Economic Growth

Poverty is not socialism. – Deng Xiaoping[1]

1 Introduction

Economic growth, or economic development, is no doubt one of the most important topics in macroeconomics. For poor countries, a stagnant economy means persistent absolute poverty. In absolute poverty, the need for survival dominates all other desires of human beings. Human lives in absolute poverty can be extremely miserable and dangerous.

In relative terms, a slight but persistent difference in growth rate would result in huge income gaps among nations. The following table illustrates how three different growth rates of income per capita (from the same level, say 100) lead to starkly different outcomes many (10, 30, 100) years later.

Scenarios\Years	0	10	30	100
1%	100	110.5	134.8	270.5
3%	100	134.4	242.7	1921.9
8%	100	215.9	1006.3	219976.1

Economic growth is important not only in terms of the outcome (that is, a wealthy society) but also the path that leads to the outcome. Economic growth is good in itself. People in a growing economy tend to be more optimistic about the future. They tend to be more open and tolerant because the pie is getting bigger. Even a wealthy nation, if it stops growing, can fell to the prey of intolerance and hostility because people are trapped in a zero-sum game.

To simplify the analysis of economic growth, we focus on the long-term trend of output potential $\bar{Y}_t$. Note that I add a time subscript to emphasize that, in this chapter, the output potential may be growing over time. We may imagine that business cycles are short-term fluctuations around the long-term trend of the output potential. More precisely, we may write,

$$Y_t = \bar{Y}_t + \left(Y_t - \bar{Y}_t\right),$$

where $\bar{Y}_t$ represents the trend of *output potential* and $\left(Y_t - \bar{Y}_t\right)$ is the *output gap*. Figure 1 illustrates the relationship between the long-term trend of output potential ($\bar{Y}_t$) and the output gap $\left(Y_t - \bar{Y}_t\right)$.

We may assume that short-term fluctuations in the output gap are independent of the long-term trend, meaning that the short-term fluctuation does not have an

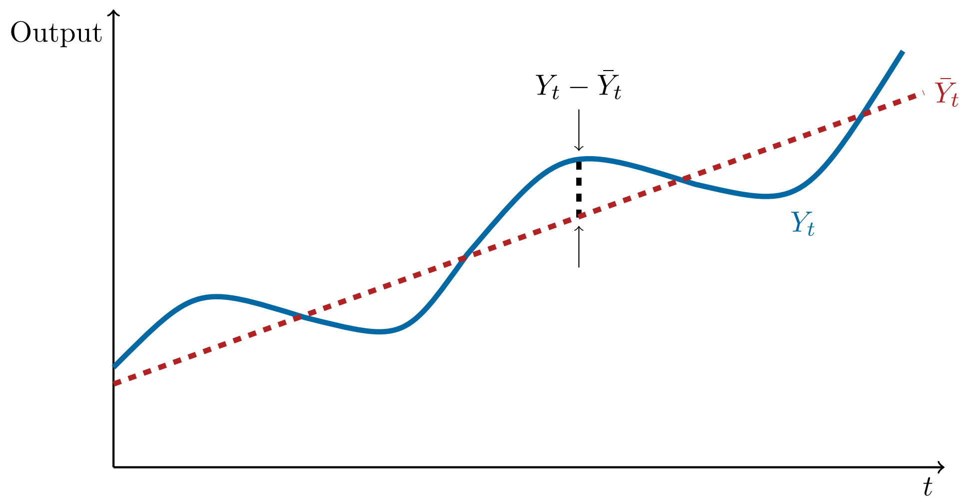


Figure 1: Short-term fluctuations in output gap around a long-term trend of output potential.

impact on the long-term trend, and vice versa. Under this assumption, we can safely disregard the fluctuations in the output gap in this chapter and focus on the long-term trend only. To justify the assumption, we may argue that the long-term trend reflects the supply-side changes such as the accumulation of capital, technological progress, etc., while the short-term fluctuation reflects the short-term changes in the aggregate demand.

Note, however, the independence of the output gap from the trend is only an assumption. It may well be that the short-term fluctuation may interact with the change in the trend. A severe recession, for example, may permanently damage the growth potential if the recession brings mass unemployment, social unrest, political instability, and so on. On the bright side, a severe downturn may also strengthen political support for reforms in the government, hence paving the way for better growth in the future. Indeed, the reform of the Chinese state sector in the late 1990s happened during a severe downturn, when the state-owned enterprises were in deep trouble.

In the rest of the chapter, we ignore the fluctuations in the output gap and assume that $Y_t = \bar{Y}_t$ for all t. We first introduce two versions of Solow models that characterize the dynamics of economic growth[2]. The first Solow model depicts a dismal picture of economic growth or, more precisely, non-growth. The first Solow model is relevant to many economies in the so-called poverty trap, or the agricultural economies before the Industrial Revolution. To model the lucky few countries that experienced sustained growth, we introduce the second Solow model that incorporates an exogenous *technological progress*, which helps to overcome the decreasing marginal product of capital, thus achieving sustained growth.

The exogenous "technological progress" includes all kinds of progress in the society that is conducive to economic prosperity. It includes not only progress in science and engineering but also the increasing capability of public-goods provision, resource allocation and mobilization, and so on. The sustained improvement in these capabilities is essential to sustainable economic growth. To assume an exogenous "technological progress," thus, is somewhat vacuous, not helpful for us to understand the causes of economic growth.

To understand how "technological progress" comes about, we first introduce an endogenous growth model that does not require an exogenous process of technological progress. Then we introduce two important theories that take directly at the causes of technological progress: the theory of creative destruction popularized by Joseph Schumpeter and the two-sector Lewis model named after W. Arthur Lewis. Both models are very relevant for the study of Chinese economic growth.

Note that since economic growth is a long-term story, we shall continue to work under the classical assumptions. As a result, the models in this chapter are all about the supply side of the economy. This chapter differs from the previous one in that we talk about a possibly expanding supply side.

2 Solow Model without Technological Progress

We first introduce a simple Solow model without technological progress, which characterizes the role of factor inputs in economic growth.

2.1 The Model

We assume that all available factor resources (e.g., labor and capital) are fully employed in production. This is a reasonable assumption since we are studying the long-term growth of the output potential $Y_t = \bar{Y}_t$. Furthermore, we make the following assumptions:

Assumptions

(a) Closed economy ($X = 0$).

(b) No government spending ($G = 0$).

(c) Fixed constant-return-to-scale technology, $Y_t = F(K_t, L_t)$.

(d) The saving rate s is a constant and $0 \leq s \leq 1$.

(e) Population grows at a constant rate n.

(f) Capital depreciates at a constant rate δ.

The assumptions (a) and (b) are for the simplification of analysis. Assumption (c) says that there is no technological progress. Assumption (d), together with (a) and (b), implies that both investment and consumption expenditures are fixed fractions of the total income,

$$I_t = sY_t, C_t = (1-s)Y_t.$$

Note that $Y_t = C_t + I_t$ for all t is a rather strong statement. It says that the aggregate demand $(C_t + I_t)$ automatically accommodates the aggregate supply, Y_t.

Assumption (e) says that the population grows by $n \times 100\%$ per unit of time (say, a year). If the population starts with L_0 at time 0, the population at time t would be $L_t = L_0 e^{nt}$. We can also characterize population growth using a differential equation,

$$\dot{L}_t = nL_t, \tag{1}$$

where $\dot{L}_t$ represents $\frac{dL_t}{dt}$. We may easily check that $L_t = L_0 e^{nt}$ solves the above equation.

