Q1: $p(x,y) = p(y|x)p(x)$ $\quad E[l(f(x),y)] = \int_x \{\int_y l(f(x),y)\, p(y|x)\} p(x)dx$

(a) $l(f(x),y) = (f(x)-y)^2$

$\Rightarrow E[l(f(x),y)] = \int_x \{\int_y (f(x)-y)^2 p(y|x)\, dy\} p(x)dx.$

SINCE FOR ANY VALVE OF $x$, $f(x)$ CAN BE CHOSEN INDEPENDENTLY

$\Rightarrow E[l(f(x),y)] = \int (f(x)-y)^2 p(y|x)dy.$

FOR OPTIMAL SOLUTION $dE/df = 0$

$\Rightarrow \frac{d\,E[l(f(x),y)]}{df(x)} = 2\int (f(x)-y)\, p(y|x)dy = 0$

$\Rightarrow \int f(x)\, p(y|x)\, dy = \int y\, p(y|x)dy.$

$\Rightarrow \int f(x) \frac{p(x,y)}{p(x)}\, dy = \int y\, p(y|x)dy$

$f(x) = \int y\, p(y|x)dy$

$f(x) = E[y|x]$

(b) $l(f(x),y) = |f(x)-y|$

$\Rightarrow E[l(f(x),y)] = \int_x \{\int_y |f(x)-y|\, p(y|x)dy\} p(x)dx$

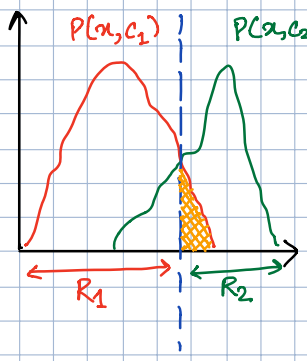
$\Rightarrow E[l(f(x),y)] = \int |f(x)-y|\, p(y|x)dy$

FOR OPTIMAL SOLUTION $dE/df = 0$

$$\Rightarrow \frac{dE[L(f(x),y)]}{df} = \int \text{sgn}(f(x)-y)\, p(y|x)\, dy$$

$$\Rightarrow \int_{f(x)}^{\infty} p(y|x)\, dy = \int_{-\infty}^{f(x)} p(y|x)\, dy$$

Q2




LETS TAKE THE EXAMPLE OF TWO CLASSES
$R_1$ IS THE REGION WHERE $P(C_1|x) > P(C_2|x)$
$R_2$ IS THE REGION WHERE $P(C_2|x) > P(C_1|x)$

ERR[y=C1] = SHADED REGION

$$= \int_{R_2} P(x, C_1)\, dx$$

$$= \int_{R_2} P(C_1|x)\, P(x)\, dx$$

$$= \int_{R_1} P(C_1|x)\, P(x)\, dx + \int_{R_2} P(C_1|x)\, P(x)\, dx - \int_{R_1} P(C_1|x)\, P(x)\, dx$$

$$= \sum_{i=1,2} \int_{R_i} P(C_1|x)\, P(x)\, dx - \int_{R_1} P(C_1|x)\, P(x)\, dx$$

GENERALIZING THE ABOVE TERM FOR N CLASSES, WE GET

$$ERR[y=C_j] = \sum_{i=1:N} \int_{R_i} P(C_j|x)\, P(x)\, dx - \int_{R_j} P(C_j|x)\, P(x)\, dx$$

Q3 (a) FISCHERS LDA FOR TWO CLASSES.

BETWEEN CLASS COVARIANCE: $S_B = (m_2 - m_1)(m_2 - m_1)^T$

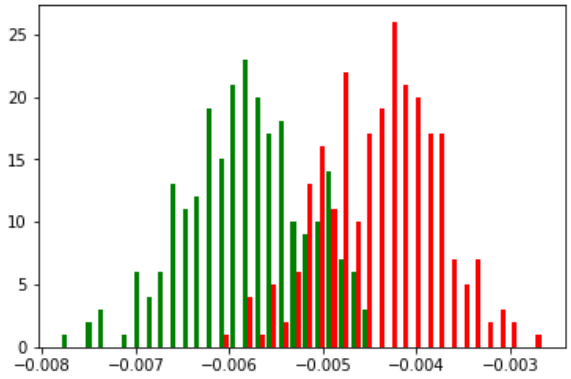
WITHIN CLASS COVARIANCE: $S_W = \sum_{x_n \in C_1} (x_n - m_1)(x_n - m_1)^T + \sum_{x_n \in C_2} (x_n - m_2)(x_n - m_2)^T$

DESIRABLE TO HAVE LOW WITHIN CLASS VARIANCE AND HIGH BETWEEN CLASS VARIANCE.

