Economic Growth

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Content

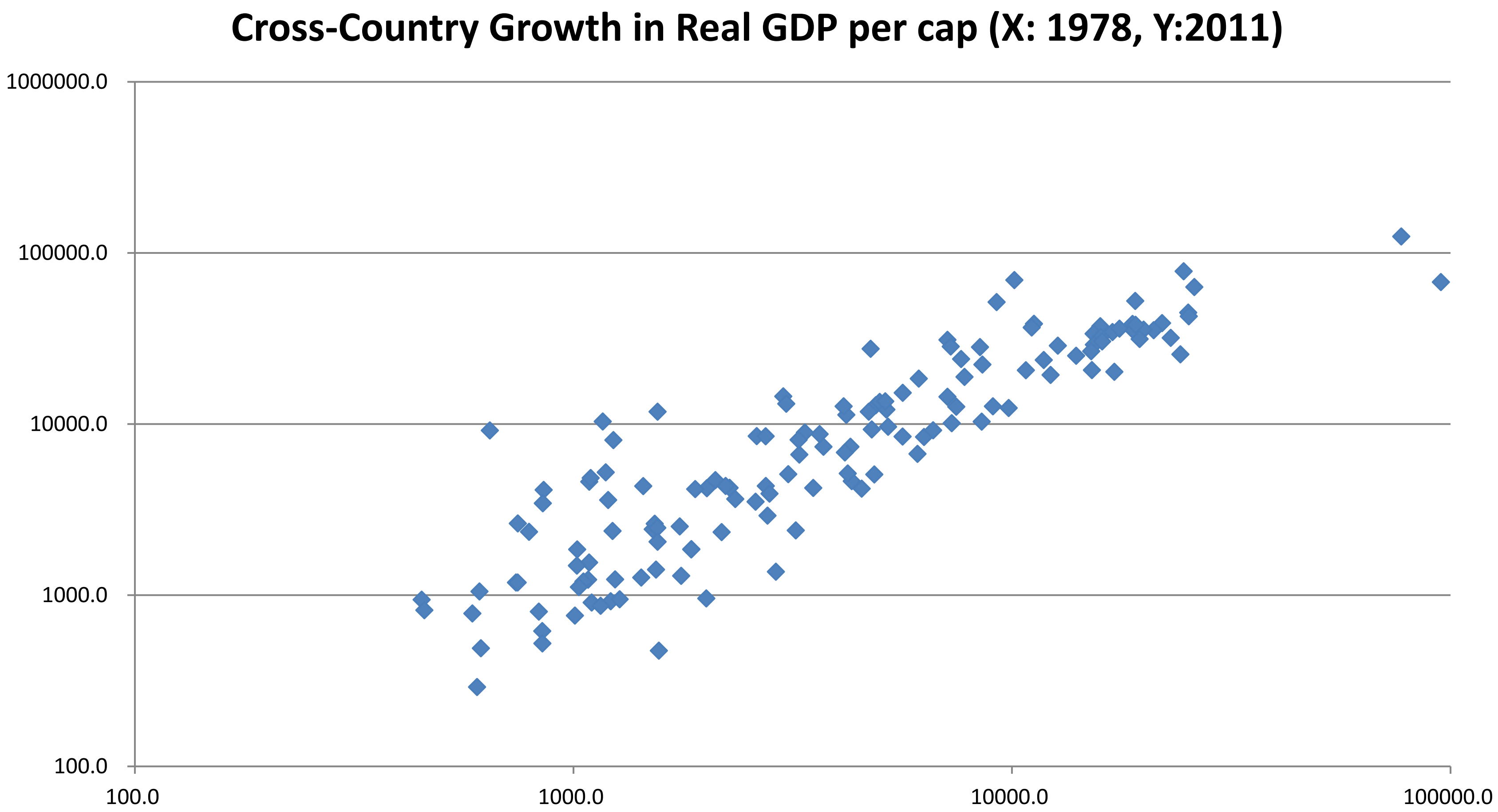
- **Overview**
- Solow Model I (accumulation of capital and population growth)
- Solow Model II (considering technological progress)
- Endogenous growth models
- Economic growth accounting
- Other growth topics

The Importance of Economic Growth

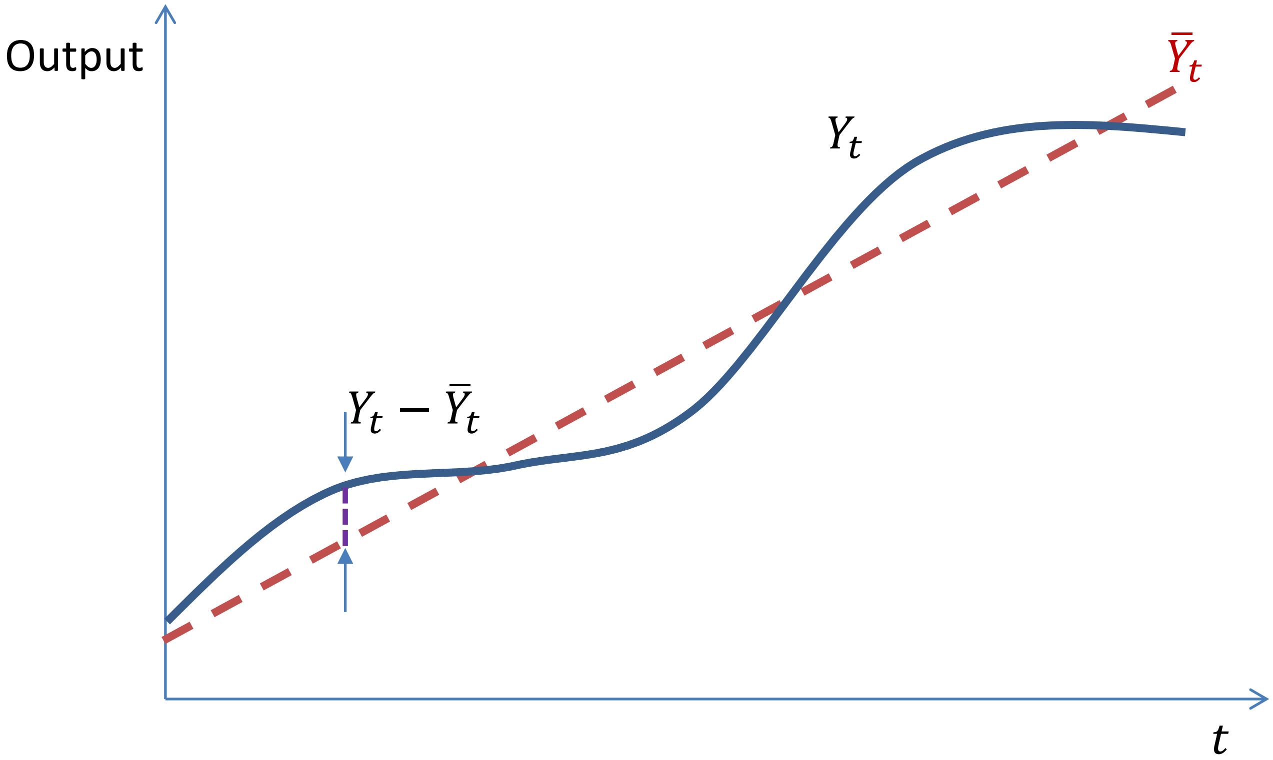
- For poor countries, a stagnant economy means a persistent absolute poverty.
- In relative terms, a slight but persistent difference in growth rate results in huge income gaps.
- The following table illustrates how three different growth rates (from the same income per cap, 100) lead to starkly different outcomes.

