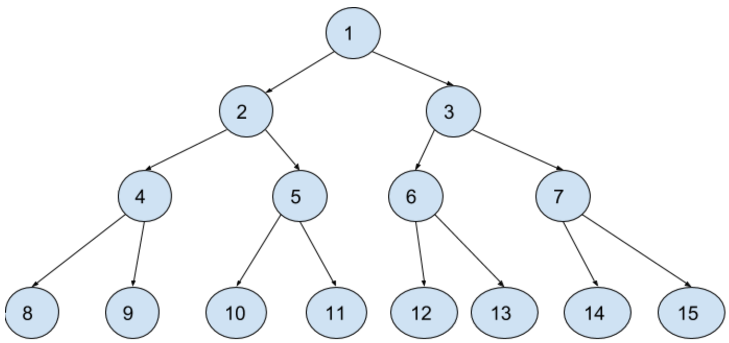
Homework 2: Solutions
CSCI 5511

Problem 1

a. 

b. BFS: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11
DLS3: 1, 2, 4, 8, 9, 5, 10, 11
IDS: 1, 1, 2, 3, 1, 2, 4, 5, 3, 6, 7, 1, 2, 4, 8, 9, 5, 10, 11

c. Yes. We define our heuristic function $h(n)$ with goal state g using pseudo-code shown below.

```
define h(n):
    x = g
    while (x > n):
        if x is even:
            x = x/2
        else:
            x = (x-1)/2
    if x=n:
        return 0
    else:
        return 1
```

This function returns 0 if the node is along the path to the goal and 1 otherwise for any goal g. Because best first search will explore the node with least cost, the next node to be explored will be the next node on the path to the goal. The order in which these nodes are explored is thus: 1, 2, 5, 11

Problem 2

a. When $w = 0$, *f(n) = 300*h(n)*. So, **Best-First Search/ Greedy Search**.

b. When $w = 150$, *f(n) = 150*g(n) + 150*h(n)*. So, **A* Search**.

c. It is optimal for all $w \in [150, 300]$. In this case the path cost is more important than the heuristic making it look for the lower cost path to the goal. This becomes obvious in the limit when $w = 300$ and the algorithm expands the lowest cost path first (uniform cost search).

Problem 3

No. An admissible heuristic never overestimates. At the goal, we will have placed all four queens in an arrangement such that they cannot attack one another. Placing 4 queens will mean we are 4 levels deep in the tree. Because our goal state must be at this level, the heuristic must have a non-zero value at the goal state and thus overestimates the cost to the goal in at least one case. Because the heuristic over-estimates, the heuristic is not admissible.

Problem 4

We represent our state as the ordered pair (i, j) representing being in location row i and column j of the 2d grid maze. For each state in the maze we know it is either wall or not, thus each state (i, j) is a member of the set $BLACK$ (wall) or $WHITE$ (not wall) where $BLACK \cup WHITE$ is the set of all states and $BLACK \cap WHITE = \emptyset$. We now define the function f that operates on sets of states $A \in BLACK \cup WHITE$ as

$$f(A) = A \cap WHITE$$

To generate the children of node (i, j) we can define the successor function of state (i, j) as

$$succ\left((i,j)\right) = f\left(\{(i+1,j),(i-1,j),(i,j+1),(i,j-1)\}\right)$$

Notice that we don't need to worry about whether we are on the edge of the maze grid or not because we can always add a ring of black squares around the edge of the grid so we wont be able to "fall off".

With these structures we can now define an admissible heuristic h to be the l_1/taxicab/Manhatten distance to the goal state. For goal state (r_g, c_g), our heuristic is $h((i,j)) = |i - r_g| + |j - c_g|$. This is the distance to our the goal state in as straight a path as is possible in our grid system and thus cannot overestimate because if there is no wall in the way the distance is exactly the cost to goal and if there are walls in the way the distance is less than the cost to the goal assuming uniform costs of 1 for any motion. Because this heuristic cannot overestimate it is admissible. This heuristic will be helpful because it will favor states closer to the goal so those will be explored first. This will cause our succession of states to be closer to a straight line to the goal when possible.

Problem 5

a. $3.5 * (h_1 + h_2)$ will be an overestimate since it will be a multiple of the sum of the two. Thus, **Not Admissible**.

b. $min(h_1, h_2)$ will be an underestimate since it will either be h_1 or h_2, *i.e.* minimum of the two. Thus, **Admissible**.

c. $max(h_1, \frac{h_2}{11})$ will be an underestimate since it will either be h_1 or $h_2/11$ which is even lesser than h_2. Thus, **Admissible**.

Problem 6

a. BFS is uniform cost search when the cost of expanding a node is the depth of that node. Because of this, all nodes at a lower depth are lower cost and are thus expanded first.

b. BFS is best first with an evaluation function equal to the node depth, thus all nodes at lower depths will be expanded first. DFS is best first with an evaluation function equal to the negative of the depth, causing nodes at a deeper depth to be expanded first. Uniform cost search is best first where the evaluation function is the path cost so far, meaning the lowest cost path is expanded forst.

c. Uniform cost search is A* with heuristic $h(n) = 0$ for all n, thus the lowest path cost node is expanded first.