# Weather Prediction And Warfare

# Introduction

Weather prediction in war has been a critical aspect of military planning of centuries. The calculus behind weather prediction involves a deep understanding of mathematics models, atmospheric physics, and numerical methods used to forecast weather patterns.

Here are some equations which were used in wars to determine the condition of weather. The Navier Stokes equations is used to know how air moves in the atmosphere and its useful because they describe the physics of many phenomena of scientific and engineering interest. They may be used to model the weather, ocean currents, water flow in a pipe and air flow around a wing.

## 1. The Navier Stokes Equations

The equations used to build climate models are, of course, partial differential equations (PDEs). The equations are non-linear and are coupled together into systems and their solutions are truly nontrivial.

By the Navier Stokes Equations we could figure out how air moves in the atmosphere

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \sum_{i=1}^{3} u_i \frac{\partial \mathbf{u}}{\partial x_i}\right) - \mu\Delta\mathbf{u} - (3\lambda + \mu)\nabla(\nabla \cdot \mathbf{u}) + \nabla p = \mathbf{f},$$

which is a formulation of Newtons second law of motion applied to fluid mechanics. Here μ and λ are physical parameters and f(x, t) is a density of force per unit volume. The functions ρ, u, p also obey the continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}),$$

which says that the mass of the fluid is conserved. If the fluid is homogeneous and incompressible as well, then ρ is independent of position and time then,

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \sum_{i=1}^{3} u_i \frac{\partial \mathbf{u}}{\partial x_i}\right) - \mu \Delta \mathbf{u} + \nabla p = \mathbf{f},$$

$$\nabla \cdot \mathbf{u} = 0.$$

Note however that in atmospheric modeling one would assume the fluid (the air) to be compressible. Below we will be concerned with the incompressible case, for simplicity. The convention is to let ρ = 1, set μ = ν

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} - \nu \Delta \mathbf{u} + \nabla p = \mathbf{f}.$$

### 1.1 A Boundary value problem for the navier stokes equations

Once again, let Ω denote a region of space filled by a fluid described by the Navier-Stokes. Let Ω be an open set of R n, n = {2, 3}, with boundary Γ. Further assume that Ω is located locally on one side of Γ and that Γ is Lipschitz or sometimes of class C k for a specified k.

Now in order to be able to use these equations, we need to determine a wellposed boundary value problem to go along with them. It is believed [7] that (2.5) and (2.4) must be augmented by the following initial and boundary conditions:

- Initial condition:

u(x, 0) = u0(x), x ∈ Ω, u0 given

- Boundary condition:

u(x, t) = Φ(x, t), x ∈ Γ, t > 0,     Ω bounded, Φ given

Therefore, for a bounded domain Ω (which is common for applications), the boundary value problem for the incompressible case can be summarized thus:

$$\begin{aligned} \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} - \nu \Delta \mathbf{u} + \nabla p = \mathbf{f}, &\qquad \mathbf{x} \in \Omega,\ t > 0,\ \mathbf{f} \text{ given}, \\ \nabla \cdot \mathbf{u} = 0, &\qquad \mathbf{x} \in \Omega,\ t > 0, \\ \mathbf{u}(\mathbf{x}, 0) = \mathbf{u}_0(\mathbf{x}), &\qquad \mathbf{x} \in \Omega,\ \mathbf{u}_0 \text{ given}, \\ \mathbf{u}(\mathbf{x}, t) = \Phi(\mathbf{x}, t), &\qquad \mathbf{x} \in \Gamma,\ t > 0,\ \Phi \text{ given}, \end{aligned}$$

where for given functions f, u0, Φ, and constant ν, we would like to solve for the unknown functions u and p. If Ω is unbounded (and in particular for Ω = R n), we add a condition at infinity:

u(x, t) → ψ(x, t) as |x| → +∞, (ψ given)

### 1.2 Approximations used in Climate Modeling

Climate Models are important tools for improving our understanding and predictability of climate behaviour on seasonal, annual, decadal and centennial time scales.