Years	0	10	30	100
1%	100	110.5	134.8	270.5
3%	100	134.4	242.7	1921.9
8%	100	215.9	1006.3	219976.1

Growth Performances of 143 Economies



Growth and Fluctuations



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Solow Model I

- The first Solow model characterizes the role of factor inputs in economic growth.
- We assume:
  - Closed economy ($NX = 0$), no government spending ($G = 0$).
  - Fixed production function, $Y = F(K, L)$, a constant-return-to-scale technology.
  - The saving rate is constant: $C = (1 - s)Y$.
  - Capital depreciates at constant rate δ.
  - Population grows at constant rate n, $L_t = L_0 e^{nt}$.

The Per Capita Output

- Let $y = \frac{Y}{L}$ and $k = \frac{K}{L}$. y is the output per cap. And k is the amount of capital per cap. We have
$$y = \frac{Y}{L} = \frac{F(K, L)}{L} = F(k, 1).$$
- Define $f(k) \equiv F(k, 1)$. We have
$$y = f(k).$$
- *f(k)* is the "individual production function," how much output one worker could produce using *k* units of capital. We assume
$$f(0) = 0, f'(k) > 0, f''(k) < 0.$$

Note that $f'(k)$ is the <u>marginal product of capital</u> (MPK).

- We also assume:
$$\lim_{k \to 0} f'(k) = \infty, \lim_{k \to \infty} f'(k) = 0$$

The Per Capita Demand

- Without government spending and net export, the demand for goods and services is composed of consumption (C) and investment (I).
- In per cap terms, we have

$$y = c + i,$$

where $c = C/L$ and $i = \frac{I}{L}$.

- The <u>per cap investment</u> is a constant fraction of the out

$$i = y - c = y - (1 - s)y = sy.$$

Accumulation of Capital

- Investment causes capital to rise and depreciation causes capital to wear out. The aggregate capital accumulation is described by

$$\dot{K}_t \equiv \frac{dK_t}{dt} = sF(K_t, L_t) - \delta K_t$$

- Similarly, the evolution of the population is characterized by

$$\dot{L_t} = nL_t.$$

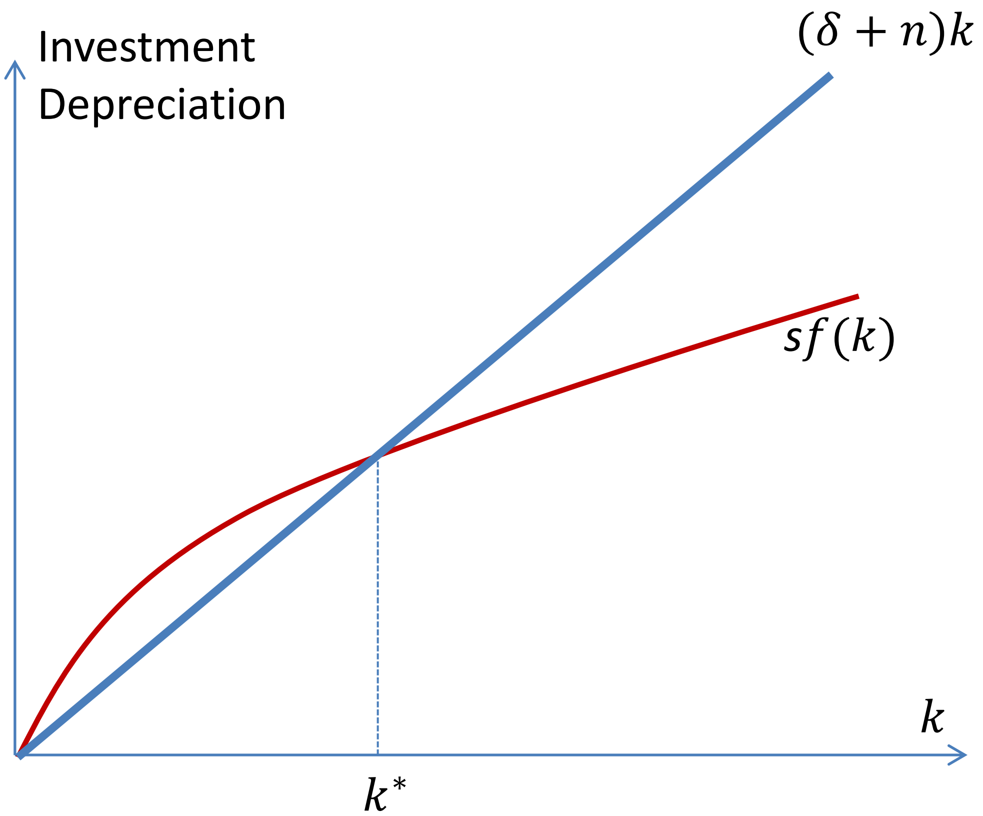
- Let $k_t = \frac{K_t}{L_t}$, the capital per cap at time t. We then characterize the per cap capital accumulation by

$$\dot{k_t} \equiv \frac{d}{dt}\left(\frac{K_t}{L_t}\right) = \frac{\dot{K}_t}{L_t} - \frac{K_t \dot{L_t}}{L_t^2} = sf(k_t) - (\delta + n)k_t.$$

Steady-State Level of Capital

- As capital accumulates, it will reach a point where new investment equals depreciation and dilution by population growth,
$$i^* = sf(k^*) = (\delta + n)k^*.$$
- At this level of capital, k^*, the economy reaches a steady state, where capital does not increase or decrease. We call k^* the stead-state level of capital.

A Graphic Illustration



Stability of Steady-State

- k^* is a *stable* steady-state.
  - k_t would get back to k^* after a perturbation.
- If a shock pushes k_t below k^*: Since the new investment is higher than the depreciation and the dilution, k_t would increase until it reaches k^*.
- if a shock pushes k_t above k^*: Since the new investment would be lower than the depreciation and the dilution, k_t would decrease until it reaches k^*.

An Example

- Suppose $F(K, L) = K^{1/2}L^{1/2}$. Then we have

$$y = \frac{Y}{L} = \frac{K^{\frac{1}{2}}L^{\frac{1}{2}}}{L} = \left(\frac{K}{L}\right)^{\frac{1}{2}} = k^{1/2}.$$

- Let $n = 0, s = 0.3, \delta = 0.1, k_0 = 4$. Each year ($\Delta t = 1$), the capital stock changes by

$$\Delta k = 0.3k_t^{\frac{1}{2}} - 0.1k_t \ .$$

- Solving $0.3k^{*\frac{1}{2}} = 0.1k^*$, we obtain $k^* = 9$.

