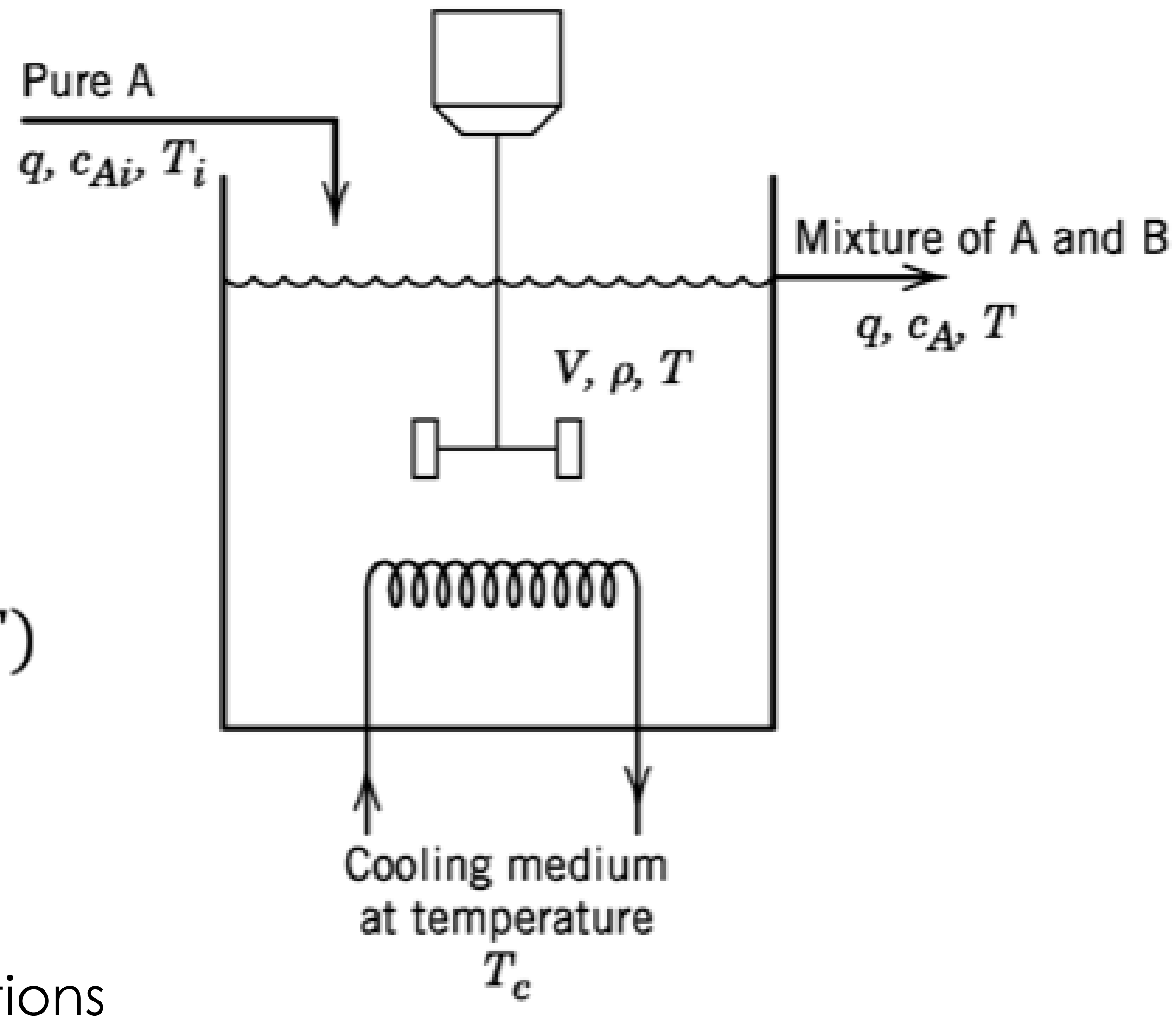
MODEL IDENTIFICATION

A ⟶ B

First order irreversible exothermic reaction

$$\frac{dC_a}{dt} = \frac{q}{V}(C_{ai} - C_a) - k_o e^{\left(\frac{-E_a}{RT}\right)} C_a$$