To understand why (1) describes population growth, we imagine that a population has constant birth and death rates, b and d, respectively, meaning that there are b births and d deaths per individual per unit of time. Let Δt be a short time interval. Then, during the interval from t to $t + \Delta t$, there would be, approximately, $(bL_t\Delta t)$ births and $(dL_t\Delta t)$ deaths. The population change is given by

$$\Delta L_t \approx bL_t\Delta t - dL_t\Delta t = (b-d)L_t\Delta t.$$

Let $n = b - d$, we have

$$\dot{L}_t = \lim_{\Delta t \to 0} \frac{\Delta L_t}{\Delta t} = nL_t.$$

Dot Notation and Differential Equation

Note that L_t is a simplified notation for $L(t)$, a function of continuous time t. And $\dot{L}_t$ represents the derivative of L with respect to t,

$$\dot{L}_t \equiv \frac{dL_t}{dt}.$$

Using differential equations to characterize L_t, K_t, and so on, we make an implicit assumption that these variables are smooth functions of t, so smooth that they are differentiable with respect to t.

Assumption (f) says that per unit of time (say, a year), the capital stock declines by $\delta \times 100\%$. If there is no new investment and we have an initial capital stock K_0, then

$$K_t = K_0 e^{-\delta t}.$$

That is, the capital stock wears out exponentially. We may easily check that this exponential function solves the following differential equation,

$$\dot{K}_t = -\delta K_t.$$

Since, at the same time, investment increases the capital stock, we can characterize capital accumulation by the following differential equation,

$$\dot{K}_t = sY_t - \delta K_t. \tag{2}$$

The left-hand side of (2) is the change in the capital stock per unit of time. The right-hand side is the additional capital stock brought by new investment (sY_t), minus the depreciation of the capital stock (δK_t).

We may also represent (1) and (2) in discrete-time form,

$$\begin{aligned} L_t - L_{t-1} &= nL_{t-1}, \\ K_t - K_{t-1} &= sY_{t-1} - \delta K_{t-1}, \\ t &= 1, 2, 3, \ldots \end{aligned}$$

The discrete-time formulation is useful in conducting simulations using spreadsheet. For theoretical analysis, however, the continuous-time formulation is more convenient.

Per Capita Production Function

Let $y_t = Y_t/L_t$ and $k_t = K_t/L_t$. Obviously, y_t is the average output, or per capita output, and k_t is the average capital, or per capita capital. Using the constant-return-to-scale property of F, we have

$$y_t = \frac{Y_t}{L_t} = \frac{F(K_t, L_t)}{L_t} = F(k_t, 1).$$

We define a *per capita production function*, $f(k_t) \equiv F(k_t, 1)$. Then we have

$$y_t = f(k_t).$$

We may also call $f(\cdot)$ the *individual production function.* We assume that

$$f(0) = 0, f'(k) > 0, f''(k) < 0. \tag{3}$$

That is, zero capital produces zero output, marginal product of capital (MPK) is positive and declining as k increases. Sometimes we may also assume that

$$\lim_{k \to 0} f'(k) = \infty, \quad \text{and} \quad \lim_{k \to \infty} f'(k) = 0. \tag{4}$$

This assumption says that MPK is very large when capital stock is very low and that MPK is close to zero when capital stock is very large.

Without government spending and net export, the aggregate demand for goods and services comes from consumption (C) and investment (I) only. In per capita terms, we have

$$y_t = c_t + i_t,$$

where $c_t = C_t/L_t$ and $i_t = I_t/L_t$. The per cap investment is a constant fraction of the per capita output,

$$i_t = y_t - c_t = sy_t = sf(k_t).$$

2.2 Steady State

To characterize the accumulation of the per capita capital, we first calculate

$$\dot{k}_t = \frac{d}{dt}\left(\frac{K_t}{L_t}\right) = \frac{\dot{K}_t}{L_t} - \frac{K_t \dot{L}_t}{L_t^2}.$$

Plug in (1) and (2), we obtain

$$\dot{k}_t = sf(k_t) - (\delta + n)k_t. \tag{5}$$

The per capita investment ($sf(k_t)$) increases the per cap capital (k_t), while depreciation and population growth make k_t decline.

The assumptions (3) and (4) ensure that the differential equation in (5) has a *steady state*. It means that, as capital accumulates from a low level, it will reach a point where new investment equals depreciation and dilution by population growth,

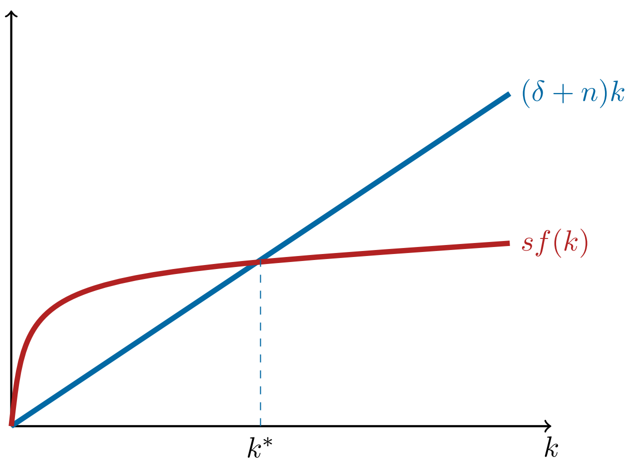
$$sf(k^*) = (\delta + n)k^*. \tag{6}$$

At this level of capital, k^*, the economy reaches a steady state, where capital per capita does not increase or decrease. We call k^* the steady-state level of capital. Note that the population growth rate (n) has a similar effect on steady-state capital stock with the depreciation rate (δ) since both population growth and depreciation reduce per capita capital stock.

Figure 2 graphically characterizes the steady-state of the model. Since $f'(k)$ is very large when k is very small, $sf(k)$ will be initially above $(\delta+n)k$ as k increases from 0. As k gets larger and larger, $f'(k)$ keeps declining and eventually goes to zero. This makes sure that $sf(k)$ (the red line) will cross $(\delta + n)k$ (the blue line) somewhere. Hence the existence of a steady state.

Note that the steady-state level of capital k^* is a *stable* steady state, meaning that k_t would get back to k^* after a perturbation. Suppose, for example, a shock pushes k_t below k^*. Since the new investment ($sf(k_t)$, red line) is higher than the depreciation and the dilution due to population growth ($(\delta + n)k_t$, blue line), k_t would rise until it reaches k^*.

Figure 2: The Solow Model without Technological Progress



Similarly, if k_t is pushed above k^*, then the new investment would be lower than the depreciation and the dilution (due to population growth). As a result, the capital stock per capita would decline until it reaches k^*.

The Solow model without technological progress allows only one type of growth, the growth from a none-steady-state with a per capita capital stock lower than k^*. If the initial level of capital is well below the steady-state level (say, due to war damage), then the new investment may be much higher than the depreciation and the dilution due to population growth, resulting in the fast accumulation of capital and fast economic recovery. We may call this *catch-up growth*. Germany and Japan, after World War Two, arguably experienced such growth.

Numerical Experiment: How to Reach a Steady-State

Suppose that $F(K, L) = K^{0.5}L^{0.5}$. Then we have

$$y = k^{0.5}.$$

Let $n = 0$, $s = 0.3$, $\delta = 0.1$, $k_0 = 4$. Using the discrete-time formulation,

$$k_t - k_{t-1} = 0.3k_{t-1}^{0.5} - 0.1k_{t-1}, \quad t = 1, 2, \ldots,$$

we can calculate k_1, k_2, ..., iteratively. The Excel Spreadsheet (Solow1.xlsx, available at the author's webpage) does this calculation. We can check how the economy, from the initial point $k_0 = 4$, reaches the steady-state $k^* = 9$, the solution to $0.3\,(k^*)^{0.5} = 0.1k^*$.

