HW 3: Numerical solutions of ODE models

Due April 21, 2023.

1. In Chapter 9 of Kot ("Elements of Mathematical Ecology"), a predator-prey model with type IV functional response is presented:

$$\frac{dN}{dT} = rN\left(1 - \frac{N}{K}\right) - \phi(N)P$$
$$\frac{dP}{dT} = b\phi(N)P - mP$$

   where

$$\phi(N) = \frac{cN}{\frac{N^2}{i} + N + a}$$

   and N represents the prey, P the predator. Substituting in $\phi(N)$ and nondimensionalizing yields the following equations

$$\frac{dx}{dt} = x\left(1 - \frac{x}{\gamma}\right) - \frac{xy}{\frac{x^2}{\alpha} + x + 1}$$
$$\frac{dy}{dt} = \frac{\beta\delta xy}{\frac{x^2}{\alpha} + x + 1} - \delta y$$

   with $x = \frac{N}{a}$, $y = \frac{c}{ra}P$, and $t = rT$.
   Numerically solve this system of equations with the following parameters:
   $\alpha = 5.2$, $\beta = 2.0$, $\gamma = 4.1$, $\delta = 2.5$, $x(0) = 1.3$, $y(0) = 1.6$.

   To visualize your results, create a figure with a two-by-two subplot grid. In the top two plots, plot both solutions, $x(t)$ and $y(t)$, in separate plots. In the bottom left subplot, plot the phase plane for this system using a quiver plot. In the bottom right subplot, plot the (x, y) solution over a chosen time period with x on the x-axis and y on the y-axis. You should see the solution spiral out to a steady oscillation (an orbit).

2. In Chapter 10 of Kot, a double mass-action chemostat model is presented including rate equations for substrate, heterotroph, and holozoic predator

$$\frac{dS}{dT} = D(S_i - S) - \frac{\mu_1}{Y_1}SH$$
$$\frac{dH}{dT} = \mu_1 SH - DH - \frac{\mu_2}{Y_2}HP$$
$$\frac{dP}{dT} = \mu_2 HP - DP.$$

Nondimensionalizing yields the following equations:

$$\frac{dx}{dt} = 1 - x - Axy$$
$$\frac{dy}{dt} = Axy - y - Byz$$
$$\frac{dz}{dt} = Byz - z$$

with $x = \frac{S}{S_i}$, $y = \frac{H}{Y_1 S_i}$, $z = \frac{P}{Y_1 Y_2 S_i}$, $t = DT$.
Numerically solve this system of equations with the following parameters,
$A = 4.0$, $B = 8.0$. Choose your own initial conditions (strictly positive, not to big... say less than 5) and plot each of x, y, and z against time in their own subplot (time on the x-axis, each variable on the y-axis) so that you can see damped oscillations. Use a one-column (three rows) subplot for this. Then, in a separate plot, create a 3D line plot of the (x, y, z) solution over the chosen time period with each variable on its respective axis.