Radiometry

- Questions:
  - how "bright" will surfaces be?
  - what is "brightness"?
    - measuring light
    - interactions between light and surfaces
- Core idea - think about light arriving at a surface
- around any point is a hemisphere of directions

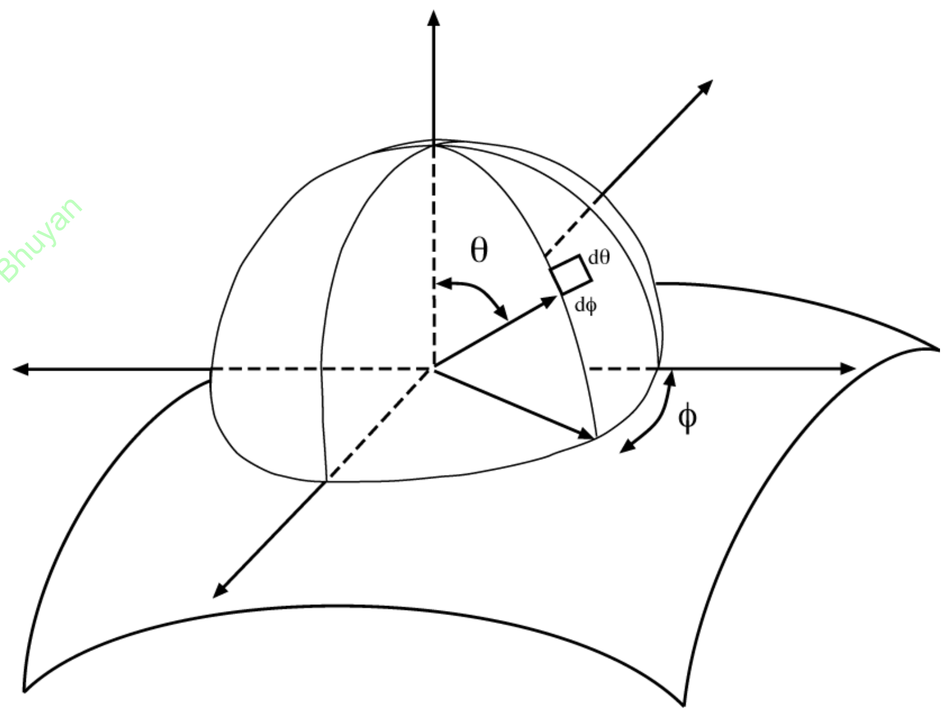
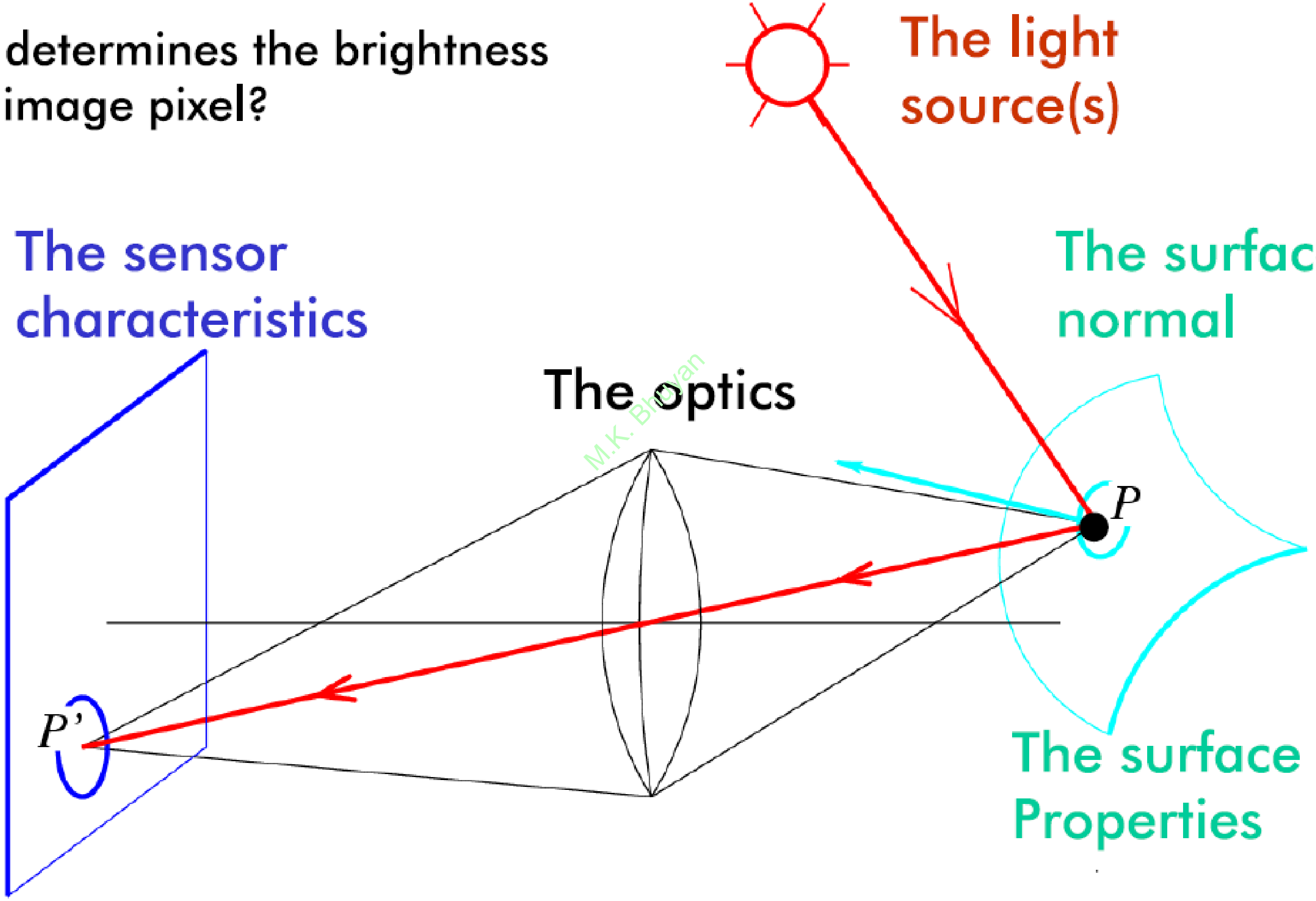
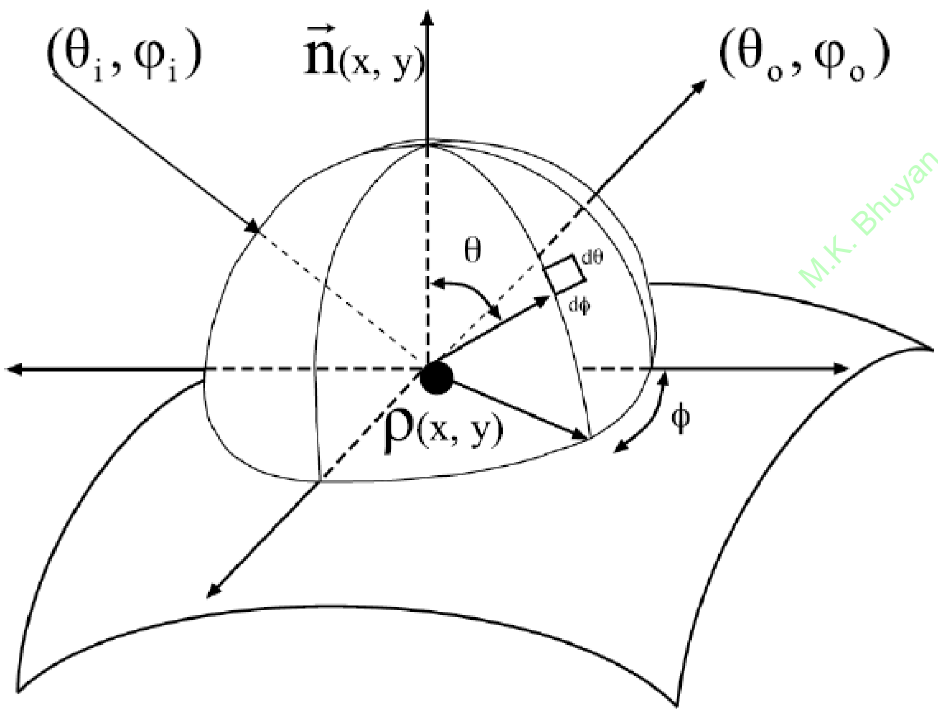


Image Formation: Radiometry

What determines the brightness of an image pixel?