$$V\rho C_p \frac{dT}{dt} = qC(T_i - T) + (-\Delta H)VkC_a + UA(T_c - T)$$



Manipulated Variable: Cooling water temperature (Tc).
Controlled Variable: Reactor temperature (T).
Disturbance Variable: Feed or ambient temperature variations

MODEL IDENTIFICATION

Transfer Function Estimation

For a **first-order system**, the continuous-time transfer function is:

$$G(s) = \frac{K_p}{\tau s + 1}$$

where:

- K_p = Process gain (steady-state output change per unit input change),
- τ = Time constant (speed of response).

For a **second-order system**:

$$G(s) = \frac{K_p}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where:

- ζ = Damping ratio,
- ω_n = Natural frequency.

MODEL IDENTIFICATION

The goal is to find K_p, τ, ζ, and ω_n that minimize the **prediction error**:

$$\epsilon(t) = y(t) - \hat{y}(t|\theta)$$

where:

- $y(t)$ = Measured output,
- $\hat{y}(t|\theta)$ = Predicted output from the model with parameters θ.

The optimization problem is:

$$\min_{\theta} \sum_{t=1}^{N} \epsilon(t)^2$$

MODEL VALIDATION

The estimated models are simulated against the test input using `lsim` (linear simulation):

$$\hat{y}_{\text{test}} = \text{lsim}(G, u_{\text{test}}, t_{\text{test}})$$

where:

- G = Transfer function model ($G(s)$),
- u_{test} = Test input data,
- t_{test} = Test time vector.

MODEL IDENTIFICATION

USING tfest

1. **Linearization**: Approximate the non-linear model $\hat{y}(t|\theta)$ using a first-order Taylor expansion around the current parameter estimate θ_k:

$$\hat{y}(t|\theta) \approx \hat{y}(t|\theta_k) + J(t, \theta_k)(\theta - \theta_k),$$

   where $J(t, \theta_k) = \frac{\partial \hat{y}(t|\theta)}{\partial \theta}\Big|_{\theta=\theta_k}$ is the **Jacobian matrix**.

2. **Update Equation**:

   The parameter update $\Delta\theta = \theta_{k+1} - \theta_k$ is found by solving:

$$(J^T J)\Delta\theta = -J^T \epsilon,$$

   where:

   - J = Jacobian matrix (size $N \times 2$ for first-order),
   - ϵ = Residual vector $[y(t) - \hat{y}(t|\theta_k)]$.

3. **Iteration**: Update $\theta_{k+1} = \theta_k + \Delta\theta$ and repeat until convergence.

MODEL IDENTIFICATION

Example working of the tfest MATLAB function

Consider a true system with $K_p = 2$, $\tau = 3$. We generate synthetic data for a step input $u(t) = 1$ at discrete times $t = 1, 2, 3, 4, 5$. The measured outputs (with noise) are:

t	$y(t)$
1	0.5
2	1.2
3	1.6
4	1.8
5	1.9

Goal: Estimate K_p and τ using the Gauss-Newton method.

MODEL IDENTIFICATION

Example working of the tfest MATLAB function

Step 1: Initial Guess

Start with an initial parameter guess: $\theta_0 = [K_p^{(0)}, \tau^{(0)}] = [1, 1]$.

