1.

(a)

Take the total differentiation of the LM-IS equation and we obtain the following set of linear equation:

$$\begin{pmatrix} C'-1 & I' \\ PL_Y & PL_r \end{pmatrix} \begin{pmatrix} \mathrm{d}Y \\ \mathrm{d}r \end{pmatrix} = \begin{pmatrix} C'\,\mathrm{d}T - \mathrm{d}G \\ \mathrm{d}M - L\,\mathrm{d}P \end{pmatrix},$$

in which $L_Y \equiv \partial L/\partial Y$, $L_r \equiv \partial L/\partial r$. Note that $\mathrm{d}P = 0$ under the sticky-price assumption. Solving the equation set and we get that

$$\mathrm{d}r = \frac{1}{P} \cdot \frac{(C'-1)\,\mathrm{d}M - PL_Y(C'\,\mathrm{d}T - \mathrm{d}G)}{L_r(C'-1) - L_Y I'}.$$

To keep the interest rate unchanged when G increases by ΔG, the central bank has to let $\mathrm{d}r$ be zero. As for the central bank, T is fixed. That is,

$$\mathrm{d}r = 0 \implies (1-C')\,\mathrm{d}M = PL_Y\,\mathrm{d}G. \tag{1}$$

Note that under our assumption, $C' < 1$ and $L_Y > 0$. Thus the central bank can increase money supply to maintain the interest rate.

(b)

Solving the equation set in (a), we obtain

$$\mathrm{d}Y = \frac{1}{P} \cdot \frac{PL_r(C'\,\mathrm{d}T - \mathrm{d}G) - I'\,\mathrm{d}M}{L_r(C'-1) - L_Y I'}.$$

With the equation (1) and $\mathrm{d}T = 0$, the government spending multiplier in this case is

$$\frac{\mathrm{d}Y}{\mathrm{d}G} = \frac{1}{1-C'}.$$

2.

(a)

The model is

$$\begin{aligned} Y &= C(Y-T) + I(r^*) + G + X(e, Y), \\ \frac{M}{P} &= L(r^*, Y). \end{aligned}$$

The LM equation determains the output, which is a vertical line in the coordinate with Y on the horizontal axis and e on the vertical axis. For the IS equation, we can compute the derivatve

$$\frac{\partial e}{\partial Y} = -\frac{C' + X_Y - 1}{X_e} = \frac{(1-C') - X_Y}{X_e}.$$

Note that $0 < C' < 1$, $X_e < 0$ and $X_Y < 0$, which means $\partial e/\partial Y < 0$. Thus, the IS curve is downward-sloping in the coordinate.

When a fiscal stimulus is conducted (say, increasing the government expenditure), Y should be larger given a fixed e. The IS curve shifts to the right and the LM curve does not change. Therefore, a fiscal stimulus results in a higher exchange rate and an unchanged output.

(b)

The model is

$$\begin{aligned} Y &= C(Y-T) + I(r^*) + G + X(e^*, Y), \\ \frac{M}{P} &= L(r^*, Y). \end{aligned}$$

In this case, the endogenous variables are Y and M. The IS curve is vertical in the coordinate with Y on the horizontal axis and M on the vertical axis. Under the assumption, $\partial M/\partial Y = P \cdot \partial L/\partial Y > 0$. Thus the LM curve is upward-sloping in the coordinate.

When there is a fiscal stimulus, the IS curve shifts to the right and the LM curve does not change. Therefore, a fiscal stimulus results in both larger output and larger money supply.

3.

(a)

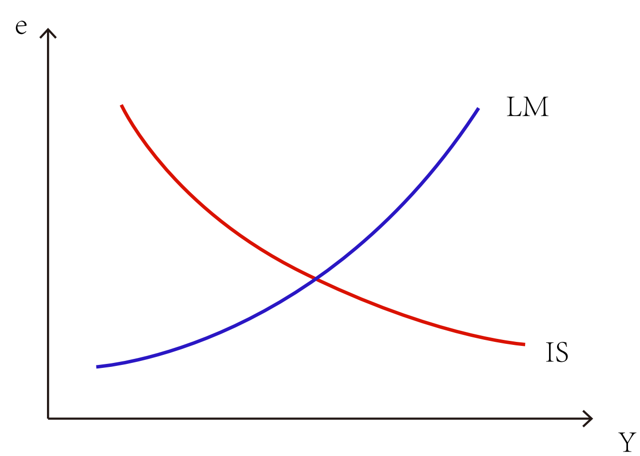

Figure 1: IS-LM curves

(b)

The model is

$$
\begin{aligned}
Y &= C(Y - T) + I(r^*) + G + X(e), \\
\frac{M}{P(e, w)} &= L(r^*, Y).
\end{aligned}
$$

When there is a negative foreign-demand shock, under the MPC-decreasing assumption,

$$\frac{\partial Y}{\partial X} = -\frac{1}{C' - 1} > 0.$$

Therefore, the IS curve in Figure 1 shifts to the left as for every e the net export declines. This results in a lower output and a currency depreciation.

(c)

Consider w exogenous. The increase in domestic price means that for every e, $P(e, w)$ increases. Because

$$\frac{\partial Y}{\partial P} = -\frac{L}{P \cdot L_Y} < 0,$$

the LM curve shifts to the left, resulting in the appreciation of the domestic currency and a lower output.