The Illumination and Viewing Hemi-sphere

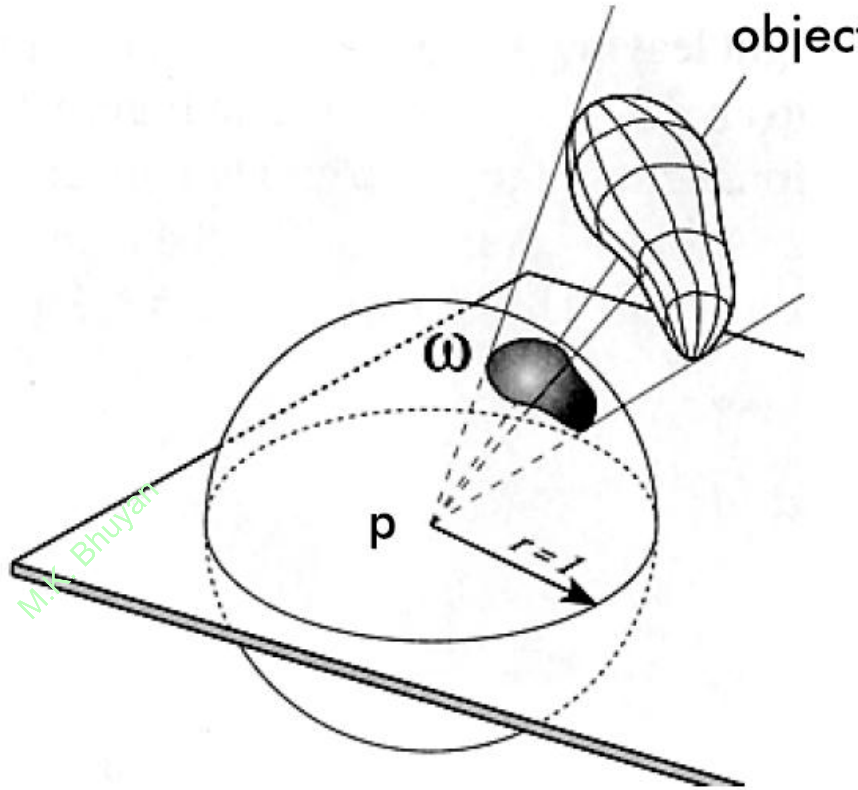


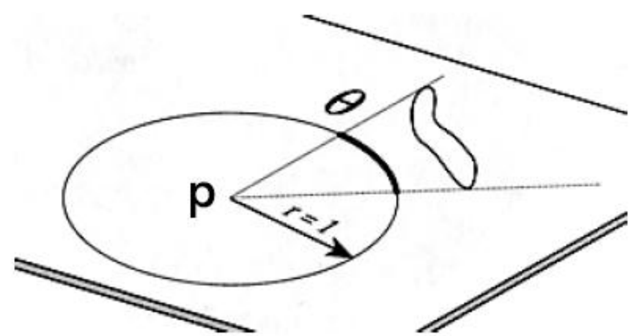
At infinitesimal, each point has a tangent plane, and thus a hemisphere W.

The ray of light is indexed by the polar coordinates (θ, φ)

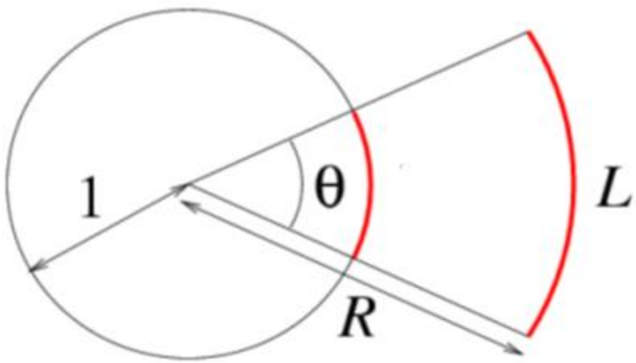
Measuring Angle

- The solid angle sub-tended by an object from a point P is the area of the projection of the object onto the unit sphere centered at P
- Definition is analogous to projected angle in 2D
- If I'm at P, and I look out, solid angle tells me how much of my view is filled with an object

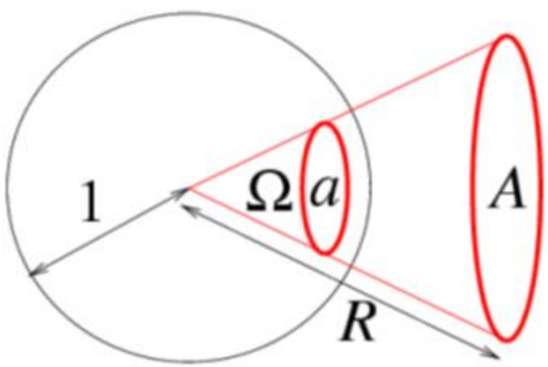




Angles and Solid Angles



$$\boxed{\theta = \frac{L}{R}}$$ (radians)

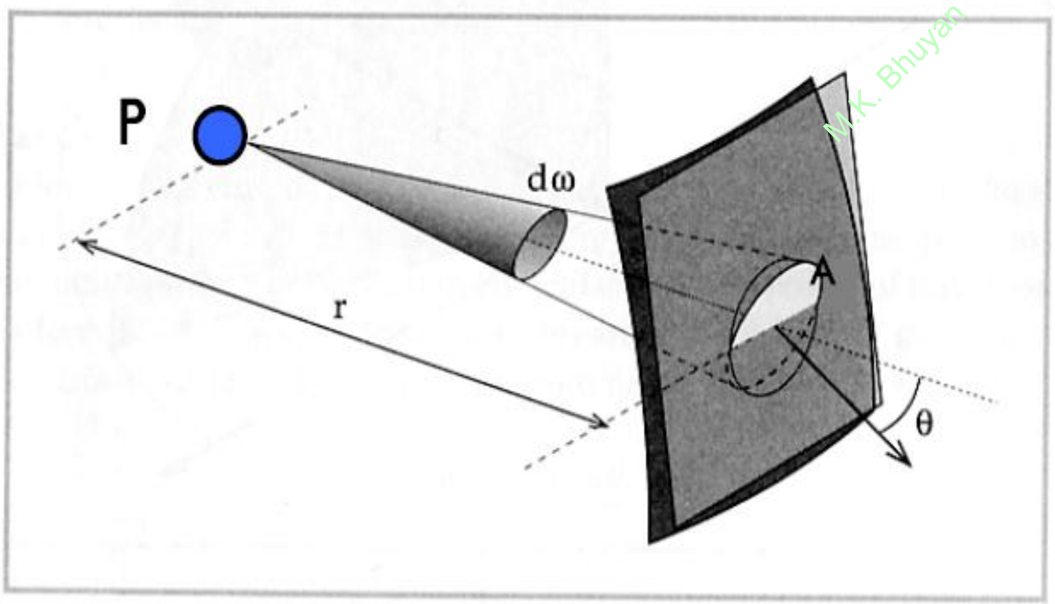


$$\boxed{\Omega = a = \frac{A}{R^2}}$$ (steradians)

By analogy with angle (in radians), the solid angle subtended by a region at a point is the area projected on a unit sphere centered at that point

Infinitesimal small solid angle

The solid angle subtended by an infinitesimal patch from a point P

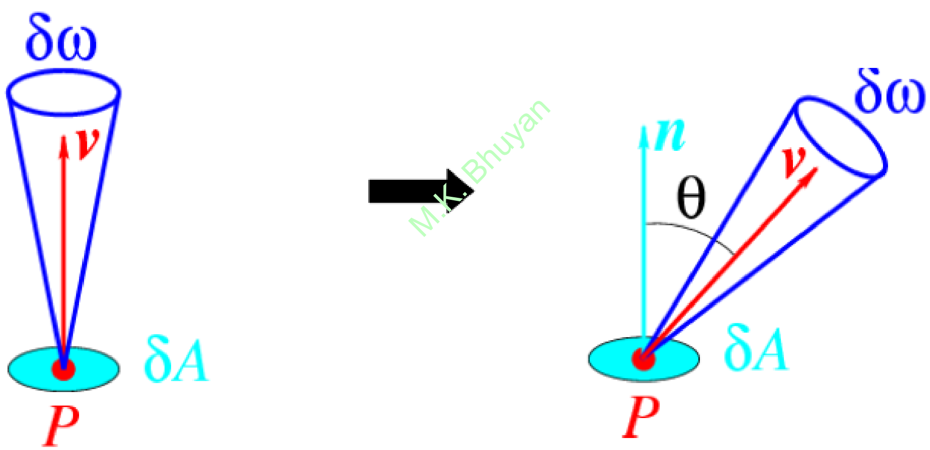


$$d\omega = \frac{dA\cos\theta}{r^2}$$

Radiance

Used to measure the distribution of light in space

DEFINITION: The radiance L(P, *v*) is the power (=energy per unit time) traveling at point P in a given direction *v* (per unit area perpendicular to this direction) (per unit solid angle)



Energy $\delta^2 P$ transmitted by a patch δA into solid angle $\delta\omega$

$$\boxed{\delta^2 P = L(P, v)\,\delta A\,\delta\omega\,\delta t} \Rightarrow \boxed{\delta^2 P = L(P, v)\cos\theta\,\delta A\,\delta\omega\,\delta t}$$

Fore shortening

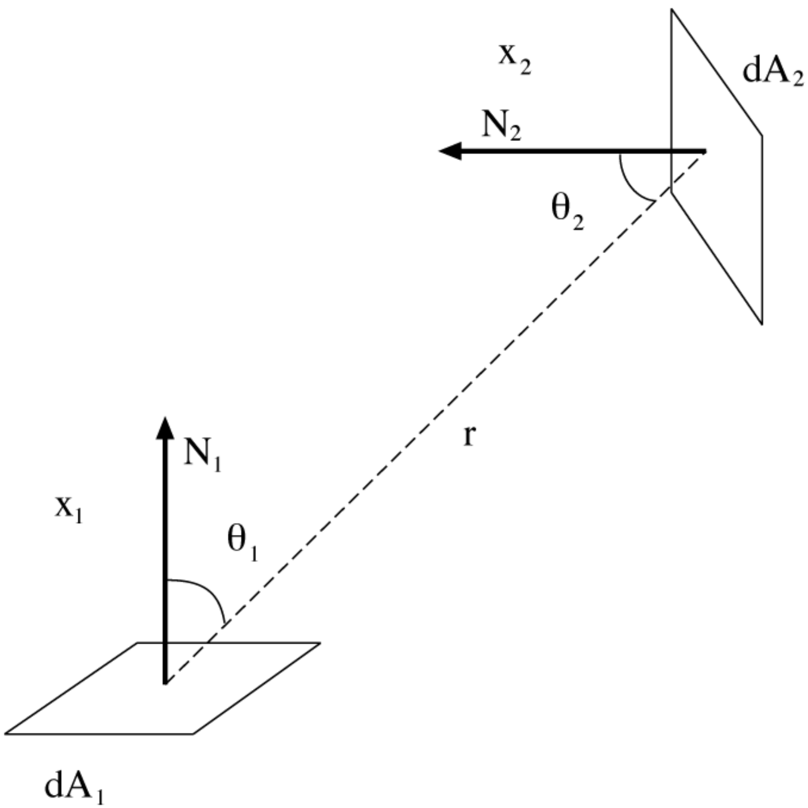
Radiance

- *Radiant power per unit foreshortened area per unit solid angle*
- This is radiance
- Units: watts per square meter per steradian ($wm^{-2}sr^{-1}$)
- Usually written as:

