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Assignment#1
CSci 5512

Question 1

According to normalization,

$$P(a|b) = \alpha P(a,b) \tag{1}$$

We know that,

$$\begin{aligned} &P(a|b) + P(\neg a|b) = 1 \\ &\alpha P(a,b) + \alpha P(\neg a,b) = 1 \qquad from(1) \\ &\alpha P(b) = 1 \\ &P(b) = \frac{1}{\alpha} \end{aligned} \tag{2}$$

Now,

$$\begin{aligned} &P(a|b) = \alpha P(a,b) \\ \implies &P(a|b) = \alpha P(b|a)P(a) \\ \implies &P(a|b) = \frac{P(b|a)P(a)}{P(b)} \qquad from(2) \end{aligned} \tag{3}$$

But equation(3) is also the Bayes Theorem. Therefore instead of dividing by $P(b)$, we can also find $P(\neg a|b)$ and normalize

Question 2

(1) $0 \le P(\omega) \le 1$

(2) $\sum_{\omega \epsilon \Omega} P(\omega) = 1$

(3) $P(a) + P(\neg a) = 1$

(4) $P(a \ or \ b) = P(a) + P(b) - P(a,b)$

(5) $P(a) = \sum_b P(a,b)$

Given: P(a) = 0.2, P(b) = 0.3, P(a or b)=0.1
Proof: Using (4) we get,

$$P(a,b) = 0.4$$

From (1), we know that $P(\omega)$ is a non-negative number.
Therefore, from (5)

$$P(a) \ge P(a,b) \ \ \& \ \ P(b) \ge P(a,b)$$

But from the values we get that,

$$P(a) < 0.4 \ \ \& \ \ P(b) < 0.4$$

Thus the probabilities are not consistent with each other.

Question 3

(1) P(a,b)

$$\begin{aligned}
P(a,b) &= \sum_c P(a,b,c) \\
&= P(a,b,c) + P(a,b,\neg c) \\
&= 0.018837 + 0.011063 \\
&= 0.0299
\end{aligned}$$

(2) P(a,b|c)

$$\begin{aligned}
P(c) &= \sum_{a,b} P(a,b,c) \\
&= P(a,b,c) + P(\neg a,b,c) + P(a,\neg b,c) + P(\neg a,\neg b,c) \\
&= 0.018837 + 0.126324 + 0.063063 + 0.160776 \\
&= 0.369 \\
P(a,b|c) &= \frac{P(a,b,c)}{P(c)} \\
&= \frac{0.018837}{0.369} \\
&= 0.051
\end{aligned}$$

(3) P(c|¬a)

$$\begin{aligned}
P(\neg a,c) &= \sum_b P(\neg a,b,c) \\
&= P(\neg a,b,c) + P(\neg a,\neg b,c) \\
&= 0.126324 + 0.160776 \\
&= 0.2871 \\
P(\neg a) &= \sum_{b,c} P(\neg a,b,c) \\
&= 0.8700 \\
P(c|\neg a) &= \frac{P(\neg a,c)}{P(\neg a)} \\
&= 0.22871/0.87 \\
&= 0.330
\end{aligned}$$

(4) P(b)

$$\begin{aligned}
P(b) &= P(a,b,c) + P(\neg a,b,c) + P(a,b,\neg c) + P(\neg a,b,\neg c) \\
&= 0.018837 + 0.126324 + 0.011063 + 0.256476 \\
&= 0.4127
\end{aligned}$$

Question 4

$$P(a) \times P(b) = 0.1300 \times 0.4127 = 0.0537 \tag{1}$$
$$P(a, b) = 0.0299 \tag{2}$$
$$P(a) \times P(c) = 0.1300 \times 0.3690 = 0.0480 \tag{3}$$
$$P(a, c) = 0.0819 \tag{4}$$
$$P(b) \times P(c) = 0.4127 \times 0.3690 = 0.1523 \tag{5}$$
$$P(b, c) = 0.1452 \tag{6}$$

Since $(1) \neq (2), (3) \neq (4)$ & $(5) \neq (6)$, none of the variables are independent.

$$P(a|c) \times P(b|c) = 0.2220 \times 0.3934 = 0.0873 \tag{7}$$
$$P(a, b|c) = 0.0510 \tag{8}$$
$$P(a|b) \times P(c|b) = 0.0724 \times 0.3517 = 0.0255 \tag{9}$$
$$P(a, c|b) = 0.0456 \tag{10}$$
$$P(b|a) \times P(c|a) = 0.2300 \times 0.6300 = 0.1449 \tag{11}$$
$$P(b, c|a) = 0.1449 \tag{12}$$

Since (11) = (12), b and c are conditionally independent.