If the economy is already at a steady state, however, then the per capita capital stock would cease to grow. The Solow model without technological progress, thus,

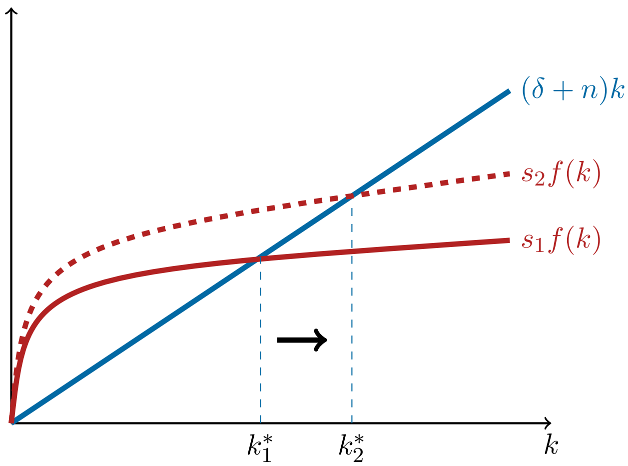


Figure 3: The effect of rising saving rate

paints a rather dismal picture of the economy. As the per capita capital stock stops growing, the per capita output and income also stagnates at $y^* = f(k^*)$. Although the total income continues to grow as the population grows, $Y_t = y^* L_t = y^* L_0 e^{nt}$, the average life quality, which is largely a function of average income, cannot improve.

2.3 The Effect of Saving Rate

To see the effect of a change in the saving rate, s, we examine the equation characterizing the steady-state in (6), which defines an implicit function $k^*(s, \delta, n)$. Applying the implicit function theorem, we have

$$\frac{\partial k^*}{\partial s} = -\frac{f(k^*)}{sf'(k^*) - (\delta + n)}.$$

We must have $sf'(k^*) < \delta + n$, otherwise the curve $sf(k)$ cannot cross with the line $(\delta + n)k$ at k^*. Hence $\frac{\partial k^*}{\partial s}$ must be positive, meaning that an increase in saving rate would lead to a higher level of steady-state capital and income (See Figure 3). However, once the economy reaches the new steady state, the income per capita stagnates once again.

2.4 Golden-Rule Level of Capital

If the saving rate is zero, the corresponding steady-state capital, income, and consumption would all be zero. And if the saving rate is one, then there would be nothing left for consumption. Hence neither too little saving nor too much saving would be desirable. And we might guess that there should be an optimal saving rate that achieves a maximum level of consumption in the steady-state.

At steady-state, the consumption is given by

$$c^* = f(k^*) - sf(k^*) = f(k^*) - (\delta + n)k^*.$$

We call the level of steady-state capital that corresponds to the maximum consumption the *golden-rule level of capital*. We may denote the golden-rule level of steady-state capital by k_g^*, which solves the following maximization problem

$$\max_{k^*} \ c(k^*) = \max_{k^*} \ f(k^*) - (\delta + n)k^*.$$

To maximize $c(k^*)$, k_g^* must satisfy the following first-order condition:

$$f'\left(k_g^*\right) = \delta + n. \tag{7}$$

The first-order condition says that, when $k^* = k_g^*$, the marginal product of capital (MPK) equals the depreciation rate plus the population growth rate.

Recall that the steady-state level of capital is an increasing function of the saving rate, $\partial k^*/\partial s > 0$. We might adjust s to achieve the golden-rule level of capital. If the initial level of capital is lower than the golden-rule level, we might increase the saving rate to achieve the golden-rule level. If the initial level of capital is higher than the golden-rule level, then we might decrease the saving rate to achieve the golden-rule level.

Numerical Experiment: Approaching the Golden Rule of Capital

Following the previous numerical experiment, we solve the steady-state condition,

$$s\left(k^*\right)^{1/2} = 0.1k^*,$$

which yields $k^*(s) = 100s^2$. Since $s = 0.3$, we obtain the steady-state level of capital in this economy, $k^* = 9$.

The golden-rule steady-state capital is obtained from,

$$1/2\left(k_g^*\right)^{-1/2} = 0.1,$$

which gives $k_g^* = 25$. Hence the steady-state level of capital is too low. We might increase the saving rate to achieve the golden rule. Which saving rate corresponds to the golden rule? We solve $100s^2 = 25$ and obtain $s_g^* = 0.5$.

The Excel spreadsheet (Solow1.xlsx, available at the author's website) shows how the economy dynamically adjusts to the increase of saving rate from 0.3 to 0.5.

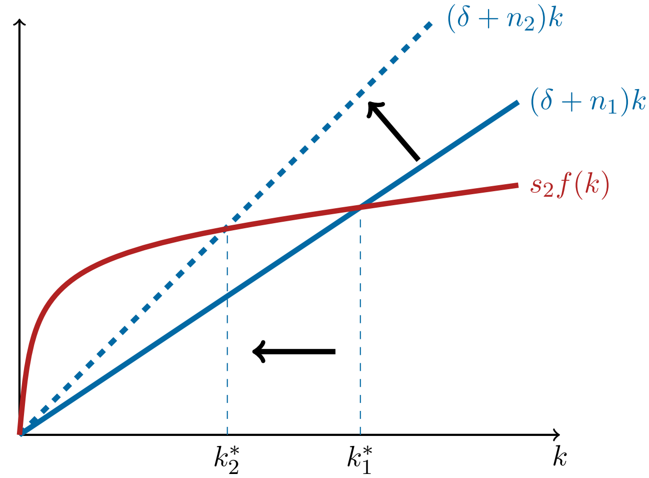


Figure 4: The effect of higher population growth rate

2.5 The Effect of Population Growth

To see how population growth affects steady-state income, we once again apply the implicit function theorem to (6), and we have

$$\frac{\partial k^*}{\partial n} = -\frac{-k^*}{sf'(k^*) - (\delta + n)} < 0.$$

Hence higher population growth leads to lower per cap capital, output, and income in steady state. Graphically, Figure 4 shows how an increase in the population growth rate reduces the steady-state per capita capital.

Empirically, we do see a negative correlation between population growth and income per capita. However, the negative correlation does not prove that higher population growth causes lower economic growth. In fact, population growth may well be endogenous. In wealthy societies, for example, costs of raising and educating children are high, making people reluctant to have more children.

3 Solow Model with Technological Progress

The Solow model without technological progress predicts that there is no sustainable growth in income per capita. The dismal prediction may be true for many poor countries in the world, or the world as a whole before the industrial revolution. But after the industrial revolution, there are a number of countries that have experienced sustained growth in the span of several decades or centuries (e.g., the United Kingdom and the United States). The existence of such countries refutes the Solow model without technological progress as a general characterization of all economies.

To accommodate such successful stories, we introduce technological progress into our model.

3.1 The Model

We assume that the economy has an expanding production function. Specifically, in this section, we assume that the economy has a *labor-augmenting* production function,

$$Y_t = F(K_t, E_t L_t),$$

where E_t represents the level of efficiency in the economy as a whole. If E_t increases over time, we say that the economy is experiencing *technological progress*. We assume that E_t is exogenous and satisfies

$$E_t = E_0 e^{gt}.$$

That is, the technology grows exponentially at a constant rate, g. The exponential technological progress has an equivalent differential-equation form,

$$\dot{E}_t = gE_t. \tag{8}$$

And we make the following assumptions:

(a) Closed economy ($X = 0$).

