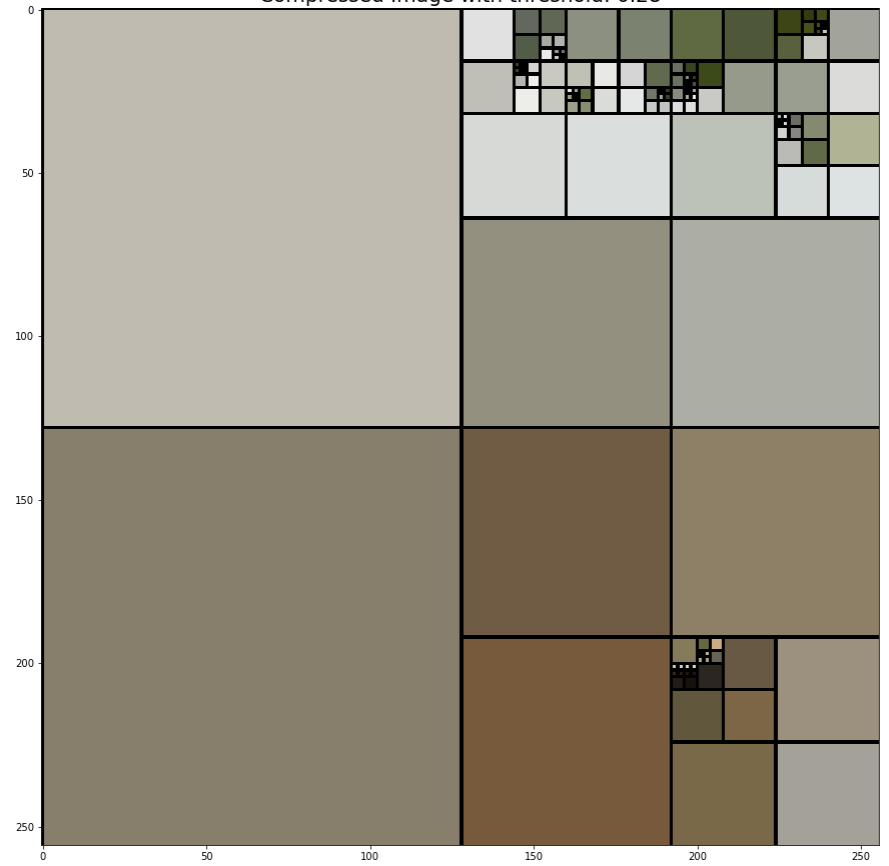
Compressed Image with threshold: 0.32



Compressed Image with threshold: 0.28



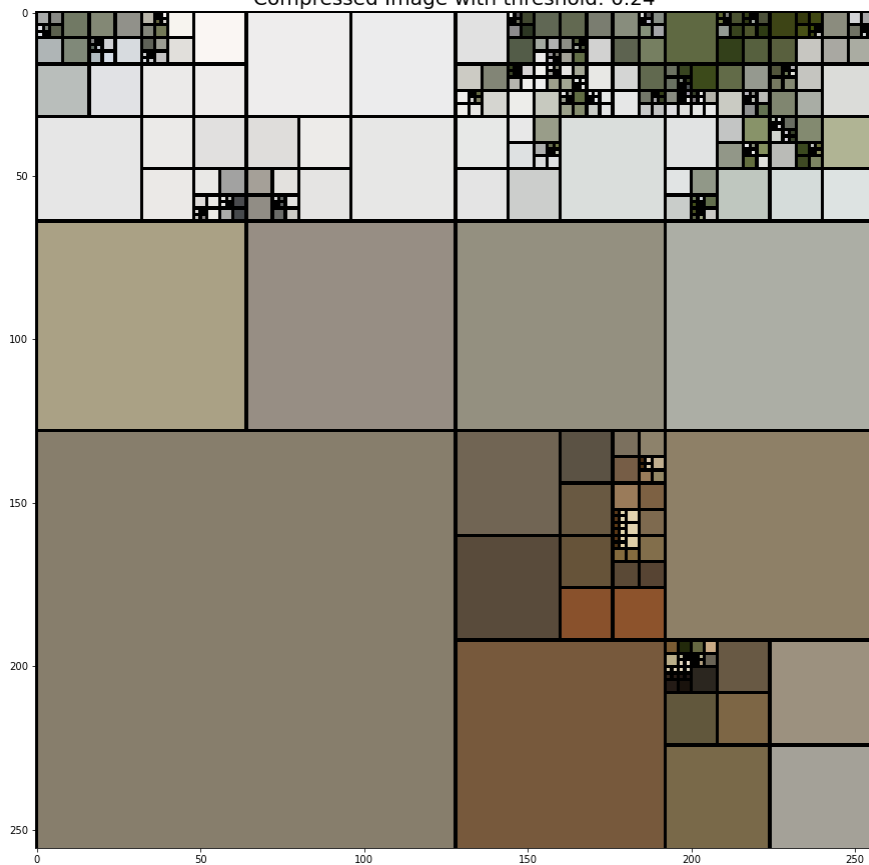
Compressed Image with threshold: 0.28



Compressed Image with threshold: 0.24



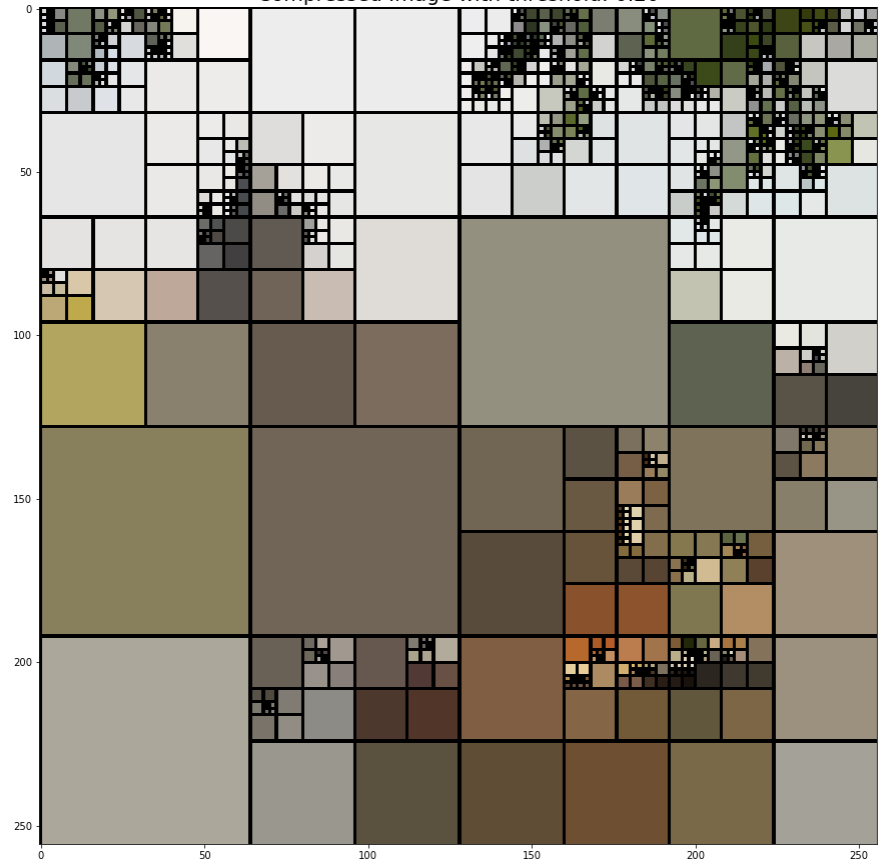
