HW0-1

Thursday, 6 September 2018 2:34 PM

Q1: YES

- READ THROUGH CLASS SYLLABUS
- NOTED IMPORTANT DATES
- READ THROUGH CLASS POLICIES

Q2:
i) INTRODUCTION TO DATA MINING
ii) EE 221 PROBABILITY AND RANDOM PROCESSES
iii) MA 101 LINEAR ALGEBRA
iv) NO, BUT I HAVE TAKEN SIMILAR COURSES LIKE
- PATTERN RECOGNITION AND MACHINE LEARNING
- DETECTION AND ESTIMATION THEORY

ALL THESE COURSES WERE TAKEN DURING MY UNDERGRADUATE AT THE "INDIAN INSTITUTE OF TECHNOLOGY - GUWAHATI" IN INDIA.

Q3: $\min_{w \in \mathbb{R}^p} \frac{1}{2} \|y - Xw\|^2 + \frac{c}{2} \|w\|^2$

ASSUMPTION: "V" IS A COLUMN VECTOR.

① $\frac{d\, x^T x}{dx} = \frac{d x^T}{dx} x + x^T \frac{dx}{dx} = 2x^T$

THE ABOVE EQUATION CAN ALSO BE WRITTEN AS:

$$\frac{1}{2} (Y - XW)^T (Y - XW) + \frac{c}{2} W^T W$$

$$\frac{1}{2} (Y^T Y - Y^T X W - W^T X^T Y + W^T X^T X W) + \frac{c}{2} W^T W$$

NOW TAKING THE DERIVATIVE OF ABOVE EQUATION WRT W:

$$\frac{\partial \left[\frac{1}{2} (Y^T Y - Y^T X W - W^T X^T Y + W^T X^T X W + \frac{c}{2} W^T W\right]}{\partial W}$$

$$= -\frac{1}{2} \frac{\partial Y^T X W}{\partial W} - \frac{1}{2} \frac{\partial W^T X^T Y}{\partial W} + \frac{1}{2} \frac{\partial (XW)^T (XW)}{\partial W} + \frac{c}{2} \frac{\partial W^T W}{\partial W}$$

FROM ① WE GET, AND ADDITIONALLY $Y^T X W = W^T X^T Y$

$$= - X^T Y + X^T X W + cW$$

EQUATING THE ABOVE EXPRESSION TO ZERO WE GET THE VALUE OF W^*

$$\Rightarrow W (X^T X + c) = X^T Y$$

$$\Rightarrow W = \frac{X^T Y}{X^T X + c}$$

$$\Rightarrow W^* = \frac{X^T Y}{X^T X + c} = (X^T X + c)^{-1} X^T Y$$

Q4.

$$\max_{w \in \mathbb{R}^n : w^T w \leq 1} w^T A w$$

$$\min_{w \in \mathbb{R}^n : w^T w \leq 1} w^T A w$$

$\boxed{\text{CONVEX QUADRATIC FORM OPTIMIZATION}}$

USING LANGRANGIAN MULTIPLIERS

$$W^T A W + \lambda (W^T W) = f(W)$$

$$\frac{\partial f(W)}{\partial W} = 2AW + 2\lambda W = 0$$

SO THE QUADRATIC FORM WILL HAVE MAXIMUM AT $W^T W = 1$ AND MINIMUM AT $W^T W = 0$.

Q5: MULTIVARIATE GAUSSIAN DISTRIBUTION

$$p(x; \mu, \Sigma) = \frac{\exp\left(-\frac{1}{2} (x-\mu)^T \Sigma^{-1} (x-\mu)\right)}{\left((2\pi)^k |\Sigma|\right)^{1/2}}$$

WHERE, Σ^{-1} = INVERSE OF Σ
$|\Sigma|$ = DETERMINANT OF Σ
k = NUMBER OF DIMENSIONS
$\exp()$ = EXPONENTIAL FUNCTION

$$p(x; \mu, \theta^{-1}) = \exp \frac{\left(-\frac{1}{2} (x-\mu)^T \Sigma^{-1} (x-\mu)\right)}{\left(\frac{(2\pi)^k}{|\theta|}\right)^{1/2}}$$