(b) No government spending ($G = 0$).

(c) The function $F(\cdot, \cdot)$ has constant return to scale.

(d) The saving rate s is a constant and $0 \leq s \leq 1$.

(e) Population grows at a constant rate n.

(f) Capital depreciates at a constant rate δ.

We let $y_t = Y_t/(E_t L_t)$ and $k_t = K_t/(E_t L_t)$. We call y_t the *output per effective worker* (p.e.w.), and k_t the p.e.w. capital stock. We have

$$y_t = \frac{F(K_t, E_t L_t)}{E_t L_t} = F(k_t, 1).$$

As in the Solow model without technological progress, we define $f(k_t) \equiv F(k_t, 1)$, and write

$$y_t = f(k_t).$$

We may call $f(\cdot)$ the *p.e.w. production function*. As in the first Solow model without technological progress, we assume (3) and (4).

3.2 Steady State

Using $\dot{E}_t = gE_t$, $\dot{L}_t = nL_t$, and $\dot{K}_t = sF(K_t, E_tL_t) - \delta K_t$, we can work out the dynamics of p.e.w. capital accumulation,

$$\begin{aligned} \dot{k}_t &= \frac{d}{dt}\left(\frac{K_t}{E_tL_t}\right) = \frac{\dot{K}_t}{E_tL_t} - \frac{K_t\dot{L}_t}{E_tL_t^2} - \frac{K_t\dot{E}_t}{L_tE_t^2} \\ &= sf(k_t) - (\delta + n + g)k_t. \end{aligned} \tag{9}$$

Note that the above differential equation has the same form with (5). The assumptions (3) and (4), once again, ensure that k_t has a steady state.

The steady-state capital p.e.w., k^*, is characterized by the following equation,

$$sf(k^*) - (\delta + n + g)k^* = 0. \tag{10}$$

At steady state, the p.e.w capital stock is a constant,

$$\frac{K_t}{E_tL_t} = k^*.$$

This implies that the total output, $Y_t = E_tL_tf(k^*)$, grows at the constant rate $n+g$ and that the per capita output, $Y_t/L_t = E_tf(k^*)$, grows at the constant rate g. Thus the Solow model with technological progress can explain sustained growth in per capita output or income.

The steady-state condition in (10) defines an implicit function $k^*(s, \delta, n, g)$. Using the same technique as in the previous section, we may analyze the effect of saving rate (s) on the steady-state p.e.w. capital stock (k^*). There is also an optimal saving rate that corresponds to the golden rule of capital, which results in maximum consumption. We leave these analyses to exercises.

3.3 Balanced-Growth Path

The steady state of the Solow model describes a balanced growth path, where income, capital stock, consumption, and investment grow at the same speed. On the balanced-growth path, many important ratios remain constant or grow at the same speed. For example, the ratio of total consumption to total income is by assumption a constant, $(1-s)$. For another example, the ~~*capital intensity* ratio~~ K_t/Y_t is a constant at the steady-state,
capital output ratio

$$\frac{K_t}{Y_t} = k^*/f(k^*).$$

The ~~capital intensity ratio~~ is a measure of the amount of capital needed for producing a unit of output. Note that the capital intensity ratio is nothing but the inverse of *capital productivity* Y_t/K_t.

On the other hand, the capital per capita K_t/L_t and the labor productivity (or the per capita income) Y_t/L_t grow at the same speed as technological progress since

$$\frac{K_t}{L_t} = k^* E_t \quad \text{and} \quad \frac{Y_t}{L_t} = f(k^*) E_t.$$

We may infer that the real wage should also grow at the same speed as labor productivity and that the real rental price of capital should be constant since capital productivity is constant. These are indeed the case, theoretically. Recall that in a competitive economy, the real wage equals the marginal product of labor (MPL), and the real rental price of capital equals the marginal product of capital (MPK). At the steady-state, we have

$$\begin{aligned} \mathrm{MPL}_t &= \frac{\partial Y_t}{\partial L_t} = \frac{\partial}{\partial L_t}\left(E_t L_t f\left(\frac{K_t}{E_t L_t}\right)\right) = E_t\left(f(k^*) - k^* f'(k^*)\right), \\ \mathrm{MPK}_t &= \frac{\partial Y_t}{\partial K_t} = \frac{\partial}{\partial K_t}\left(E_t L_t f\left(\frac{K_t}{E_t L_t}\right)\right) = f'\left(\frac{K_t}{E_t L_t}\right) = f'(k^*). \end{aligned}$$

Thus the Solow model with technological progress implies that the real wage grows at the same speed with labor productivity and that the real return to capital remains constant.

3.4 Optimism of Growth

In contrast to the Solow model without technological progress, the Solow model with technological progress paints a much more optimistic picture of economic growth. It implies that all countries, as long as they embrace the same "technology" in the world, would achieve sustainable growth.

Note that $k^* = 0$ is also a steady state in (10). We may call it the *subsistence steady state*. At the subsistence steady state, people can barely feed themselves, and nothing remains for investment. But the subsistence steady state is not stable. Any positive perturbation, which gives people some capital stock, would push the economy into a virtuous cycle: higher income, more investment, more capital stock, higher income, and so on. Eventually, the economy would settle into the balanced-growth steady state ($k^* > 0$).

And, importantly, the steady-state capital and income have nothing to do with the initial level of capital and income. Note that the balanced-growth steady-state capital (k^*) is a function of saving rate, rate of technological progress, the rate of population growth, and the depreciation rate. That is, $k^* = k^*(s, g, n, \delta)$, which is implicitly defined in (10). If a poor country has the same saving rate, the same growth rate of population, the same depreciation rate, and enjoys the same technology as an advanced high-income country, then the Solow model with technological progress predicts that the poor country would converge to the high-income country in terms of average living standards.

The prediction of convergence, however, has very limited empirical support. Many poor countries remain poor in the past half-century. And only a few countries in East Asia, notably the Asian Tigers, have grown from low-income countries to achieve high-income status. It remains to be seen whether China, full of potential, can become a high-income country.

To reconcile theory and facts, note that the "technology" in the Solow model encompasses not only science and engineering, but also the quality of government and market institutions, transportation and communication infrastructures, social trust, and so on. And, accordingly, technological progress has multiple meanings. It means not only scientific or engineering advances, but also improvement in infrastructures, and most importantly, the improvement of governance. While scientific and engineering know-how does not have national borders, all the other "technology" has national borders. To improve the "technology" within borders, the government should continuously reform itself. Most emerging countries, however, either do not have a strong government or have a strong government without incentives to reform itself. Hence the rarity of successful stories about economic growth.

4 Endogenous Growth

A major criticism of the Solow model is on the assumption that "technological progress" is exogenous. And since technological progress is the most important variable that makes the Solow model predict sustainable growth, one must ask how nations can achieve technological progress, whatever it means. Assuming the existence of an essential element without further explanations may remind serious readers of the famous can-opener joke about economists.

The Can-Opener Joke

There is a story that has been going around about a physicist, a chemist, and an economist who were stranded on a desert island with no implements and a can of food. The physicist and the chemist each devised an ingenious mechanism for getting the can open; the economist merely said, "Assume we have a can opener!"

in *Economics as a Science* (1970) by Kenneth E. Boulding.

Thus economists start to come up with models of *endogenous growth*, which either makes technological progress endogenous or discards it all together. In this section, we introduce the famous AK model that follows the latter approach. It generates sustainable growth without using the exogenous device of technological progress.

