AI- Assignment - I (Theory)

Q1 (a) A = The universe Exists as it is
B = or will end in a death /heat death
C = ~~If~~ there was ~~a~~ big bang
D = Universe is Expanding.
E = Universe is accelerating

$\neg C \rightarrow A$ = If there was no big bang then the universe simply existed

$A \vee B$ = The universe exists as it is or will end in a heat death.

$C \Leftrightarrow D$ = If and only if the universe is expanding then there was a big bang

$E \wedge D \rightarrow B$ = If universe is expanding and accelerating then it will end in a heat death. $(E \wedge D \rightarrow B)$

(b) Contrapositive of $\neg C \rightarrow A$ will be $\neg A \rightarrow C$
which means :-
If Universe doesn't exist as it is. ~~it~~ ~~fo~~ then there was a big bang

Contrapositive of $E \wedge D \rightarrow B$ will be $\neg B \rightarrow \neg(E \wedge D)$
$\Rightarrow \neg B \rightarrow (\neg E \vee \neg D)$

which means:-
if Universe will not end in a heat death then it is not expanding or not accelerating

$A \vee B \equiv \neg A \rightarrow B$

contrapositive will be $\neg B \rightarrow A$ which means
If universe ~~will not~~ doesn't end in a heat death then it Existed as it is

CNF forms –

1 (c) $A \vee B$

$A \vee C$ $(\neg C \to A)$ — (ii)

$\neg E \vee \neg D \vee B$ (from $E \wedge D \to B$)

$\neg C \vee D$ — (iii)

$\neg D \vee C$ (from $C \Leftrightarrow D$)

By resolution we can say that we cannot individually infer A, B, C, D, E.

we can infer all the statements in given knowledge base as well as we can also infer $\neg A \to D$ from (ii) and (iii), if universe will not exist as it is then universe is expanding

By resolving (ii), (iii) we get $A \vee D$ ($C, \neg C$ get eliminated)

$A \vee D \equiv \neg A \to D$

(d) And-OR graph:-

$A \vee B$ can be written as $(\neg A \rightarrow B)$

Means $A \vee B \equiv \neg A \rightarrow B$

~~$\Rightarrow$ $\neg A$~~

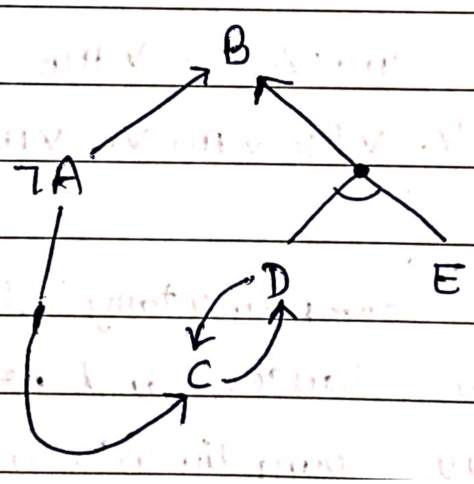
and, $\neg C \rightarrow A \equiv C \vee A$

$\Rightarrow$ $C \vee A \equiv A \vee C$

$\Rightarrow$ $A \vee C \equiv \neg A \rightarrow C$, also, $E \wedge D \rightarrow B$

overall $(\neg C \rightarrow A \equiv \neg A \rightarrow C)$ and $C \leftrightarrow D$

Hence, AND-OR graph would be:-



The fact $\neg A \rightarrow C \equiv \neg C \rightarrow A$ can be verfied from truth table also as below:-

A	C	$\neg A$	$\neg C$	$\neg A \rightarrow C$	$\neg C \rightarrow A$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	F	F

Q3 # Soundness:-

Unit resolution Inference rule is :-

$$\frac{l_1 \vee \ldots \vee l_k, \quad m}{l_1 \vee \ldots \vee l_{i-1} \vee l_{i+1} \vee \ldots \vee l_k}$$

where each l is a literal and li and m are complementary literals, this take a clause which is a disjunction of literals and a literal and it produces a new clause.

A single literal can be viewed as a disjunction of one literal, also known as a unit clause.

Unit Resolution rule can be generalized to the full resolution rule:-

$$\frac{l_1 \vee \ldots \vee l_k, \quad m_1 \vee \ldots \vee m_n}{l_1 \vee \ldots \vee l_{i-1} \vee l_{i+1} \vee \ldots \vee l_k \vee m_1 \vee \ldots \vee m_{j-1} \vee m_{j+1} \vee \ldots \vee m_n}$$

where li and mj are complementary literals. So resolution takes two clauses and produces a new clause except the two complimentary literals

If l_i is true, then m_j is false, and Hence $m_1 \vee \dots \vee m_{j-1} \vee m_{j+1} \vee \dots \vee m_n$ must be true, because $m_1 \vee \dots \vee m_n$ is given

Similarly if l_i is false then, $l_1 \vee \dots \vee l_{i-1} \vee l_{i+1} \vee \dots \vee l_k$ must be true because $l_1 \vee \dots \vee l_k$ is given

So, now l_i is either true or false, so one or other of these conclusion holds Exactly as the resolution rule states, which easily proves the soundness of the resolution.

Completeness of resolution:-

Resolution closure RC(S) represents a set of clauses S, which is the set of all clauses derivable by repeated application of the resolution rule to clauses in S or their derivatives.
There are only finitely many distinct clauses that can be constructed out of the symbols $P_1, \dots P_k$ that appears in S.

Completeness theorem:-
If a set of clauses is unsatisfiable, then the resolution closure of those clauses contains empty clause.
Theorem is proved by demonstrating its contrapositive it closure RC(S) does not contain the empty clause then S is satisfiable. In fact we can construct a model for S with suitable truth for $P_1, \dots, P_k$.

from $i=1$ to $i=K$

→ If a clause in $RC(S)$ contains the literal $\neg P_i$ and all its other literals are false under the assignment chosen for $P_1, \ldots P_{i-1}$, then assign false to P_i

→ otherwise, assign true to P_i

Assignment to $P_1, P_2, \ldots P_K$ is a model of S. To see this assume the opposite that, at some stage i in the sequence, assigning symbol P_i causes some clause C to become false.

For this to happen, it must be the case that all the other literals in C must already have been falsified by assignment to $P_1, \ldots, P_{i-1}$. Thus C must ~~took~~ now look like either.

(false v false v --- false $\vee P_i$) or like (false v false v--- false $\vee \neg P_i$)

If just one of these two is in $RC(S)$, then the algorithm will assign the appropriate truth value to P_i to make c true, so c can only be falsified if both of these clauses are in $RC(S)$.

Now, since $RC(S)$ is closed under resolution, it will contain the resolvent of these two clauses, and that resolvent will have all of its literals already falsified by the assignments to $P_1, \ldots P_{i-1}$

so, this contradicts our assumption that the first falsified clause appears at stage i,

Hence we have proved that the construction never falsifies a clause in $RC(S)$; that is it produces a model of $RC(S)$. Finally because S is contained in $RC(S)$, any model of $RC(S)$ is a model of S itself.