Step 2: Compute Predicted Output

For a first-order system, the predicted output is:

$$\hat{y}(t|\theta) = K_p \left(1 - e^{-t/\tau}\right).$$

At $\theta_0 = [1, 1]$, the predictions are:

$$\hat{y}(1|1,1) \approx 0.6321, \quad \hat{y}(2|1,1) \approx 0.8647, \quad \text{etc.}$$

Step 3: Compute Residuals

Residuals $\epsilon = y(t) - \hat{y}(t|\theta_0)$:

$$\epsilon = [-0.1321, 0.3353, 0.6498, 0.8183, 0.9067].$$

Step 4: Compute Jacobian Matrix

The Jacobian J contains partial derivatives of $\hat{y}(t|\theta)$ w.r.t. K_p and τ:

$$\frac{\partial \hat{y}}{\partial K_p} = 1 - e^{-t/\tau}, \quad \frac{\partial \hat{y}}{\partial \tau} = \frac{K_p t}{\tau^2} e^{-t/\tau}.$$

MODEL IDENTIFICATION

Example working of the tfest MATLAB function

At $\theta_0 = [1, 1]$:

$$J = \begin{bmatrix} 0.6321 & 0.3679 \\ 0.8647 & 0.2706 \\ 0.9502 & 0.1494 \\ 0.9817 & 0.0733 \\ 0.9933 & 0.0337 \end{bmatrix}.$$

Step 5: Solve for Parameter Update

Solve $(J^T J)\Delta\theta = -J^T \epsilon$:

$$J^T J = \begin{bmatrix} 4.0006 & 0.7136 \\ 0.7136 & 0.2373 \end{bmatrix}, \quad -J^T \epsilon = \begin{bmatrix} -2.5281 \\ -0.2297 \end{bmatrix}.$$

The solution gives:

$$\Delta\theta = \begin{bmatrix} -0.990 \\ 2.003 \end{bmatrix}.$$

Step 6: Update Parameters

$$\theta_1 = \theta_0 + \Delta\theta = [1 - 0.990, 1 + 2.003] = [0.01, 3.003].$$

MODEL IDENTIFICATION RESULTS

NON LINEAR

```
First Order Model - MSE: 1587.401398, RMSE: 39.842206
Second Order Model - MSE: 434.483852, RMSE: 20.844276
``
```

LINEAR

```
>> codetask2
First Order Model - MSE: 1851.358004, RMSE: 43.027410
Second Order Model - MSE: 433.086467, RMSE: 20.810730
>>
```

Your paragraph text

MODEL IDENTIFICATION RESULTS

NON LINEAR

Positive Step: K_p = 0.3568, τ = 0.1231
Negative Step: K_p = 0.1233, τ = 0.1231
Averaged: K_{p_avg} = 0.2401, τ_{avg} = 0.1231

LINEAR

Positive Step: K_p = 1.4518, τ = 0.0385
Negative Step: K_p = 0.0388, τ = 0.0385
Averaged: K_{p_avg} = 0.7453, τ_{avg} = 0.0385