$J(W) = \frac{W^T S_B W}{W^T S_W W}$. FISCHER'S CRITERION

MAXIMISING THIS GIVES $W \propto S_W^{-1}(m_2 - m_1)$

PLOT OF BOSTON DATA ON BEING PROJECTED TO R1 DIMENSION




(b) $S_B$ IS AN OUTER PRODUCT OF TWO VECTORS, THEREFORE IT HAS RANK 1. FOR MULTICLASS CASE, $S_B$ IS WRITTEN AS

$S_B = \sum_{k=1}^{K} N_k (\mu_k - \mu)(\mu_k - \mu)^T$ (WHERE K IS THE # CLASSES)

$\Rightarrow$ $S_B$ IS COMPOSED OF K MATRICES EACH OF WHICH ARE OF RANK 1 MOREOVER, OUT OF THIS ONLY (K-1) ARE INDEPENDENT AND THEREFORE THERE ARE ATMOST (K-1) NONZERO EIGENVALUES.

THUS FOR TWO CLASS PROBLEM WE CANNOT PROJECT IT TO R2 DIMENSIONS.

(c) FISCHER'S LDA FOR MULTICLASS

$$S_B = \sum_{k=1}^{K} N_k (\mu_k - \mu)(\mu_k - \mu)^T$$

$$S_W = \sum_{k=1}^{K} S_k \qquad \text{WHERE} \quad S_k = \sum_{n \in C_k} (x_n - m_k)(x_n - m_k)^T$$

$$m_k = \frac{1}{N_k} \sum_{n \in C_k} x_n$$

$$J(W) = Tr\{S_W^{-1} S_B\} = Tr\{(W S_W W^T)^{-1} (W S_B W^T)\}$$

THE WEIGHT VALUES ARE THE EIGENVECTORS WITH THE D LARGEST EIGENVALUES.

GAUSSIAN GENERATIVE MODELLING:

$$p(y|x) = \frac{p(x|y)\,p(y)}{p(x)} \leftarrow \text{CLASS PRIOR}$$

↑ CLASS CONDITIONAL

$$p(x|C_k) = \frac{1}{(2\pi)^{d/2}} \frac{1}{|\Sigma|^{1/2}} \exp\left\{-\frac{1}{2}(x - \mu_k)^T \Sigma^{-1} (x - \mu_k)\right\}$$

$\mu_k$ IS THE CLASS MEAN & $\Sigma$ IS THE SAME FOR ALL

GAUSSIAN GENERATIVE MODELLING ON DIGITS DATASET PROJECTED TO 2 DIMENSIONS
Mean Accuracy: 0.7177777777777778
Standard Deviation in Accuracy: 0.017838438175558125

Q4 (a) LOGISTIC REGRESSION (MULTI CLASS)

CLASS POSTERIER $P(C_k|k) = \pi_k(w_k; x) = \frac{\exp(a_k)}{\sum_j \exp(a_j)}$

NEGATIVE LOG LIKELIHOOD $= -\sum_{n=1}^{N}\sum_{k=1}^{K} y_{nk} \log \pi_{nk}$

GRADIENT $= \sum_{n=1}^{N} (\pi_{nk} - y_{nk}) x_n$

FOR TWO CLASS

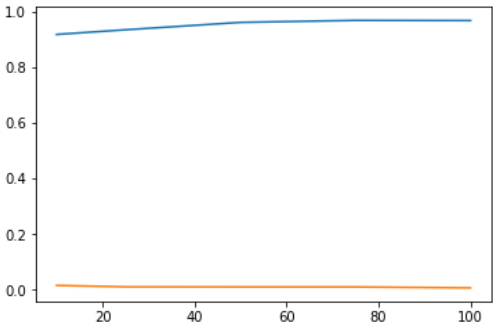
$P(1|x) = \frac{\exp(w^T x)}{1+\exp(w^T x)} = \sigma(w^T x)$

LOG-LIKELIHOOD $= \sum_{n=1}^{N} y_n \log P(1|x_n) + (1-y_n) \log(1-P(1|x_n))$

GRADIENT : $\sum_{n=1}^{N} (\pi(w_t; x_n) - y_n) x_n = x^T(\pi(w_t; x) - y)$

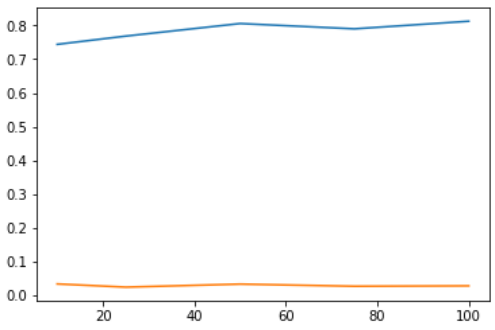
$w_{t+1} = w_t - \alpha_t \nabla E(w_t)$

