1.

(a)

As the Solow Model without technological progrss, in which

$$\dot{L}_t = nL_t \qquad \dot{K}_t = sY - \delta K_t,$$

in the model with technological progrss, we also have

$$\dot{E}_t = gE_t$$

and define

$$f(k_t) \equiv y_t = \frac{Y_t}{E_t L_t} = K_t^{\alpha}(E_t L_t)^{-\alpha} = k_t^{\alpha}.$$

Therefore,

$$\dot{k}_t = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{K_t}{E_t L_t}\right) = sf(k_t) - (\delta + g + n)k_t = sk_t^{\alpha} - (\delta + g + n)k_t.$$

When the steady state is reached,

$$\dot{k}_t|_{k_t = k^*} = 0 \Longrightarrow k^* = \left(\frac{\delta + g + n}{s}\right)^{\frac{1}{\alpha - 1}}.$$

Plug the k^* into the expressions, we have

$$y^* = f(k^*) = \left(\frac{\delta + g + n}{s}\right)^{\frac{\alpha}{\alpha - 1}},$$

$$c^* = (1 - s)y^* = (1 - s)\left(\frac{\delta + g + n}{s}\right)^{\frac{\alpha}{\alpha - 1}}.$$

(b)

To maximize the $c^* = y^* - sy^* = (k^*)^{\alpha} - (\delta + g + n)k^*$, employ the FOC and get

$$\left.\frac{\partial c^*}{\partial k^*}\right|_{k^* = k_g^*} = 0 \Longrightarrow k_g^* = \left(\frac{\delta + g + n}{\alpha}\right)^{\frac{1}{\alpha - 1}}.$$

(c)

To find the the golden-rule saving rate, just let $k^* = k_g^*$ and solve it. Since $\partial k^* / \partial s > 0$,

$$s_g = \alpha.$$

2.

(a)

$$
\begin{aligned}
MPK = \frac{\partial Y_t}{\partial K_t} &= \frac{\partial}{\partial K_t}\left(E_t L_t \cdot f\Big(\frac{K_t}{E_t L_t}\Big)\right) \\
&= \frac{\partial}{\partial (K_t/(E_t L_t))} f\Big(\frac{K_t}{E_t L_t}\Big) \\
&= f'\left(\frac{K_t}{E_t L_t}\right) = f'(k)
\end{aligned}
$$

When at the steady state, $k = k^*$. That is, $MPK\big|_{k=k^*} = f'(k^*)$.

(b)

Despite the change of distribution, the equation of the dynamics of p.e.w. capital accumulation still works,

$$
\dot{k}_t = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{K_t}{E_t L_t}\right) = \frac{\dot{K}_t E_t L_t - K_t(\dot{E}_t L_t + E_t \dot{L}_t)}{(E_t L_t)^2}.
$$

Plugging the new expression of $\dot{K}_t$ into the equation, we have

$$
\dot{k}_t = (MPK - \delta)k_t - (g+n)k_t = k_t \cdot f'(k_t) - (\delta + g + n)k_t. \tag{1}
$$

(c)

According to (1), at he steady state,

$$
\dot{k}_t = 0 \implies f'(k_t) = \delta + g + n.
$$

Meanwhile, $c^* = y^* - sy^* = f(k^*) - (\delta + g + n)k^*$. Employ the FOC and we can compute that the golden-rule level of capital per capita satisfies

$$
f'(k_g^*) = \delta + g + n.
$$

In the model assumption, $f''(k) < 0$. Therefore, $k^* = k_g^*$.