Compressed Image with threshold: 0.24



Compressed Image with threshold: 0.20



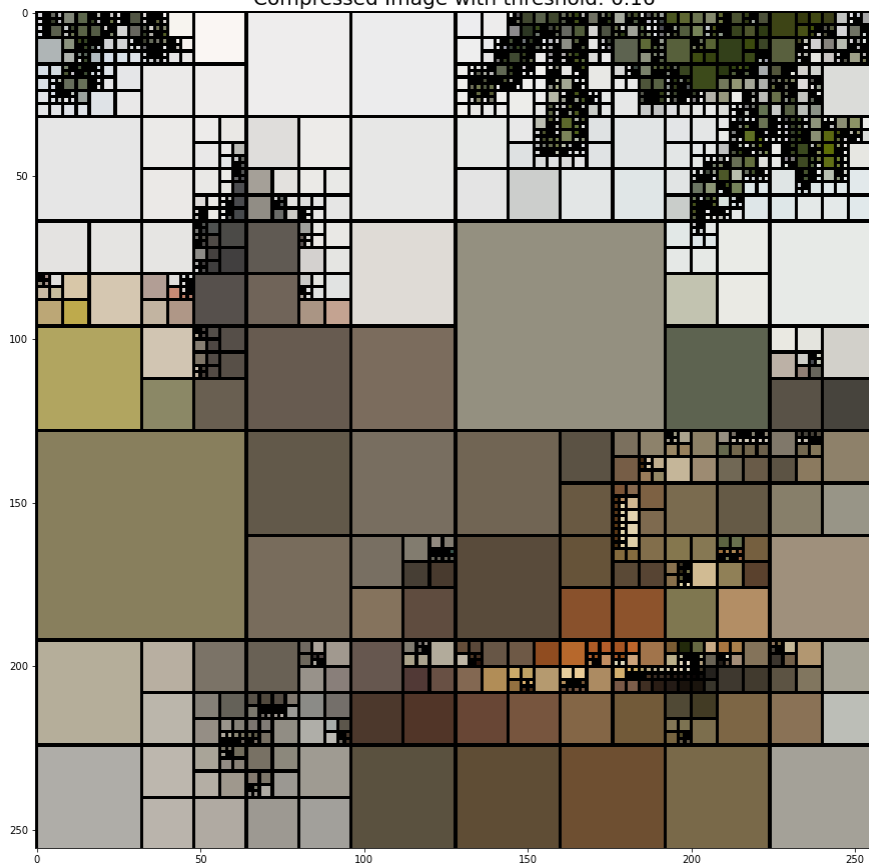
Compressed Image with threshold: 0.20



Compressed Image with threshold: 0.16



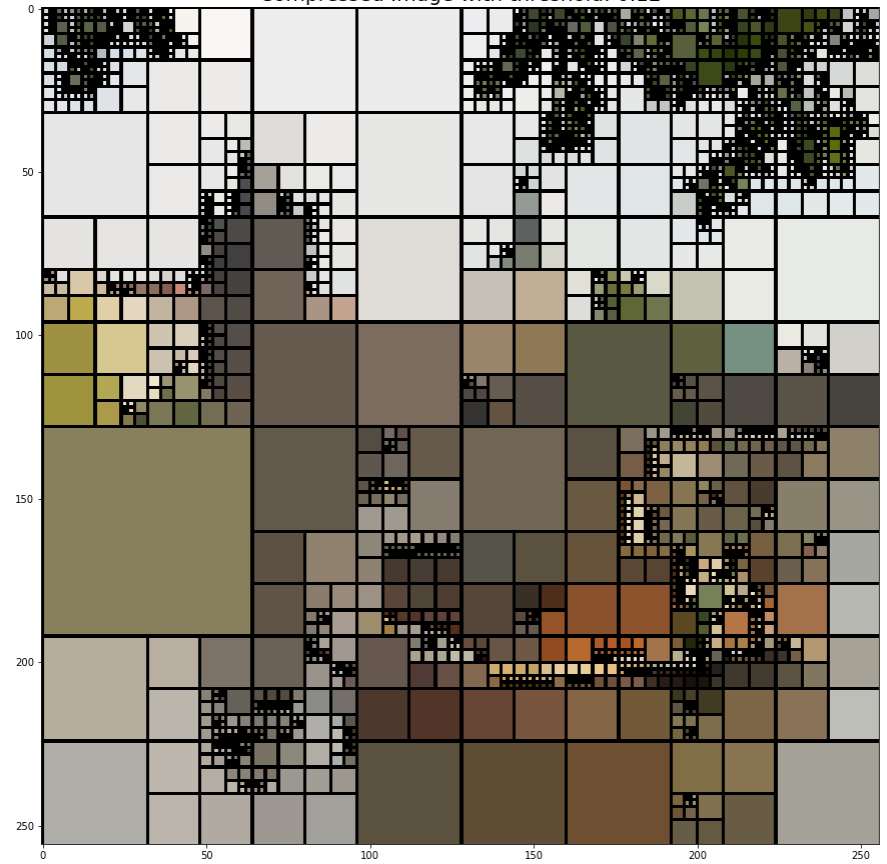
Compressed Image with threshold: 0.16