Approaching the Steady State: A Numerical Illustration

Assumptions: $y = k^{1/2}$, s=0.3, $\delta = 0.1$, $k_0 = 4$.

k	y	c	i	depreciation	change in k			
4.000	2.000	1.400	0.600	0.400	0.200			
4.200	2.049	1.435	0.615	0.420	0.195			
4.395	2.096	1.467	0.629	0.439	0.189			
4.584	2.141	1.499	0.642	0.458	0.184			

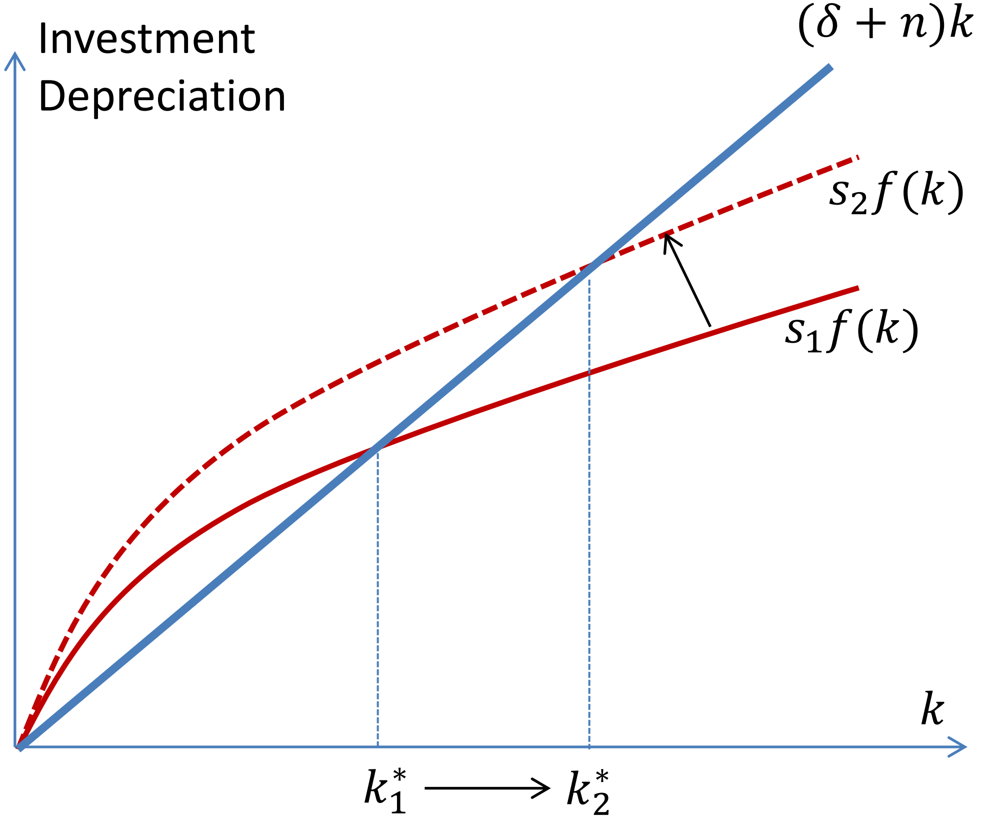
Implications of Solow Model I

- If the economy is already at the steady state, per capita income ($y^* = f(k^*)$) ceases to grow. However, the total income continues to grow as the population grows,
$$Y_t = y^* L_t = y^* L_0 e^{nt}.$$
- If the initial level of capital is below the steady-state level, there will be a convergence (or, catch-up) period to the steady-state level.

The Role of Saving

- To see the effect of a change in the saving rate, s, we examine the equation characterizing the steady state,
$$sf(k^*) = (\delta + n)k^*.$$
- Fix δ and n. Using the implicit function theorem, we have
$$\frac{dk^*}{ds} = -\frac{f(k^*)}{sf'(k^*) - (\delta + n)}.$$
- Due to the assumption $\lim_{k\to 0} f'(k) = \infty$, $\lim_{k\to\infty} f'(k) = 0$, we must have $sf'(k^*) < \delta + n$, otherwise the curve $sf(k)$ cannot cross with the line $(\delta + n)k$ at $k = k^*$. Hence $\frac{dk^*}{ds}$ must be positive.
- An increase in saving rate would lead to a higher level of steady-state capital and income.

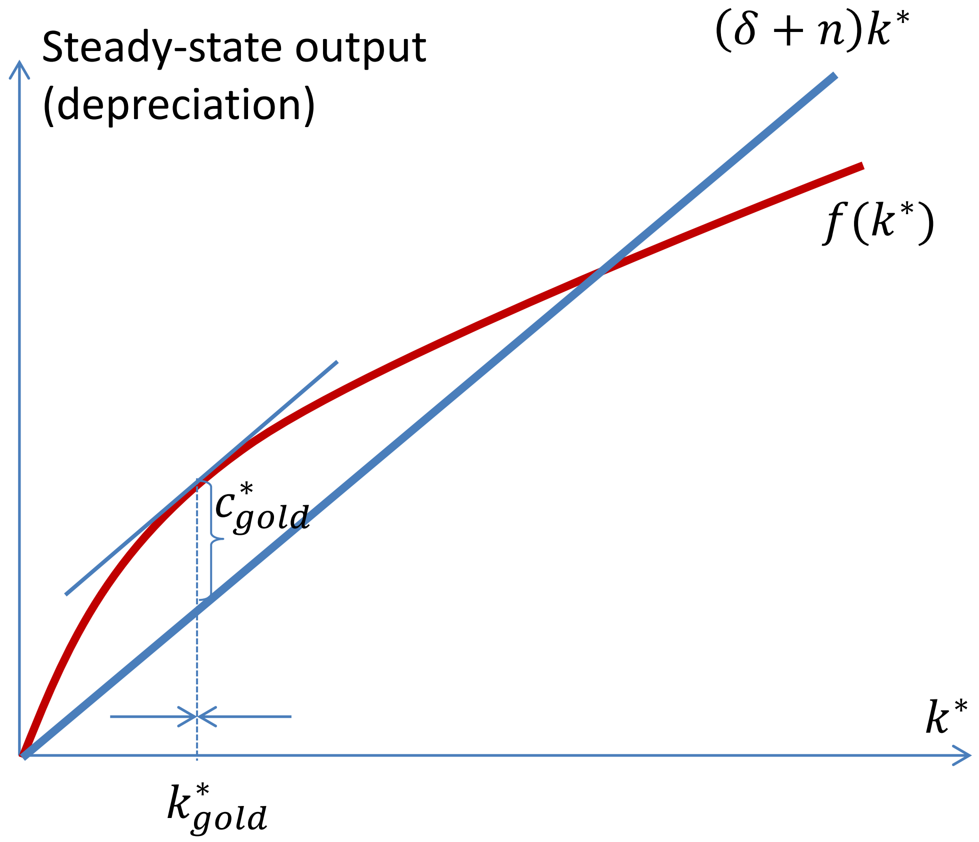
A Graphic Illustration



The Golden-Rule Level of Capital

- At steady states, the consumption is given by
$$c^* = f(k^*) - sf(k^*) = f(k^*) - (\delta + n)k^*.$$
- The level of capital that corresponds to the maximum consumption, which we call the <u>golden-rule level of capital</u>, must satisfy the first-order condition:
$$f'\left(k^*_{gold}\right) = \delta + n.$$
- At the golden-rule level, marginal product of capital (MPK) equals the depreciation rate plus the population growth rate.