- Crucial property:
  In a vacuum, radiance leaving p in the direction of q is the same as radiance arriving at q from p

$$L\left(\underline{x},\theta,\varphi\right)$$

Radiance is constant along straight lines



- Power 1->2, leaving 1:

$$L(\underline{x}_1, \theta, \varphi)(dA_1 \cos\theta_1)\left(\frac{dA_2 \cos\theta_2}{r^2}\right)$$

- Power 1->2, arriving at 2:

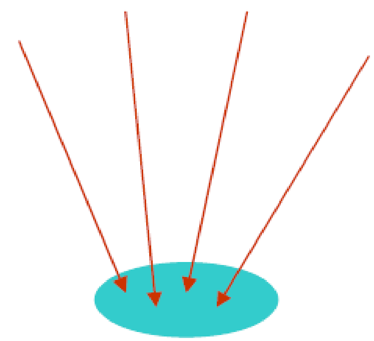
$$L(\underline{x}_2, \theta, \varphi)(dA_2 \cos\theta_2)\left(\frac{dA_1 \cos\theta_1}{r^2}\right)$$

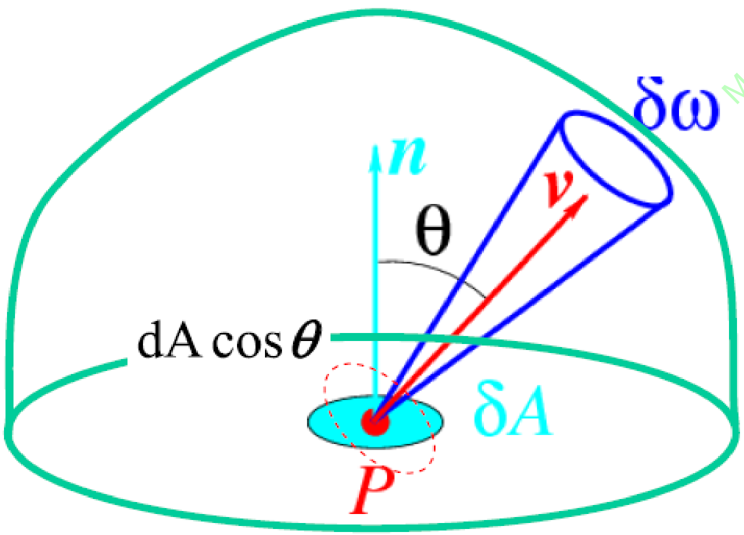
- But these must be the same, so that the two radiances are equal

Irradiance

Used for representing incoming power

The irradiance $\mathrm{E}(\theta,\phi)$ is the power (per unit area incident on a surface).





$$\delta E = \boxed{L_i(\,P\,,\ v_i\,)\ \delta\omega_i}\cos\theta_i$$

Radiance in a region of solid angle $d\omega_i$

$$E = \int_H L_i\ (P,\ v_i)\ \cos\theta_i\ d\omega_i$$

$$\delta^2 P = \delta E\ \delta A = L_i(\,P\,,\ v_i\,)\ \cos\theta_i \delta\omega_i\ \delta A\ \delta t$$

Irradiance

- How much light is arriving at a surface?
- Sensible unit is *Irradiance*
- A surface experiencing radiance $L(x,\theta,\phi)$ coming in from $d\omega$ experiences irradiance

$$L(\underline{x},\theta,\varphi)\cos\theta d\omega$$

- Crucial property:
Total power arriving at the surface is given by adding irradiance over all incoming angles Total power is

$$\int_{\Omega} L(\underline{x},\theta,\varphi)\cos\theta d\omega$$

To sum up

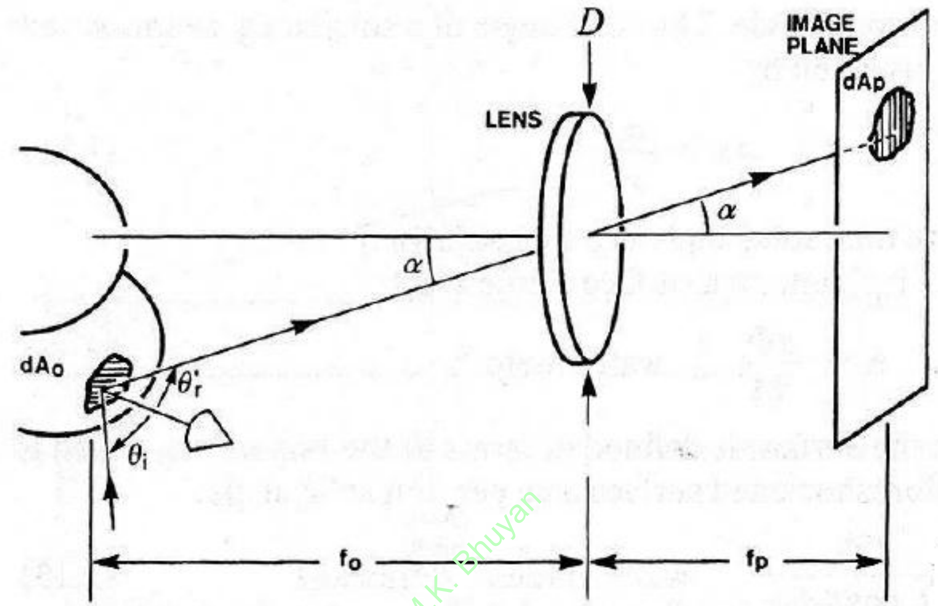
Radiance (L): energy carried by a ray

- Power per unit area perpendicular to the direction of travel, per unit solid angle
- Units: Watts per square meter per steradian ($W\ m^{-2}\ sr^{-1}$)
- PROPERTY: Radiance is constant along straight lines (in vacuum)

Irradiance (E): energy arriving at a surface

- Incident power in a given direction per unit area
- Units: $W\ m^{-2}$

Radiometry of thin lenses



$$E = \frac{1}{4}\left(\frac{D}{f_p}\right)^2 \cos^4\alpha \, \pi L$$

Image irradiance is linearly related to scene radiance

- Irradiance is proportional to the area of the lens and inversely proportional to the squared distance between the lens and the image plane [remember pin- hole model]

- The irradiance falls off as the angle between the viewing ray and the optical axis increases

Light at surfaces

- Many effects when light strikes a surface -- could be:
  - absorbed
  - transmitted
    - skin
  - reflected
    - mirror
  - scattered
    - milk
  - travel along the surface and leave at some other point
    - sweaty skin

- Assume that
  - surfaces don’t fluoresce
    - e.g. scorpions, washing powder
  - surfaces don’t emit light (i.e. are cool)
  - all the light leaving a point is due to that arriving at that point

Bidirectional Reflectance Distribution Function

BRDF

- Assuming that
  - surfaces don’t fluoresce
  - surfaces don’t emit light (i.e. are cool)
  - all the light leaving a point is due to that arriving at that point

- Can model this situation with the Bidirectional Reflectance Distribution Function (BRDF)
- **the ratio of the radiance in the outgoing direction to the incident irradiance**

$$\rho_{bd}\left(\underline{x}, \theta_o, \varphi_o, \theta_i, \varphi_i,\right) = \frac{L_o\left(\underline{x}, \theta_o, \varphi_o\right)}{L_i\left(\underline{x}, \theta_i, \varphi_i\right)\cos\theta_i d\omega}$$

Model of local reflection that tells how bright a surface appears when viewed from one direction when light falls on it from another

BRDF

- Units: inverse steradians (sr^{-1})
- Radiance leaving a surface in a particular direction:

$$L_o(P,\theta_o,\varphi_o) = \rho_{bd}\left(\underline{x},\theta_o,\varphi_o,\theta_i,\varphi_i,\right)L_i\left(\underline{x},\theta_i,\varphi_i\right)\cos\theta_i d\omega_i$$

- Radiance leaving a surface due to its irradiance:
  - add contributions from every incoming direction

$$\int_{\Omega} \rho_{bd}\left(\underline{x},\theta_o,\varphi_o,\theta_i,\varphi_i,\right)L_i\left(\underline{x},\theta_i,\varphi_i\right)\cos\theta_i d\omega_i$$

Suppressing Angles - Radiosity

- In many situations, we do not really need angle coordinates
  - e.g. cotton cloth, where the reflected light is not dependent on angle
- Appropriate radiometric unit is radiosity
  - total power leaving a point on the surface, per unit area on the surface (Wm^{-2})
  - note that this is independent of the direction
- Radiosity from radiance?
  - sum radiance leaving surface over all exit directions, multiplying by a cosine because this is per unit area not per unit foreshortened area

$$B\left(\underline{x}\right) = \int_{\Omega} L_o\left(\underline{x}, \theta, \varphi\right) \cos\theta d\omega$$

