1. (a)
  - 주어진 밀도 함수 f(x)와 제안 분포 g(x)~BETA(2,3)를 사용하여 중요도 샘플링을 진행
  - 중요도 추출의 기본적인 방법은 제안 분포에서 샘플을 생성한 후, 이 샘플을 이용해 기댓값을 추정하는 것
  - 알고리듬
    1) $x_i \sim BETA(2,3)$에서 샘플 $x_1, x_{2,} \dots x_n$을 생성
    2) $w_i = \frac{f(x_i)}{g(x_i)}$를 계산하여 각 샘플의 중요도 가중치를 구함
    3) 기댓값 $E[f(x)]$는 중요도 가중치로 평균을 계산하여 구함

$$E[f(x)] \approx \frac{\sum_{n}^{i=1} w_i x_i}{\sum_{n}^{i=1} w_i}$$

(b) R로 구현하여 기댓값 구하기

```
library(stats)

# f(x) 정의
f <- function(x) {
+       exp(-x) * x^(2 - 1) * (1 - x)^(3 - 1)
+ }

# 제안 분포 g(x) : BETA(2, 3)에서 샘플링
n <- 10000
x_samples <- rbeta(n, 2, 3)

# 중요도 비율 계산
w <- f(x_samples) / dbeta(x_samples, 2, 3)

# 기댓값 계산
expectation <- sum(w * x_samples) / sum(w)
print(expectation)
[1] 0.363246
```

2. (a)

```
 # R 코드에서 필요한 패키지 로드
 library(rstan)

# 데이터 준비
data_list <- list(N = 10, x = c(3, 5, 3, 4, 4, 2, 3, 5, 7, 4))

 # Stan 모델 컴파일 및 샘플링
 fit <- stan(model_code = stan_model_code, data = data_list, iter = 15000, warmup = 
5000, chains = 4)

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 1).
Chain 1: 
Chain 1: Gradient evaluation took 3.6e-05 seconds
Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.36 
seconds.
Chain 1: Adjust your expectations accordingly!
Chain 1: 
Chain 1: 
Chain 1: Iteration:     1 / 15000 [  0%]  (Warmup)
Chain 1: Iteration:  1500 / 15000 [ 10%]  (Warmup)
Chain 1: Iteration:  3000 / 15000 [ 20%]  (Warmup)
Chain 1: Iteration:  4500 / 15000 [ 30%]  (Warmup)
Chain 1: Iteration:  5001 / 15000 [ 33%]  (Sampling)
Chain 1: Iteration:  6500 / 15000 [ 43%]  (Sampling)
Chain 1: Iteration:  8000 / 15000 [ 53%]  (Sampling)
Chain 1: Iteration:  9500 / 15000 [ 63%]  (Sampling)
Chain 1: Iteration: 11000 / 15000 [ 73%]  (Sampling)
Chain 1: Iteration: 12500 / 15000 [ 83%]  (Sampling)
Chain 1: Iteration: 14000 / 15000 [ 93%]  (Sampling)
Chain 1: Iteration: 15000 / 15000 [100%]  (Sampling)
Chain 1: 
Chain 1:  Elapsed Time: 0.053 seconds (Warm-up)
Chain 1:                0.112 seconds (Sampling)
Chain 1:                0.165 seconds (Total)
Chain 1: 
```

```
SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 2).
Chain 2:
Chain 2: Gradient evaluation took 7e-06 seconds
Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.07 seconds.
Chain 2: Adjust your expectations accordingly!
Chain 2:
Chain 2:
Chain 2: Iteration:     1 / 15000 [  0%]  (Warmup)
Chain 2: Iteration:  1500 / 15000 [ 10%]  (Warmup)
Chain 2: Iteration:  3000 / 15000 [ 20%]  (Warmup)
Chain 2: Iteration:  4500 / 15000 [ 30%]  (Warmup)
Chain 2: Iteration:  5001 / 15000 [ 33%]  (Sampling)
Chain 2: Iteration:  6500 / 15000 [ 43%]  (Sampling)
Chain 2: Iteration:  8000 / 15000 [ 53%]  (Sampling)
Chain 2: Iteration:  9500 / 15000 [ 63%]  (Sampling)
Chain 2: Iteration: 11000 / 15000 [ 73%]  (Sampling)
Chain 2: Iteration: 12500 / 15000 [ 83%]  (Sampling)
Chain 2: Iteration: 14000 / 15000 [ 93%]  (Sampling)
Chain 2: Iteration: 15000 / 15000 [100%]  (Sampling)
Chain 2:
Chain 2:  Elapsed Time: 0.057 seconds (Warm-up)
Chain 2:                0.123 seconds (Sampling)
Chain 2:                0.18 seconds (Total)
Chain 2:

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 3).
Chain 3:
Chain 3: Gradient evaluation took 4e-06 seconds
Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.04 seconds.
Chain 3: Adjust your expectations accordingly!
Chain 3:
Chain 3:
Chain 3: Iteration:     1 / 15000 [  0%]  (Warmup)
```

```
Chain 3: Iteration:  1500 / 15000 [ 10%]  (Warmup)
Chain 3: Iteration:  3000 / 15000 [ 20%]  (Warmup)
Chain 3: Iteration:  4500 / 15000 [ 30%]  (Warmup)
Chain 3: Iteration:  5001 / 15000 [ 33%]  (Sampling)
Chain 3: Iteration:  6500 / 15000 [ 43%]  (Sampling)
Chain 3: Iteration:  8000 / 15000 [ 53%]  (Sampling)
Chain 3: Iteration:  9500 / 15000 [ 63%]  (Sampling)
Chain 3: Iteration: 11000 / 15000 [ 73%]  (Sampling)
Chain 3: Iteration: 12500 / 15000 [ 83%]  (Sampling)
Chain 3: Iteration: 14000 / 15000 [ 93%]  (Sampling)
Chain 3: Iteration: 15000 / 15000 [100%]  (Sampling)
Chain 3: 
Chain 3:  Elapsed Time: 0.05 seconds (Warm-up)
Chain 3:                0.124 seconds (Sampling)
Chain 3:                0.174 seconds (Total)
Chain 3: 

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 4).
Chain 4: 
Chain 4: Gradient evaluation took 4e-06 seconds
Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.04 seconds.
Chain 4: Adjust your expectations accordingly!
