Homework 3

1. Suppose that the output of an economy can be characterized by the Cobb-Douglas function,

$$F(K, L) = EK^{\alpha}L^{1-\alpha}, 0 < \alpha < 1.$$

(1) Calculate the marginal product of labor (MPL) and the marginal product of capital (MPK). Check whether they are positive.
(2) Calculate the second derivatives. Check that MPL is decreasing as L increases and that MPK is decreasing as K increases.
(3) Verify that the Cobb-Douglas function satisfies constant-return-to-scale.

2. Apply the classical theory of income distribution to predict the effect on the real wage and the real rental price of capital if the following events happen:
   (1) An earthquake damages part of the capital stock.
   (2) The government raises the retirement age.
   (3) Inflation raises all prices (output price and factor-input prices) by 10%.
   (4) Suppose that the production function is labor-augmenting. A technological breakthrough improves the production function. (hint: on labor-augmenting and capital augmenting technologies, read page 5 of my notes "Classical Theories" macro03.pdf.)
   (5) Following (4), what if the production function is capital-augmenting.

3. Consider a closed economy characterized by the following equilibrium condition and specifications:

$$\mathrm{Y} = \mathrm{C}(\mathrm{Y} - \mathrm{T}) + \mathrm{I}(\mathrm{r}) + \mathrm{G},$$
$$\mathrm{Y} = 8000, \mathrm{G} = 2500, \mathrm{T} = 2000,$$
$$\mathrm{C}(\mathrm{Y} - \mathrm{T}) = 1000 + \frac{2}{3}(Y - T),$$
$$\mathrm{I}(\mathrm{r}) = 1200 - 100\mathrm{r}.$$

   (1) Calculate private saving, public saving, and national saving.
   (2) Calculate the equilibrium real interest rate.
   (3) Suppose that the government reduces its expenditure to achieve a balanced budget. Calculate private saving, public saving, and national saving. And calculate the new equilibrium real interest rate.