ZJU-ADS-HQM2020-WK11

判断题 3 | A. 单选题 4

1-1 Greedy method is a special case of local search. (2分)

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○ T ◉ F

1-1 答案正确 (2 分) 创建提问

1-2 For the graph given in the following figure, if we start from deleting the black vertex, then local search can always find the minimum vertex cover. (2分)

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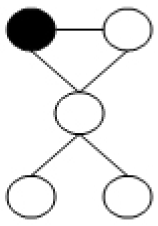

◉ T ○ F

1-2 答案正确 (2 分) 创建提问

1-3 Since finding a locally optimal solution is presumably easier than finding an optimal solution, we can claim that for any local search algorithm, one step of searching in neighborhoods can always be done in polynomial time. (2分)

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1-3 答案正确 (2 分) 创建提问

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2-1 A bipartite graph G is one whose vertex set can be partitioned into two sets A and B, such that each edge in the graph goes between a vertex in A and a vertex in B. Matching M in G is a set of edges that have no end points in common. Maximum Bipartite Matching Problem finds a matching with the greatest number of edges (over all matching).

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Consider the following **Gradient Ascent Algorithm**:

`As long as there is an edge whose endpoints are unmatched, add it to the current matching. When there is no longer such an edge, terminate with a locally optimal matching.`

Let M_1 and M_2 be matchings in a bipartite graph G. Which of the following statements is true? (3分)

- A. This gradient ascent algorithm never returns the maximum matching.
- B. Suppose that $|M_1| > 2|M_2|$. Then there must be an edge e in M_1 such that $M_2 \cup \{e\}$ is a matching in G.
- C. Any locally optimal matching returned by the gradient ascent algorithm in a bipartite graph G is at most half as large as a maximum matching in G.
- D. All of the above

2-1 答案正确 (3 分) 创建提问

2-2 **Spanning Tree Problem**: Given an undirected graph $G = (V, E)$, where $|V| = n$ and $|E| = m$. Let F be the set of all spanning trees of G. Define $d(u)$ to be the degree of a vertex $u \in V$. Define $w(e)$ to be the weight of an edge $e \in E$. We have the following three variants of spanning tree problems:

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- (1) Max Leaf Spanning Tree: find a spanning tree $T \in F$ with a maximum number of leaves.
- (2) Minimum Spanning Tree: find a spanning tree $T \in F$ with a minimum total weight of all the edges in T.
- (3) Minimum Degree Spanning Tree: find a spanning tree $T \in F$ such that its maximum degree of all the vertices is the smallest.

For a pair of edges (e, e') where $e \in T$ and $e' \in (G - T)$ such that e belongs to the unique cycle of $T \cup e'$, we define **edge-swap**(e, e') to be $(T - e) \cup e'$.

Here is a local search algorithm:

```
T = any spanning tree in F;
while (there is an edge-swap(e, e') which reduces Cost(T)) {
    T = T - e + e';
}
return T;
```

Here `Cost(T)` is the number of leaves in `T` in Max Leaf Spanning Tree; or is the total weight of `T` in Minimum Spanning Tree; or else is the minimum degree of `T` in Minimum Degree Spanning Tree.

Which of the following statements is TRUE? (4分)

- A. The local search always return an optimal solution for Max Leaf Spanning Tree
- B. The local search always return an optimal solution for Minimum Spanning Tree
- C. The local search always return an optimal solution for Minimum Degree Spanning Tree
- D. For neither of the problems that this local search always return an optimal solution

2-2 答案正确 (4 分) 创建提问

2-3 Max-cut problem: Given an undirected graph $G = (V, E)$ with positive integer edge weights w_e, find a node partition (A, B) such that $w(A, B)$, the total weight of edges crossing the cut, is maximized. Let us define S' be the neighbor of S such that S' can be obtained from S by moving one node from A to B, or one from B to A. We only choose a node which, when flipped, increases the cut value by at least $w(A, B)/|V|$. Then which of the following is true? (3分)

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- A. Upon the termination of the algorithm, the algorithm returns a cut (A, B) so that $2.5w(A, B) \geq w(A^*, B^*)$, where (A^*, B^*) is an optimal partition.
- B. The algorithm terminates after at most $O(\log |V| \log W)$ flips, where W is the total weight of edges.
- C. Upon the termination of the algorithm, the algorithm returns a cut (A, B) so that $2w(A, B) \geq w(A^*, B^*)$.
- D. The algorithm terminates after at most $O(|V|^2)$ flips.

2-3 答案正确 (3 分) 创建提问

2-4 There are n jobs, and each job j has a processing time t_j. We will use a local search algorithm to partition the jobs into two groups A and B, where set A is assigned to machine M_1 and set B to M_2. The time needed to process all of the jobs on the two machines is $T_1 = \sum_{j \in A} t_j$, $T_2 = \sum_{j \in B} t_j$. The problem is to minimize $|T_1 - T_2|$.

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Local search: Start by assigning jobs $1, \ldots, n/2$ to M_1, and the rest to M_2. The local moves are to move a single job from one machine to the other, and we only move a job if the move decreases the absolute difference in the processing times. Which of the following statement is true? (4分)

- A. The problem is NP-hard and the local search algorithm will not terminate.
- B. When there are many candidate jobs that can be moved to reduce the absolute difference, if we always move the job j with maximum t_j, then the local search terminates in at most n moves.

- C. The local search algorithm always returns an optimal solution.
- D. The local search algorithm always returns a local solution with $\frac{1}{2}T_1 \le T_2 \le 2T_1$.   n=2, t1=1, t2=10

2-4 答案正确 （4 分） 创建提问