

RSA

RSA algorithm is based on factoring assumption which states that,

Let genmodulus be a polynomial time algorithm that, outputs (N, p, q) where $N = pq$, and p, q are n -bit primes except negligible probability. Then, we define an experiment such that:

- 1) Run GenModulus to obtain (N, p, q)
- 2) Adversary is given N , and outputs $p', q' > 1$
- 3) The output is defined to be 1 if $p' \cdot q' = N$, 0 otherwise.

We say that factoring is hard for all probabilistic polynomial time adversaries and that there exists a negligible function negl such that:

$$\Pr [\text{Factor}(n) = 1] \leq \text{negl}(n)$$

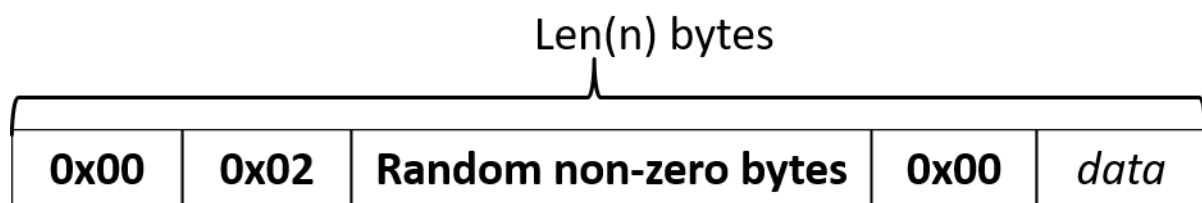
RSA can now be defined as, given N, e, y find x such that $x^e = y \pmod N$.

RSA experiment,

- 1) Run $\text{GenRSA}()$ to obtain (N, e, d)
- 2) Choose $y \leftarrow \mathbb{Z}^*$
- 3) Adversary is given N, e, y and outputs x belonging to \mathbb{Z}^*
- 4) Output of the experiment is 1 if $x^e = y \pmod N$. 0 otherwise.

Therefore, we say that RSA problem is hard if for all probabilistic polynomial time algorithms, there exists a negligible function negl such that

$$\Pr[\text{RSA-invA}; \text{GenRSA}(n) = 1] \leq \text{negl}(n)$$



Padding used for RSA encryption



Padding used for RSA signature

Build Algorithm

- 1) Generate p, q such that they are n -bit prime numbers.
- 2) Calculate $N = p \cdot q$ and $\phi_N = (p-1) \cdot (q-1)$
- 3) Calculate public and private keys by choosing e and d , such that e is co-prime with N and ϕ_N , similarly, d is chosen such that $d \cdot e \bmod \phi_N = 1$.
- 4) Define NULL byte, fixed Bytes as well as the bytes to be padded for padded RSA.
- 5) Message from the user is taken and converted into a list of ascii values per word.
- 6) The integer list is then encrypted using public key of the receiver.
- 7) Convert the encrypted list into binary and pad the bytes. We finally have the cipher we can send over a public channel.
- 8) At receiver's end, padded bytes are first removed.
- 9) Then the encrypted message list is decrypted and finally converted back to the original format.

Theorem: If the RSA problem is hard relative to GenRSA then the above algorithm has indistinguishable encryptions under chosen-plaintext attacks.