Ballistic missiles using calculus

INTRODUCTION:

Ballistics, the scientific investigation of projectile motion, is fundamental to law enforcement, military strategy, and attacking sports or war. For accurate targeting, weapon design, and overall combat effectiveness, it is essential to understand the precise trajectories and behaviors of projectiles. Calculus is essential to ballistics because it provides tools for modeling dynamic systems and analyzing rates of change.

Basically, ballistics is the study of projectile motion under the influence of air resistance, gravity, and initial propulsion. Examples of projectiles include bullets, artillery shells, and missiles. The mathematical foundation required to simulate and examine the complex and dynamic nature of projectile motion is provided by calculus.

We look into projectile motion equations, the impact of external factors on trajectories, and the optimization of launch conditions for maximum accuracy and range in our study of ballistics estimates using calculus. Ballistics researchers can improve weaponry, targeting systems, and the general effectiveness of military and law enforcement operations by using the power of calculus.

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A BALLISTIC PROBLEM:

The next task, ballistics calculations, is a classic in scientific computing as well as in some computer gaming genres. We will use a shooting method to solve this problem, which means we'll base our approach on initial value solvers and try to select initial conditions that will satisfy the limits of the problem.

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Model without air drag:

Let us begin with an intermediate model, which most of you have likely encountered in an introductory physics curriculum. When a projectile is shot from a launcher at a set speed, where will it rest depending on the launch angle? Newton's law indicates that in the most basic version of this model, gravity is the only force acting on the ball following launch.

$ma = -mg e_y$

Where :

e_y is a unit vector in the vertical direction

m is the particle mass

$a = x'' = (x'', y'')$ is the acceleration vector.

If our launcher is positioned at the origin, then the initial conditions for a launch with speed s at angle θ are :

$\mathbf{x(0) = 0,}$ $\quad\quad$ $\mathbf{x'(0) = s\cos(\theta) = v_{0,x},}$

$\mathbf{y(0) = 0,}$ $\quad\quad$ $\mathbf{y'(0) = s\sin(\theta) = v_{0,y}.}$

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Subject to these initial conditions, we can compute the solution analytically:

$x(t) = v_{0,x}t$
$y(t) = v_{0,y}t - gt^2/2.$

The trajectory returns to the ground at time $T_{final} = 2v_{0,y}/g$ and at position

$$x_{\text{final}} = x(t_{\text{final}}) = \frac{2}{g} v_{0,y} v_{0,x} = \frac{s^2}{g} \sin(2\theta).$$

Therefore, we can reach a target at distances $d \le s^2/g$ with launch angles θ that satisfy $\sin(2\theta) = gd/s^2$. In general, if we can hit the target at all there will be two trajectories that work. One will have an angle between 0 and $\pi/4$, while the other has an angle between $\pi/4$ and $\pi/2$.

Model with air drag:

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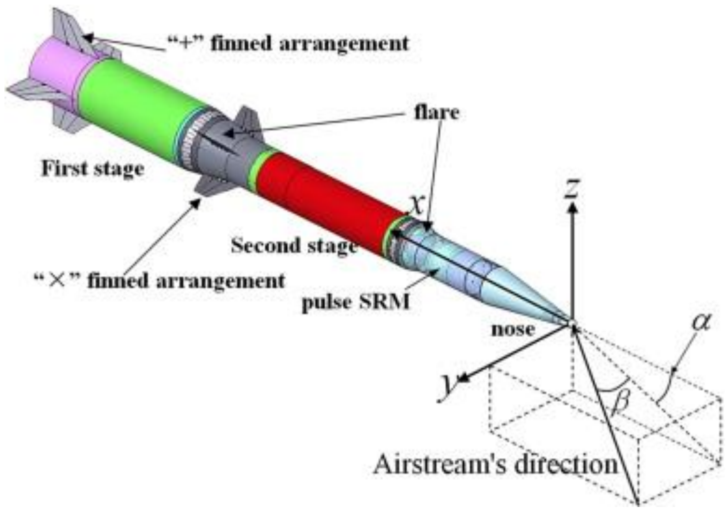
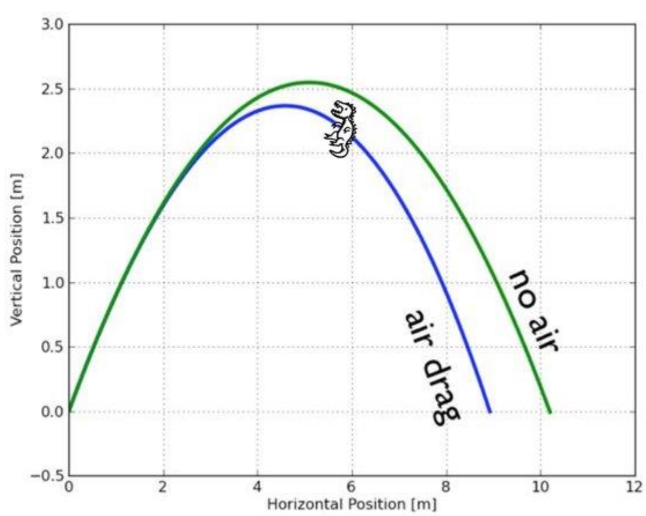


In actuality, aerodynamic resistance has an impact on projectiles in addition to gravity. The drag force induced by air resistance acts in the exact opposite direction of the velocity and has a magnitude equal to the square of the velocity for projectiles with an applicable range, provided that the projectile cannot reach so high as to create troubles with differences in atmospheric pressure.