Chain 4: 
Chain 4: 
Chain 4: Iteration:     1 / 15000 [  0%]  (Warmup)
Chain 4: Iteration:  1500 / 15000 [ 10%]  (Warmup)
Chain 4: Iteration:  3000 / 15000 [ 20%]  (Warmup)
Chain 4: Iteration:  4500 / 15000 [ 30%]  (Warmup)
Chain 4: Iteration:  5001 / 15000 [ 33%]  (Sampling)
Chain 4: Iteration:  6500 / 15000 [ 43%]  (Sampling)
Chain 4: Iteration:  8000 / 15000 [ 53%]  (Sampling)
Chain 4: Iteration:  9500 / 15000 [ 63%]  (Sampling)
Chain 4: Iteration: 11000 / 15000 [ 73%]  (Sampling)
Chain 4: Iteration: 12500 / 15000 [ 83%]  (Sampling)
Chain 4: Iteration: 14000 / 15000 [ 93%]  (Sampling)
```

```
Chain 4: Iteration: 15000 / 15000 [100%]  (Sampling)
Chain 4:
Chain 4:  Elapsed Time: 0.06 seconds (Warm-up)
Chain 4:                0.124 seconds (Sampling)
Chain 4:                0.184 seconds (Total)
Chain 4:

 # 사후표본 추출
 posterior_samples <- extract(fit)
 print(posterior_samples)
$theta
   [1] 3.445603 5.207336 4.254783 3.819930 3.201131 3.595814 3.273514 3.407784
3.827176 4.339007 3.603158 2.978973
  [13] 3.440691 4.423264 4.107595 3.783178 4.716802 3.142380 4.762579 4.096370
4.276408 5.372437 3.899219 4.122407
  [25] 4.392978 4.366621 4.104879 4.122928 3.960988 3.222817 3.637218 3.349401
4.281940 4.370535 3.663038 3.960539
  [37] 4.171915 3.649671 3.542375 4.077354 4.516309 5.250884 4.567541 4.032664
3.548034 4.024496 3.767693 3.295317
  [49] 3.814808 3.866771 4.781169 3.921401 3.228510 2.674731 4.314803 2.844673
4.361176 3.744820 4.071879 3.710208
  [61] 3.357638 3.818976 5.493548 4.681274 4.239540 3.782264 3.301155 3.587947
5.154232 3.341497 3.896728 4.089677
  [73] 4.216479 4.175259 4.819318 4.358551 3.627441 5.300617 3.146992 3.872979
6.105203 4.731770 3.290179 4.085995
  [85] 3.828614 3.631023 3.540945 3.997197 3.559003 3.574714 3.562595 5.554293
3.455027 3.573720 4.268640 4.524286
  [97] 2.669963 2.810806 4.587997 4.489633 3.845256 3.726410 3.564610 3.171271
4.180055 3.455291 4.487555 3.739379
 [109] 2.988388 4.699321 3.608142 4.213024 4.073908 3.531686 3.218229 3.903656
3.748002 3.945296 3.408384 3.525859
 [121] 4.314633 2.855916 4.005181 3.954305 3.435033 4.982527 3.105177 3.818553
2.999924 4.151980 4.326920 5.135261
 [133] 3.547843 3.667110 3.850102 4.269439 3.696120 4.392461 3.574146 4.054863
3.265823 4.280409 4.414008 3.911059
 [145] 4.337885 3.647732 3.865178 4.067710 3.380308 4.284483 3.617804 4.698800
```

```
4.325499 4.146744 3.519605 3.607631
 [157] 3.731715 4.193185 4.008435 3.678536 4.408858 3.864409 5.352564 4.563393
4.182073 4.850411 3.543646 4.701311
 [169] 5.151207 3.024383 3.841657 3.453834 3.154843 4.088774 2.621154 4.592767
4.879142 3.293824 2.999957 5.376830
 [181] 3.471391 4.664998 3.965342 3.685470 4.661695 3.635868 3.859753 5.073102
4.518269 3.664766 4.348782 4.218566
 [193] 4.131278 5.066454 3.448145 2.962775 3.790307 2.916612 4.253973 3.713281
4.073113 3.507187 3.025342 3.783585
 [205] 3.259262 3.445045 4.102156 4.216807 4.140176 4.129154 3.232228 4.855982
4.092906 3.354646 3.532393 3.511497
 [217] 3.755590 3.736038 4.139076 3.985853 3.810536 3.057436 2.722586 4.235963
4.309240 3.637168 4.122470 3.685636
 [229] 3.673617 3.775298 4.367298 3.691967 3.833333 3.990498 3.286831 5.548553
3.362173 4.238626 3.691779 4.556483
 [241] 4.022853 3.088716 3.448988 3.743916 4.209312 3.252270 3.779162 4.152547
4.495232 3.882873 3.302715 4.323920
 [253] 3.507666 3.975143 3.232760 4.353873 4.132943 3.615922 3.963419 3.022152
3.528277 4.556471 4.341276 3.193755
 [265] 4.319992 5.369560 3.916467 4.404114 5.643721 3.796109 3.995490 2.486842
3.762157 3.830322 3.642266 4.758303
 [277] 3.902635 3.655016 3.580861 3.108144 3.880056 4.244926 4.676574 3.266523
3.486573 4.491597 4.360396 3.028552
 [289] 3.371208 3.369003 3.058218 5.656088 3.579876 3.220326 3.868837 3.829594
3.753405 5.116004 4.734329 3.590530
 [301] 4.785482 4.088433 3.607822 3.583965 4.298708 4.524428 3.752016 5.131427
2.924915 4.096235 4.285494 4.130452
 [313] 3.989485 4.085301 3.732976 4.414424 4.639556 3.631545 3.373032 3.912479
4.693490 4.077969 5.696336 3.938052
 [325] 3.936697 4.918489 3.186897 4.079969 3.646024 3.961033 4.759285 2.234773
4.062160 4.070986 3.688917 4.716438
 [337] 4.996449 3.130898 5.164714 3.464073 3.888947 4.160782 4.293231 4.314679
3.051081 3.647830 3.473899 4.804839
 [349] 4.269601 4.175879 4.149234 3.420257 3.986099 3.968768 4.730603 3.991303
3.360730 3.555299 4.222154 4.526108
 [361] 4.940802 4.242847 4.368013 2.717403 3.998216 3.526120 3.810868 4.525012
```

```
3.748564 4.659227 4.895136 3.770841
 [373] 3.106395 4.899525 3.627288 3.466213 3.869709 5.008112 2.818539 3.545297
4.757301 3.750602 3.992973 3.969066
 [385] 3.849136 4.099241 3.164204 4.513799 2.994653 4.783060 4.313905 2.279741
3.832864 3.368467 3.952375 3.602333
 [397] 3.774363 3.694523 3.845826 3.957448 3.141176 3.661604 3.704950 3.765101
4.188203 3.091955 3.929200 3.621423
 [409] 2.902573 3.911925 3.971486 4.449271 4.348190 3.395538 4.503119 3.395053
3.257923 2.479215 3.797880 4.388664
 [421] 3.101921 3.588636 3.113177 4.185222 2.719249 4.728657 4.814387 3.306252
3.707309 4.292747 3.880112 3.036038
 [433] 3.781337 3.910804 4.882995 3.510848 4.338974 4.197290 4.615131 3.965732
3.991142 3.514648 4.136936 3.070112
 [445] 3.250724 3.956619 3.482234 3.525254 3.904608 3.505035 3.257901 2.932981
3.833793 3.397565 3.994993 3.454244
 [457] 3.119021 4.206480 3.466747 3.203230 4.680901 4.705949 3.519654 5.107239
3.857458 3.472450 3.671587 5.152120
 [469] 3.823618 3.831148 3.283611 3.477328 4.774353 3.926887 3.658222 4.232465
4.205393 3.830618 3.822011 3.168703
 [481] 3.931853 3.233244 4.066568 3.766247 4.257958 2.771094 2.644177 4.417283
4.278973 3.201379 4.358161 4.387809