MODEL IDENTIFICATION RESULTS

NON LINEAR

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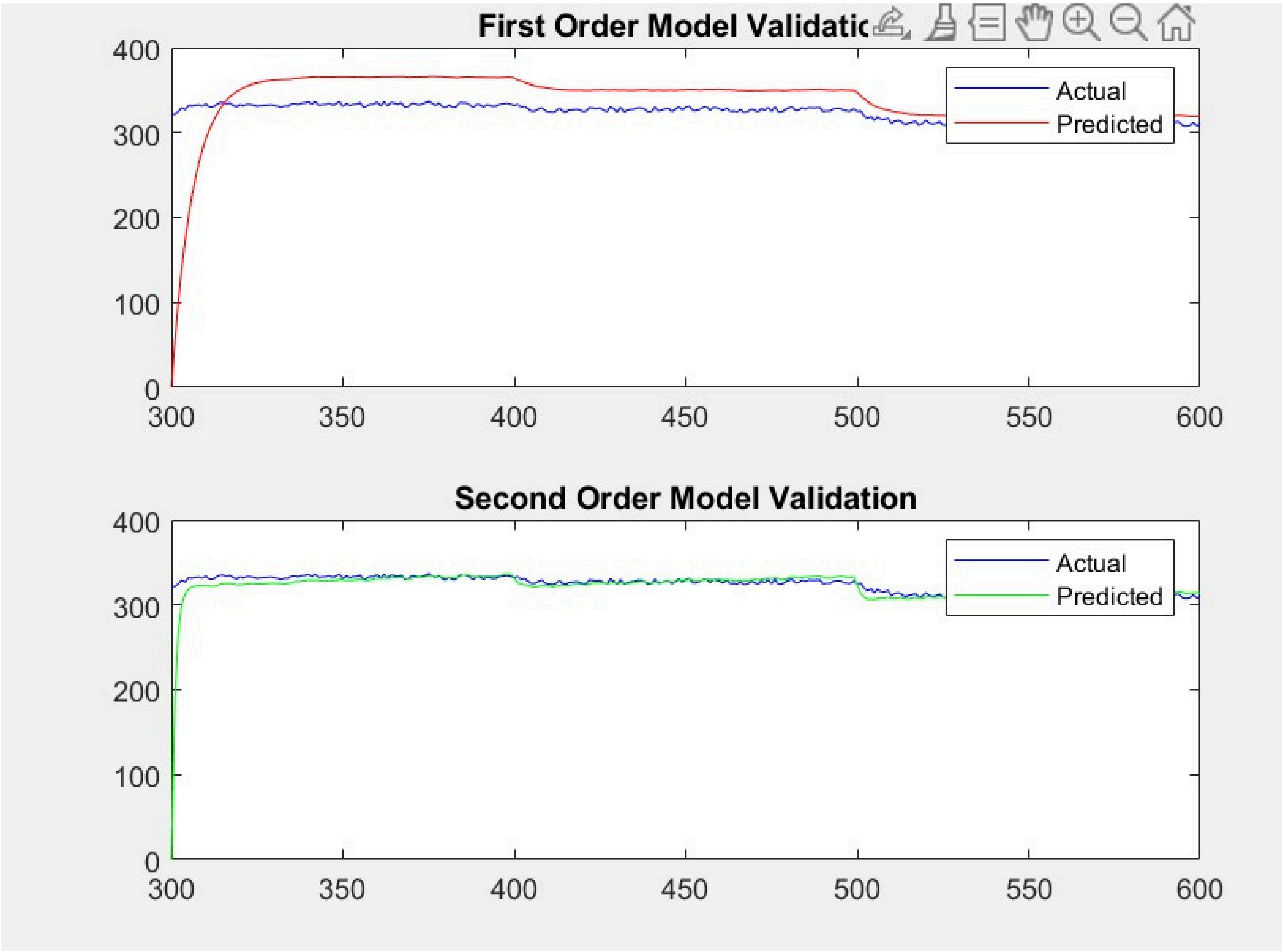
MODEL IDENTIFICATION RESULTS