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$ma = -mge_y - mc\|v\|v$

If we also consider a constant horizontal wind velocity we_x, we arrive at

$ma = -mge_y - mc\|v - we_x\|(v - we_x)$

The size, shape, and mass of the projectile as well as the air's temperature and pressure all have an importance when establishing the coefficient c. Let's say for the time being that it is a given.

We could analyze the differential equation by hand given that it lacked any air drag. As this model is more complicated, we resort to numerical techniques. We need to transform the model into first-order form in order to employ Mat lab's Ordinary differential equation solvers.

An example trajectory:

As an example, let's consider a concrete example with a high drag coefficient ($c = 0.05\ m^{-1}$) and some wind ($w = -2.5$ m/s). We want to hit a target at distance $d = 10$ m. The trajectories computed for the two solution angles the residual error $f_{target}(\theta)$ is

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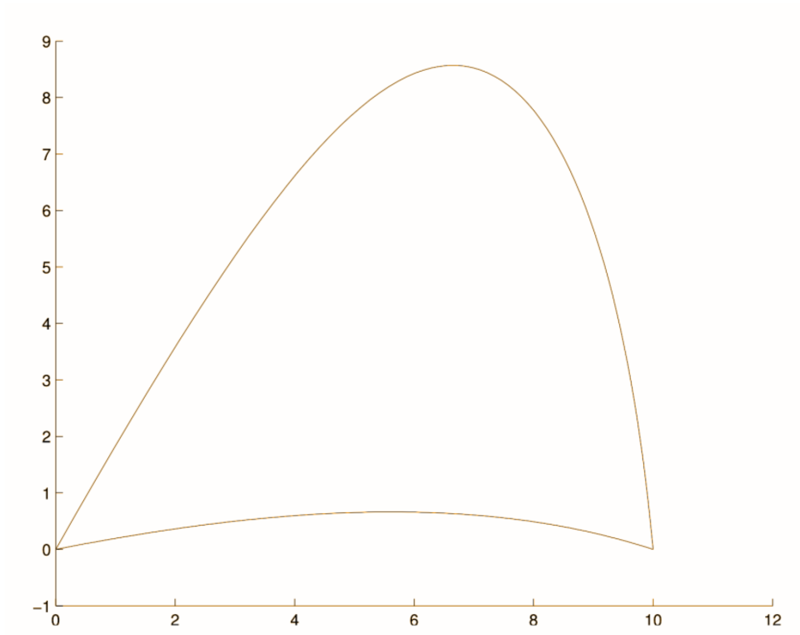
on the order of 10^{-7} for this problem, which is almost certainly smaller than errors due to uncertainty in the model parameters.



DERIVATION OF FORCE EQUATION OF BALLISTIC MISSILES:

A booster or ballistic missile is a variable-mass weapon that makes power through its evacuation of quickly speed particles. This section contains a brief, non-rigorous derivation of the linear force equation. Any system of particles' rate of change in linear momentum is equal to the sum of all external forces acting on it.

$$\sum \mathbf{F} = m\left(\frac{d\mathbf{V}}{dt}\right) + m_g \mathbf{V}_g,$$

Where

m=mass of missile,

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v=velocity of missile

m_g=mass of escaping gas,

v_g=velocity of escaping gas,

$$m\mathbf{g} = m\left(\frac{d\mathbf{V}}{dt}\right) + \left(\frac{dm}{dt}\right)\mathbf{V} + m\left(\frac{d\mathbf{V}_g}{dt}\right) + \left(\frac{dm_g}{dt}\right)\mathbf{V}_g.$$

Now, ($d\mathbf{V}_g/dt$) = 0 when the gas exits into free space, and (dm/dt) = ?(dm_g/dt), since the total system mass is a constant. Thus,

$$\begin{aligned} m\mathbf{g} &= m\left(\frac{d\mathbf{V}}{dt}\right) + \left(\frac{dm_g}{dt}\right)(\mathbf{V}_g - \mathbf{V}) \\ &= m\left(\frac{d\mathbf{V}}{dt}\right) + \left(\frac{dm_g}{dt}\right)\mathbf{c} = m\left(\frac{d\mathbf{V}}{dt}\right) - \mathbf{T}, \end{aligned}$$

where

c = escape gas velocity (or ejection velocity) with respect to the missile; also called specific impulse,

T = missile thrust vector $\left(\frac{dm_g}{dt}\right)$.

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$$\frac{d^2\mathbf{R}}{dt^2} = \left(\frac{d\mathbf{V}}{dt}\right) = \mathbf{g} + \mathbf{a}_T,$$

where $\mathbf{a}_T$ is the thus acceleration and is given by

$a_{T=}T/mt$