Compressed Image with threshold: 0.12



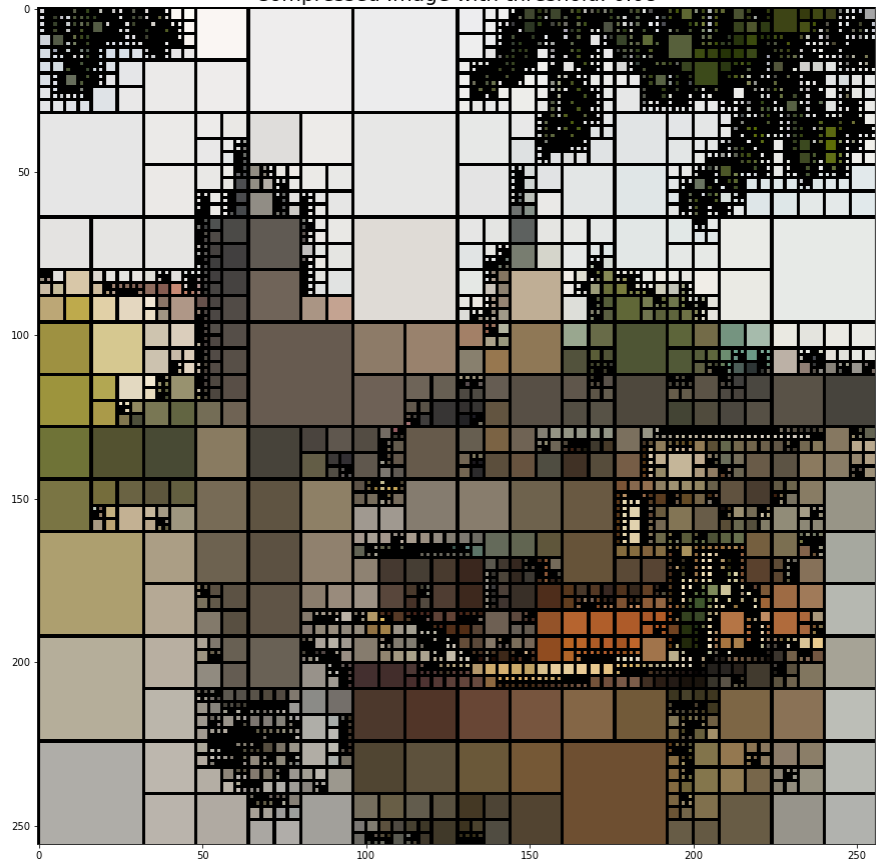
Compressed Image with threshold: 0.12



Compressed Image with threshold: 0.08



Compressed Image with threshold: 0.08



Compressed Image with threshold: 0.04