NON LINEAR



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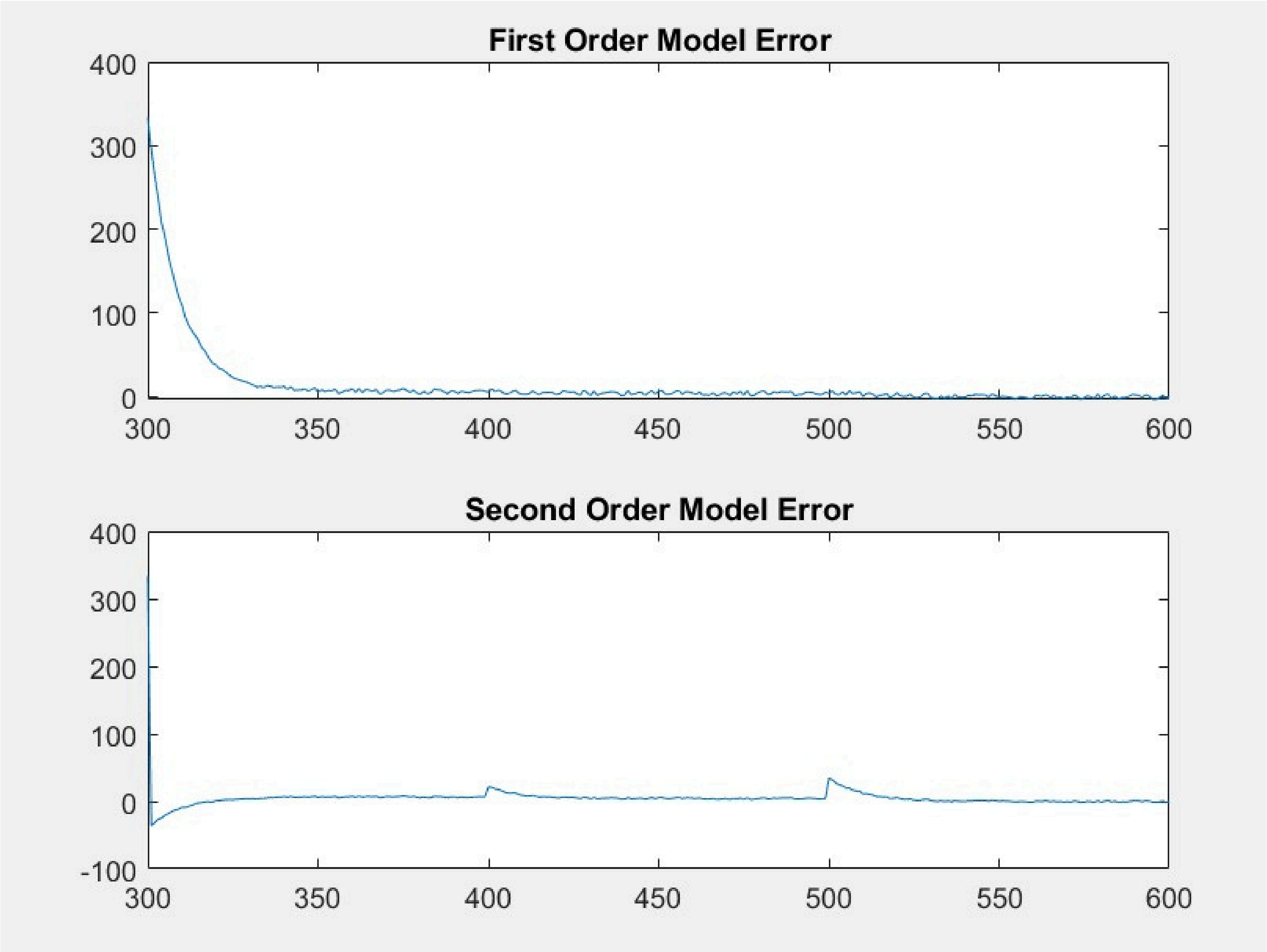
MODEL IDENTIFICATION RESULTS