Radiosity

- Important relationship:
  - radiosity of a surface whose radiance is independent of angle (e.g. that cotton cloth)

$$
\begin{aligned}
B(\underline{x}) &= \int_{\Omega} L_o(\underline{x}, \theta, \varphi) \cos\theta \, d\omega \\
&= L_o(\underline{x}) \int_{\Omega} \cos\theta \, d\omega \\
&= L_o(\underline{x}) \int_0^{\frac{\pi}{2}} \int_0^{2\pi} \cos\theta \sin\theta \, d\varphi \, d\theta \\
&= \pi L_o(\underline{x})
\end{aligned}
$$

Suppressing the angles in the BRDF

- BRDF is a very general notion
  - some surfaces need it (underside of a CD; tiger eye; etc)
  - very hard to measure
    - ,illuminate from one direction, view from another, repeat
  - very unstable
    - minor surface damage can change the BRDF
    - e.g. ridges of oil left by contact with the skin can act as lenses
- for many surfaces, light leaving the surface is largely independent of exit angle
  - surface roughness is one source of this property

Directional hemispheric reflectance

- Directional hemispheric reflectance:
  - the fraction of the incident irradiance in a given direction that is reflected by the surface (whatever the direction of reflection)
  - unitless, range is 0-1

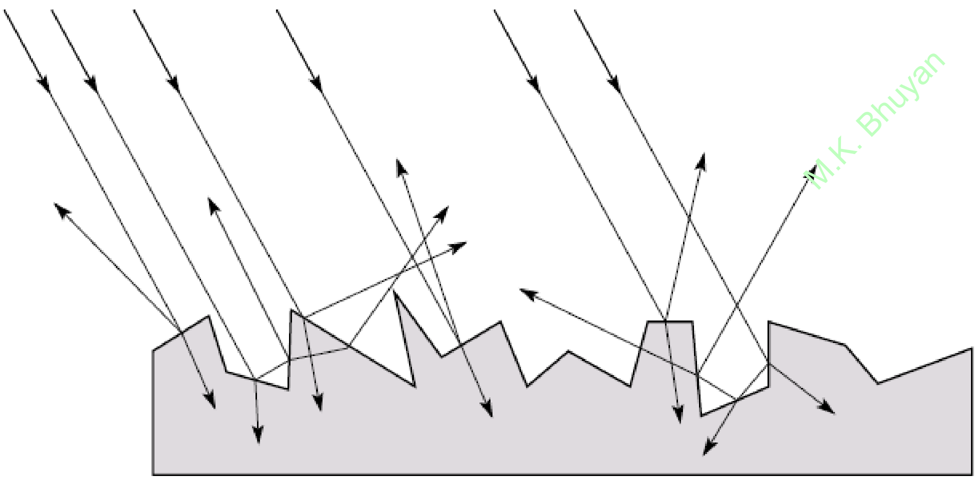
- Note that DHR varies with incoming direction

$$\rho_{dh}\left(\theta_i,\varphi_i\right)=\frac{\int_{\Omega} L_o\left(\underline{x},\theta_o,\varphi_o\right)\cos\theta_o d\omega_o}{L_i\left(\underline{x},\theta_i,\varphi_i\right)\cos\theta_i d\omega_i}$$

$$=\int_{\Omega}\rho_{bd}\left(\underline{x},\theta_o,\varphi_o,\theta_i,\varphi_i,\right)\cos\theta_o d\omega_o$$

Lambertian (or Matte) Surfaces

A Lambertian surface is a surface whose BRDF is independent of the outgoing direction



Microfacets scatter incoming light randomly

→ Radiance leaving the surface is independent of angle

Lambertian surfaces and albedo

- For some surfaces, the DHR is independent of illumination direction too
  - cotton cloth, carpets, matte paper, matte paints, etc.
- For such surfaces, radiance leaving the surface is independent of angle
- Called **Lambertian surfaces** (same Lambert) or **ideal diffuse surfaces**

- Use radiosity as a unit to describe light leaving the surface
- DHR is often called **diffuse reflectance**, or **albedo** ρ_d
- for a Lambertian surface, BRDF is independent of angle, too.

For a Lambertian surface $\rho_{bd}\left(\theta_o,\varphi_o,\theta_i,\varphi_i,\right)=\rho$

$$\rho_d=\int_{\Omega}\rho_{bd}\left(\theta_o,\varphi_o,\theta_i,\varphi_i,\right)\cos\theta_o d\omega_o$$

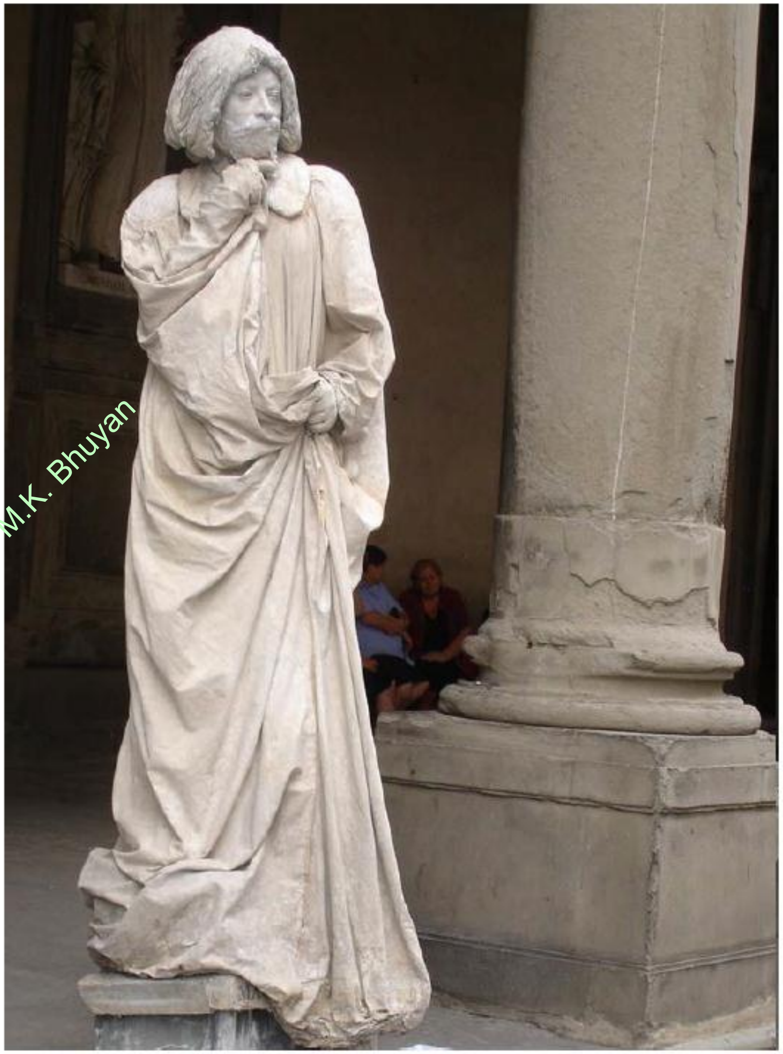
$$=\int_{\Omega}\rho\cos\theta_o d\omega_o$$

$$=\rho\int_0^{\frac{\pi}{2}}\int_0^{2\pi}\cos\theta_o\sin\theta_o d\theta_o d\varphi_o$$

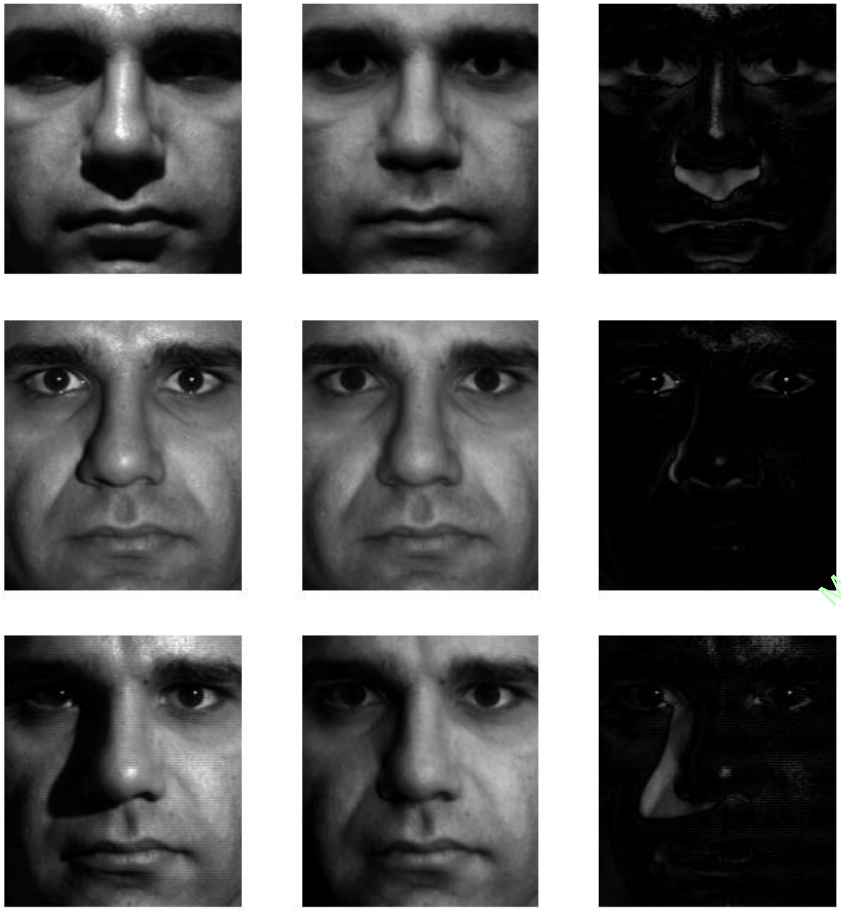
$$=\pi\rho=\text{Albedo}$$

In general, $\rho_{brdf}=\dfrac{\rho_d}{\pi}$

Lambertian surfaces



Non-lambertian surfaces



(a) M (b) $\hat{L}$ (c) $\hat{S}$

(a) M (b) $\hat{L}$ (c) $\hat{S}$

Lambertian + specular

- Widespread model
  – all surfaces are Lambertian plus specular component
- Advantages
  – easy to manipulate
  – very often quite close true
- Disadvantages
  – some surfaces are not
    - e.g. underside of CD’s, feathers of many birds, blue spots on many marine crustaceans and fish, most rough surfaces, oil films (skin!), wet surfaces
  – Generally, very little advantage in modelling behaviour of light at a surface in more detail -- it is quite difficult to understand behaviour of L+S surfaces