 [493] 3.859104 3.403010 3.151900 4.719428 4.355332 4.709170 2.841260 3.493771
4.184976 5.006693 3.667198 3.933286
 [505] 4.126003 4.382708 4.349247 3.503336 3.845774 4.943080 4.819690 3.408273
3.052148 3.334267 2.914517 4.515140
 [517] 3.257788 3.895021 4.189081 4.589734 5.296177 4.169119 4.144541 4.647262
3.212481 2.380122 5.113155 3.248593
 [529] 4.381787 3.479459 4.733711 3.933306 4.018856 3.730739 3.401164 4.215087
3.506123 4.644768 4.119083 3.739792
 [541] 4.760921 3.172994 4.073991 3.782601 4.595033 3.459370 3.661793 4.608928
2.666837 3.457061 3.819071 2.815847
 [553] 3.427411 4.516295 3.562463 3.288571 5.005015 4.957061 4.213014 2.991828
3.444546 4.234174 3.754741 4.070104
 [565] 3.777984 3.193496 3.956240 5.318497 5.254254 4.099397 3.712526 4.385468
3.956590 4.029743 3.705058 4.470682
 [577] 3.223379 3.248369 3.435667 4.243596 3.935353 3.614808 5.398563 3.272414
```

4.087884 3.732856 4.170477 3.186911
[589] 4.489737 4.065118 3.076061 3.759963 4.277146 3.673840 5.621061 4.313148
4.709170 3.616043 2.512365 3.413684
[601] 4.347829 3.139247 3.655763 4.027120 5.038062 4.287248 4.613550 4.322044
4.148320 2.833577 4.494212 3.722937
[613] 3.145632 3.331005 4.468689 3.733730 4.071974 5.448839 4.327752 4.810466
3.714213 3.742962 4.291976 4.201570
[625] 3.974872 4.532858 4.575511 3.495469 5.124978 3.293997 3.500916 5.174043
4.735397 3.652968 4.752117 3.257181
[637] 4.078492 4.395299 3.373753 4.070508 4.137553 2.709572 4.208208 4.050121
2.593086 3.992512 3.619460 3.626971
[649] 4.379538 3.906402 4.059711 4.198903 3.018627 4.294360 3.186734 4.620376
4.081790 5.191752 3.793704 4.409974
[661] 3.264193 3.317725 3.794766 4.068924 4.914662 4.043045 2.989785 5.191062
2.767956 3.968788 3.096503 4.701878
[673] 5.022790 5.276950 4.290232 3.570496 3.702393 4.312176 3.704353 4.380643
3.715562 3.317745 4.129821 4.412247
[685] 3.708587 3.384499 4.117653 3.392566 4.443056 3.915159 4.984407 3.949258
3.150869 4.946258 4.383171 3.597955
[697] 3.560713 3.905895 3.201512 2.931565 3.155798 3.254789 2.808027 2.988380
4.352797 4.598697 3.810320 3.645012
[709] 3.630710 3.212461 3.654493 4.552686 3.525377 3.151716 5.540824 4.059804
3.440187 2.948219 5.409585 4.984821
[721] 4.983255 5.091542 4.523082 4.189460 3.979228 3.793193 2.801287 3.190868
3.683470 3.393209 3.502879 3.813433
[733] 3.322845 4.543912 3.625178 3.514782 2.470164 2.902386 3.606959 3.717947
3.364596 3.706377 4.282323 4.373305
[745] 4.132693 3.925568 3.918517 4.421722 3.462424 3.851556 3.866939 3.337001
3.415556 3.957232 3.731037 3.536487
[757] 4.892131 5.304381 4.058929 4.227972 3.517120 4.437163 4.569652 4.770237
3.898950 3.915921 5.031453 3.463853
[769] 3.622633 3.651432 3.980062 4.120601 4.256820 3.142372 4.927291 4.418804
3.608486 4.750148 3.969210 2.932178
[781] 3.364703 4.501698 3.347466 3.975535 3.808277 3.916384 4.185442 3.972009
4.297009 4.288616 5.192981 4.559794
[793] 4.356828 3.364927 3.722885 3.156921 4.475804 4.171862 5.656088 3.249289

```
4.132085 4.190634 5.439627 3.482163
 [805] 4.271590 3.331176 4.132566 3.553451 3.574015 3.575982 4.518817 3.380621
3.948162 3.291496 4.007597 4.583726
 [817] 5.593824 3.731035 4.266460 3.682447 3.856660 3.380273 4.270305 4.019036
4.514068 3.636911 3.385229 3.759536
 [829] 3.309228 2.549239 3.720356 2.784846 3.963564 4.250329 4.386511 4.762335
4.589168 5.103193 4.514068 3.625905
 [841] 2.628187 4.097688 3.663452 3.654744 3.100371 3.725289 3.041542 3.308143
4.005035 2.906012 3.896926 4.164573
 [853] 4.058926 3.820280 3.977771 3.451624 3.625633 3.673793 3.433519 3.611486
3.663308 2.633778 4.466967 4.425959
 [865] 3.846736 4.581320 5.880800 3.733507 3.834016 4.022110 4.502745 4.387981
3.772412 4.711367 4.232267 5.508991
 [877] 4.167995 3.901514 3.802471 3.663430 4.438512 3.160628 3.751017 3.795948
2.707722 3.743355 3.542926 5.562813
 [889] 4.254704 5.354228 4.222571 3.411958 4.374058 3.697865 3.384087 3.998711
3.093068 4.638691 4.441904 5.123270
 [901] 4.491059 3.009537 3.553579 3.161269 4.450970 4.151870 4.141726 3.699384
3.335310 4.503478 3.291635 5.182784
 [913] 4.016789 3.133504 4.071760 4.697639 3.356086 4.639293 3.951242 2.939686
3.136348 4.497698 3.565500 2.777639
 [925] 3.530261 3.430091 4.890037 3.965842 3.756235 4.498402 3.768037 4.234591
4.259612 4.699867 4.049181 4.713610
 [937] 3.988199 3.608631 3.691669 4.119184 3.577530 3.637749 4.471358 4.106001
5.048490 3.652862 3.799667 4.115542
 [949] 4.694384 4.160618 4.142260 3.417824 3.190288 3.436409 3.839280 4.991734
5.330359 3.131379 4.932893 4.007258
 [961] 2.324301 3.878188 3.655520 4.516334 2.913999 3.441925 3.736924 4.755310
4.555632 2.807766 3.545576 4.203148
 [973] 3.561079 3.005226 3.486367 3.695726 3.182825 4.086555 5.023725 3.309692
3.925438 3.646976 4.400564 4.733336
 [985] 3.727769 4.039062 2.890445 3.779200 2.964567 3.803391 5.098732 3.059499
3.238146 4.267006 3.426818 5.034179
 [997] 3.505985 5.284166 3.943432 4.360186
 [ reached getOption("max.print") -- omitted 39000 entries ]
```

```
$lp__
    [1]  13.500603  12.087272  13.690847  13.807967  13.052964  13.674804  13.208075
13.445140 13.809911 13.617638 13.681502
   [12]  12.446150  13.493671  13.529520  13.781269  13.795904  13.114082  12.912210
13.035023 13.786129 13.673501 11.659479
   [23]  13.821623  13.774404  13.562871  13.590370  13.782472  13.774154  13.820904
13.101519 13.710429 13.349908 13.668902
   [34]  13.586377  13.730044  13.820944  13.747783  13.720137  13.621053  13.793685
13.415598 11.978178 13.345678 13.808046
   [45]  13.627166  13.810149  13.789708  13.250961  13.806507  13.818048  13.001875
13.822490 13.113969 11.248926 13.640224
   [56]  11.974212  13.595872  13.779333  13.795701  13.760823  13.364064  13.807700
