1

(1)

$$\text{MPL} = \frac{\partial F}{\partial L} = (1-\alpha)EK^{\alpha}L^{-\alpha} = (1-\alpha)\frac{F(K,L)}{L}.$$

Similarly,

$$\text{MPK} = \frac{\partial F}{\partial K} = \alpha EK^{\alpha-1}L^{1-\alpha} = \alpha\frac{F(K,L)}{K}.$$

As $0 < \alpha < 1$ and naturally $F(K,L), K, L > 0$, both MPL and MPK are positive.

(2)

$$\frac{\partial^2 F}{\partial L^2} = -\alpha(1-\alpha)EK^{\alpha}L^{-\alpha-1} = -\alpha(1-\alpha)\frac{F(K,L)}{L^2} < 0,$$

$$\frac{\partial^2 F}{\partial K^2} = \alpha(\alpha-1)EK^{\alpha-2}L^{1-\alpha} = -\alpha(1-\alpha)\frac{F(K,L)}{K^2} < 0.$$

Therefore, MPL is decreasing as M increases and that MPK is decreasing as L increases.

(3)

For all $z > 0$,

$$F(zK, zL) = E(zK)^{\alpha}(zL)^{1-\alpha} = Ez^{\alpha}K^{\alpha}z^{1-\alpha}L^{1-\alpha} = zEK^{\alpha}L^{1-\alpha} = zF(K,L).$$

The constant-return-to-scale is satisfied.

2

Use the assumption on the production function $F(K,L)$ that

$$F_1 \equiv \frac{\partial F}{\partial K} > 0,\ F_2 \equiv \frac{\partial F}{\partial L} > 0,\ F_{11} \equiv \frac{\partial^2 F}{\partial K^2} < 0,\ F_{22} \equiv \frac{\partial^2 F}{\partial L^2} < 0,\ F_{12} \equiv \frac{\partial^2 F}{\partial K \partial L} > 0,$$

and the classical theory of income distribution that the economy fully employs the total capital and labor and the two variables give the answer of distribution in the following way:

$$\text{Real Rental Price of Capital} = \frac{R}{P} = F_1,\ \text{Real Wage} = \frac{W}{P} = F_2.$$

(1)

As part of the capital stock is damaged, the total capital K decreases. Apply the assumption and the classical theory of income distribution, the real rental price of capital $R/P = F_1$ increases and the real wage $W/P = F_2$ decreases.

(2)

The rise of retirement age means that the total L increases. Therefore, the real rental price of capital $R/P = F_1$ increases and the real wage $W/P = F_2$ decreases.

(3)

Since the inflation does not affect the total capital and total labor, both the real rental price of capital and the real wage keep the same.

(4)

We can rewrite the production function as $F(K, E_t \cdot L)$ to make it labor-augmenting. In this way, to maximize the economic profit, we solve the new problem:

$$\max_{K,L} P \cdot F(K, E_t \cdot L) - R \cdot K - W \cdot L.$$

We draw the conclution that when

$$F_1 = \frac{R}{P} \text{ and } E_t \cdot F_2 = \frac{W}{P},$$

the maximum is achieved. That is,

$$\text{Real Rental Price of Capital} = \frac{R}{P} = F_1, \text{ Real Wage} = \frac{W}{P} = E_t \cdot F_2.$$

When a technological breakthrough is made, E_t increases and $E_t \cdot L$ increases. Applying the assumption on production function and using the new form above, we know the real rental price of capital $R/P = F_1$ increases as $E_t \cdot L$ increases and the change of the real wage $W/P = E_t \cdot F_2$ is uncertain as F_2 decreases and E_t increases.

(5)

Similarly, if the production function is capital-augmenting and a technological breakthrough is made, the change of the real rental price of capital $R/P = E_T \cdot F_1$ is uncertain as F_1 decreases and E_t increases and the real wage $W/P = F_2$ increases as $E_t K$ increases.

3

(1)

Plug in the Y and T given,

$$C = 1000 + \frac{2}{3}(Y - T) = 5000.$$

Then,

$$\begin{aligned} S_{ng} &= Y - C - T = 8000 - 5000 - 2000 = 1000, \\ S_g &= T - G = 2000 - 2500 = -500, \\ S &= S_{ng} + S_g = 1000 - 500 = 500. \end{aligned}$$

(2)

In the equilibrium,

$$1200 - 100r = I(r) = Y - C - G = S = 500.$$

Solving the equation, we have the equilibrium interest rate $r = 7$.

(3)

When a balanced budget is achieved by reducing expenditure, $S'_g = 0$. Thus $G' = T = 2000$. Therefore,

$$S'_{ng} = Y - C - T = 1000,$$
$$S' = S'_{ng} + S'_g = 1000.$$

Accordingly, with

$$1200 - 100r' = I(r') = S' = 1000,$$

the new equilibrium real interest rate is $r' = 2$.