Sources and shading

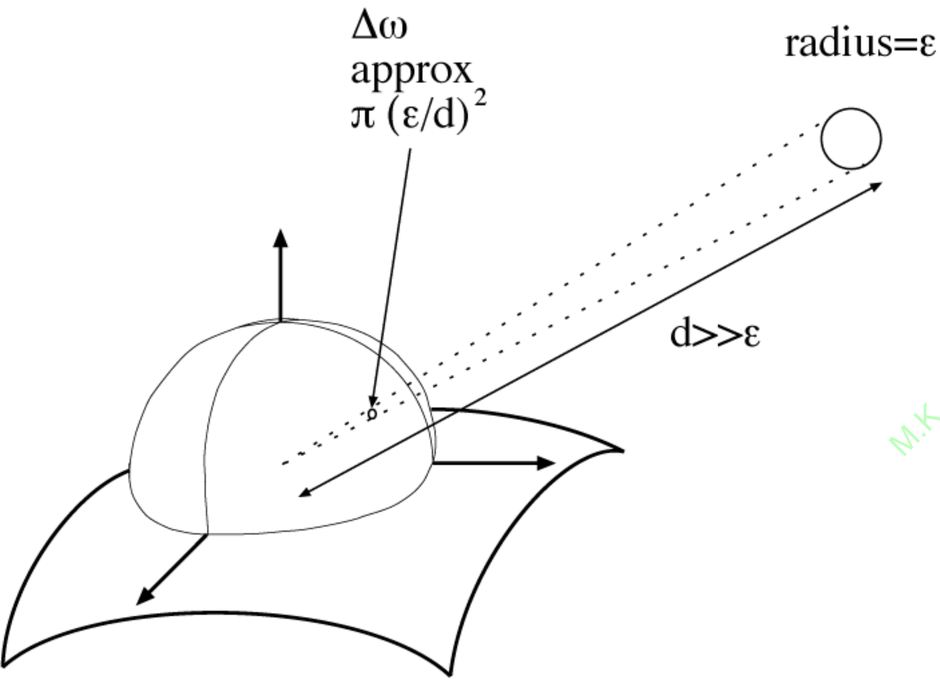
- How bright (or what colour) are objects?
- One more definition: Exitance of a source is
  - the internally generated power radiated per unit area on the radiating surface
- similar to radiosity: a source can have both
  - radiosity, because it reflects
  - exitance, because it emits

- General idea:

$$B(x) = E(x) + \int_{\Omega} \left\{ \begin{array}{l} \text{radiosity due to} \\ \text{incoming radiance} \end{array} \right\} d\omega$$

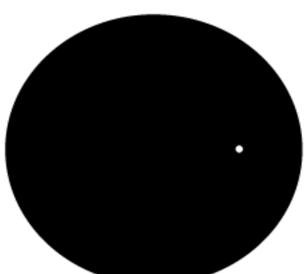
- But what aspects of the incoming radiance will we model?

Radiosity due to a point sources



- small, distant sphere radius ε and exitance E, which is far away subtends solid angle of about

$$\pi\left(\frac{\varepsilon}{d}\right)^2$$



Constant radiance patch due to source

Radiosity due to a point source

- Radiosity is

$$
\begin{aligned}
B(x) &= \pi L_o(x) \\
&= \rho_d(x) \int_{\Omega} L_i(x,\omega)\cos\theta_i d\omega \\
&= \rho_d(x) \int_{D} L_i(x,\omega)\cos\theta_i d\omega \\
&\approx \rho_d(x)(\text{solid angle})(\text{Exitance term})\cos\theta_i \\
&= \frac{\rho_d(x)\cos\theta_i}{r(x)^2}(\text{Exitance term and some constants})
\end{aligned}
$$

Standard nearby point source model

$$\rho_d(x)\left(\frac{N(x)\bullet S(x)}{r(x)^2}\right)$$

- N is the surface normal
- rho is diffuse albedo
- S is source vector - a vector from x to the source, whose length is the intensity term
  - works because a dot-product is basically a cosine

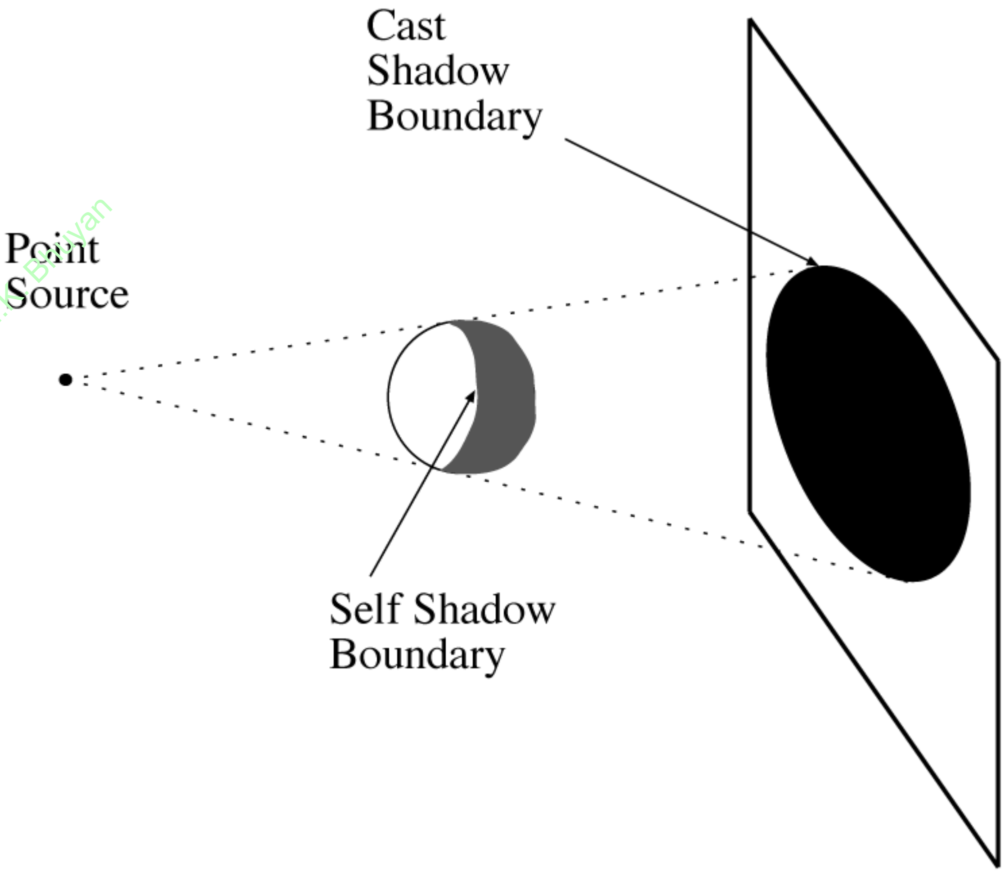
Standard distant point source model

- Issue: nearby point source gets bigger if one gets closer
  - the sun doesn't for any reasonable binding of closer
- Assume that all points in the model are close to each other with respect to the distance to the source. Then the source vector doesn't vary much, and the distance doesn't vary much either, and we can roll the constants together to get:

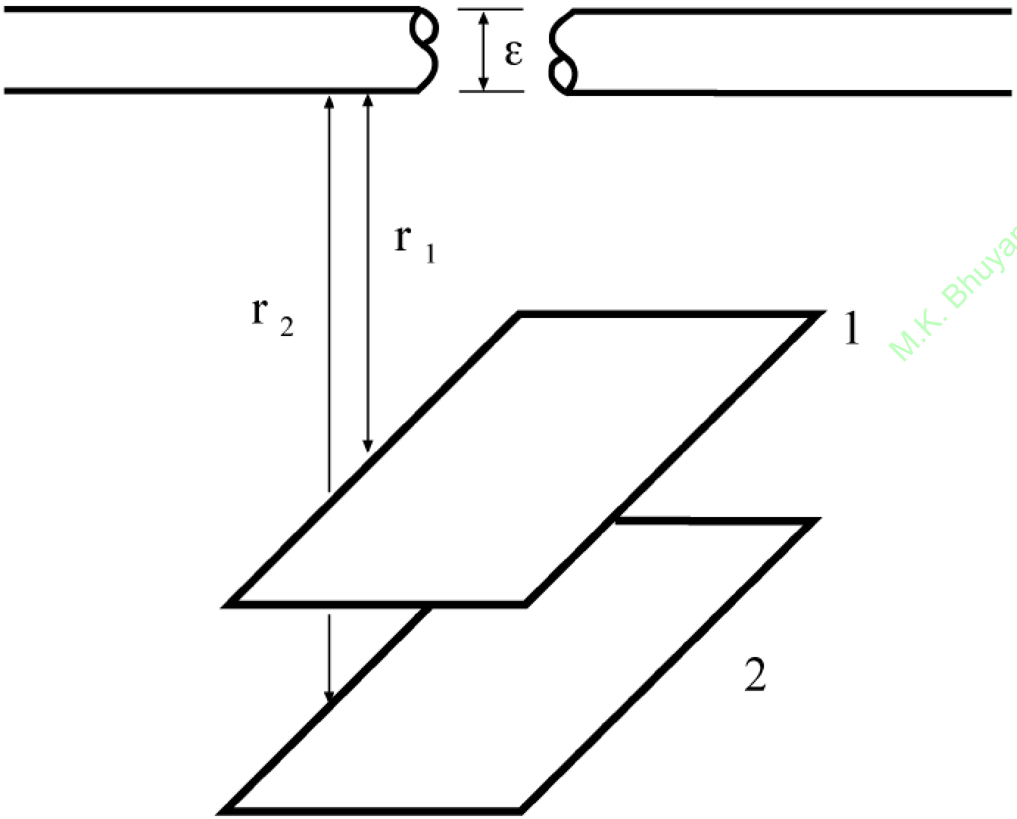
$$\rho_d(x)\left(N(x)\bullet S_d(x)\right)$$