A Graphic Illustration



How The Golden-Rule Level of Capital Might Be Achieved

- Recall that the steady-state level of capital is an increasing function of the saving rate, $k^*(s)$. We might adjust s to achieve the golden-rule level of capital.
- Suppose that $k^*(s) < k^*_{gold}$. Since $\frac{dk^*}{ds} > 0$, we might increase the saving rate to achieve the golden-rule level.
- If the initial level of capital is higher than the golden-rule level, then we might decrease the saving rate to achieve the golden-rule level.

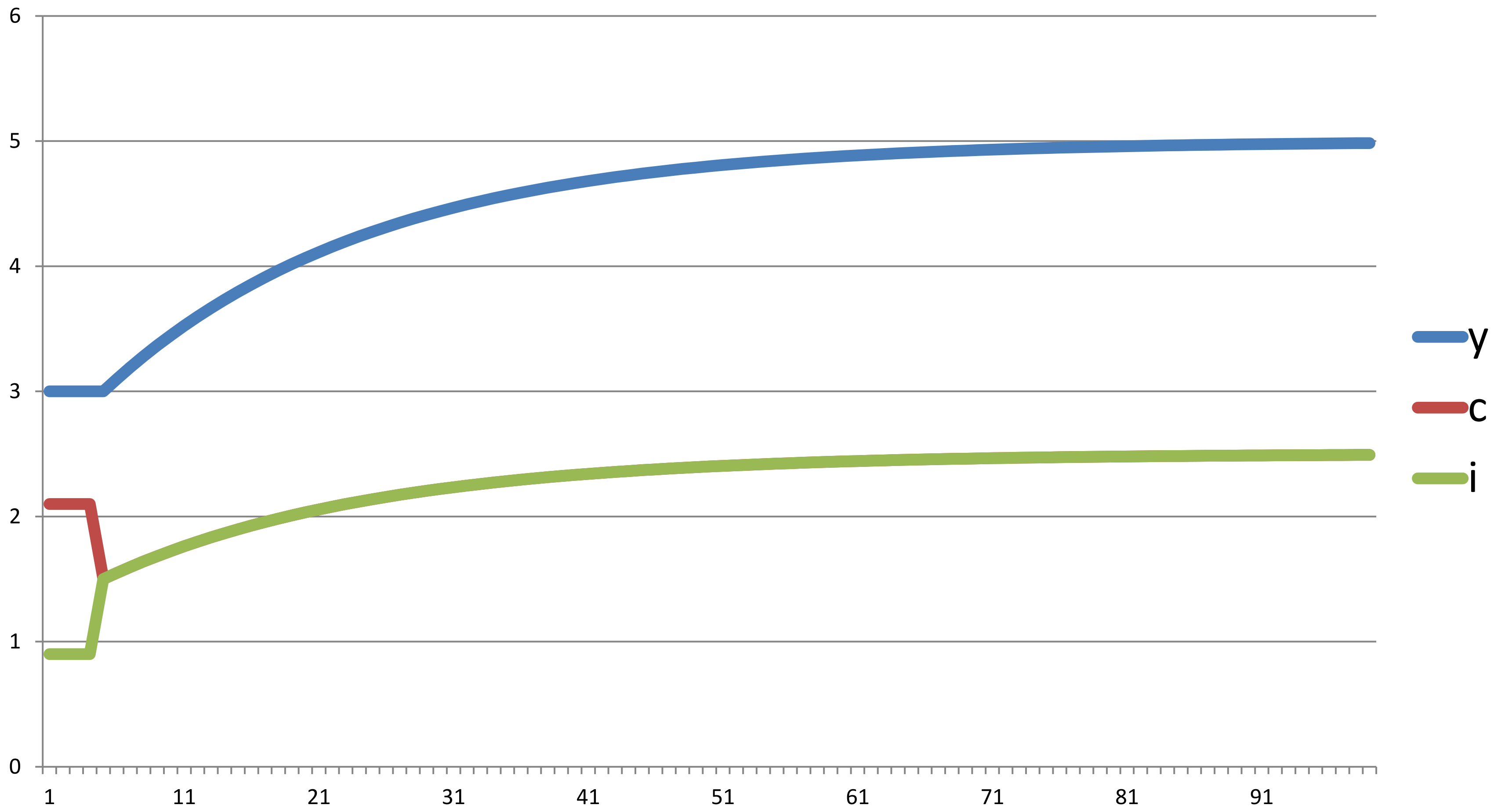
An Example

- Suppose $n = 0, \delta = 0.1, f(k) = k^{1/2}$. Then solving for the steady-state level
$$s{k^*}^{1/2} = 0.1k^*,$$
we obtain the function $k^*(s) = 100s^2$.
- Suppose $s = 0.3$, we obtain the steady-state level of capital in this economy.
$$k^* = 9.$$
- The golden-rule level, however, is obtained from
$$\frac{1}{2}{k_{gold}^*}^{-1/2} = 0.1,$$
which gives k_{gold}^*=25.
- In this economy, the steady-state level of capital is too low. We might increase the saving rate to achieve the golden rule. Which saving rate corresponds to the golden rule?
- We solve $100s_{gold}^2 = 25$ and obtain $s_{gold} = 0.5$.

Dealing with Too Little Capital

s	delta	k	y	c	i	Assumption: y=sqrt(k)		
0.3	0.1	9	3	2.1	0.9			
0.3	0.1	9	3	2.1	0.9			
0.3	0.1	9	3	2.1	0.9			
0.3	0.1	9	3	2.1	0.9			
0.5	0.1	9.000	3.000	1.500	1.500			
0.5	0.1	9.600	3.098	1.549	1.549			
0.5	0.1	10.189	3.192	1.596	1.596			
0.5	0.1	10.766	3.281	1.641	1.641			
0.5	0.1	11.330	3.366	1.683	1.683			
0.5	0.1	11.880	3.447	1.723	1.723			
0.5	0.1	12.416	3.524	1.762	1.762			
0.5	0.1	12.936	3.597	1.798	1.798			
0.5	0.1	13.441	3.666	1.833	1.833			
0.5	0.1	13.930	3.732	1.866	1.866			
0.5	0.1	14.403	3.795	1.898	1.898			
0.5	0.1	14.860	3.855	1.927	1.927			
0.5	0.1	15.301	3.912	1.956	1.956			
0.5	0.1	15.727	3.966	1.983	1.983			
0.5	0.1	16.137	4.017	2.009	2.009			
0.5	0.1	16.532	4.066	2.033	2.033			
0.5	0.1	16.912	4.112	2.056	2.056			
0.5	0.1	17.277	4.157	2.078	2.078			
0.5	0.1	17.628	4.199	2.099	2.099			

