1

(i)

$$
\begin{aligned}
D &= \begin{vmatrix} 1 & 0.5 & 0 \\ -0.5 & 1 & 1.5 \\ 0 & 0 & 1 \end{vmatrix} = 1.25, \\
D_1 &= \begin{vmatrix} b_1 & 0.5 & 0 \\ b_2 & 1 & 1.5 \\ b_3 & 0 & 1 \end{vmatrix} = b_1 - 0.5b_2 + 0.75b_3, \\
D_2 &= \begin{vmatrix} 1 & b_1 & 0 \\ -0.5 & b_2 & 1.5 \\ 0 & b_3 & 1 \end{vmatrix} = 0.5b_1 + b_2 - 1.5b_3, \\
D_3 &= \begin{vmatrix} 1 & 0.5 & b_1 \\ -0.5 & 1 & b_2 \\ 0 & 0 & b_3 \end{vmatrix} = 1.25b_3.
\end{aligned}
$$

Therefore, the solution is

$$
\begin{aligned}
x_1 &= \frac{D_1}{D} = 0.8b_1 - 0.4b_2 + 0.6b_3, \\
x_2 &= \frac{D_2}{D} = 0.4b_1 + 0.8b_2 - 1.2b_3, \\
x_3 &= \frac{D_3}{D} = b_3.
\end{aligned}
$$

(ii)

$$
\begin{pmatrix} 1 & 0.5 & 0 \\ -0.5 & 1 & 1.5 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}
$$

(iii)

$$
A^{-1} = \frac{1}{5} \begin{pmatrix} 4 & -2 & 3 \\ 2 & 4 & -6 \\ 0 & 0 & 5 \end{pmatrix}
$$

It is easy to get $A^{-1}\boldsymbol{b} = \boldsymbol{x}$.

2

$$f_x(x,y) = 4x^{-\frac{1}{3}}y^{\frac{1}{2}} \qquad\qquad f_y(x,y) = 3x^{\frac{2}{3}}y^{-\frac{1}{2}}$$

Let $x_0 = 1000, y_0 = 100$, we can estimate the function value

$$\begin{aligned} f(998, 101.5) &= f(x_0, y_0) + f_x(x_0, y_0)(998 - x_0) + f_y(x_0, y_0)(101.5 - y_0) \\ &= 6000 - 4 \times 2 + 30 \times 1.5 = 6037 \end{aligned}$$

3

Let $f(x,y) = x^2 - 3xy + y^3 - 7$.

(i)

$f(4,3) = 0$, so the point $(4,3)$ is on the curve defined by the equation.

(ii)

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{f_x}{f_y} = \frac{2x - 3y}{3x - 3y^2}\ ,\ \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{(4,3)} = \frac{1}{15}$$

(iii)

Suppose the implicit function decided by the equation is $y = g(x)$, and $g(4) = 3$. Therefore, we have

$$\Delta g_1 = g(4.1) - g(4) \approx g(4) + \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=4}(4.1 - 4) - g(4) = \frac{1}{150}$$

Using the computer, I get that $\Delta y_2 = 0.0061 > \Delta y_1$.

4

Let $F(x,y) = f(x,y) + g(y) - y + z$.

(i)

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{F_x}{F_y} = -\frac{f_x}{f_y + g' - 1}$$

$$\begin{cases} \Delta y = \dfrac{\mathrm{d}y}{\mathrm{d}x}\Delta x \\ \Delta m = m_x \Delta x + m_y \Delta y \end{cases} \implies \Delta x = \frac{f_y + g' - 1}{m_x(f_y + g' - 1) - m_y f_x}\Delta m$$

(ii)

Consider $Z(x,y) = y - f(x,y) - g(y)$, then

$$\begin{cases} \Delta z = -f_x \Delta x + (1 - f_y - g')\Delta y \\ 0 = \Delta m = m_x \Delta x + m_y \Delta y \end{cases} \implies \Delta y = \frac{m_x}{m_x(1 - f_y - g') + m_y f_x} \Delta z$$