11.321886 13.172885 13.702461 13.795556
   [67]  13.262150  13.667446  12.216571  13.336100  13.821446  13.788886  13.719062
13.745783 12.931987 13.598502 13.702484
   [78]  11.850280  12.923753  13.818946   9.340687  13.088637  13.241012  13.790357
13.810280 13.705428 13.619492 13.815996
   [89]  13.638730  13.654639  13.642435  11.145304  13.513678  13.653656  13.679849
13.405041 11.226354 11.841548 13.316369
  [100]  13.450002  13.814146  13.769912  13.644496  12.983124  13.742871  13.514040
13.452624 13.776649 12.476120 13.143297
  [111]  13.685954  13.721448  13.794962  13.609229  13.091397  13.821898  13.780864
13.822009 13.446059 13.602631 13.640379
  [122]  12.017001  13.814476  13.821451  13.485586  12.605580  12.815946  13.807581
12.512308 13.759179 13.629072 12.261754
  [133]  13.626961  13.732957  13.815133  13.679202  13.752308  13.563424  13.654077
13.801512 13.192529 13.670182 13.539910
  [144]  13.822244  13.618712  13.718656  13.817801  13.797189  13.401795  13.666766
13.694370 13.144158 13.630396 13.762022
  [155]  13.595430  13.685502  13.772725  13.734631  13.813811  13.740872  13.545616
13.817680 11.712987 13.351525 13.741629
  [166]  12.873197  13.622434  13.139997  12.223813  12.587100  13.813373  13.512038
12.943207 13.789250 10.987923 13.309420
  [177]  12.817431  13.248079  12.512412  11.647576  13.535681  13.199066  13.820487
13.745488 13.204322 13.709350 13.816911
  [188]  12.406008  13.413016  13.731286  13.608167  13.717608  13.770049  12.421089
```

13.504158 12.393656 13.798533 12.237499
[199] 13.691477 13.762605 13.795253 13.580759 12.589980 13.796057 13.179092
13.499819 13.783661 13.718834 13.765498
[210] 13.771109 13.122032 12.862490 13.787569 13.358950 13.610022 13.585907
13.784401 13.774959 13.766071 13.817885
[221] 13.805234 12.684057 11.468545 13.705113 13.645241 13.710390 13.774374
13.745597 13.737512 13.792834 13.589681
[232] 13.749687 13.811452 13.817150 13.234477 11.162193 13.371756 13.703141
13.749568 13.361194 13.810553 12.771543
[243] 13.505334 13.778893 13.723981 13.164598 13.794361 13.758867 13.442897
13.820167 13.265118 13.631862 13.581333
[254] 13.819375 13.123182 13.603155 13.769212 13.692753 13.820677 12.580387
13.605382 13.361210 13.615457 13.036036
[265] 13.635487 11.667258 13.822407 13.550825 10.876802 13.800569 13.816301
10.258992 13.787331 13.810711 13.714420
[276] 13.042564 13.821840 13.724162 13.660655 12.823829 13.819846 13.698416
13.180495 13.193952 13.555309 13.447516
[287] 13.596656 12.599590 13.386865 13.383204 12.686297 10.838887 13.659699
13.096034 13.818358 13.810529 13.783399
[298] 12.307067 13.084246 13.669883 12.994097 13.789387 13.685670 13.663649
13.654558 13.404853 13.782755 12.270819
[309] 12.266312 13.786186 13.665912 13.770462 13.817315 13.790631 13.773382
13.539447 13.239030 13.705854 13.389880
[320] 13.822294 13.152921 13.793454 10.714191 13.822309 13.822350 12.738850
13.020107 13.792696 13.717342 13.820900
[331] 13.040835 8.566242 13.799105 13.796023 13.747732 13.114697 12.575740
12.883097 12.191377 13.525950 13.820790
[342] 13.754259 13.659310 13.640337 12.665779 13.718730 13.538975 12.958804
13.679071 13.745410 13.760678 13.463966
[353] 13.817847 13.820126 13.090635 13.817017 13.369316 13.634867 13.715086
13.402614 12.693162 13.699985 13.588953
[364] 11.445359 13.815811 13.602928 13.805335 13.404075 13.781132 13.208234
12.785795 13.791021 12.819187 12.777040
[375] 13.702357 13.528813 13.818486 12.550505 11.872343 13.624222 13.044326
13.782094 13.816737 13.820093 13.814941
[386] 13.784914 12.966083 13.418894 12.495847 12.998469 13.641039 8.902352

```
13.811338 13.382312 13.821587 13.680758
 [397]  13.792458  13.751306  13.814266  13.821207  12.909182  13.729007  13.757711
13.788606 13.737803 12.780369 13.822493
 [408]  13.697450  12.188053  13.822275  13.819818  13.499404  13.608746  13.426143
13.432782 13.425379 13.176332 10.214233
 [419]  13.801173  13.567470  12.807251  13.668098  12.837121  13.739674  11.453634
13.093961 12.941159 13.271817 13.759118
 [430]  13.659726  13.819852  12.621824  13.795202  13.822234  12.809850  13.585135
13.617669 13.731975 13.276281 13.820447
 [441]  13.817044  13.589632  13.767176  12.720008  13.161369  13.821274  13.549776
13.601940 13.821951 13.578166 13.176287
 [452]  12.293992  13.811563  13.429322  13.816388  13.512602  12.852423  13.725893
13.529526 13.057743 13.173490 13.132284
 [463]  13.595486  12.327509  13.816510  13.537074  13.736104  12.221629  13.808974
13.810917 13.228153 13.543445 13.014097
 [474]  13.822508  13.726535  13.707679  13.726622  13.810785  13.808540  12.976954
13.822459 13.124225 13.797589 13.789095
 [485]  13.688364  11.678638  11.102076  13.536254  13.671377  13.053529  13.598892
13.568377 13.816799 13.437796 12.935944
 [496]  13.109647  13.601710  13.126904  11.961100  13.564352  13.739827  12.553586
13.733020 13.822433 13.772661 13.573756
 [507]  13.607712  13.576108  13.814255  12.688452  12.931292  13.445889  12.668860
13.323276 12.230179 13.417135 13.176053
 [518]  13.821316  13.737248  13.313843  11.861838  13.749436  13.763199  13.227050
13.078601  9.597554 12.313725 13.156902
 [529]  13.574722  13.546205  13.085309  13.822432  13.811507  13.772214  13.434935
13.720026 13.579479 13.230939 13.775990
 [540]  13.776855  13.037951  12.987250  13.794932  13.795684  13.306106  13.519603
13.729144 13.285566 11.211486 13.516460
 [551]  13.807727  11.861657  13.474526  13.415617  13.642299  13.237878  12.557226
12.659365 13.721454 12.486973 13.499119
 [562]  13.706429  13.784013  13.796340  13.793899  13.035438  13.821304  11.803445
11.969623 13.784847 13.762169 13.570851