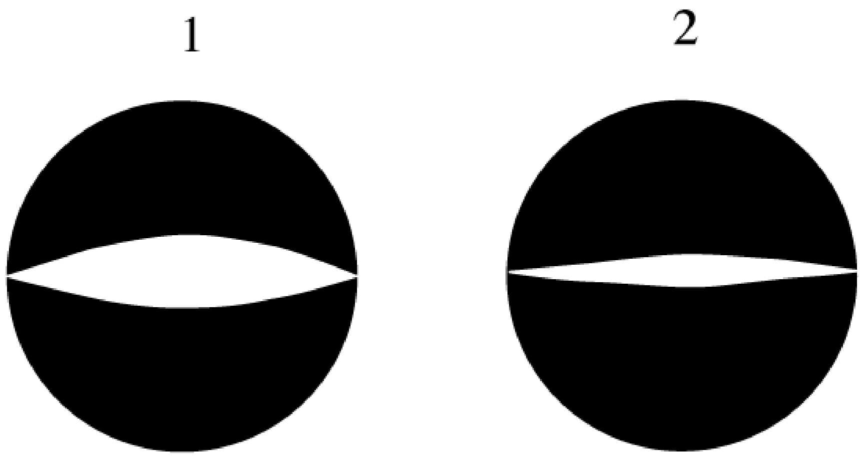
Shadows cast by a point source

- A point that can't see the source is in shadow
- For point sources, the geometry is simple



Line sources

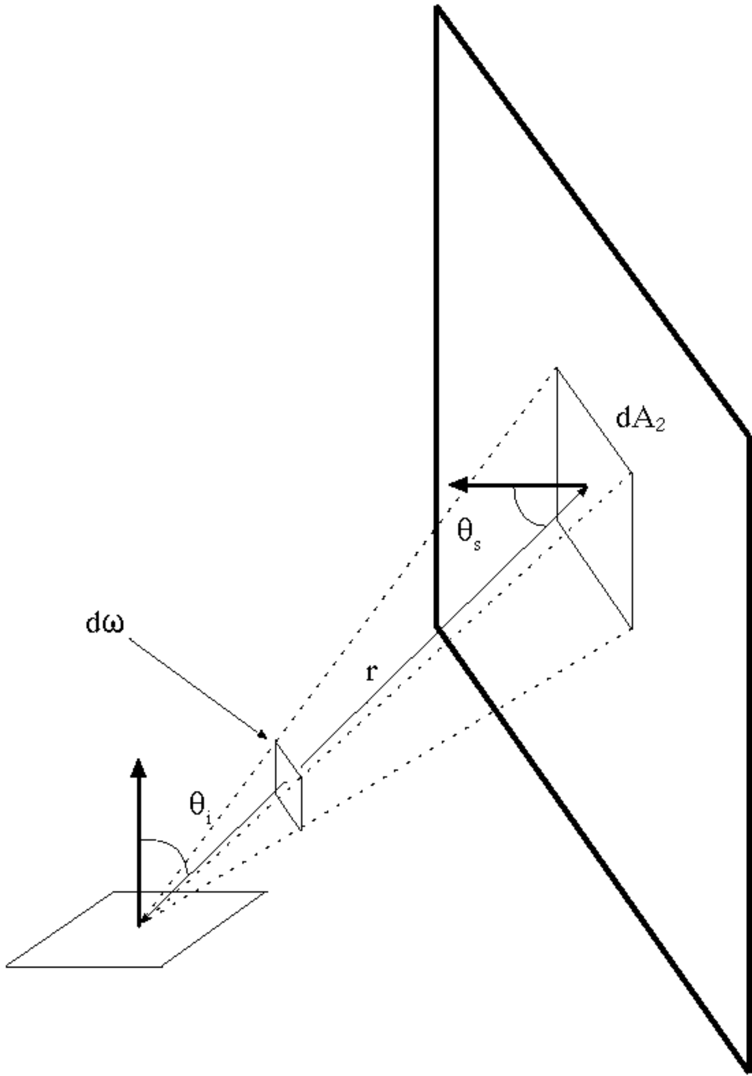




radiosity due to line source varies with inverse distance, if the source is long enough

Area sources

- Examples: diffuser boxes, white walls.
- The radiosity at a point due to an area source is obtained by adding up the contribution over the section of view hemisphere subtended by the source
  - change variables and add up over the source



Radiosity due to an area source

- rho is albedo
- E is exitance
- r(x, u) is distance between points
- u is a coordinate on the source

$$B(x) = \rho_d(x) \int_{\Omega} L_i(x, u \to x) \cos\theta_i \, d\omega$$

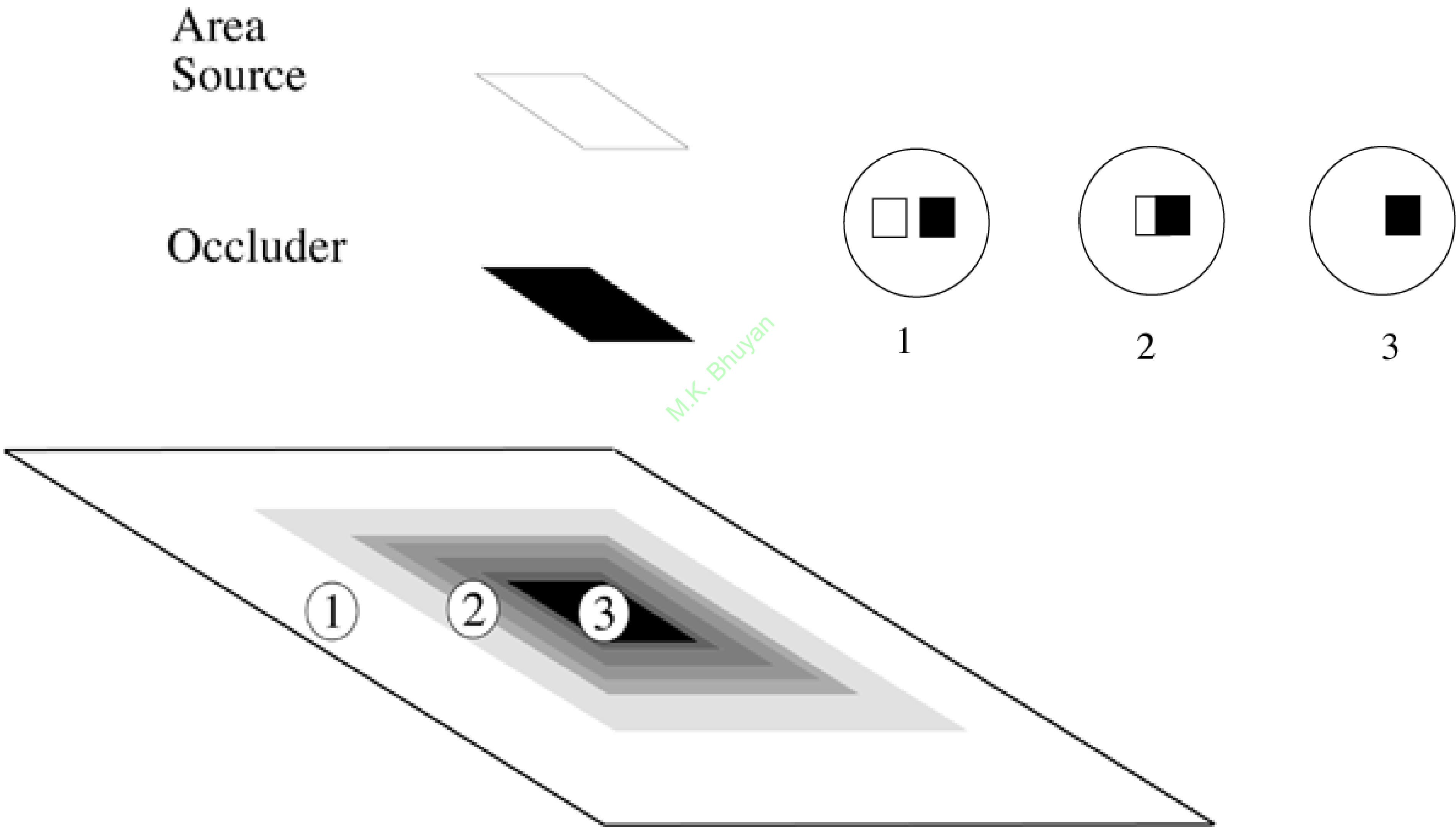
$$= \rho_d(x) \int_{\Omega} L_e(x, u \to x) \cos\theta_i \, d\omega$$

$$= \rho_d(x) \int_{\Omega} \left(\frac{E(u)}{\pi} \right) \cos\theta_i \, d\omega$$

$$= \rho_d(x) \int_{source} \left(\frac{E(u)}{\pi} \right) \cos\theta_i \left(\cos\theta_s \frac{dA_u}{r(x,u)^2} \right)$$

$$= \rho_d(x) \int_{source} E(u) \frac{\cos\theta_i \cos\theta_s}{\pi r(x,u)^2} \, dA_u$$