The AK model assumes that the population is constant and that the technology of the economy is linear. Specifically, we assume

$$Y_t = AK_t,$$

where Y_t is output, K_t is capital stock that includes "knowledge" or human capital, and $A > 0$ is a constant, representing both the *marginal product of capital* and the *average product of capital.* Capital accumulation follows

$$\dot{K}_t = sY_t - \delta K_t,$$

where s is saving rate, δ is the depreciation rate. It is obvious that

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{K}_t}{K_t} = sA - \delta. \tag{11}$$

As long as $sA > \delta$, the AK model produces sustained growth without making an exogenous assumption on technological progress. And the growth in the AK model is driven by investment or accumulation of capital. The linear technology, which has a constant return to capital, is the crucial assumption that makes investment-driven growth viable. In contrast, the Solow model without technological progress assumes diminishing return to capital, making investment-based growth unsustainable. To make a case for constant-return-to-capital, we may understand that the capital stock in the AK model includes "knowledge" or human capital. Here, knowledge includes scientific understanding, engineering know-how, managerial and marketing skills, the ability of artistic design, and so on. Knowledge arguably has increasing returns: more knowledge makes better applications of knowledge. If knowledge makes a substantial part of the total capital stock, then we may have an increasing return to capital for the whole economy.

According to Equation (11), the growth rate of the AK economy at the steady-state depends on the saving rate (s), the average product of capital (A), and the depreciation rate (δ). The more saving, the more investment, especially in human capital, the better chance of sustainable growth. And higher growth rate requires a higher saving rate and higher investment. A higher average product of capital means better *quality* of the existing capital stock, which in turn depends on the quality of past investment. Thus the AK model indicates that economic growth relies on not only the quantity of investment but also the quality of investment. Finally, a lower depreciation rate would be good for growth. Human capital arguably has a lower depreciation rate than physical capital. The investment in human capital increases the share of human capital in the capital stock, lowering the overall depreciation rate.

5 Growth Accounting

A nation can achieve economic growth, either by accumulating factor inputs (e.g., labor and capital), or by increasing efficiency ("technology" of the aggregate economy). The job of growth accounting is to assess the contribution of factor inputs and efficiency gain to economic growth. Note that since the marginal product of (physical) capital is generally declining as the capital stock increases, the economic growth that relies on capital accumulation is unsustainable. And the growth that relies on population growth is not particularly attractive since it does not raise the average income. In contrast, if a substantial part of economic growth comes from efficiency gain, then the growth is sustainable and good for improving the average well-being.

For the simplicity of accounting for contributions to growth, we assume that the economy can be characterized by

$$Y_t = E_t F(K_t, L_t),$$

where $F(\cdot, \cdot)$ is a constant-return-to-scale production function and E_t is a positive process that measures the technological progress of the economy. Note that, here, technological progress augments not only labor (as in the Solow model with technological progress), but also capital. In this sense we also call E_t the *total factor productivity*.

Taking total differential and divide both sides by Y_t,

$$\frac{\dot{Y}_t}{Y_t} = \frac{E_t F_{1t} \times K_t}{Y_t} \times \frac{\dot{K}_t}{K_t} + \frac{E_t F_{2t} \times L_t}{Y_t} \times \frac{\dot{L}_t}{L_t} + \frac{\dot{E}_t}{E_t},$$

where $F_{1t} = \partial F(K_t, L_t)/\partial K_t$ and $F_{2t} = \partial F(K_t, L_t)/\partial L_t$. Note that $E_t F_{1t}$ is the marginal product of capital and $E_t F_{2t}$ is the marginal product of labor. Denote

$$\alpha_t = \frac{E_t F_{1t} \times K_t}{Y_t}, \quad \text{and} \quad \beta_t = \frac{E_t F_{2t} \times L_t}{Y_t}.$$

If the markets for factor inputs are competitive, then α_t and β_t are the income shares of capital and labor, respectively. We then have

$$\frac{\dot{Y}_t}{Y_t} = \alpha_t \frac{\dot{K}_t}{K_t} + \beta_t \frac{\dot{L}_t}{L_t} + \frac{\dot{E}_t}{E_t}. \tag{12}$$

In this equation, the growth rate of the total output $\frac{\dot{Y}_t}{Y_t}$ is decomposed into three components: the growth of capital stock $\frac{\dot{K}_t}{K_t}$, the growth of labor $\frac{\dot{L}_t}{L_t}$, and technological progress $\frac{\dot{E}_t}{E_t}$. Since $\frac{\dot{E}_t}{E_t}$ is unobservable, this term has to be estimated in empirical analyses. Specifically, to estimate $\frac{\dot{E}_t}{E_t}$ in practice, we can assume that $\alpha_t = \alpha$ and $\beta_t = \beta$ are constant and run a linear regression of $\frac{\dot{Y}_t}{Y_t}$ on $\frac{\dot{K}_t}{K_t}$ and $\frac{\dot{L}_t}{L_t}$, both of which are observable. The residual term from this regression gives an estimate of $\frac{\dot{E}_t}{E_t}$. Hence we often call $\frac{\dot{E}_t}{E_t}$ the *Solow residual*. Technically speaking, the Solow residual is the growth in output that cannot be explained by growth in factor inputs. The contribution of the Solow residual is thus believed to be the contribution of the total factor productivity or technological progress.

6 Understanding Growth

Some theories do not have nice mathematical formulations. But they are as powerful as formal theories. In this section, we present two such theories of growth, *creative destruction* popularized by Joseph Schumpeter[3] and the Lewis model named after W. Arthur Lewis[4].

6.1 Creative Destruction

Creative destruction is a dynamic evolutionary process in a market economy, by which creative entrepreneurs drive incumbents out of businesses so that the "technology" of the whole economy makes continuous progress. Entrepreneurs come up with new products, new technology, new managerial and marketing ideas, and other innovations. Their entry would ultimately drive uncreative incumbents out of the market, hence the term of creative destruction. These entrepreneurs would then become the new incumbents, trying hard to protect their market power. But a new generation of entrepreneurs would enter the market nonetheless, with even better products or ideas. The dynamic process of creative destruction goes on.

Entrepreneurs

Entrepreneurs are people who start businesses and who strive for profits by taking initiatives and risks. Entrepreneurship is the act of being an entrepreneur, the dynamic process by which entrepreneurs identify business opportunities, acquire the necessary resources, and manage the resources to realize profits.

Entrepreneurship is a commendatory term. Although entrepreneurs may conduct their business solely out of personal motives, their actions must bring gains to society. For example, a successful start-up company would create new job opportunities and new products for consumers, as well as profit to its owners. And to beat the incumbents, entrepreneurs must offer higher pay to attract productive employees, must produce higher-quality products, and must make production more cost-efficient. As the French economist Jean-Baptiste Say puts it, entrepreneurs "shifts economic resources out of an area of lower and into an area of higher productivity and greater yield." With millions of entrepreneurs working tirelessly for their own interests, the productivity of the economy as a whole improves. In other words, using the Solow model's terminology, entrepreneurs drive the "technological progress".

Although we only discuss business entrepreneurship, entrepreneurship can be more general. Anyone who takes initiatives and risks to realize social gains can be called an entrepreneur. For example, a writer taking the initiative and risks to write a novel is an entrepreneur. Entrepreneurship is part of human nature, and it manifests in all areas of work. Those who have strong entrepreneurship become leaders in business, scientific research, arts, and so on.

Market Economy

The market economy is essential for creative destruction to happen. More precisely, the market must play a dominant role in picking winners, rewarding success, and bankrupting losers. If it is some government agencies that pick winners, even using some ostensible measure of "innovation", then the true innovative entrepreneurs would generally lose out. Those who specialize in winning political favors do not typically have an edge in innovation. Nor do they have incentives to invest in research and development.