 [573]  13.821276  13.808817  13.757777  13.473598  13.102755  13.156431  13.486498
13.699420 13.822386 13.691791 11.588314
 [584]  13.205865  13.789606  13.773320  13.748636  13.020138  13.449870  13.798093
```

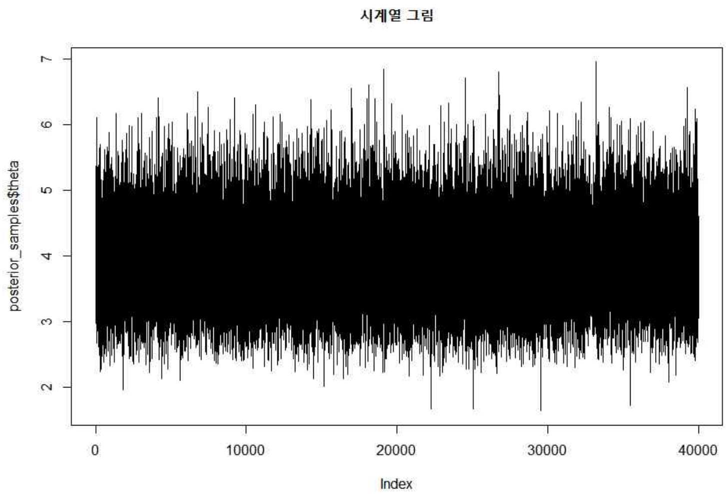
```
12.736644 13.786365 13.672892 13.737665
 [595]  10.945785  13.641724  13.126904  13.692857  10.406077  13.454112  13.609099
12.904318 13.724717 13.809491 12.484729
 [606]  13.664427  13.278654  13.633598  13.761172  11.931375  13.444195  13.768027
12.920356 13.317429 13.476038 13.773773
 [617]  13.795667  11.448784  13.628295  12.948425  13.763140  13.778426  13.660389
13.729166 13.819409 13.393562 13.334352
 [628]  13.566461  12.286021  13.248415  13.573162  12.168818  13.082412  13.722629
13.053415 13.174797 13.793257 13.560381
 [639]  13.391069  13.796195  13.766859  11.410057  13.724729  13.803007  10.844597
13.816815 13.695784 13.702095 13.577072
 [650]  13.822043  13.799927  13.730921  12.569736  13.658335  13.019726  13.268375
13.791998 12.125644 13.799737 13.544384
 [661]  13.189206  13.293232  13.800106  13.796760  12.746604  13.805138  12.480535
12.127335 11.665419 13.820123 12.792687
 [672]  13.139057  12.518443  11.911578  13.661884  13.650443  13.756169  13.642602
13.757353 13.575918 13.763910 13.293269
 [683]  13.770777  13.541867  13.759872  13.408575  13.776663  13.421454  13.506724
13.822374 12.601567 13.821789 12.933392
 [694]  12.681870  13.573269  13.676774  13.640499  13.822018  13.053833  12.289156
12.945558 13.169842 11.830412 12.476096
 [705]  13.604219  13.300724  13.805169  13.716559  13.705172  13.078557  13.723772
13.366467 13.602080 12.935487 11.184866
 [716]  13.799896  13.492955  12.345472  11.558017  12.600683  12.604027  12.363824
13.406643 13.737008 13.818840 13.799557
 [727]  11.803230  13.029351  13.744172  13.422470  13.575553  13.806103  13.302636
13.378550 13.700604 13.589790 10.160627
 [738]  12.187388  13.684905  13.765258  13.375836  13.758564  13.668581  13.583531
13.769338 13.822511 13.822449 13.531263
 [749]  13.523732  13.815416  13.818074  13.328147  13.456934  13.821225  13.772370
13.614584 12.791772 11.840457 13.800186
 [760]  13.710935  13.592533  13.513594  13.342690  13.021440  13.821605  13.822394
12.499364 13.525655 13.698471 13.721472
 [771]  13.818726  13.775269  13.689256  12.912188  12.720923  13.534548  13.686259
13.056855 13.820077 12.291251 13.376016
 [782]  13.434614  13.346548  13.819325  13.804542  13.822405  13.739537  13.819757
```

```
13.656039 13.663263 12.122631 13.356573
 [793]  13.600222  13.376392  13.767998  12.948315  13.467290  13.747814  10.838887
13.158363 13.769644 13.736262 11.474603
 [804]  13.549684  13.677454  13.317736  13.769402  13.632924  13.653947  13.655890
13.412292 13.402304 13.821854 13.243572
 [815]  13.813984  13.322553  11.027855  13.772369  13.681607  13.743493  13.816368
13.401738 13.678500 13.811465 13.418541
 [826]  13.710185  13.409749  13.786176  13.277416  10.611335  13.766604  11.735963
13.820663 13.694293 13.569750 13.035455
 [837]  13.314667  12.336904  13.418541  13.701210  11.023117  13.785574  13.730342
13.723959 12.803097 13.769307 12.638021
 [848]  13.275379  13.814505  12.200250  13.821461  13.752086  13.800187  13.808064
13.819036 13.508990 13.700983 13.737633
 [859]  13.483405  13.688899  13.730239  11.050888  13.478141  13.526462  13.814454
13.326022 10.117975 13.773658 13.811617
 [870]  13.810733  13.433265  13.568194  13.791666  13.123224  13.707824  11.277446
13.750095 13.821772 13.802697 13.730327
 [881]  13.512027  12.957385  13.782289  13.800514  11.401667  13.778618  13.621652
11.120157 13.690908 11.708525 13.714790
 [892]  13.451499  13.582756  13.753393  13.407911  13.815720  12.783392  13.240368
13.508073 12.290036 13.448198 12.542015
 [903]  13.633059  12.958949  13.497390  13.759239  13.764686  13.754331  13.325138
13.432319 13.243840 12.147567 13.811985
 [914]  12.889752  13.795744  13.146079  13.361415  13.239438  13.821663  12.316775
12.896982 13.439746 13.645402 11.706041
 [925]  13.607625  13.478438  12.795927  13.820436  13.784694  13.438845  13.789853
13.706123 13.687062 13.142393 13.803297
 [936]  13.119456  13.817521  13.686387  13.749497  13.775942  13.657410  13.710853
13.472769 13.781977 12.461498 13.722550
 [947]  13.801773  13.777649  13.151449  13.754352  13.764406  13.460335  13.028005
13.487563 13.812844 12.585879 11.772130
 [958]  12.884327  12.709448  13.814054   9.220104  13.819621  13.724537  13.415565
12.228368 13.495420 13.775410 13.047823
 [969]  13.362378  11.829364  13.624523  13.728120  13.640876  12.528745  13.555046
13.752061 13.010562 13.790135 12.516391
 [980]  13.278287  13.822511  13.718075  13.554693  13.085951  13.770640  13.806285
```

```
12.144592 13.794375 12.399521 13.802995
 [991] 12.347236 12.689956 13.134759 13.681167 13.473658 12.493335 13.579312
11.892974 13.822099 13.596865
 [ reached getOption("max.print") -- omitted 39000 entries ]
```

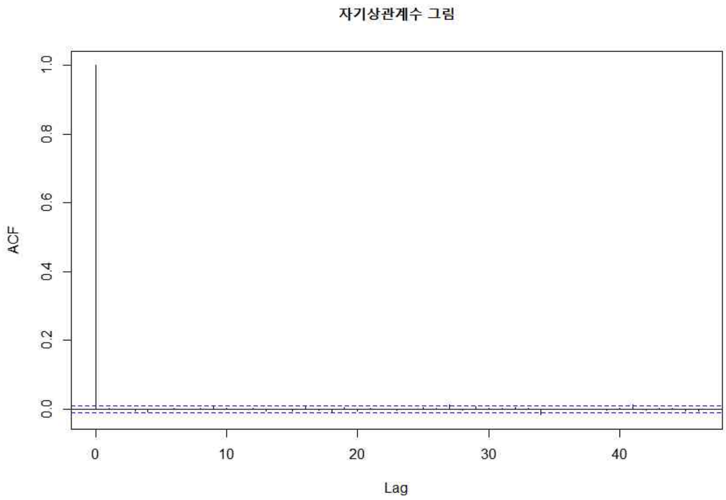
(b)

```
# 시계열 그림
plot(posterior_samples$theta, type = "l", main = "시계열 그림")
```



- 각 체인이 3~5 사이의 특정범위에서 집중적으로 변동

```
# 자기상관계수 그림
acf(posterior_samples$theta, main = "자기상관계수 그림")
```



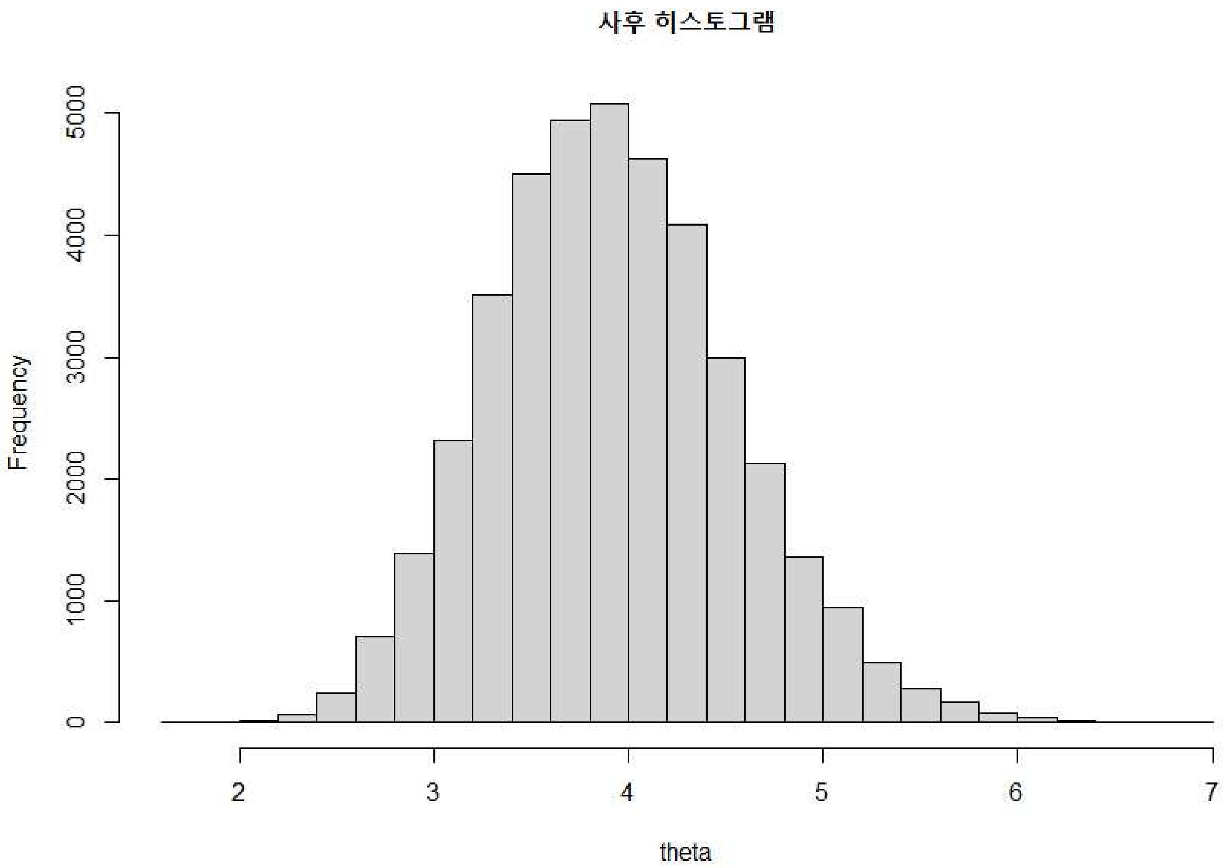
- 자기상관이 0에 가까워지면서 체인이 독립적으로 잘 섞이고 있음

```
# Stan 결과를 각 체인별로 `mcmc` 객체로 변환
# 체인별 추출 후 `coda::mcmc` 객체로 변환
chains <- extract(fit, permuted = FALSE)
mcmc_chains <- lapply(1:dim(chains)[2], function(chain) mcmc(as.matrix(chains[, chain, ])))

# `mcmc.list` 객체로 결합
mcmc_list <- mcmc.list(mcmc_chains)

# Gelman-Rubin 진단
gelman_diag <- gelman.diag(mcmc_list)
print(gelman_diag)

Potential scale reduction factors:

      Point est. Upper C.I.