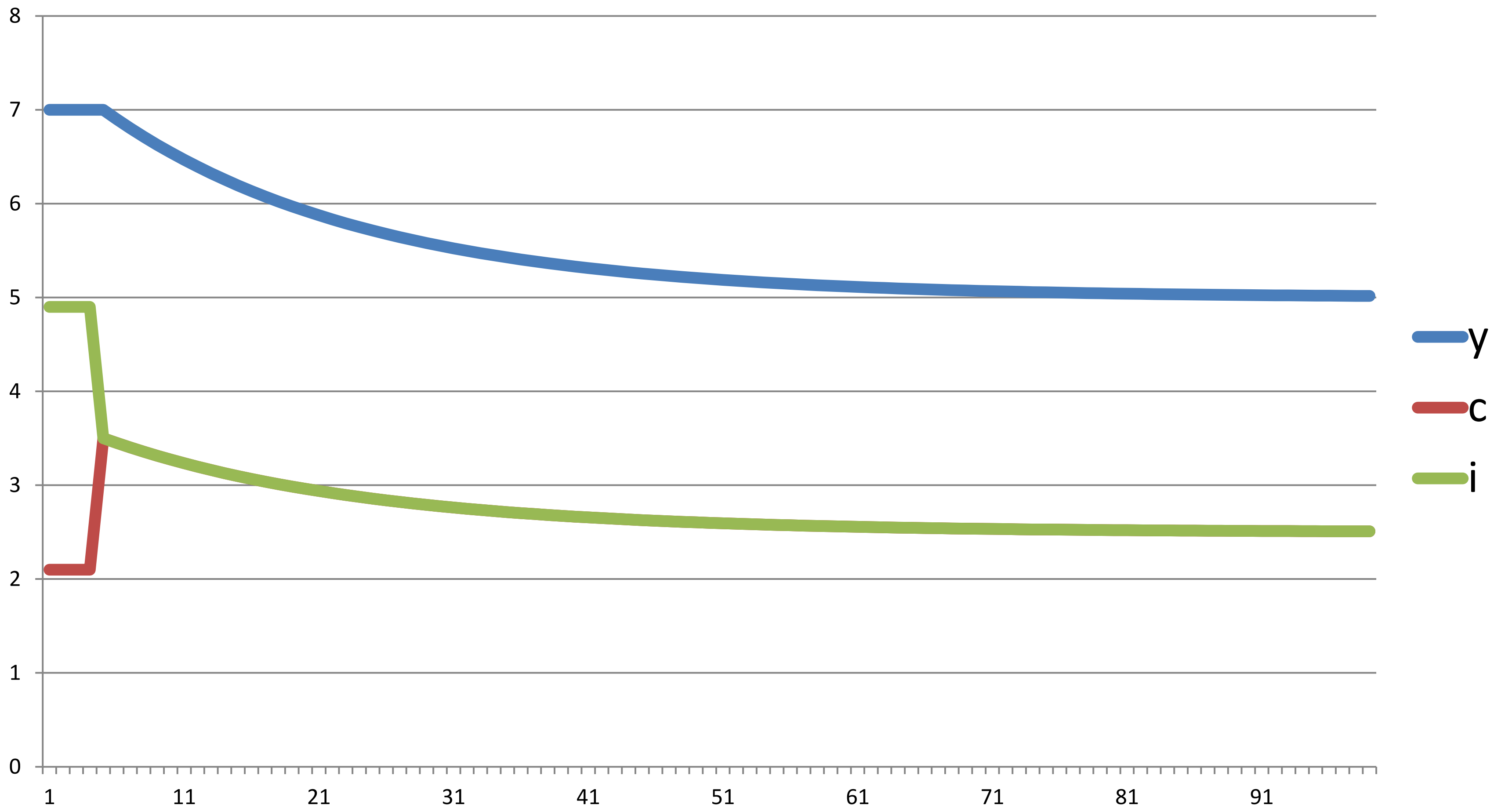
Approaching The Golden Rule



Dealing with Too Much Capital

s	delta	k	y	c	i	Assumption: y=sqrt(k)		
0.7	0.1	49	7	2.1	4.9			
0.7	0.1	49	7	2.1	4.9			
0.7	0.1	49	7	2.1	4.9			
0.7	0.1	49	7	2.1	4.9			
0.5	0.1	49.000	7.000	3.500	3.500			
0.5	0.1	47.600	6.899	3.450	3.450			
0.5	0.1	46.290	6.804	3.402	3.402			
0.5	0.1	45.062	6.713	3.356	3.356			
0.5	0.1	43.913	6.627	3.313	3.313			
0.5	0.1	42.835	6.545	3.272	3.272			
0.5	0.1	41.824	6.467	3.234	3.234			
0.5	0.1	40.875	6.393	3.197	3.197			
0.5	0.1	39.984	6.323	3.162	3.162			
0.5	0.1	39.147	6.257	3.128	3.128			
0.5	0.1	38.361	6.194	3.097	3.097			
0.5	0.1	37.622	6.134	3.067	3.067			
0.5	0.1	36.926	6.077	3.038	3.038			
0.5	0.1	36.272	6.023	3.011	3.011			
0.5	0.1	35.656	5.971	2.986	2.986			
0.5	0.1	35.076	5.923	2.961	2.961			
0.5	0.1	34.530	5.876	2.938	2.938			
0.5	0.1	34.015	5.832	2.916	2.916			
0.5	0.1	33.530	5.790	2.895	2.895			

Approaching The Golden Rule



Case Studies

- The economic miracle of Japan and Germany since the end of the World War II.
- The economic miracle of China since 1978.

The Effect of Population Growth

- To see the effect of a change in n, we still examine the equation characterizing the steady state,