More generally, the rule of law (in contrast to "the rule of man") is essential for the market to pick winners, reward success, and bankrupt losers in a fair manner. The rule of law represents the quality of the market and the quality of market matters. The incumbents typically have more money and thus political influence. If they can buy "help" from government officials, law-makers, police officers, and judges, then small entrepreneurs would have no chance of success in competing with large incumbents. For entrepreneurs to challenge the incumbents, the playing ground must be level for everyone. This is possible only if all players, including the government, are equally and predictably bounded by the law.

The size of the market also matters. A bigger market has bigger rewards for innovation. Bigger rewards bring more entrepreneurs who challenge the status quo. Note that the market size is not the same thing as the size of an economy, measured by GDP or population. A small nation can enjoy a big market size if the nation is an integral part of the world market. Countries like Singapore and Israel are examples of such successful small open economies.

A large nation, on the other hand, can enjoy no market-size dividend, if the large nation has a segmented domestic market. The segmentation can be due to the poor infrastructure of transportation and communications. More importantly, the segmentation can be due to various forms of local protectionism. The local governments often have incentives to protect their local business and employment. Or, more sinisterly, the local officials have incentives to impose local tax and regulations for rent-seeking opportunities.

One thousand segmented local markets do not make one large market. Potential entrepreneurs in each of these local markets can only expect a small reward that a small market can afford. Many densely populated developing countries suffer from market segmentation, either due to local protectionism or poor infrastructure or both. They have a huge population, but they have small markets. China, before it joins WTO, was an example.

Limitations

Even under the rule of law and with substantial market size, creative destruction is not a perfect process. It may not bring about "technological progress", which underpins economic growth. And it is even less certain that creative destruction would bring a better society.

First, at least some industries exhibit increasing returns to scale. Monopolies, as a result, can easily take hold in such industries. Compared to smaller potential competitors, they enjoy tremendous cost advantages simply because of their scale. More creative entrepreneurs, who may potentially produce better goods, may fail to challenge the less efficient incumbents because they have to start from small. And the incumbents, facing no existential threat, have little incentive to upgrade their technology or management. The "technology" of the industry, as a result, may stagnate.

Second, the social gain from creative destruction is not guaranteed. Creativity can be used in the wrong place. For example, entrepreneurs who are shameless and creative in evading environmental laws would win over those with social responsibilities since the shameless ones enjoy cost advantages. For another example, entrepreneurs in the nutrition industry may be very creative in marketing their useless or hazardous products to incredulous consumers. Those who produce truly helpful and safe products, which require expensive R&D spending, may not compete with the fraud.

Third, the gain from creative destruction is necessarily unevenly distributed. The process of creative destruction creates losers as well as winners. Although creative destruction brings overall welfare to society, the welfare may be reaped by a small percentage of the population, i.e., the successful entrepreneurs. The displaced workers in failed firms would find their skills too specific to find comparable jobs in other firms. They would have to accept a deep wage cut to find new jobs, and they would have to lower their living standards to make ends meet. Sometimes, a whole town of jobs may be lost due to the failure of a firm. The old way of life would be gone for all people in town.

Any responsible government, thus, cannot let creative destruction run its own course. The responsible government would ensure competition by breaking up monopolies. The responsible government would vigorously play the cat-and-mouse games with law evaders and make sure a level playing ground for all entrepreneurs. Finally, the responsible government must establish meaningful welfare programs to help losers from creative destruction. Without doing this, relentless creative destruction may destroy the institutional framework that underpins creative destruction. This dismal prospect is exactly what both Schumpeter and Marx, who first raised the idea of creative destruction, predicted.

6.2 The Lewis Model

W. Arthur Lewis's 1954 paper, *Economic Development with Unlimited Supplies of Labour*, was instrumental in developing the field of development economics. His model characterizes how a developing country transforms its predominantly subsistence economy into a predominantly industrial one. The Lewis model is particularly relevant to the experience of China's growth. Chinese economists started to use the *Lewis turning point* to explain the emergence of labor shortages from as early as 2005. This rekindled general interest, not restricted to academic circles, in W. Arthur Lewis's theory.

Assumptions

The Lewis model assumes that the developing country has two sectors, a small industrial sector in a few cities and the agricultural sector in the vast land around the cities. The agricultural sector supports so huge a population relative to the land that the marginal product of labor is around zero and that farmers can barely feed themselves. For this reason, we may also call the agricultural sector the *subsistence sector*. The industrial sector, in contrast, employs only a fraction of the population and sustains a high level of marginal productivity and, thus, the real wage.

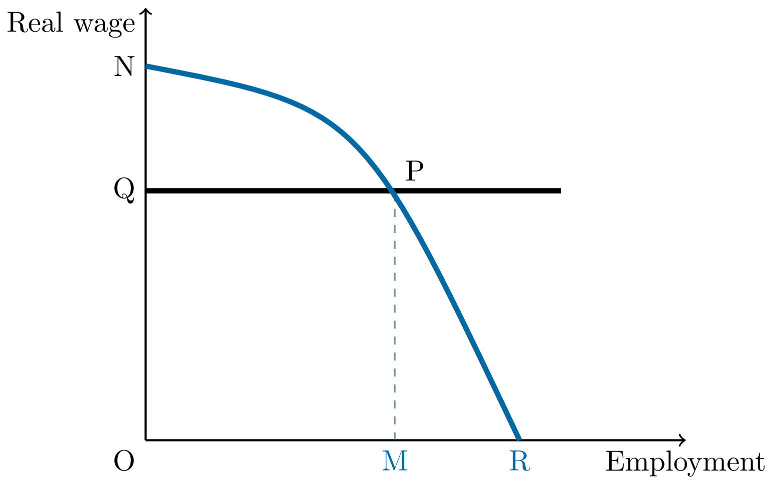
The reason why the industrial sector does not immediately expand employment until the marginal productivity of labor reaches zero is that, in reality, labor is simply not available at a zero wage. To attract peasants from their accustomed way of life in the countryside, the industrialists must offer a high wage. The wage premium in the industrial sector works partly to offset the higher living cost in the city. But more importantly, the wage premium works to elevate the social image of industrial workers so that the industrial sector can continue to attract workers from the countryside. The industrial wage may also be much higher than the income of petty traders and casual laborers in the city so that industrial workers would have better morale and discipline.

In contrast to what a naive theory would predict, industrial managers are willing to pay wages higher than the marginal labor productivity. In the modern age, economists may call it *efficiency wage*. In ancient times, the grand seigneurs were also willing to pay high wages to their servants, even though the marginal productivity of the army of servants might be close to zero. The grand seigneurs are, of course, not stupid. A loyal army of handsome or beautiful servants boosts the social prestige of the grand seigneurs.

The above discussions imply that, in modeling, we may regard the real wage in the industrial sector as fixed in the initial stage of development. And we also assume that, as long as the marginal labor productivity in the subsistence sector stays around zero, the real wage in the industrial sector will remain fixed. Note that the real wage, although much higher than the marginal product of labor, should be very low, especially compared with the level in the high-income countries. The real wage would eventually rise when the labor migration from the countryside to the industrial sector starts to cause strains in agricultural production, which starts to offer higher and higher real wages. We may conjecture that, for a densely populated subsistence economy, it would take many years of industrial development to "digest" all of the under-employed labor in the subsistence sector.

Figure 5 gives a snapshot of a developing economy under the Lewis assumptions.