Area Source Shadows



Shading models

- Local shading model
  - Surface has radiosity due only to sources visible at each point
  - Advantages:
    - often easy to manipulate, expressions easy
    - supports quite simple theories of how shape information can be extracted from shading

- Global shading model
  - surface radiosity is due to radiance reflected from other surfaces as well as from surfaces
  - Advantages:
    - usually very accurate
  - Disadvantage:
    - extremely difficult to infer anything from shading values

Photometric stereo

- Assume:
  - a local shading model
  - a set of point sources that are infinitely distant
  - a set of pictures of an object, obtained in exactly the same camera/object configuration but using different sources
  - A Lambertian object (or the specular component has been identified and removed)

Projection model for surface recovery - usually called a Monge patch

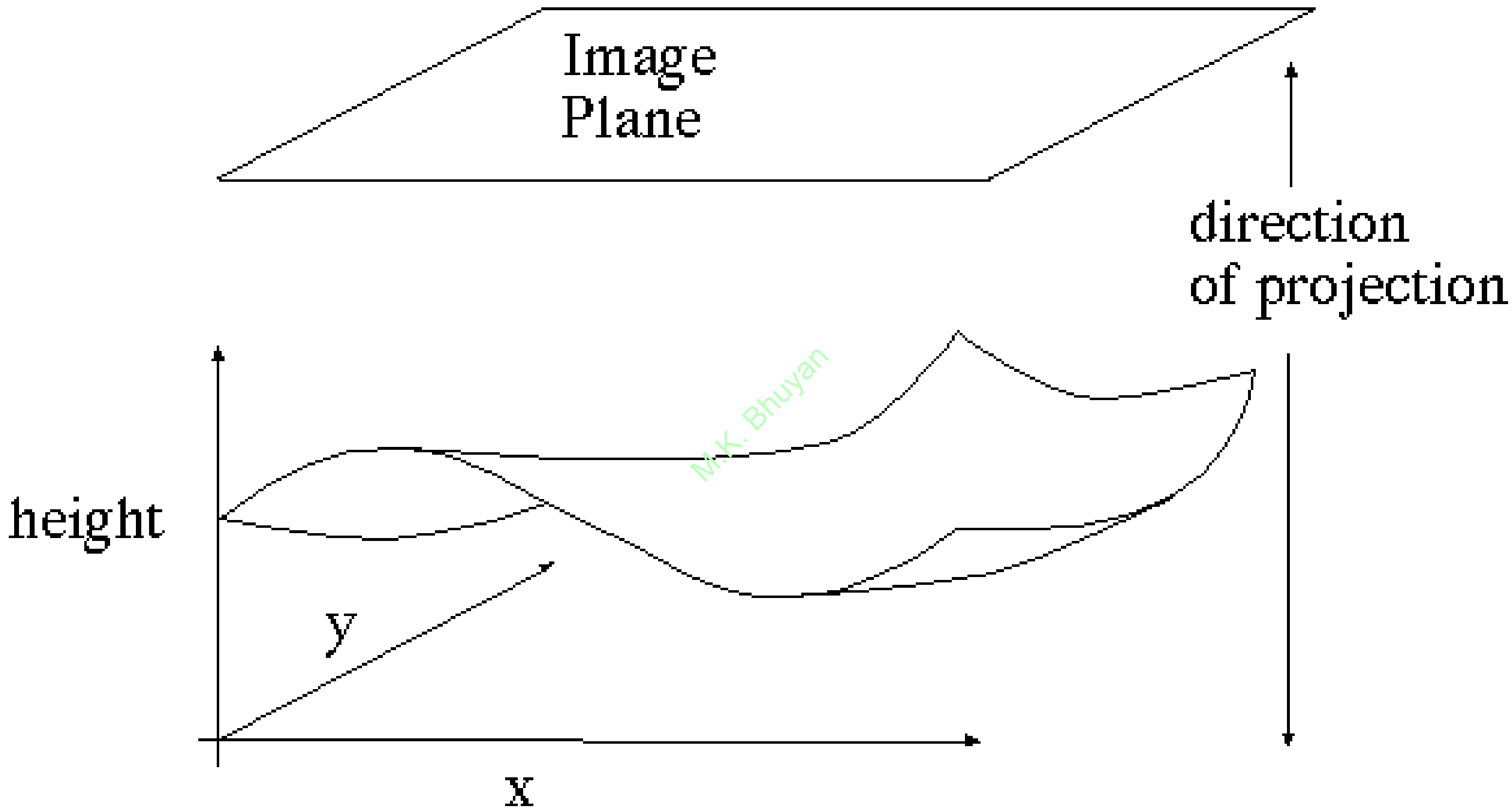


Image model

- For each point source, we know the source vector (by assumption). We assume we know the scaling constant of the linear camera. Fold the normal and the reflectance into one vector g, and the scaling constant and source vector into another Vj

- Out of shadow:

$$
\begin{aligned}
I_j(x,y) &= kB(x,y) \\
&= k\rho(x,y)\left(\mathbf{N}(x,y)\bullet\mathbf{S}_j\right) \\
&= \mathbf{g}(x,y)\bullet\mathbf{V}_j
\end{aligned}
$$

- In shadow:

$$
\mathrm{I}_{\mathrm{j}}(\mathrm{x},\mathrm{y}) = 0
$$

Dealing with shadows

$$\begin{pmatrix} I_1^2(x,y) \\ I_2^2(x,y) \\ .. \\ I_n^2(x,y) \end{pmatrix} = \begin{pmatrix} I_1(x,y) & 0 & .. & 0 \\ 0 & I_2(x,y) & .. & .. \\ .. & .. & .. & 0 \\ 0 & .. & 0 & I_n(x,y) \end{pmatrix} \begin{pmatrix} \mathbf{V}_1^T \\ \mathbf{V}_2^T \\ .. \\ \mathbf{V}_n^T \end{pmatrix} \mathbf{g}(x,y)$$

Known Known Known Unknown

Recovering normal and reflectance

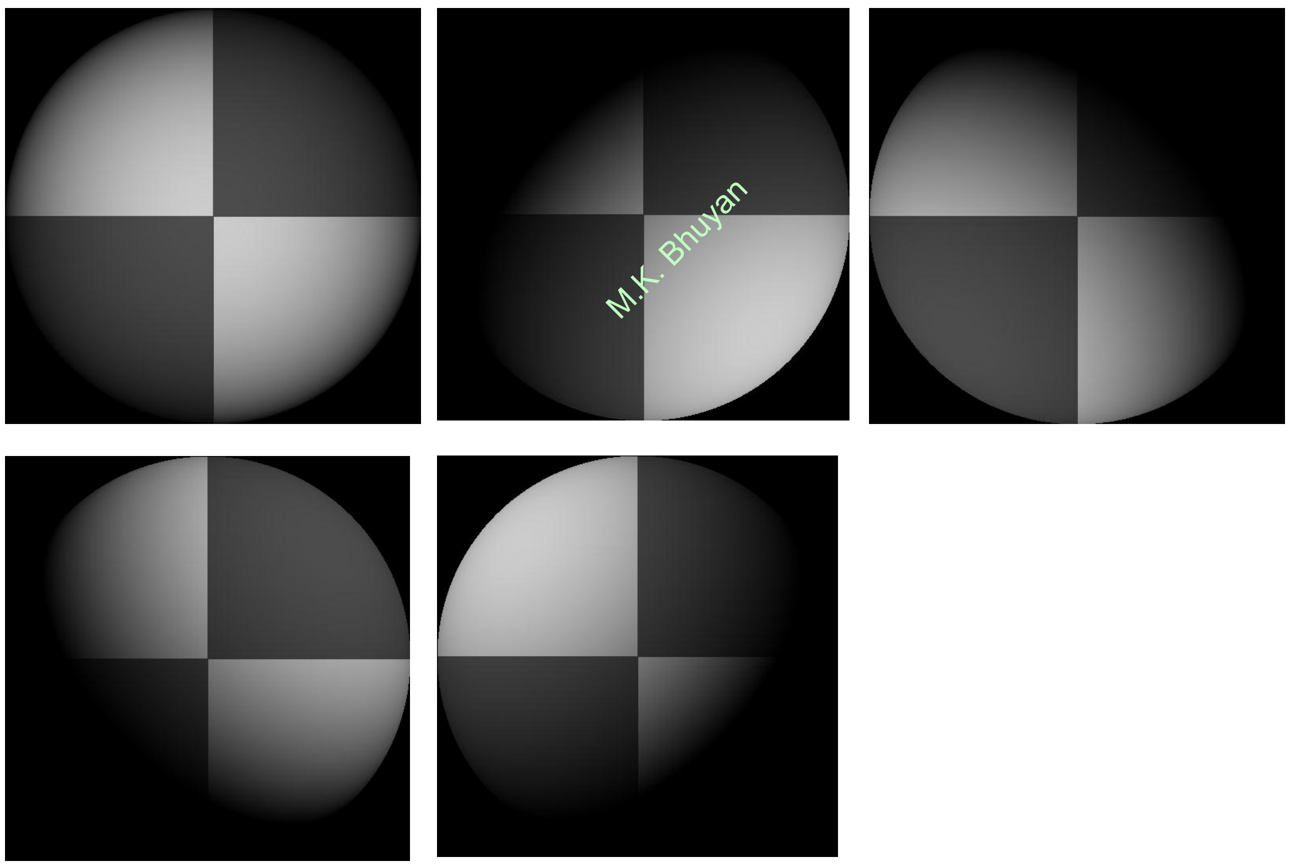
- Given sufficient sources, we can solve the previous equation (most likely need a least squares solution) for

