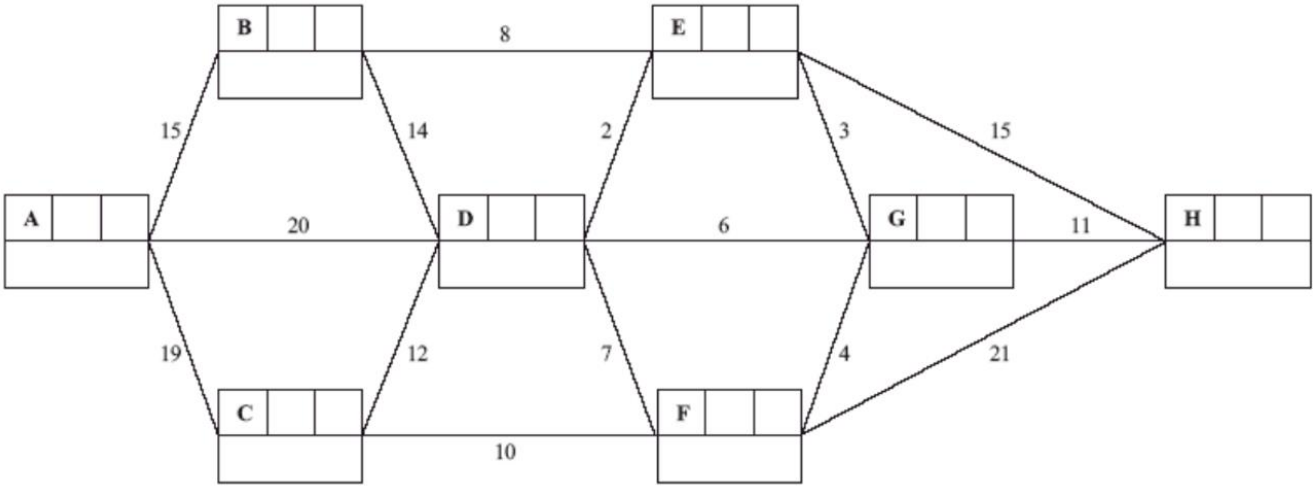
CSE 3500 – Algorithms and Complexity
Homework 4

(30 points) Question 1

Peter wants to minimize his driving time from his residence at point A to the university located at point H. The provided diagram displays the towns along with the respective driving times, measured in minutes, between them.

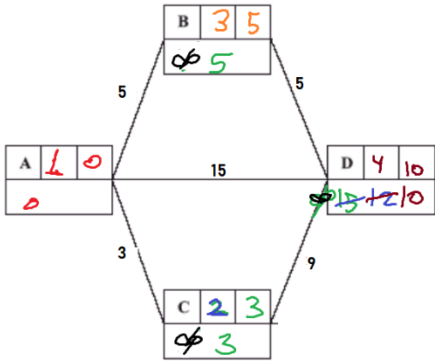


a. Use Dijkstra's algorithm to determine the optimal routes from point A to point H that minimize the total travel time.
b. What is the shortest path that should be taken from point A to point H.

Use the following key table to fill in the values in the graph.

Vertex	Order of extraction from the queue	Final distance
Changing of the distance		

For example



(20 points) Question 2

During our discussions in class, we learned that Dijkstra's algorithm reliably computes the shortest paths in a graph where all edge weights are positive.

Why does Dijkstra's algorithm encounter issues or produce inaccurate results when negative edge weights are introduced into the graph? Discuss the reason behind this limitation and provide an example to illustrate the scenario.

(50 points) Question 3

You've joined a game where you're given a box with a maximum weight limit, denoted as W Inside the room, there's an assortment of items, each identified by its weight W_i and value v_i where $i = 1,2,3 \ldots\ldots n$. The main objective is to carefully select items to place inside your box, aiming to maximize the total value of the chosen items while ensuring the total weight doesn't exceed the box's capacity. If the weight limit is surpassed, the box will break, leaving you with nothing.

An interesting feature of this game is the option to take fractional parts of items. For instance, if your box has a maximum weight capacity of 5 lbs and you've already loaded 3 lbs of items, you could opt to take half of an item weighing 4 lbs with a value of $14, resulting in 2 lbs and a corresponding value of $7.

You were considering three different greedy strategies to maximize the value:

1. The first approach involves selecting the item with the highest value.
2. The second strategy focuses on selecting the item with the lowest weight.
3. The third approach revolves around calculating the ratio of value to weight.

a. Do you believe the first strategy will yield an optimal solution? Justify your answer and illustrate with an example.
b. Do you believe the second strategy will yield an optimal solution? Justify your answer and illustrate with an example.
c. Do you believe the third strategy will yield an optimal solution? Justify your answer and illustrate with an example.
d. Write an algorithm based on the chosen greedy approach.
e. Analyze the time complexity of your algorithm.
f. Prove why the proposed algorithm consistently provides the optimal solution.