Figure 5: The Lewis Model



NPR represents the labor demand curve. If the industrial sector increases employment of labor until the marginal product of labor reaches zeros, then the industrial employment would be OR. The level of real wage, however, is exogenously given. That is, OQ. As a result, the industrial employment stands at OM. The industrial sector is profitable as a whole, with its profit (or surplus) equal to the area of QNP. The area of OQPM is the income of industrial labor.

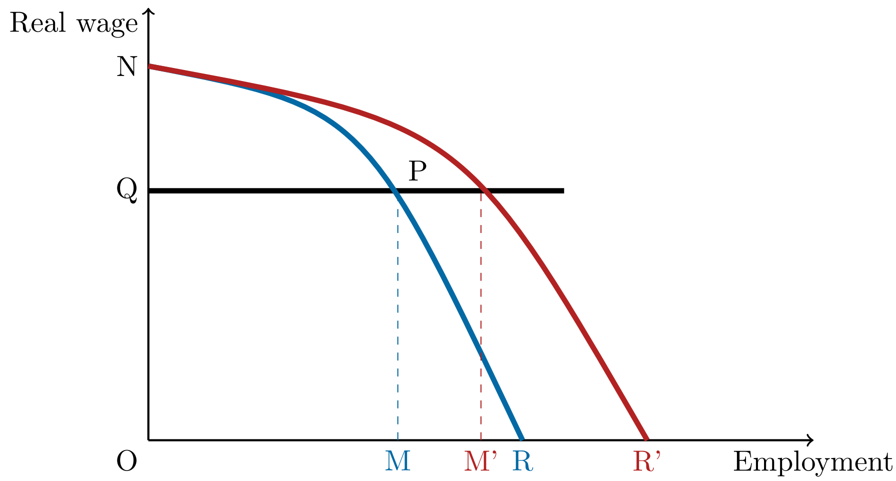
Industrialization and Urbanization

Since the marginal product of labor in the countryside was around zero, the migration of some people to the industrial sector would not affect the agricultural output. People in the countryside might become less hungry since they have to share food with fewer people, but they would stay at the subsistence level for a prolonged time. Here, we may also invoke the Malthusian argument that people would have more children when more food is available, keeping rural households at a subsistence level of living. Rural households would then accumulate no surplus, which ensures no new investment, tying down the marginal labor productivity in the countryside.

The industrial sector, on the other hand, re-invests the profit and expands the capital stock. Since the marginal product of labor increases when more capital is available, the labor demand curve would shift to the right. Thus the industrial employment expands from OM to OM' (Figure 6).

Meanwhile, the industrial profit expands to QNP'. Thanks to the fixed (or slowly increasing) real wage, which is held down by the army of underemployed labor in the subsistence sector, the return to new capital investment can be sustained at a high level. A high level of profit attracts more investment in the capital stock and, thus, more industrial employment. The dynamic process goes on, continuously shifting the labor demand curve to the right.

Figure 6: Industrialization in the Lewis model



This dynamic process may take the name "industrialization". As more and more people work in the industrial sector, the average labor productivity increases. Here the "technological progress" comes not from advances in science and engineering, but the improvement of (labor) resource allocation.

We may also conjecture that, as more and more people migrate to the city for industrial jobs, "urbanization" takes place. If we measure urbanization by calculating the percentage of people living in the urban area, then urbanization may progress faster than industrialization. Many people may go to the city first, looking for jobs. When they cannot find one since job opportunities are inherently scarce, they may choose to settle in slums and keep looking, doing some petty trade or casual labor to get by. They, in effect, join the army of underemployed labor in the city. Their existence contributes directly to the persistently low level of the real wage.

The Lewis Turning Point

When industrialization eventually exhausts the redundant labor supply in the subsistence sector, the industrial wage will have to rise to attract more workers to the industrial sector. If the real wage does not rise, or not rapidly enough, the industrial sector will face labor shortages. During China's economic development, labor shortages occurred as early as 2004. At this point, we may say that the economy reaches the Lewis turning point.

The economic development does not stop at the Lewis turning point, though. If anything, the economic growth after reaching the Lewis turning point may be more "balanced", meaning that the share of domestic consumption will rise and the economy will become less dependent on the foreign demand.

After the turning point, the investment growth would decline as the return to new investment declines (thanks to the rising labor costs). But the growth rate remains positive. The rising labor costs are not purely bad news for the capitalists, after all. Labor costs are incomes for workers. The rising labor costs imply a booming domestic consumption market for the capitalists. As the result of continued investment, the capital stock continues to accumulate, pushing up marginal labor of productivity and, thus, real wage. As pay goes up, labor's share of income would rise. Since workers' marginal propensity to consume is generally higher than that of the capitalists, the growth of total consumption expenditure may outpace the total investment expenditure. As a result, the consumption share of total expenditure would rise.

Before the Lewis turning point, the fast expansion of the industrial sector may depend on foreign demand since the growth of domestic consumption cannot match that of the domestic production, thanks to the stagnating real wage. The economy has to run a substantial trade surplus, which may lead to international trade disputes. But after the turning point, the growth of domestic consumption may outpace that of export, given that income growth is higher than the world average. As a result, the share of the net export would shrink.

Both predictions, that of rising consumption share and that of shrinking share of the net export, have proved true for China. In China, problems of labor shortage started to emerge around 2004, suggesting the advent of the Lewis turning point. The share of trade surplus topped in 2007, after which it staged a secular decline (Figure 7). The share of consumption in GDP found a bottom in 2008, the year when the Global Financial Crisis happened. Then it found a second bottom in 2010, thanks to a surge in investment spending after the Four-Trillion Stimulus Program enacted in 2009. After 2010, the consumption share started to climb back (Figure 7).

The Kuznets Curve

The celebrated Kuznets curve, named after Simon Kuznets (1901-1985), is the hypothesis that as an economy develops, the economic inequality first rises and then falls (Figure 8). The Kuznets curve hypothesis may be formulated as a prediction of the Lewis model. When a predominantly subsistence economy starts to develop, capitalists rapidly accumulate and reinvest wealth, while the rest of the population either live in the subsistence sector or receive low wages in the industrial sector. As a result, income inequality increases. At the same time, the average income of the economy rises, thanks to, first, the surging income to capitalists and, second, the migration of workers from the subsistence sector to the industrial sector, where wages are higher.

As the economy reaches the Lewis turning point, real wages in both industrial and agricultural sectors start to rise rapidly. At the same time, return to capital stagnates or declines. Consequently, the Kuznets curve also turns around at some level of average income. The inequality starts to decline as the average income

