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Assignment#1
CSci 5512

Question 1

$$
\begin{aligned}
P(d|\neg b) &= \alpha P(d, \neg b) \\
&= \alpha \sum_a \sum_c \sum_e \sum_f P(a, \neg b, c, d, e, f) \\
&= \alpha \sum_a \sum_c \sum_e \sum_f P(a)P(\neg b)P(c|a, \neg b)P(d|c)P(e|c)P(f|e, d) \\
&= \alpha P(\neg b) \sum_c P(d|c) \sum_a P(a)P(c|a, \neg b) \cancelto{1}{\sum_e P(e|c)} \cancelto{1}{\sum_f P(f|e, d)} \\
&= \alpha P(\neg b) \sum_c P(d|c) \sum_a P(a)P(c|a, \neg b) \\
&= \alpha f_1() \sum_c f_2(C) \sum_a f_3(A) f_4(A, C)
\end{aligned}
$$

where, $f_1() = P(\neg b),\ f_2(C) = P(d|c),\ f_3(A) = P(a),\ f_4(A, C) = P(c|a, \neg b)$
$f_{34}(a, c) = P(a) \times P(c|a, \neg b) = 0.5 \times 0.7 = 0.35$
$f_{34}(a, \neg c) = P(a) \times P(\neg c|a, \neg b) = 0.5 \times 0.3 = 0.15$
$f_{34}(\neg a, c) = P(\neg a) \times P(c|\neg a, \neg b) = 0.5 \times 0.2 = 0.1$
$f_{34}(\neg a, \neg c) = P(\neg a) \times P(\neg c|\neg a, \neg b) = 0.5 \times 0.8 = 0.4$

$$
\begin{aligned}
&= \alpha f_1() \sum_c f_2(C) \sum_a f_3(A) f_4(A, C) \\
&= \alpha f_1() \sum_c f_2(C) \sum_a f_{34}(A, C)
\end{aligned}
$$

$f_{34a}(c) = f_{34}(a, c) + f_{34}(\neg a, c) = 0.35 + 0.1 = 0.45$
$f_{34a}(\neg c) = f_{34}(a, \neg c) + f_{34}(\neg a, \neg c) = 0.15 + 0.4 = 0.55$

$$= \alpha f_1() \sum_c f_2(C) f_{34a}(C)$$

$f_{234a}(c) = f_2(c) \times f_{34a}(c) = P(d|c) \times f_{34a}(c) = 0.3 \times 0.45 = 0.135$
$f_{234a}(\neg c) = f_2(\neg c) \times f_{34a}(\neg c) = P(d|\neg c) \times f_{34a}(\neg c) = 0.6 \times 0.55 = 0.33$

$$= \alpha f_1() \sum_c f_{234a}(C)$$

$f_{234ac}() = f_{234a}(c) + f_{234a}(\neg c) = 0.135 + 0.33 = 0.465$

$$
\begin{aligned}
&= \alpha f_1() f_{234ac}() \\
&= \alpha (0.3)(0.465) \\
P(d|\neg b) &= \alpha 0.1395
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
P(\neg d|\neg b) &= \alpha 0.1605 \\
\implies P(d|\neg b) &= 0.465
\end{aligned}
$$

Question 2

$$
\begin{aligned}
&P\left((1-\epsilon)\mu \leq LikelihoodEstimate \leq (1+\epsilon)\mu)\right) > 1-\delta \\
&N \geq \frac{4}{\mu\epsilon^2}\ln\frac{2}{\delta}
\end{aligned}
$$

Now, $\delta = 0.05$ & $\epsilon = 0.01$
Moreover, $(1+\epsilon)\mu \leq 1 \implies \mu \leq 0.9901$
Therefore,

$$
\begin{aligned}
N &\geq \frac{4}{0.9901 \times 0.0001}\ln\frac{2}{0.05} \\
&\geq 149030
\end{aligned}
$$