The expressions that meteorologists work with to create forecasts generally have the form

$$\mathbf{u}_t + \sum_{i=1}^{n} \nabla \cdot F_i(\mathbf{u}) = P(\mathbf{u}),$$

- The spherical geopotential approximation

The first main approximation used in current operational climate models is known as the spherical geopotential approximation. The assumption here is that the Earth can be modeled by a perfect sphere. The common practice is to use spherical coordinates, which do not provide the flexibility of adjusting at will for the potato-like shape of the actual Earth.

Expressed in spherical polar coordinates (r, λ, ϕ), the equations of motion become:

$$\frac{Du}{Dt} - f_r v + f_\phi w - \frac{uv\tan\phi}{r} + \frac{uw}{r} + \frac{\alpha}{r\cos\phi}\frac{\partial p}{\partial\lambda} = P^u,$$
$$\frac{Dv}{Dt} + f_r u - f_\lambda w + \frac{u^2\tan\phi}{r} + \frac{vw}{r} + \frac{\alpha}{r}\frac{\partial p}{\partial\phi} = P^v,$$
$$\frac{Dw}{Dt} - f_\phi u + f_\lambda v - \frac{(u^2+v^2)}{r} + g + \alpha\frac{\partial p}{\partial r} = P^w,$$

where (u, v, w) = (λr˙ cos ϕ, ϕr, ˙ r˙) are the longitudinal, latitudinal, and vertical wind components, (fλ, fϕ, fr) = (0, 2ω cos ϕ, 2ω sin ϕ) are the Coriolis terms arising from the rotation of the Earth (the angular speed of rotation given by ω), α is specific volume, and the P terms are the various physical parametrizations. See [11] for more detailed definitions. The material derivative is given by

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \frac{u}{r\cos\phi}\frac{\partial}{\partial\lambda} + \frac{v}{r}\frac{\partial}{\partial\phi} + w\frac{\partial}{\partial r}.$$

- The shallow atmosphere approximation

The shallow atmosphere approximation makes two simplifications to the NHM equations. The idea is to replace the radial distance r from the center of the Earth with a mean radius a, and to omit the four metric terms that do not explicitly depend on latitude, as well as the two Coriolis terms associated with the horizontal component of planetary vorticity ω. The wind components are redefined as (u, v, w) = (λa˙ cos ϕ, ϕa, ˙ z˙) and the material derivative becomes:

$$\frac{D_a}{Dt} = \frac{\partial}{\partial t} + \frac{u}{a\cos\phi}\frac{\partial}{\partial\lambda} + \frac{v}{a}\frac{\partial}{\partial\phi} + w\frac{\partial}{\partial z}.$$

$$
\begin{aligned}
\frac{D_a u}{Dt} - f_r v - \frac{uv\tan\phi}{a} + \frac{uw}{a} + \frac{\alpha}{a\cos\phi}\frac{\partial p}{\partial\lambda} &= P^u, \\
\frac{D_a v}{Dt} + f_r u + \frac{u^2\tan\phi}{a} + \frac{vw}{a} + \frac{\alpha}{r}\frac{\partial p}{\partial\phi} &= P^v, \\
\frac{D_a w}{Dt} + g + \alpha\frac{\partial p}{\partial z} &= P^w.
\end{aligned}
$$

- Primitive equations models

Those models which use the spherical geoid, shallow atmosphere and hydrostatic approximations are known as primitive equations models or hydrostatic PEMs. Combining all the approximations together, becomes:

$$
\begin{aligned}
\frac{D_a u}{Dt} - f_r v - \frac{uv\tan\phi}{a} + \frac{uw}{a} + \frac{\alpha}{a\cos\phi}\frac{\partial p}{\partial\lambda} &= P^u, \\
\frac{D_a v}{Dt} + f_r u + \frac{u^2\tan\phi}{a} + \frac{vw}{a} + \frac{\alpha}{r}\frac{\partial p}{\partial\phi} &= P^v, \\
g + \alpha\frac{\partial p}{\partial z} &= 0.
\end{aligned}
$$

For example, The European Centre for Medium Range Weather Forecasts uses an operational PEM.