Figure 7: China's Share of Consumption and Net Export

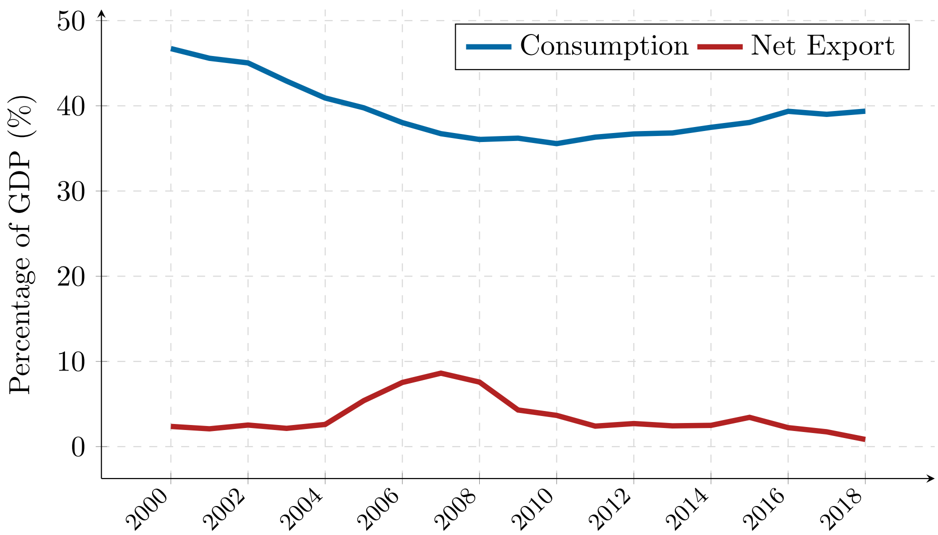
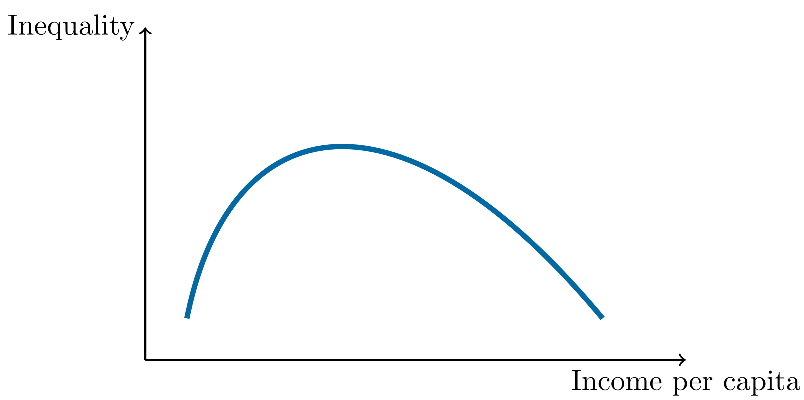


Figure 8: The Kuznets Curve



increases beyond the turning point.

7 Concluding Remarks

The Solow models make it clear that sustainable growth has to come from technological progress. However, we should understand technological progress in broad terms. It is not only about the progress of science and engineering. It is also about improvement in the overall ability of a society to mobilize, organize, and manage enterprises. For developing countries, the Lewis model illuminates the point that the improvement of resource allocation may be the key to the development, at least during the initial period.

The human society is not without engines for growth. People desire better lives, and they innovate and compete. However, sustainable growth is not easy. Among all nations in the world, those that have achieved moderate growth for at least thirty years are in the minority. And the club of high-income countries remains small and exclusive. If the market is over-burdened with taxes and regulations, the economy would be stagnant. If the market force rules all and everything is for sale, the economy would also be stagnant or worse. Economists have yet to agree on the set of dos and don'ts the government must obey to bring sustainable growth. But there is no controversy that the government plays a decisive role in the nation's fortune.

Notes

[1]On April 26, 1980, Deng told foreign guests, "To build socialism, we must achieve higher productivity. Poverty is not socialism." （搞社会主义，一定要使生产力发达，贫穷不是社会主义。）

[2]Solow, Robert M., 1956. A contribution to the theory of economic growth. Quarterly Journal of Economics. 70 (1): 65–94.

[3]Schumpeter, Joseph A., 1942. Capitalism, Socialism and Democracy.

[4]Lewis, W. Arthur, 1954. Economic development with unlimited supplies of labour. The Manchester School. 22 (2): 139–91.

Exercises:

1. Suppose that the production function of the economy is Cobb-Douglas, $Y = K^\alpha L^{1-\alpha}$ and that there is no technological progress.

(a) Find the expressions for k^*,y^*, and c^* as functions of α, s, n, δ.

(b) What is the golden-rule level for k^*?

(c) What is the golden-rule saving rate?

2. Suppose that the production function is the constant-elasticity-of-substitution (CES), $Y = \left[K^{\sigma/(\sigma-1)} + L^{\sigma/(\sigma-1)}\right]^{\sigma/(\sigma-1)}$, where $0 < \sigma < \infty$ and $\sigma \neq 1$. Note that σ is the elasticity of substitution between capital and labor. If $\sigma \to 1$, the CES function becomes the Cobb-Douglas.

(a) Show that the CES production function has constant returns to scale.

(b) Derive the individual production function $f(k)$.

(c) Under what conditions does $f(k)$ satisfy $f'(k) > 0$ and $f''(k) < 0$?

3. Suppose that differential equation characterizing the accumulation of per capita capital is $\dot{k}_t = h(k_t)$, where h is a differentiable function. If a steady-state k^* is stable, then $h'(k^*)$ should be negative or positive? Why?

4. Let β be the fraction of working-age population (say, those who are aged between 15 and 65). Assume a 100% labor force participation rate and a 0% natural unemployment rate. Suppose that there is no technological progress, the population growth is zero, the saving rate is s, the depreciation rate is δ, and that the production function of the economy is Cobb-Douglas, $Y = K^{\alpha}(\beta L)^{1-\alpha}$, where L is population.

(a) Write the equation characterizing the steady state.

(b) Analyze the effect of population aging on the income per capita, Y/L.

5. Suppose that the production function of the economy is Cobb-Douglas, $Y = K^{\alpha}(EL)^{1-\alpha}$ and that there is a constant rate technological progress, g.

(a) Find the expressions for k^*,y^*, and c^* as functions of α, s, n, δ, g.

(b) What is the golden-rule level for k^*?

(c) What is the golden-rule saving rate?

6. Assume that, in the Solow model with technological progress, both labor and capital are paid their marginal products.

(a) Show that MPL$= E(f(k^*) - k^* f'(k^*))$.

(b) Suppose that the economy starts with a level of per capita capital less than k^*. As k_t moves toward k^*, does the real wage grow faster, slower, or equal to the technological progress? What about the real rental price of capital?

7. Suppose that in the Solow model with technological progress, all capital income is saved and all labor income is consumed. Thus, $\dot{K}_t = \text{MPK}_t \cdot K_t - \delta K_t$.

(a) Derive the equation characterizing the steady-state.

(b) Is the steady-state capital per capita larger than, less than, or equal to the golden-rule level?

8. Suppose that the individual production function is given by $f(k) = \max(Ak^{0.5} - k_0, 0)$, where $A > 0$ and $k_0 > 0$. k_0 may be interpreted as the minimum fixed cost of production.

(a) If there exists a unique steady state, then express k_0 as a function of A, n, g, δ, and s.

(b) If there exist two steady states, then derive the steady-state capital per effective labor. Which one of these two steady states is stable?

9. Assume that the Solow model with technological progress is at the steady state and that the production function is Cobb-Douglas, $Y = K^{\alpha}(EL)^{1-\alpha}$.

(a) Calculate the partial effect of a unit change in the saving rate s on k^* (hint: calculate $\partial k^*/\partial s$.)

(b) Calculate the elasticity of steady-state per capita output $\left(\frac{\partial y^*}{\partial s} \cdot \frac{s}{y^*}\right)$.

10. Suppose that the economy has two sectors, the manufacturing sector that produces goods and the university sector that produces knowledge. The production function in manufacturing is given by $Y_t = F(K_t, (1-u)L_tE_t)$, where u is the fraction of the labor force in universities. The production function in research universities is given by $\dot{E}_t/E_t = g(u)$, where $g(u)$ describes how the growth in knowledge depends on the fraction of labor force in universities. The saving rate and the depreciation rate are s and δ, respectively.

(a) Characterize the steady state of the model.

(b) Analyze the effect of a one-time university expansion on the economy.

(c) Is there an optimal u that yields highest income per capita?