Question 3

$P(g|k, \neg b, c) = 0.2628$

Question 4

$P(g|k, \neg b, c) = 0.256$

Question 5

(5.1) MarkovBlanket(G) = {C, D, E, I, J}

(5.2)

$$
\begin{aligned}
P(g|MarkovBlanket(G)) &= \alpha P(g|Parent(G)) \prod_{y \in Children(G)} P(y|Parent(Y)) \\
&= \alpha P(g|c, d, e)P(i|f, g)P(j|g, h) \\
&= \alpha(0.1)(0.8)(0.2) \\
&= \alpha 0.016 \\
P(\neg g|MarkovBlanket(G)) &= \alpha P(\neg g|Parent(G)) \prod_{y \in Children(G)} P(y|Parent(Y)) \\
&= \alpha P(\neg g|c, d, e)P(i|f, \neg g)P(j|\neg g, h) \\
&= \alpha(0.9)(0.6)(0.9) \\
&= \alpha 0.486 \\
\implies P(g|MarkovBlanket(G)) &= \frac{0.016}{0.016 + 0.486} = 0.0319
\end{aligned}
$$

(5.3) We use Gibbs sampling to find $P(g|c, d, e, f)$ and the value obtained (0.1066) is very close to 0.1 i.e. $P(g|c, d, e)$.

$$
\begin{aligned}
P(g|c,d,e,f) =& \alpha P(c,d,e,f,g) \\
=& \alpha \sum_a \sum_b \sum_h \sum_i \sum_j \sum_k P(a)P(b)P(c|a)P(d|a)P(e|b)P(f)P(g|c,d,e)P(h|e)P(i|f,g) \\
& P(j|g,h)P(k|i) \\
=& \alpha P(f)P(g|c,d,e) \sum_a P(a)P(c|a)P(d|a) \sum_b P(b)P(e|b) \\
=& \alpha P(f)P(g|c,d,e)((0.3)(0.2)(0.8)+(0.7)(0.5)(0.4))((0.6)(0.8)+(0.4)(0.1)) \\
=& \alpha P(f)P(g|c,d,e)0.09776 \\
=& \alpha' P(g|c,d,e) \\
=& P(g|c,d,e)
\end{aligned}
$$

$\implies$ g and f are conditionally independent.

(5.4) Again we use Gibbs sampling to find $P(g|f)$ and $P(g)$. The values obtained (0.5122 and 0.4975 respectively) are almost similar.

$$
\begin{aligned}
P(g) =& \sum_a \sum_b \sum_c \sum_d \sum_e \sum_f \sum_h \sum_i \sum_j \sum_k P(a,b,c,d,e,f,g,h,i,j,k) \\
=& \sum_a \sum_b \sum_c \sum_d \sum_e \sum_f \sum_h \sum_i \sum_j \sum_k P(a)P(b)P(c|a)P(d|a)P(e|b)P(f)P(g|c,d,e)P(h|e)P(i|f,g) \\
& P(j|g,h)P(k|i) \\
=& \sum_a P(a) \sum_c P(c|a) \sum_d P(d|a) \sum_b P(b) \sum_e P(e|b)P(g|c,d,e) \sum_f \cancelto{1}{P(f)} \sum_h \cancelto{1}{P(h|e)} \sum_j \cancelto{1}{P(j|g,h)} \\
& \sum_i \cancelto{1}{P(i|f,g)} \sum_k \cancelto{1}{P(k|i)} \\
P(g|f) =& \alpha \sum_a \sum_b \sum_c \sum_d \sum_e \sum_h \sum_i \sum_j \sum_k P(a,b,c,d,e,f,g,h,i,j,k) \\
=& \alpha \sum_a \sum_b \sum_c \sum_d \sum_e \sum_h \sum_i \sum_j \sum_k P(a)P(b)P(c|a)P(d|a)P(e|b)P(f)P(g|c,d,e)P(h|e)P(i|f,g) \\
& P(j|g,h)P(k|i) \\
=& \alpha P(f) \sum_a P(a) \sum_c P(c|a) \sum_d P(d|a) \sum_b P(b) \sum_e P(e|b)P(g|c,d,e) \sum_h \cancelto{1}{P(h|e)} \sum_j \cancelto{1}{P(j|g,h)} \\
& \sum_i \cancelto{1}{P(i|f,g)} \sum_k \cancelto{1}{P(k|i)} \\
=& \alpha P(f)P(g) \\
P(\neg g|f) =& \alpha P(f)P(\neg g) \implies P(g|f) = P(g)
\end{aligned}
$$

$\implies$ g and f are independent.