LOGISTIC REGRESSION CLASSIFIER ON DIGITS DATASET
10 percent : Mean Accuracy: 0.9184065934065935
Standard Deviation: 0.015199534278706649
25 percent : Mean Accuracy: 0.9351648351648352
Standard Deviation: 0.010220371009746293
50 percent : Mean Accuracy: 0.9618131868131867
Standard Deviation: 0.009863364324734196
75 percent : Mean Accuracy: 0.968956043956044
Standard Deviation: 0.009832708883611678
100 percent : Mean Accuracy: 0.9684065934065934
Standard Deviation: 0.006413526115345445




Logistic Regression on Digits Dataset

LOGISTIC REGRESSION CLASSIFIER ON BOSTON DATASET
10 percent : Mean Accuracy: 0.8107142857142857
Standard Deviation: 0.021189494723377656
25 percent : Mean Accuracy: 0.8741758241758241
Standard Deviation: 0.02080669915248514
50 percent : Mean Accuracy: 0.8832417582417582
Standard Deviation: 0.011006167925280028
75 percent : Mean Accuracy: 0.9013736263736263
Standard Deviation: 0.012735524160988195
100 percent : Mean Accuracy: 0.9030219780219781
Standard Deviation: 0.012592490809193247



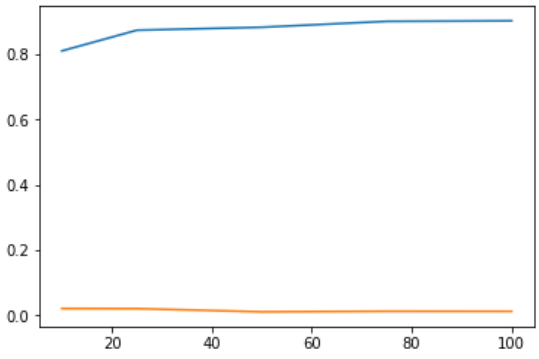

Logistic Regression on Boston Dataset

(b) NAIVE BAYES

$$P(x_i|C_k) = \mathcal{N}(\mu_{ik}, \sigma^2_{ik})$$

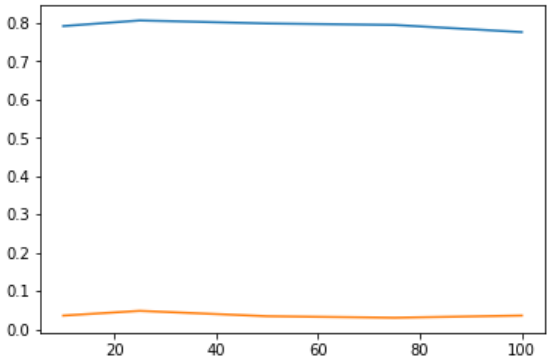
$$P(x|C_k) = \prod_{i=1}^{D} p(x_i|C_k) = \frac{1}{(2\pi)^{D/2}\left(\prod_{i=1}^{D}\sigma_{ik}\right)} \exp\left\{-\sum_{i=1}^{D}\frac{(x_i-\mu_{ik})^2}{2\sigma_{ik}^2}\right\}$$

NAIVE BAYES CLASSIFIER ON DIGITS DATASET
10 percent : Mean Accuracy: 0.8107142857142857
Standard Deviation: 0.021189494723377656
25 percent : Mean Accuracy: 0.8741758241758241
Standard Deviation: 0.02080669915248514
50 percent : Mean Accuracy: 0.8832417582417582
Standard Deviation: 0.01100616792528028
75 percent : Mean Accuracy: 0.9013736263736263
Standard Deviation: 0.012735524160988195
100 percent : Mean Accuracy: 0.9030219780219781
Standard Deviation: 0.012592490809193247




Naive Bayes on Digits Dataset

NAIVE BAYES CLASSIFIER ON BOSTON DATASET
10 percent : Mean Accuracy: 0.7911764705882354
Standard Deviation: 0.03563290557918874
25 percent : Mean Accuracy: 0.8058823529411765
Standard Deviation: 0.04798916960988398
50 percent : Mean Accuracy: 0.7980392156862746
Standard Deviation: 0.034018336417446005
75 percent : Mean Accuracy: 0.7941176470588234
Standard Deviation: 0.0300582543465802
100 percent : Mean Accuracy: 0.7754901960784313
Standard Deviation: 0.03574063947756507




Naive Bayes on Boston Dataset