# Homework 4

1. To test for any significant difference in the number of hours between breakdowns for four machines, the following data were obtained.

| Machine 1 | Machine 2 | Machine 3 | Machine 4 |
|---|---|---|---|
| 6.1 | 8.4 | 11.1 | 9.8 |
| 7.5 | 7.8 | 10.8 | 12.1 |
| 5.8 | 9.9 | 9.5 | 12.5 |
| 7.4 | 10.1 | 10.2 | 10.8 |
| 8.2 | 9.6 | 9.2 | 11.1 |
| 7.6 | 9.5 | 8.9 | 11.6 |

(a). Show the ANOVA Table and use $\alpha = .05$ to test whether there are significant differences in the population mean times among the four machines.

(b). Use Fisher's LSD procedure to test for the equality of the means for machines 2 and 4, where level of significance $\alpha = .05$.

(c). Recall the problem (b), if we use Fisher's LSD procedure for all pairs of four machines with $\alpha = .05$, what is the experiment-wise Type I error rate? If desired experiment-wise Type I error rate $\alpha_{EW}$ is .05 and we use the Bonferroni adjustment, what is the achieved experiment-wise Type I error rate?

2. The Scholastic Aptitude Test (SAT) contains three areas: critical reading, mathematics, and writing. Each area is scored on an 800-point scale. A sample of SAT scores for six students follows.

| Student | Critical Reading | Mathematics | Writing |
|---|---|---|---|
| 1 | 525 | 535 | 520 |
| 2 | 590 | 580 | 585 |
| 3 | 465 | 460 | 445 |
| 4 | 560 | 565 | 550 |
| 5 | 435 | 480 | 430 |
| 6 | 430 | 460 | 420 |

(a). Using a .05 level of significance, do the three areas of the SAT have different degree of challenge for students? (Exclude the heterogeneity of different students)
(b). If we do not exclude the heterogeneity of different students and run an one-way ANOVA, does your answer for Problem (a) change? If change, could you explain the reason why the conclusion is different after excluding the heterogeneity of different students.

3. A factorial experiment was designed to test for any significant differences in the time needed to translate English to foreign language among two translators. The type of language translated was also considered a significant factor, translations were made with both translators for three different languages: Spanish, French, and German. Use the following data for translation time in hours.

| | Spanish | French | German |
|---|---|---|---|
| Translator 1 | 10<br>8 | 12<br>10 | 16<br>12 |
| Translator 2 | 6<br>8 | 10<br>8 | 14<br>18 |

Test for any significant differences due to translator, type of language, and interaction with $\alpha = .05$.

4. Recall the setting of one-way ANOVA where we construct F-statistic based on

$F = \dfrac{MSTR}{MSE}$ and the null hypothesis is that the means of all populations are the same.

(a). Explain why the rejection rule is $F > F_\alpha$ rather than $F > F_{\frac{\alpha}{2}}$ or $F < F_{1-\frac{\alpha}{2}}$.

(b). If we define a new statistic $\check{F} = \dfrac{MSE}{MSTR}$ with level of significance $\alpha$, then what is the rejection rule based on $\check{F}$?

5. **Bonus**

Denote $A_i, i = 1, 2, \cdots, N$ as the $i$th random event and $A_i^c$ is the complementary event of $A_i$.

(a). Prove the Bonferroni inequality below

$$\sum_{i=1}^{N} \Pr\{A_i^c\} \geq 1 - \Pr\left\{\bigcap_{i=1}^{N} A_i\right\}$$

(b). Recall the question (c) of Problem 1, given desired experiment-wise Type I error rate $\alpha_{EW} = .05$, dose the achieved experiment-wise Type I error rate $\bar{\alpha}_{EW} = 1 - (1 - \alpha_{EW}/C_k^2)^{C_k^2}$ is exactly .05 after Bonferroni adjustment? If not, prove the relationship between $\bar{\alpha}_{EW}$ and $\alpha_{EW}$ using Bonferroni inequality.

中文题意：记 $A_i, i = 1, 2, \cdots, N$ 为第 $i$ 个随机事件，且 $A_i^c$ 是 $A_i$ 的互补事件。

(a). 证明下述的 Bonferroni inequality：

$$\sum_{i=1}^{N} \Pr\{A_i^c\} \geq 1 - \Pr\left\{\bigcap_{i=1}^{N} A_i\right\}$$

(b). 对于本次作业第 1 题的 (c) 小问，给定想要达到的实验误差率 $\alpha_{EW} = .05$，使用 Bonferroni adjustment 之后，实际能达到的实验误差率 $\bar{\alpha}_{EW} = 1 - [(1 - \alpha_{EW}/C_k^2)]^{C_k^2}$ 正好是.05 吗?

如果不是正好为.05，请使用上述 Bonferroni inequality 来证明 $\bar{\alpha}_{EW}$ 与 $\alpha_{EW}$ 的大小关系。