Compressed Image with threshold: 0.04



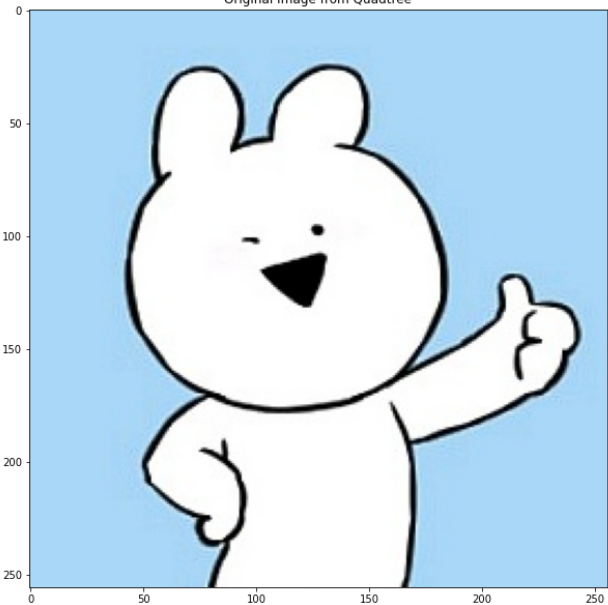
Compressed Image with threshold: 0.00



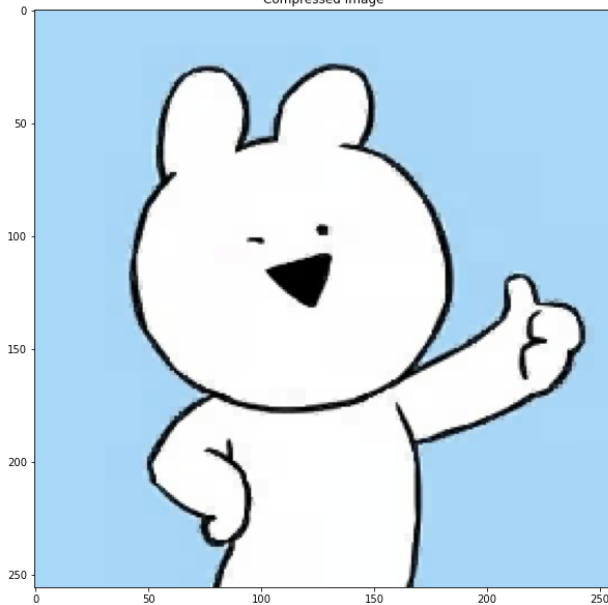
Compressed Image with threshold: 0.00



Original Image from Quadtree



Compressed Image



Compressed Image with area boundaries

