Random numbers can simplify Error analysis

Steve Spicklemire
University of Indianapolis

Problem:

- Students have little intuition about uncertainty
- Significant figures are a poor substitute for actually tracking uncertainty
- Mathematical formalism can be a barrier to understanding

Empirical Approach: Monte Carlo

- Use random number generators to "simulate" uncertainty
- Use histograms or other means to visualize variability
- Perform straightforward calculations using random numbers to see the effect on calculated results

Example

- Calculate the volume of a cylinder
- measure length 3cm +/- 0.1cm
- measure radius 1cm +/- 0.1cm
- What is the uncertainty in the volume?

Analytical Approach

- Analytical approach doesn't require a computer
- Provides some intuition with practice
- Can be useful in simple situations

Analytical Approach

$$f_{\mathrm{val}} = f(a, b, c)$$

$$\sigma_f^2 = \left(\frac{\partial f}{\partial a}\sigma_a\right)^2 + \left(\frac{\partial f}{\partial b}\sigma_b\right)^2 + \left(\frac{\partial f}{\partial c}\sigma_c\right)^2$$

$$V = \pi r^2 l$$

Analytical Approach

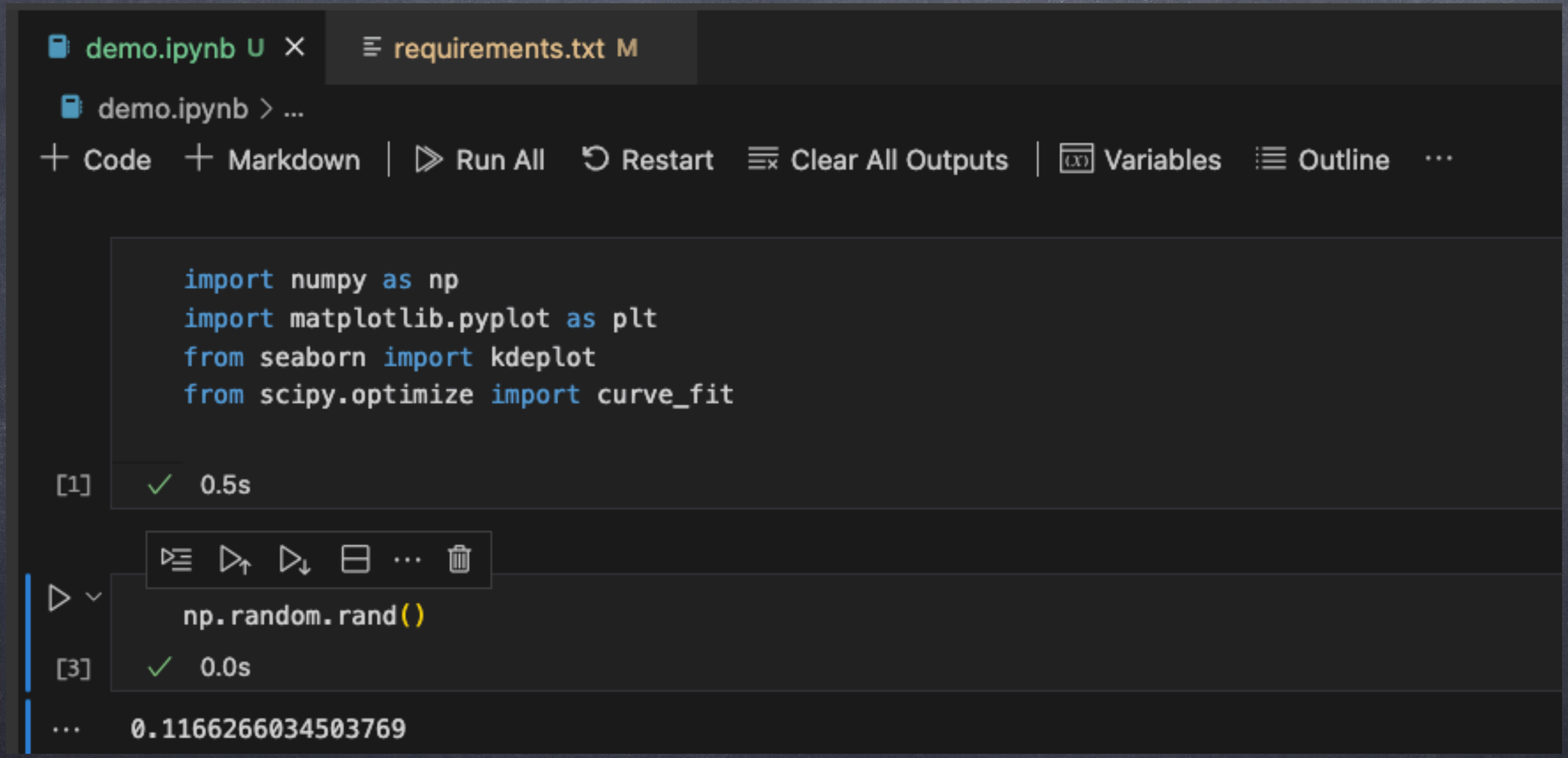
$$V = \pi r^2 l$$

$$\sigma_V^2 = \left(\frac{\partial V}{\partial r}\sigma_r\right)^2 + \left(\frac{\partial V}{\partial l}\sigma_l\right)^2$$

$$\left(\frac{\sigma_V}{V}\right)^2 = \left(2\frac{\sigma_r}{r}\right)^2 + \left(\frac{\sigma_l}{l}\right)^2$$

$$\sigma_V = V\sqrt{\left(2\frac{0.1\text{cm}}{1\text{cm}}\right)^2 + \left(\frac{0.1\text{cm}}{3\text{cm}}\right)^2}$$

Demo


demo.ipynb U
requirements.txt M
demo.ipynb > ...
Code
Markdown
Run All
Restart
Clear All Outputs
Variables
Outline
import numpy as np
import matplotlib.pyplot as plt
from seaborn import kdeplot
from scipy.optimize import curve_fit
[1]
0.5s
np.random.rand()
[3]
0.0s
0.1166266034503769

Pros/Cons

- Con: More for students to learn/manage new skills
- Pro: Students have an easy/robust way to handle uncertainty
- Pro: Lays groundwork for uncertainty in parameters, etc.

Summary

- Teach students to use random numbers (monte carlo, MC) to simulate uncertainty
- Makes it easy to visualize the effect of uncertainty in measurements on calculuated results
- Prepares students to be able to apply MC in other contexts

The End!