Question 5

(1) Let the variable order be A, B and C.

- First loop iteration, we need to find, P(A|Parent(A)). But since i=1, no parents needed to be found.

$$P(A|nothing) = P(A)$$

- Next loop iteration, We find P(B|Parent(B)) for which we have two options,

$$P(B|A) = \begin{cases} P(B|A) \\ P(B) \end{cases}$$

  But since B and A are not independent, the second options is not possible. So Parent(B)=A

- Last iteration we find, P(C|Parent(C)),

$$P(C|Parent(C)) = P(C|B, A)$$

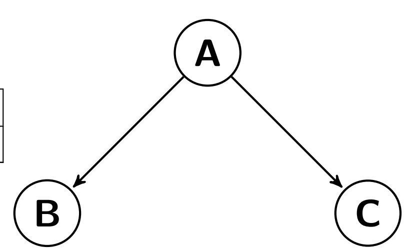
  but since C and B are conditionally independent,

$$\implies P(C|Parent(C)) = P(C|B, A) = P(C|A)$$

  So Parent(C)=A

So depending on this network we have,

P(a)=0.1300	P(b|a) = 0.2300	P(c|a) = 0.6300
	P(b|¬a) = 0.4400	P(c|¬a) = 0.3300



(2) Let the variable order be C, B and A.

- First loop iteration, we need to find, P(C|Parent(C)). But since i=1, no parents needed to be found.

$$P(C|nothing) = P(C)$$

- Next loop iteration, We find P(B|Parent(B)) for which we have two options,

$$P(B|C) = \begin{cases} P(B|C) \\ P(B) \end{cases}$$

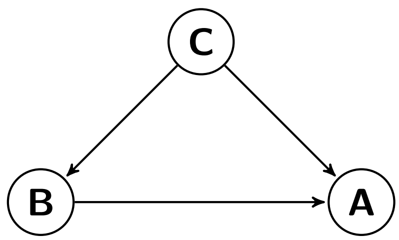
  But since B and C are not independent, the second options is not possible. So Parent(B)=C

- Last iteration we find, P(A|Parent(A)),

$$P(A|Parent(A)) = P(A|B, C)$$

So depending on this network we have,

P(c)=0.3690	P(b|c) = 0.3934	P(a|b,c) = 0.1298
		P(a|¬b,c) = 0.2817
	P(b|¬c) = 0.4240	P(a|b,¬c)=0.0414
		P(a|¬b,¬c)=0.1019



Question 6

(1) let the "slot reward" be the random variable. According to the given information,

$P(slot\_reward = 100) = 0.1$
$P(slot\_reward = 30) = 0.3$
$P(slot\_reward = 5) = 0.5$
$P(slot\_reward = 0) = 0.1$

(2) Using the above probabilities the expected reward from the slot machine is,

$$\begin{aligned} E[reward] &= \sum reward * P(reward) \\ &= (100 * 0.1) + (30 * 0.3) + (5 * 0.5) + (0 * 0.1) \\ &= 10 + 9 + 2.5 \\ &= 21.5 \end{aligned}$$

Therefore the Casino should attach \$21.5 to play at this machine.

(3)

$$\begin{aligned} P(atleast\ 1\ reward) &= 1 - P(no\ reward) \\ &= 1 - (0.1)^5 \\ &= 1 - 0.00001 \\ &= 0.99999 \end{aligned}$$

Question 7

$$
\begin{aligned}
E[reward] =& \sum reward * P(reward) \\
=&(100+0)(0.1)(0.5)+(100+5)(0.1)(0.5)+ \\
&(30+0)(0.3)(0.1)+(30+5)(0.3)(0.5)+(30+30)(0.3)(0.3)+(30+100)(0.3)(0.1)+ \\
&(5+0)(0.5)(0.1)+(5+5)(0.5)(0.5)+(5+30)(0.5)(0.3)+(5+100)(0.5)(0.1)+ \\
&(0+0)(0.1)(0.1)+(0+5)(0.1)(0.5)+(0+30)(0.1)(0.3)+(0+100)(0.1)(0.1) \\
=&5+5.25+0.9+5.25+5.4+3.9+0.25+2.5+5.25+5.25+0+0.25+0.9+1 \\
=&41.1
\end{aligned}
$$