theta          1          1
lp__           1          1

Multivariate psrf

1
```

=> Point estimate와 Upper C.I.가 모두 1: 이는 체인이 수렴했음을 나타냄. 일반적으로 PSRF 값이 1에 가까울수록, 특히 1.1 이하일 때 체인이 수렴했다고 간주할 수 있음.
여기서는 theta와 lp__ 두 변수 모두 1로 나타나므로 수렴한 것으로 판단할 수 있음.
Multivariate PSRF 값이 1: 다변량 진단에서도 체인들이 수렴했음을 보여줌.
따라서, 이 결과에 따르면 마르코프 체인이 수렴했다고 결론지을 수 있음.

(c)

```
# 히스토그램
hist(posterior_samples$theta, breaks = 30, main = "사후 히스토그램", xlab = "theta")
```



(d)

```
 # 통계량 계산
 posterior_mean <- mean(posterior_samples$theta)
 posterior_sd <- sd(posterior_samples$theta)
 credible_interval <- quantile(posterior_samples$theta, probs = c(0.025, 0.975))
 print(list(mean = posterior_mean, sd = posterior_sd, CI = credible_interval))
$mean
[1] 3.920841

$sd
[1] 0.6242959

$CI
    2.5%     97.5%
2.789188 5.227749
```

3.

(a)

```
# 필요한 패키지 로드
library(rstan)
library(coda)

# 데이터
data_list <- list(
  N = 10,
  x = c(1, 2, 1, 0, 1, 0, 0, 2, 3, 3),
  y = c(0, 1, 0, 0, 0, 2, 1, 2, 3, 1)

# Stan 모델 정의
stan_model <- "
data {
  int<lower=0> N;
  int<lower=0> x[N];
  int<lower=0> y[N];
}parameters {
  real<lower=0> theta1;
  real<lower=0> theta2;
}model {
  theta1 ~ exponential(1);
  theta2 ~ exponential(1);
  x ~ poisson(theta1);
  y ~ poisson(theta2);
}generated quantities {
  real diff = theta1 - theta2;
}"
# Stan 모델 컴파일 및 샘플링
fit <- stan(model_code = stan_model, data = data_list, iter = 15000, warmup = 5000,
chains = 4)
SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 1).
Chain 1:
Chain 1: Gradient evaluation took 2.6e-05 seconds
Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.26
seconds.
Chain 1: Adjust your expectations accordingly!
```

```
Chain 1:
Chain 1:
Chain 1: Iteration:     1 / 15000 [  0%]  (Warmup)
Chain 1: Iteration:  1500 / 15000 [ 10%]  (Warmup)
Chain 1: Iteration:  3000 / 15000 [ 20%]  (Warmup)
Chain 1: Iteration:  4500 / 15000 [ 30%]  (Warmup)
Chain 1: Iteration:  5001 / 15000 [ 33%]  (Sampling)
Chain 1: Iteration:  6500 / 15000 [ 43%]  (Sampling)
Chain 1: Iteration:  8000 / 15000 [ 53%]  (Sampling)
Chain 1: Iteration:  9500 / 15000 [ 63%]  (Sampling)
Chain 1: Iteration: 11000 / 15000 [ 73%]  (Sampling)
Chain 1: Iteration: 12500 / 15000 [ 83%]  (Sampling)
Chain 1: Iteration: 14000 / 15000 [ 93%]  (Sampling)
Chain 1: Iteration: 15000 / 15000 [100%]  (Sampling)
Chain 1:
Chain 1:  Elapsed Time: 0.072 seconds (Warm-up)
Chain 1:                0.161 seconds (Sampling)
Chain 1:                0.233 seconds (Total)
Chain 1:

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 2).
Chain 2:
Chain 2: Gradient evaluation took 4e-06 seconds
Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.04 seconds.
Chain 2: Adjust your expectations accordingly!
Chain 2:
Chain 2:
Chain 2: Iteration:     1 / 15000 [  0%]  (Warmup)
Chain 2: Iteration:  1500 / 15000 [ 10%]  (Warmup)
Chain 2: Iteration:  3000 / 15000 [ 20%]  (Warmup)
Chain 2: Iteration:  4500 / 15000 [ 30%]  (Warmup)
Chain 2: Iteration:  5001 / 15000 [ 33%]  (Sampling)
Chain 2: Iteration:  6500 / 15000 [ 43%]  (Sampling)
Chain 2: Iteration:  8000 / 15000 [ 53%]  (Sampling)
Chain 2: Iteration:  9500 / 15000 [ 63%]  (Sampling)
Chain 2: Iteration: 11000 / 15000 [ 73%]  (Sampling)
Chain 2: Iteration: 12500 / 15000 [ 83%]  (Sampling)
```

```
Chain 2: Iteration: 14000 / 15000 [ 93%]  (Sampling)
Chain 2: Iteration: 15000 / 15000 [100%]  (Sampling)
Chain 2: 
Chain 2:  Elapsed Time: 0.083 seconds (Warm-up)
Chain 2:                0.22 seconds (Sampling)
Chain 2:                0.303 seconds (Total)
Chain 2: 

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 3).
Chain 3: 
Chain 3: Gradient evaluation took 6e-06 seconds
Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.06 seconds.
Chain 3: Adjust your expectations accordingly!
Chain 3: 
Chain 3: 
Chain 3: Iteration:     1 / 15000 [  0%]  (Warmup)
Chain 3: Iteration:  1500 / 15000 [ 10%]  (Warmup)
Chain 3: Iteration:  3000 / 15000 [ 20%]  (Warmup)
Chain 3: Iteration:  4500 / 15000 [ 30%]  (Warmup)
Chain 3: Iteration:  5001 / 15000 [ 33%]  (Sampling)
Chain 3: Iteration:  6500 / 15000 [ 43%]  (Sampling)
Chain 3: Iteration:  8000 / 15000 [ 53%]  (Sampling)
Chain 3: Iteration:  9500 / 15000 [ 63%]  (Sampling)
Chain 3: Iteration: 11000 / 15000 [ 73%]  (Sampling)
Chain 3: Iteration: 12500 / 15000 [ 83%]  (Sampling)
Chain 3: Iteration: 14000 / 15000 [ 93%]  (Sampling)
Chain 3: Iteration: 15000 / 15000 [100%]  (Sampling)
Chain 3: 
Chain 3:  Elapsed Time: 0.089 seconds (Warm-up)
Chain 3:                0.228 seconds (Sampling)
Chain 3:                0.317 seconds (Total)
Chain 3: 

SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 4).
Chain 4: 
Chain 4: Gradient evaluation took 5e-06 seconds
Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.05
```

```
seconds.
Chain 4: Adjust your expectations accordingly!