(5.5)

$$\begin{aligned}
P(a|c,d,e) =& \alpha \sum_b P(a)P(b)P(c|a)P(d|a)P(e|b) \\
=& \alpha P(a)P(c|a)P(d|a) \sum_b P(b)P(e|b) \\
P(g,a|c,d,e) =& \alpha \sum_b P(a)P(b)P(c|a)P(d|a)P(e|b)P(g|c,d,e) \\
=& \alpha P(a)P(c|a)P(d|a)P(g|c,d,e) \sum_b P(b)P(e|b) \\
=& P(g|c,d,e)P(a|c,d,e)
\end{aligned}$$

$\implies$ A and G are conditionally independent given C,D and E.

(5.6) J and G are not conditionally independent.

Question 6

Let Z = A+B = $A \cup B$,

$$P(b) = \sum_a P(a,b) = \sum_a P(b|a)P(a) = P(b|a)P(a) + P(b|\neg a)P(\neg a) = 1(0.3) + (0.5)(0.7) = 0.65$$

$$P(z) = P(a) + P(b) - P(a,b) = P(a) + \cancel{P(b|a)P(a)} + P(b|\neg a)P(\neg a) - \cancel{P(b|a)P(a)} = 0.3 + (0.5)(0.7) = 0.65$$

$$P(a|z) = \frac{P(a,b) + P(a,\neg b)}{P(z)} = \frac{P(b|a)P(a) + P(\neg b|a)P(a)}{P(z)} = \frac{(1)(0.3) + (0)(0.3)}{0.65} = \frac{0.3}{0.65} = 0.4615$$

$$P(a|\neg z) = P(b|\neg z) = 0$$

$$P(b|z) = \frac{P(a,b) + P(\neg a,b)}{P(z)} = \frac{P(b|a)P(a) + P(b|\neg a)P(\neg a)}{P(z)} = \frac{(1)(0.3) + (0.5)(0.7)}{0.65} = \frac{0.65}{0.65} = 1$$

$$P(c|z) = \frac{P(c,z)}{P(z)} = \frac{P(c,a,b) + P(c,\neg a,b) + P(c,a,\neg b)}{P(z)} = \frac{(0.3)(1)(0.4) + (0.7)(0.5)(0.9) + (0.3)(0)(0.4)}{0.65} = 0.67$$

$$P(c|\neg z) = \frac{P(c,\neg z)}{P(\neg z)} = \frac{P(c,\neg a,\neg b)}{P(\neg z)} = \frac{(0.7)(0.5)(0.9)}{(0.5)(0.7)} = 0.9$$

$$P(d|z,c) = \frac{P(d,z,c)}{P(z,c)} = \frac{P(a,b,c,d) + P(a,\neg b,c,d) + P(\neg a,b,c,d)}{P(a,b,c) + P(a,\neg b,c) + P(\neg a,b,c)}$$

$$= \frac{(0.3)(1)(0.4)(0.4) + (0.3)(0)(0.4)(0.3) + (0.7)(0.5)(0.9)(0.4)}{(0.3)(1)(0.4) + (0.3)(0)(0.4) + (0.7)(0.5)(0.9)} = \frac{0.174}{0.435} = 0.4$$

$$P(d|z,\neg c) = \frac{P(d,z,\neg c)}{P(z,\neg c)} = \frac{P(a,b,\neg c,d) + P(a,\neg b,\neg c,d) + P(\neg a,b,\neg c,d)}{P(a,b,\neg c) + P(a,\neg b,\neg c) + P(\neg a,b,\neg c)}$$

$$= \frac{(0.3)(1)(0.6)(0.9) + (0.3)(0)(0.6)(0.5) + (0.7)(0.5)(0.1)(0.9)}{(0.3)(1)(0.6) + (0.3)(0)(0.6) + (0.7)(0.5)(0.1)} = \frac{0.1935}{0.215} = 0.9$$

$$P(d|\neg z,c) = \frac{P(d,\neg z,c)}{P(\neg z,c)} = \frac{P(\neg a,\neg b,c,d)}{P(\neg a,\neg b,c)} = \frac{(0.7)(0.5)(0.9)(0.3)}{(0.7)(0.5)(0.9)} = 0.3$$

$$P(d|\neg z,\neg c) = \frac{P(d,\neg z,\neg c)}{P(\neg z,\neg c)} = \frac{P(\neg a,\neg b,\neg c,d)}{P(\neg a,\neg b,\neg c)} = \frac{(0.7)(0.5)(0.1)(0.5)}{(0.7)(0.5)(0.1)} = 0.5$$

Given Z, we have the probabilities $P(A|Z)$ and $P(B|Z)$ to get the values of A and B.