$$\mathbf{g}(x, y)$$

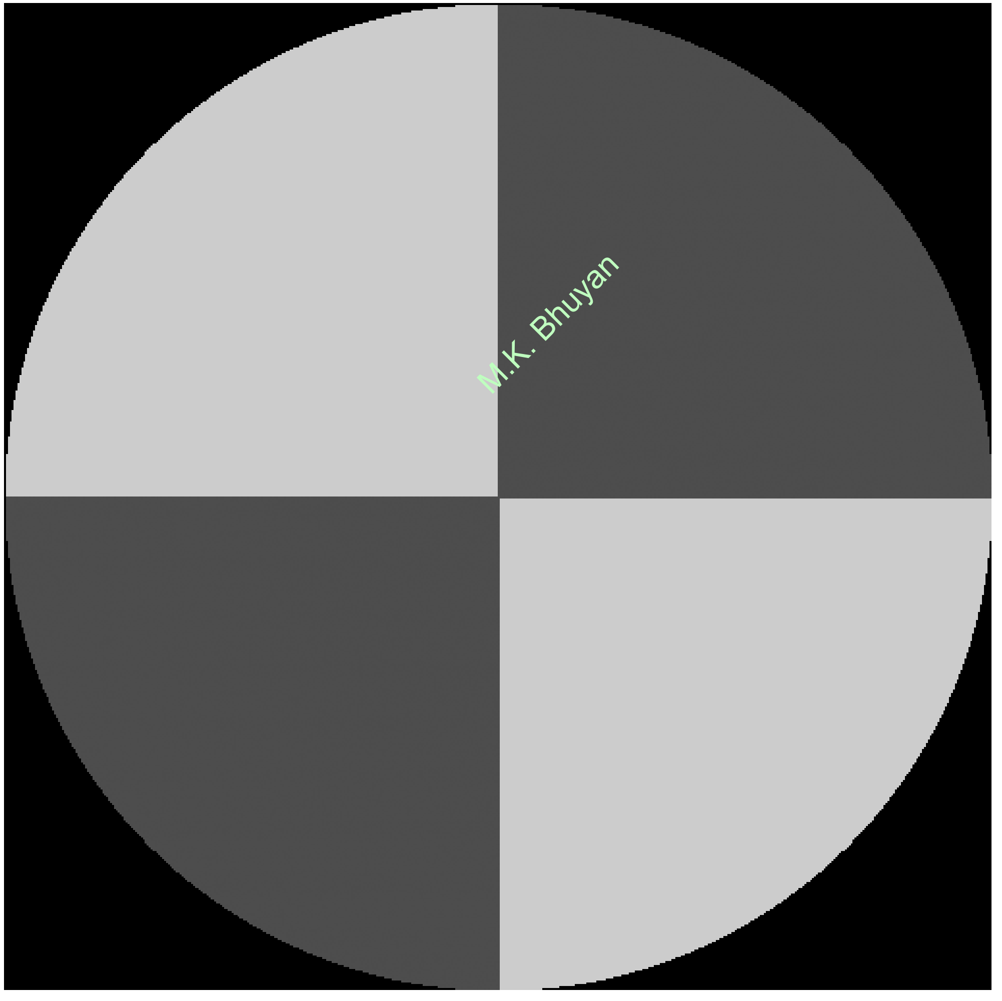
- Recall that $\mathbf{N}(x, y)$ is the unit normal
- This means that $\rho(x,y)$ is the magnitude of $\mathbf{g}(x, y)$
- This yields a check
  - If the magnitude of $\mathbf{g}(x, y)$ is greater than 1, there's a problem
- And

$$\mathbf{N}(x, y) = \mathbf{g}(x, y) / \rho(x,y)$$

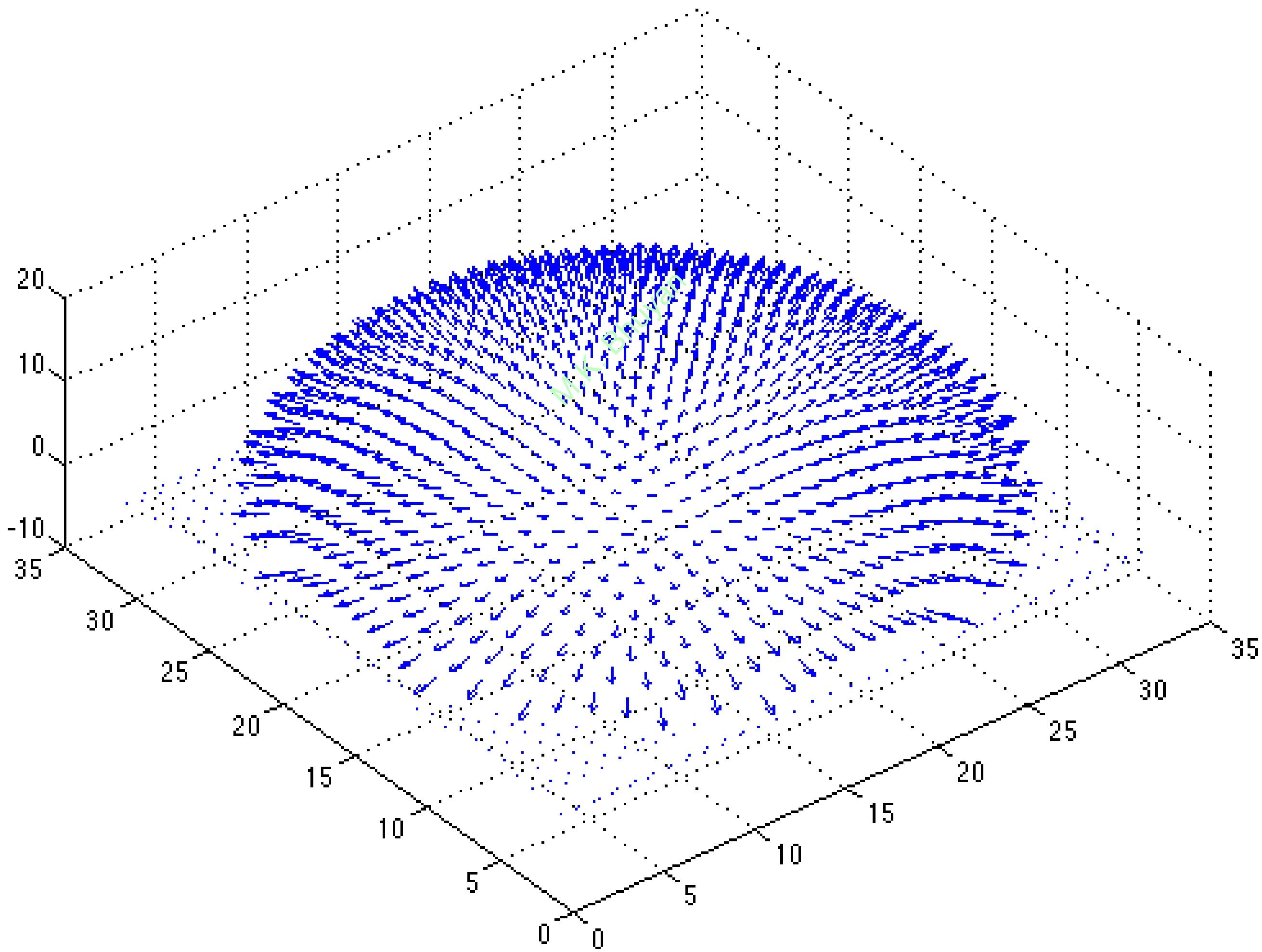
Example figures



Recovered reflectance



Recovered normal field



Recovering a surface from normals - 1

- Recall the surface is written as

$$(x, y, f(x, y))$$

- This means the normal has the form:

$$N(x,y) = \left(\frac{1}{\sqrt{f_x^2 + f_y^2 + 1}}\right)\begin{pmatrix} -f_x \\ -f_y \\ 1 \end{pmatrix}$$

- If we write the known vector **g** as

$$\mathbf{g}(x,y) = \begin{pmatrix} g_1(x,y) \\ g_2(x,y) \\ g_3(x,y) \end{pmatrix}$$

- Then we obtain values for the partial derivatives of the surface:

$$f_x(x,y) = \left(g_1(x,y)/g_3(x,y)\right)$$

$$f_y(x,y) = \left(g_2(x,y)/g_3(x,y)\right)$$

Recovering a surface from normals - 2

- Recall that mixed second partials are equal --- this gives us a **check.** We must have:

$$\frac{\partial\left(g_1(x,y)/g_3(x,y)\right)}{\partial y} = \frac{\partial\left(g_2(x,y)/g_3(x,y)\right)}{\partial x}$$

(or they should be similar, at least)

- We can now recover the surface height at any point by integration along some path, e.g.

$$f(x,y) = \int_0^x f_x(s,y)\,ds + \int_0^y f_y(x,t)\,dt + c$$

Surface recovered by integration