Chain 4:
Chain 4:
Chain 4: Iteration:     1 / 15000 [  0%]  (Warmup)
Chain 4: Iteration:  1500 / 15000 [ 10%]  (Warmup)
Chain 4: Iteration:  3000 / 15000 [ 20%]  (Warmup)
Chain 4: Iteration:  4500 / 15000 [ 30%]  (Warmup)
Chain 4: Iteration:  5001 / 15000 [ 33%]  (Sampling)
Chain 4: Iteration:  6500 / 15000 [ 43%]  (Sampling)
Chain 4: Iteration:  8000 / 15000 [ 53%]  (Sampling)
Chain 4: Iteration:  9500 / 15000 [ 63%]  (Sampling)
Chain 4: Iteration: 11000 / 15000 [ 73%]  (Sampling)
Chain 4: Iteration: 12500 / 15000 [ 83%]  (Sampling)
Chain 4: Iteration: 14000 / 15000 [ 93%]  (Sampling)
Chain 4: Iteration: 15000 / 15000 [100%]  (Sampling)
Chain 4:
Chain 4:  Elapsed Time: 0.093 seconds (Warm-up)
Chain 4:                0.182 seconds (Sampling)
Chain 4:                0.275 seconds (Total)
Chain 4:

# 사후표본 추출
posterior_samples <- extract(fit)
print(posterior_samples)

$theta1
   [1] 1.2668631 0.9731773 1.8738613 1.6145547 1.8354870 0.8144773 1.5930936
0.7378882 1.3575845 1.3410056 1.1689440
  [12] 1.2336804 1.1203088 1.6578888 1.0450051 1.1271575 0.9089330 1.1546596
1.2478028 1.3841022 1.2977756 2.0417244
  [23] 1.0152349 1.1265221 1.0880690 1.0277726 1.5290695 1.0362229 1.2778133
0.7360973 1.6959353 1.6413268 1.0552767
  [34] 1.0407748 1.7079826 0.8244508 1.1569542 1.3878469 1.2206447 1.2374660
1.7206365 1.2977368 1.0812016 1.5701236
  [45] 0.7633375 1.8481562 0.9337708 1.4867301 1.5938793 0.8216459 1.0162456
1.3131414 1.7944806 1.6038961 1.2434235
  [56] 1.2205768 1.0368313 0.9738308 1.7835393 1.0649060 0.9742341 1.2951844
```

```
1.9752971 2.0535916 1.4725410 0.8367019
  [67]  0.9020565  1.3810321  0.6918161  1.1021510  1.3699514  1.2783553  1.3172772
0.8546070 0.7692252 0.9822657 1.7344512
  [78]  0.9756285  0.7837877  1.2926695  0.9921817  0.9853500  1.2044925  0.8584834
1.1673813 0.9761475 1.5254130 1.6402381
  [89]  0.8139369  0.7643350  1.7047254  1.4658326  1.0674242  1.3756989  0.9269461
0.8047735 1.1094800 0.8640304 1.2988338
 [100]  1.6831902  1.4718982  1.3873373  0.9438492  1.5471149  0.6305330  1.1750871
0.8556096 1.0016541 0.7286500 1.6538655
 [111]  1.1725288  1.3147777  1.7603476  1.3565938  1.5524007  0.8971626  1.1439511
1.4989646 1.5879278 1.8256709 0.8920750
 [122]  1.2879956  1.0002390  0.8753255  1.3989410  1.6412030  1.8073886  1.1609687
1.0853394 1.3899120 1.1355043 1.0317581
 [133]  0.8464954  1.1497645  1.5102181  1.1541059  1.1583752  1.3449541  1.5333905
0.8934996 1.4322828 1.0182181 0.6677236
 [144]  1.1411634  0.8803808  1.6967058  1.2226883  0.9610834  1.0420060  1.3558761
0.5743871 1.8492716 1.0339517 0.8602741
 [155]  1.4654681  1.3770381  1.6138800  0.7984075  1.3025373  1.2715635  1.8264112
1.3960996 1.4903779 1.3973127 1.0583331
 [166]  1.5023672  1.4833868  1.7346971  1.2176068  0.9985982  1.6928476  0.9974914
1.1003767 1.3882278 1.4461010 1.0249091
 [177]  1.1496560  0.9693457  1.4650279  1.9067315  1.0083759  1.8798123  1.0281270
1.3085237 1.5032613 1.0869458 1.0690169
 [188]  0.5908742  1.2564807  1.2573195  0.9735214  1.4197706  1.4852846  0.9055557
0.8533280 0.9143868 1.3811119 1.5659144
 [199]  1.5105953  1.3160177  1.0607685  1.6533563  1.0823224  1.9003441  1.0264459
1.7128509 1.5755438 1.2255076 1.4174318
 [210]  1.1309292  1.5279528  1.2193396  0.9923342  1.0099683  1.0672153  1.0434634
1.3586791 0.7562699 1.4892416 1.5954201
 [221]  1.1428017  1.1430759  1.6455899  1.9066765  0.8297477  1.3154849  1.3433508
1.3193633 0.9198219 1.1104257 1.2799011
 [232]  1.5116776  1.5942697  0.7909841  1.0631405  1.3049580  2.0067379  1.5182005
1.2313439 1.2764316 1.4664411 1.3993492
 [243]  1.1577785  0.8612199  1.3904996  1.2307459  1.5361676  1.3498612  1.5930258
1.7029133 0.7367539 1.3473252 1.6411764
 [254]  0.9772452  0.8267058  0.5582641  1.2083751  1.2940101  0.7087651  1.2420602
1.2604142 0.9568819 0.7597697 1.2854121
 [265]  1.3510281  0.9812110  1.2805927  1.2633584  1.1287906  0.6247318  1.1009271
```

```
1.4841539 0.9641299 0.8168958 1.9406858
 [276] 1.4460286 0.9733915 1.6207091 0.8789638 0.7391189 1.4353673 1.0212612
1.2667015 0.9207435 1.7781483 1.1663618
 [287] 1.6626237 1.2208478 1.4987740 1.5019021 1.2326714 0.7160716 1.2999233
0.7472101 1.5500502 1.4869978 1.0130906
 [298] 1.4694018 1.2616402 1.0108075 1.4821591 1.1232710 1.7707400 1.1015289
1.4152720 1.1851458 1.3530437 1.1270080
 [309] 1.3320771 0.9522912 1.2458900 1.2735199 1.6747507 1.4420705 1.3414170
1.6577216 1.0530065 1.4814919 2.0468739
 [320] 1.6250280 0.9787287 0.8606984 1.4121566 1.0648148 0.7854934 0.7137506
1.2230247 1.0117753 1.1385779 1.4418715
 [331] 1.2505501 1.1830374 0.7886758 1.3836399 1.5989696 1.0516980 1.5435950
1.7858259 1.5054620 0.9679237 1.0032862
 [342] 1.3125889 0.9460054 1.3823164 1.5157576 1.1796401 1.4154563 1.6869668
1.5519990 1.0560573 1.4477924 1.1392566
 [353] 1.3435563 1.1651717 1.7749558 1.4415598 2.2443655 1.3008893 1.5488470
1.3531116 1.7986917 1.4414823 1.2362519
 [364] 1.3427286 1.3981052 1.1067966 1.1796282 0.9526723 1.3918353 1.0156910
1.8972240 0.9931545 0.9908183 1.0673064
 [375] 1.2631761 1.7765178 1.3438857 1.1790422 1.1272530 0.9155022 1.6565961
1.0444137 1.7992090 1.3840575 1.1738074
 [386] 1.3786302 1.6541267 0.8397860 1.5497089 1.5095612 1.2947139 1.3669355
1.2746740 1.0147367 0.6040192 1.4734852
 [397] 1.2570173 1.2635011 1.7523562 1.4648502 1.0847772 1.1773552 0.8830981
1.7214231 1.7455768 0.7301367 1.4977912
 [408] 1.7967860 1.3688985 1.0626956 1.1007776 1.4114345 1.1147567 0.7983430
1.2193992 1.5009027 1.1670377 1.3081653
 [419] 0.9797883 0.9886584 1.5912510 1.4033665 1.1058464 0.8027547 0.5549735
1.0891153 1.3545069 0.9347992 1.3057646
 [430] 1.0517929 1.1857053 1.4098133 2.1818042 1.4336292 1.2051555 0.9366623