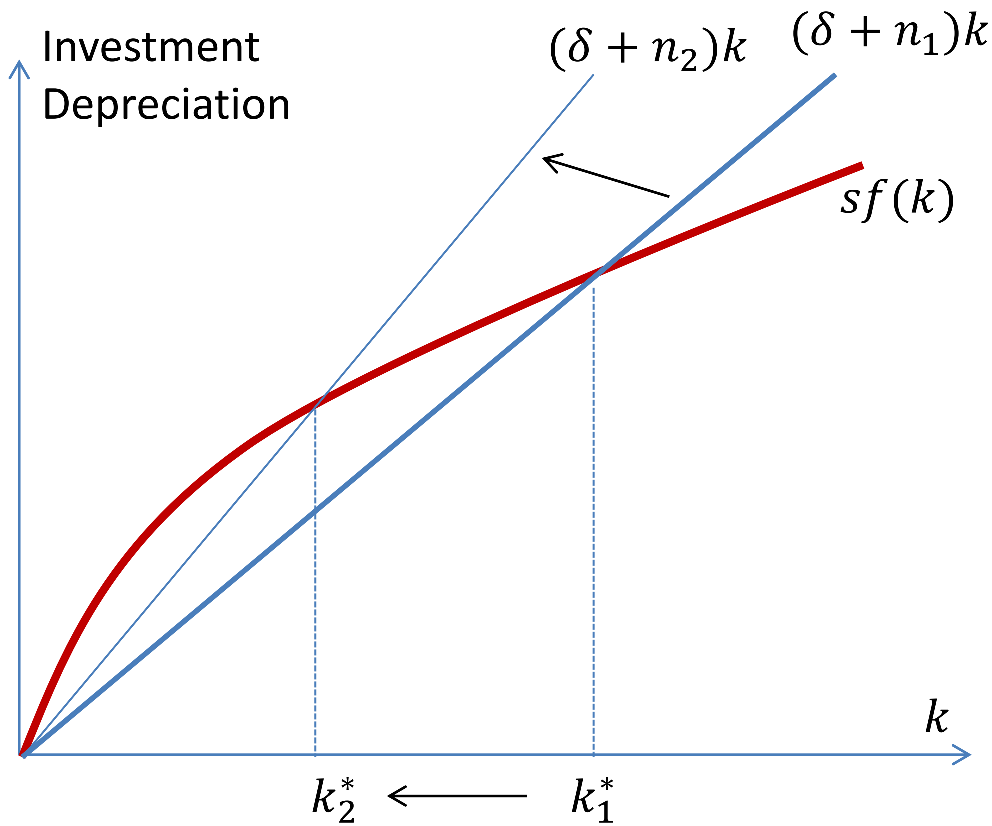
$$sf(k^*) = (\delta + n)k^*.$$

- Fixing s and δ and using the implicit function theorem, we have

$$\frac{dk^*}{dn} = -\frac{-k^*}{sf'(k^*) - (\delta + n)} < 0.$$

- Hence higher population growth leads to lower steady-state capital per cap.

A Graphic Illustration



Case Study: Population Growth v.s. Per Capita Income

International Evidence on Population Growth and Income per Person This figure is a scatterplot of data from 96 countries. It shows that countries with high rates of population growth tend to have low levels of income per person, as the Solow model predicts.

Source: Alan Heston, Robert Summers, and Bettina Aten, Penn World Table Version 6.2, Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania, September 2006.

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Solow Model II

- The first Solow model fails to allow sustainable growth in per capita output/income.
- The second Solow model introduce technological progress as the engine for sustainable growth.
- We assume:
  - Closed economy ($NX = 0$), no government spending ($G = 0$).
  - Labor-augmenting production function, $Y_t = F(K_t, E_t L_t)$, where
    - F is a constant-return-to-scale function,
    - $E_t = E_0 e^{gt}$ is technology level, which grows at a constant rate g, and
    - $L_t = L_0 e^{nt}$ denotes population, which grows at a constant rate n.
  - The saving rate is constant: $C = (1 - s)Y$.
  - Capital depreciates at constant rate δ.

Output Per Effective Worker

- Let $y = \frac{Y}{EL}$ and $k = \frac{K}{EL}$. y is called the output per effective worker. And k is the amount of capital per effective worker. We have
$$y = \frac{Y}{EL} = \frac{F(K, EL)}{EL} = F(k, 1).$$
- As in the first Solow model, we define $f(k) \equiv F(k, 1)$, and write
$$y = f(k).$$
- *f(k)* is the *"effective individual production function"* or "per effective worker production function," how much output one effective worker could produce using *k* units of capital. We assume
$$f(0) = 0, f'(k) > 0, f''(k) < 0.$$

Note that $f'(k)$ is the <u>marginal product of capital</u> (MPK).

- We also assume:
$$\lim_{k \to 0} f'(k) = \infty, \lim_{k \to \infty} f'(k) = 0$$

The Accumulation of Capital

- Note that $\dot{E}_t = gE_t$. And as previously, we have

$$\dot{K}_t = sF(K_t, E_t L_t) - \delta K_t$$
$$\dot{L}_t = nL_t.$$

- The the dynamics of capital accumulation is characterized by

$$\dot{k}_t \equiv \frac{d}{dt}\left(\frac{K_t}{E_t L_t}\right) = \frac{\dot{K}_t}{E_t L_t} - \frac{K_t \dot{L}_t}{E_t L_t^2} - \frac{K_t \dot{E}_t}{L_t E_t^2}$$
$$= sf(k_t) - (\delta + n + g)k_t.$$

The Steady State

- The steady state capital per effective worker, k^*, is then characterized by the following equation,
$$sf(k^*) - (\delta + n + g)k^* = 0.$$
- At steady state, the capital per effective worker is a constant,
$$\frac{K_t}{E_t L_t} = k^*.$$
- We have the following implications:
  - Total output, $Y_t = E_t L_t f(k^*)$, grows at the constant rate $n + g$.
  - Per capita output, $\frac{Y_t}{L_\mathrm{t}} = E_t f(k^*)$, grows at the constant rate g.
- Thus the Solow Model II, incorporating the factor of technological progress, is able to explain sustained growth in per capita terms.

Balanced Growth Path

- The steady state of the Solow model describes a "balanced growth" path.
- Many important ratios remain constant
  - Investment ratio (s)
  - Capital intensity (K_t/Y_t)
- Many grow together at the same speed
  - Capital per capita (K_t/L_t)
  - Labor productivity (Y_t/L_t)

Real Wage and Real Rental Price of Capital

- Recall that in a competitive economy, the real wage equals marginal product of labor (MPL) and the real rental price of capital equals the marginal product of capital (MPK). At the steady state, we have

$$MPL = \frac{\partial Y}{\partial L} = \frac{\partial}{\partial L}\left(ELf\left(\frac{K}{EL}\right)\right) = E\big(f(k^*) - k^* f'(k^*)\big).$$

$$MPK = \frac{\partial Y}{\partial K} = \frac{\partial}{\partial K}\left(ELf\left(\frac{K}{EL}\right)\right) = f'(k^*).$$

- Hence the Solow Model II implies that the real wage grows with the technological progress and that the real return to capital remains constant.

Exercises

- What would be the impact of an increase in saving rate on the steady-state capital per capita?
- What would be the Golden-rule level of capital per effective worker?

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What Endogenous Models Do

- In the Solow model, technological progress is assumed.
- Endogenous models treat the technology progress as an outcome of economic activities, or in other words, as an endogenous process.
- In this part of the lecture, we introduce two simple endogenous models.
  - A basic model
  - Two-sector model

A Basic Model

- The basic model assumes
  - Constant population.
  - A linear technology,

$$Y_t = AK_t,$$

where Y_t is output, K_t is capital stock that includes "knowledge", and A is a constant measuring <u>average product of capital</u> or <u>marginal product of capital</u>.

  - The capital accumulation follows

$$\dot{K}_t = sY_t - \delta K_t.$$

- It is obvious that

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{K}_t}{K_t} = sA - \delta.$$

The Implication

- As long as $sA > \delta$, sustained growth is achieved. The sustained growth depends on high saving and investment.
- This is possible because this model enjoys the constant return to capital, while the Solow model assumes diminishing return to capital.
- The capital stock in this model should be understood to include “knowledge” or human capital.

What Determines Growth Rate

- Saving rate (s)
- Average product of capital (A)
  - Quality of capital
- Depreciation rate (δ)
  - The importance of human capital

A Two-Sector Model

- The two-sector model assumes that the economy has two sectors, the manufacturing sector that produces goods and the university sector that produces knowledge.
- The production function in manufacturing

$$Y_t = F(K_t, (1-u)L_t E_t),$$

where u is the fraction of the labor force in universities.