LINEAR



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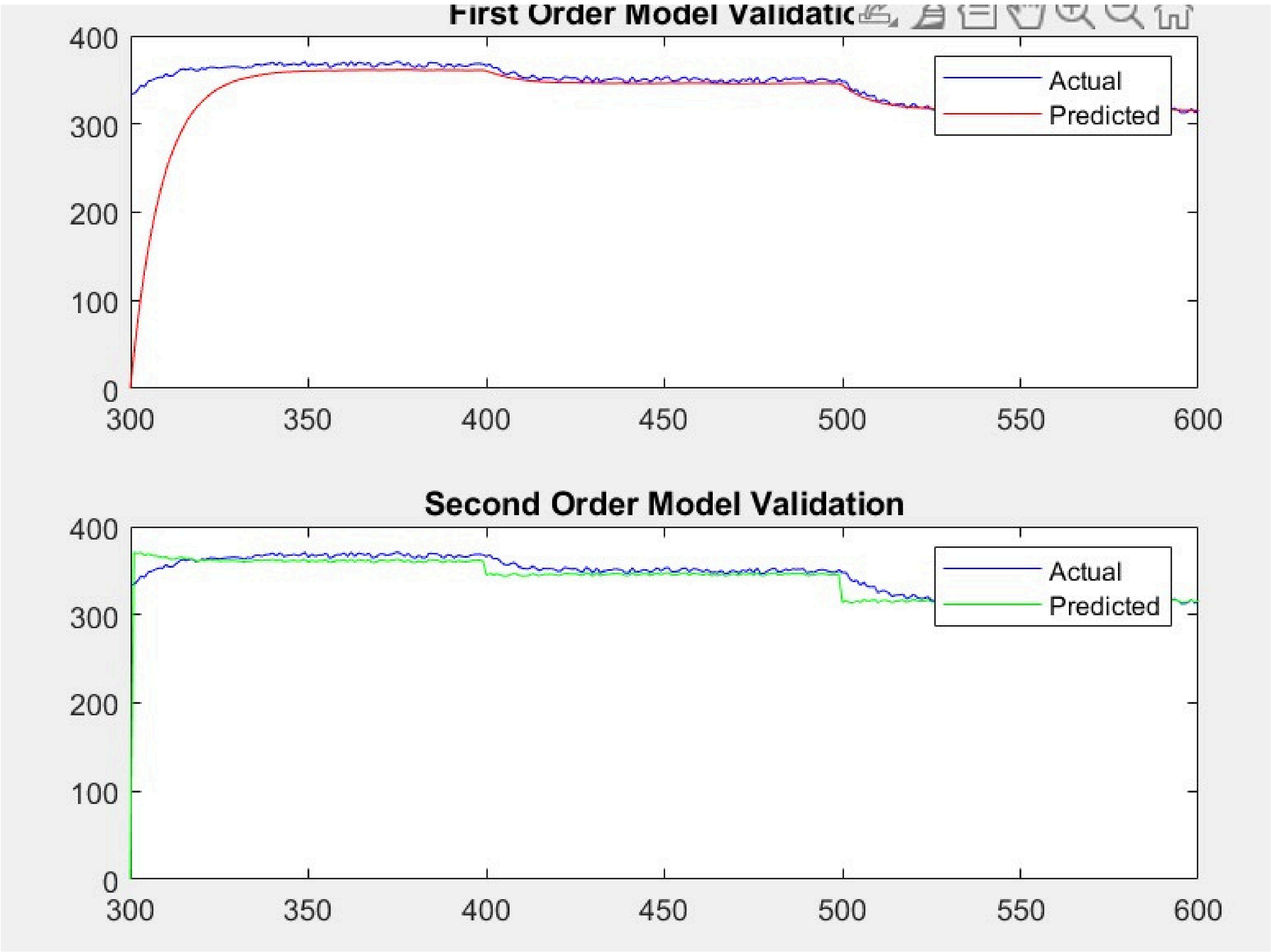
MODEL IDENTIFICATION RESULTS

LINEAR



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OBSERVATIONS

- THE MODELS PERFROM RELATIVELY WELL AND ARE ABLE TO CAPTURE THE ESSENCE OF THE INPUT STEP CHANGE
- MORE ACCURACY IS SEEN FROM THE LINEAR MODEL AS COMPARED TO THE NON LINEAR ONE.
- tfest PROVIDES A ROBUST LINEAR REGRESSION OPTIMIZER AND IS ABLE TO SOLVE BOTH THE PROBLEMS RELATIVELY WELL.
- THERE IS NOT MUCH ERROR IN VALIDATION PART EXCEPT THE TIME , THE GRAPH REACHES ITS ABSOULTE SATURATION VALUE (INITIALLY).,

THANK YOU