用分层样本的均值、方差估计总体的均值、方差

1.若一个总体划分为两层，通过按样本量比例分配分层随机抽样，各层抽取的样本量、样本平均数和样本方差分别为：m，$\bar{x}$，S_1^2；n，$\bar{y}$，S_2^2．记总的样本平均数为$\bar{w}$，样本方差为S^2，证明：

① $\bar{w}=\dfrac{m}{m+n}\bar{x}+\dfrac{n}{m+n}\bar{y}$；

② $S^2=\dfrac{1}{m+n}\left\{m\left[S_1^2+\left(\bar{x}-\bar{w}\right)^2\right]+n\left[S_2^2+\left(\bar{y}-\bar{w}\right)^2\right]\right\}$．

$$S^2=\frac{1}{m+n}\left[\sum_{i=1}^{m}(x_i-\bar{w})^2+\sum_{j=1}^{n}(\mathrm{y}_j-\bar{w})^2\right]=\frac{1}{m+n}\left[\sum_{i=1}^{m}(x_i-\bar{x}+\bar{x}-\bar{w})^2+\sum_{j=1}^{n}(\mathrm{y}_j-\bar{y}+\bar{y}-\bar{w})^2\right]$$

$$=\frac{1}{m+n}\left[\sum_{m}^{i=1}(x_i-\bar{x})^2+2\sum_{m}^{i=1}(x_i-\bar{x})(\bar{x}-\bar{w})+m(\bar{x}-\bar{w})^2+\right.$$

$$\left.\sum_{n}^{j=1}(y_j-\bar{y})^2+2\sum_{n}^{j=1}(y_j-\bar{y})(\bar{y}-\bar{w})+n(\bar{y}-\bar{w})^2\right]$$

又$\sum\limits_{i=1}^{m}(x_i-\bar{x})(\bar{x}-\bar{w})=\sum\limits_{i=1}^{m}x_i(\bar{x}-\bar{w})-m\bar{x}(\bar{x}-\bar{w})=m\bar{x}(\bar{x}-\bar{w})-m\bar{x}(\bar{x}-\bar{w})=0$

同理$\sum\limits_{j=1}^{n}(\mathrm{y}_j-\bar{y})(\bar{y}-\bar{w})=0$，

$$\therefore S^2=\frac{1}{m+n}\left[\sum_{i=1}^{m}(x_i-\bar{x})^2+m(\bar{x}-\bar{w})^2+\sum_{j=1}^{n}(\mathrm{y}_j-\bar{y})^2+n(\bar{y}-\bar{w})^2\right]$$

$$=\frac{1}{m+n}\left[mS_1^2+m(\bar{x}-\bar{w})^2+\mathrm{n}S_2^2+n(\bar{y}-\bar{w})^2\right]$$

$$=\frac{1}{m+n}\left\{m\left[S_1^2+(\bar{x}-\bar{w})^2\right]+n\left[S_2^2+\left(\bar{y}-\bar{w}\right)^2\right]\right\}.$$

2.三层的结论