- Production function in research universities

$$\frac{\dot{E}_t}{E_t} = g(u),$$

where $g(u)$ is describes how the growth in knowledge depends on the fraction of labor force in universities.

- Capital accumulation

$$\dot{K}_t = sY_t - \delta K_t.$$

The Steady State

- Let $k = \frac{K}{EL}$. The steady-state is characterized by
$$sf(k^*, u) = \big(\delta + n + g(u)\big)k^*,$$
where $f(k, u) = F(k, 1 - u)$.
- Obviously, the two-sector model allows sustainable growth.
- At the steady state, the growth rate of the economy is not exogenously given, but dependent on u and how knowledge and innovations are produced in “universities” ($g(u)$).

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Accounting for Economic Growth

- Where does economic growth come from?
  - Increases in the factors of production
    - Capital (K)
    - Labor (L)
  - Technological progress
- How much does each element contribute to economic growth?
- In particular, if the growth comes not from technological progress, it would be considered “bad” growth, which is not sustainable.

Technological Progress

- Assume $Y_t = A_t F(K_t, L_t)$, where A_t is called **total factor productivity**. A_t measures current level of technology.
- Some derivation leads to **the equation of growth accounting**:

$$\frac{\dot{Y}_t}{Y_t} = \alpha_t \frac{\dot{K}_t}{K_t} + \beta_t \frac{\dot{L}_t}{L_t} + \frac{\dot{A}_t}{A_t},$$

- $\frac{\dot{A}_t}{A_t}$, which is called the **Solow residual**, measures technological progress. It is the changes in output that cannot be explained by changes in inputs.

Case Study: Source of Growth in US

Accounting for Economic Growth in the United States

			SOURCE OF GROWTH				
Years	Output Growth $\Delta Y/Y$	=	Capital $\alpha \Delta K/K$	+	Labor $(1-\alpha)\Delta L/L$	+	Total Factor Productivity $\Delta A/A$
			(average percentage increase per year)				
1948–2007	3.6		1.2		1.2		1.2
1948–1972	4.0		1.2		0.9		1.9
1972–1995	3.4		1.3		1.5		0.6
1995–2007	3.5		1.3		1.0		1.3

Source: U.S. Department of Labor. Data are for the non-farm business sector.

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Creative Destruction

- In *Capitalism, Socialism, and Democracy* by Joseph Schumpeter (1883-1950), economic growth is achieved in a process of creative destruction.
- Entrepreneurs come up with new products, new technology, new managerial and marketing ideas, and other innovations, which would drive uncreative incumbents out of the market.
- These entrepreneurs would become the new incumbents. A new generation of entrepreneurs enter the market with new ideas…

Necessary Conditions for Creative Destruction

- Entrepreneurs
  - Economic freedom
  - Inclusive society
- Market economy
  - Let market pick “winners”
  - Ensure a level “playground”
  - Market should be big enough

Limits of Creative Destruction

- Monopoly
- Social gain is not guaranteed.
- Gain is not evenly distributed.

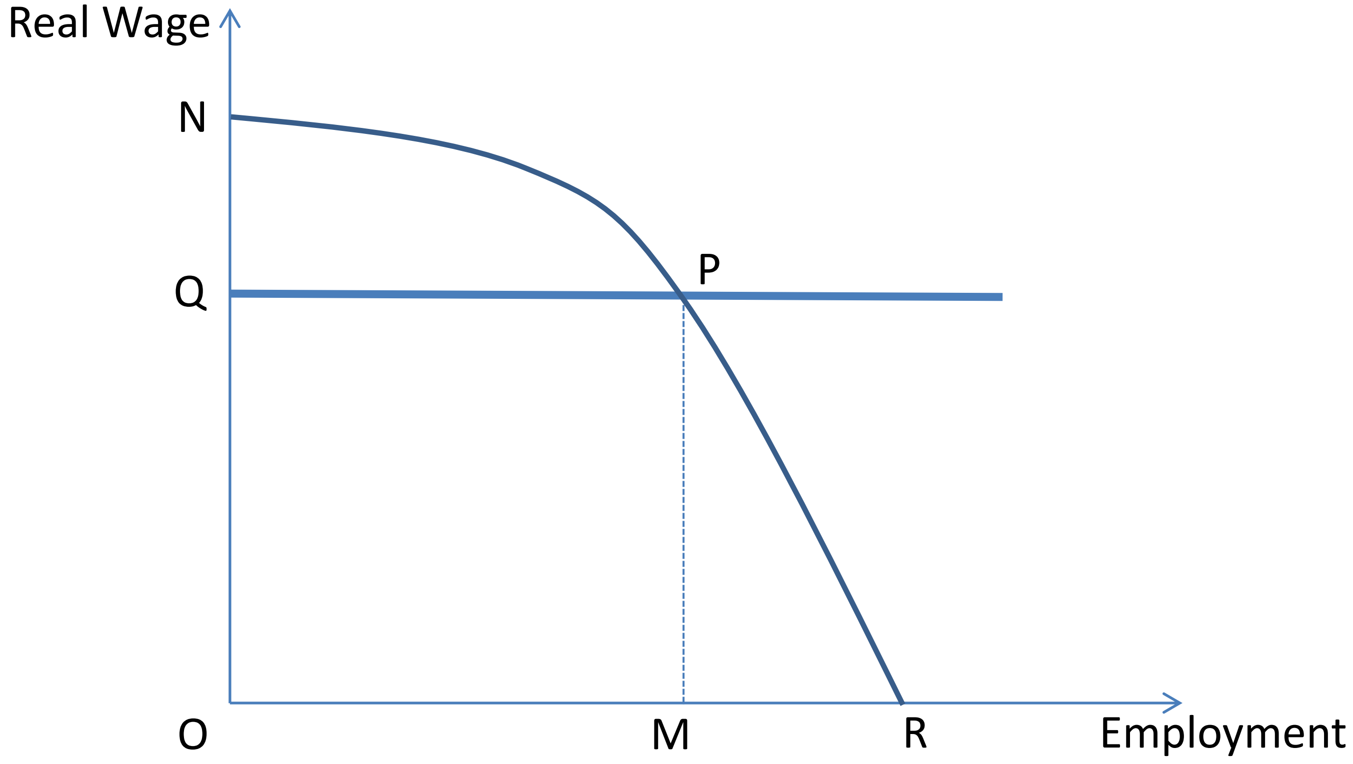
To Rein In Creative Destruction

- The government has responsibility to rein in creative destruction and avoid its excesses.
  - Anti-monopoly
  - Actively update and enforce laws to ensure a level playing ground
  - Welfare programs to help losers

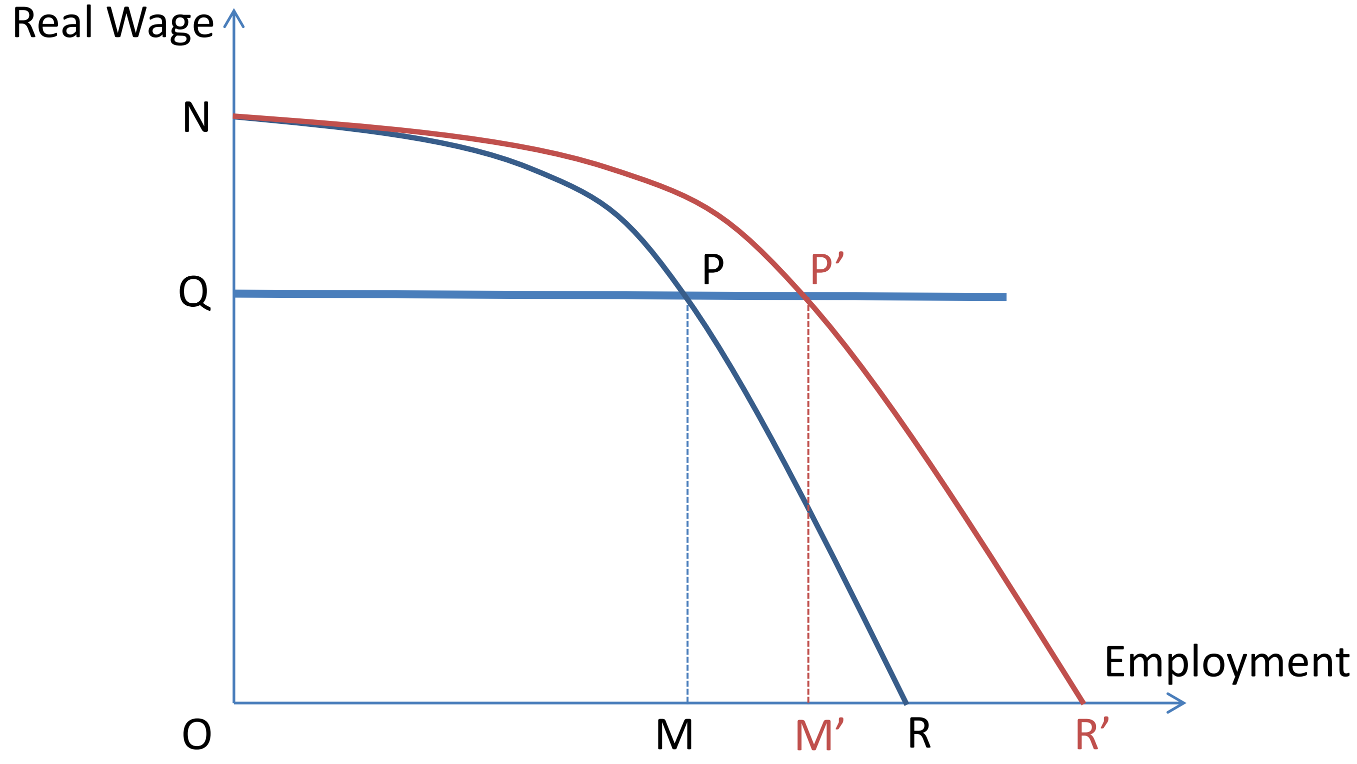
The Lewis Model

- This is named after British economist Arthur Lewis (1915-1991), who was best known for his theory of economic development.
- In the Lewis model, there are two sectors in an under-developed economy: *capitalist* and *subsistence*. The latter is endowed with an unlimited supply of labor.
- In the early stage of development, the capitalist sector attracts labor from the subsistence sector at low wage, resulting in high return to capital, reinvestment, capital accumulation, and higher demand for labor. The economic development goes into a virtual cycle.
- As soon as the excess labor in the subsistence sector is fully absorbed in the capitalist sector, wages start to rise. This is called the Lewis Turning Point.

The Lewis Model



Industrialization in the Lewis Model



Migration, Outsourcing, and International Trade

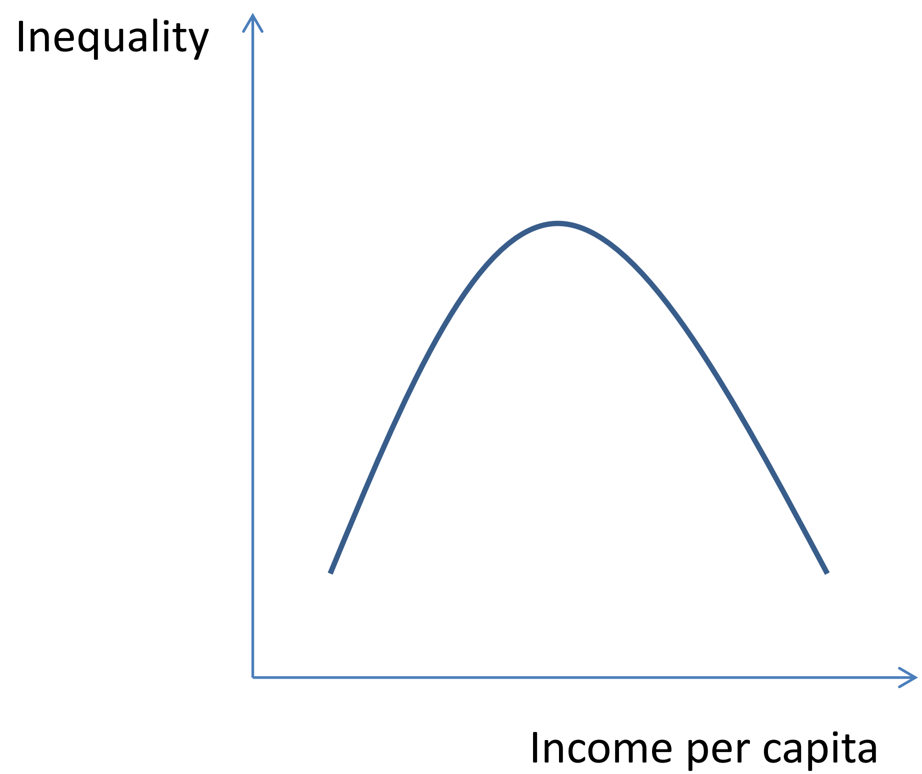
- The Lewis model describes a non-steady state, where return to labor is not equalized (in capitalist and subsistence sectors).
- To equalize the return to labor:
  - labor to capital (migration to industrialized countries or regions)
  - capital to labor (outsourcing, globalization of supply chain, industrialization of the third-world)
- International trade is a substitute of factor mobility.

Industrialization, De-industrialization, and Urbanization

- The Lewis model implies both urbanization and industrialization for one developing economy.
- Internationally, industrialization may peak in some advanced countries.
  - Manufacturing may be outsourced to developing countries
  - Service is an increasingly important sector.
- Urbanization realizes the scale of economy in cities, both during the industrialization and de-industrialization.

Kuznets Curve

- This is named after Jewish-American economist Simon Kuznets (1901-1985).
- The Kuznets curve is a conjecture that as an economy grows, the market force first increases and then decreases inequality (e.g., Gini coefficient).



Environmental Kuznets Curve

- The Environment Kuznets curve is a conjecture that as an economy grows, the natural environment first deteriorates, due to industrial pollution, and then get better, due to an increasing public awareness and state regulation.