0.8020653 1.5427435 1.3561358 1.4225569
 [441] 1.0773362 1.3175601 1.8514554 0.9463269 0.5432315 0.8415280 0.8977168
1.0590932 0.9761054 0.7635604 1.6543078
 [452] 1.3151155 0.8530370 1.8121109 0.7966441 0.7958343 1.3195879 1.4239193
0.9537243 0.7316521 1.5034208 0.6163449
 [463] 1.5404006 0.8820989 0.8340221 0.6424110 0.9738196 1.2404622 1.1966016
1.5658013 0.9699570 1.8806958 2.0178165
 [474] 1.5543780 1.5255197 1.1775628 1.6862738 1.5799214 1.0540423 0.9871264
```

```
1.5550556 0.8822928 1.0466085 1.2746579
 [485]  1.5812379  0.9014849  1.5098732  1.1425499  1.1296620  1.4962124  1.4528008
0.9248540 1.4378870 1.2588808 1.0001695
 [496]  0.9658172  1.6541328  1.0671043  1.5215362  1.5649935  0.8847135  1.2259228
1.0358863 1.1950242 1.3498612 1.1492609
 [507]  0.9832263  1.2914988  0.7991693  1.5272650  1.2833263  1.6770479  1.3715552
1.3128107 2.0591307 1.5045216 1.0404826
 [518]  0.6775137  1.1437908  1.0413158  0.8399461  0.7985234  0.8649404  0.9530991
1.1663582 1.0344871 1.4844422 0.8930064
 [529]  1.1783165  1.5661322  1.1272928  0.7551545  1.2013360  1.0608800  0.9862392
1.3306424 1.3949855 1.9497210 1.4530712
 [540]  1.4774107  0.9429638  1.3565479  1.6745297  2.0098927  1.1446048  0.8831230
1.8131072 0.9476348 1.0763501 1.2962483
 [551]  1.2070403  0.6875580  1.0154912  1.0072128  1.2309017  0.8117341  1.6721325
1.6328843 1.2995451 1.1821345 1.0512697
 [562]  2.0570166  1.0878037  1.4765972  1.4758303  1.3668078  1.0154706  1.6838215
0.9843729 0.7816411 1.0007259 0.9801194
 [573]  1.0195827  1.5284889  1.4255107  1.2539877  1.1050829  1.1679803  1.4976641
1.6812862 1.0606587 2.0661435 1.4572161
 [584]  0.9863520  1.9852884  1.2341489  1.7057601  1.4360555  0.9905068  0.8819872
0.9457496 0.8536976 1.5754570 1.1072550
 [595]  1.3530774  1.2320066  1.3837015  1.3777906  1.1522187  0.9887912  1.0887849
0.9661888 1.2296188 1.5008860 1.7394893
 [606]  1.2543533  0.9957488  1.0540168  1.3738162  1.3386577  1.2026288  1.7158215
0.7684411 1.9536925 1.4943866 1.2471226
 [617]  1.2863924  1.2059110  1.2260086  1.0104520  1.6681590  0.7292368  1.4768006
1.3947618 1.1319598 0.8570935 1.4132976
 [628]  0.9350954  0.8299793  1.0890067  1.6663901  1.4237498  1.1569714  1.6543170
1.1867226 1.5864516 1.1920460 1.8833938
 [639]  1.0656622  1.5310935  1.1603202  1.3222670  1.0564734  1.0784727  1.4121577
0.7850352 1.0219143 0.9325953 1.1603759
 [650]  1.3914518  1.5606724  0.6705584  1.0181835  1.5804377  0.8558101  1.4196499
0.8266446 1.4660186 1.4214752 1.0662330
 [661]  1.3010499  1.2697424  1.1217362  1.1258387  1.4485065  1.2461985  1.1150358
0.9368869 1.2076457 0.9809657 1.3234879
 [672]  1.3023244  1.1035856  0.8776993  1.1958317  0.9461841  1.6171964  1.6254291
0.9151044 1.0702503 1.1875002 1.3674356
 [683]  1.4629923  1.5518140  2.1124930  1.5121927  1.5739909  1.4481715  1.8562265
```

```
1.3492395 1.0992345 1.1947011 0.9248494
 [694] 1.9022401 0.7286872 0.6482025 1.0540152 0.6634171 1.5890340 1.5723402
0.5571107 1.0730229 1.0841618 1.1991478
 [705] 1.4013622 1.6691780 1.3452017 1.7984402 0.7926696 1.6784356 1.0461414
1.5871625 1.4734534 1.0707361 1.9389539
 [716] 1.6066217 2.0218260 1.8662416 1.0024107 1.1842160 1.0443257 1.5041497
1.5761574 0.9943835 1.3325903 1.6885261
 [727] 1.2772431 1.0716248 1.2205768 1.1991575 1.3016276 2.0302606 2.0264512
1.3756080 1.9616722 1.4748271 0.9768294
 [738] 1.3458601 1.3349425 1.2734480 1.5821248 1.2534131 1.7004069 1.7980691
0.7550481 1.4277450 1.1184070 1.4745087
 [749] 1.5134528 0.7980114 1.2574305 1.1704570 2.6721353 1.4328210 1.4090047
1.6222088 1.8816749 1.1551919 1.0801109
 [760] 1.2365819 0.9734981 1.0988048 1.3255984 1.6803755 1.4302350 1.2691002
1.1949643 1.2166780 1.1039936 1.1163610
 [771] 1.1074852 1.6686563 1.4795972 1.6001912 1.4589923 1.3371449 1.2114904
2.1185625 1.0110853 1.3339402 0.9282452
 [782] 1.5802496 1.2270310 1.3491615 1.0476980 1.2330798 0.7695244 0.7048809
1.1337617 0.6372615 1.6009913 1.3836766
 [793] 0.9993113 0.8471644 1.2104494 1.4397208 1.3542246 1.6386687 1.4575104
1.4078477 1.3743668 1.0062252 1.0873204
 [804] 1.2566855 1.5358196 1.1978043 1.2960730 1.3303424 1.9970951 1.8466826
1.3769983 1.9048363 1.0562117 0.8947158
 [815] 1.4525923 1.2198704 0.5252390 0.8719974 0.8496127 1.0814401 1.4334156
0.8220327 1.4309883 1.5840838 0.7845089
 [826] 1.7217188 1.8993846 1.5450606 1.5270825 1.1412793 1.5792084 1.4560603
1.9775643 0.7121189 0.9581869 1.1586060
 [837] 1.7106674 1.1071193 0.8492735 1.3788870 0.9512942 1.9636532 1.0937591
1.7161268 2.1050518 0.9832592 1.2400942
 [848] 1.2429077 1.0584102 1.2444394 0.9670490 0.8158672 1.6250681 1.3972503
1.0695690 1.0043268 1.2549703 1.1539712
 [859] 1.4045647 1.4353606 1.8643455 0.9464852 1.4906225 2.0440594 2.0989489
1.0502970 0.6478685 1.0431572 1.2676542
 [870] 1.1666852 0.9541429 1.5207823 0.9812380 1.9781152 1.3752815 0.9285099
0.6627766 1.3577603 1.6363634 1.3666335
 [881] 0.9373429 1.3498118 1.6779303 1.0580008 1.0637817 0.9367497 1.3437581
1.3598494 1.2662022 1.2799960 1.5304052
 [892] 1.0856351 1.3711481 1.4147310 0.6529828 1.5989279 1.2143785 1.5595202
```

```
1.1509733 1.3989458 1.2728482 1.2689225
 [903] 1.0762596 1.5978576 1.0879659 1.3803043 1.7619791 1.2372889 0.9279037
1.1411685 1.8614581 1.0202764 1.1896531
 [914] 1.7659186 0.9234074 1.7710456 0.7717874 0.9492829 1.3303051 1.0821085
0.9215945 1.4790611 1.3208970 1.7745790
 [925] 0.7472932 1.0760016 1.3972977 1.7510227 1.0948477 0.8308701 1.3423751
1.1896841 1.2506030 1.2482005 1.0713214
 [936] 1.0654408 1.5919686 1.7869501 1.5065139 1.8826270 1.3515513 1.1431329
1.6133117 1.2060824 1.3508777 0.8415191
 [947] 0.9525427 1.4597268 1.5889731 1.7696266 1.0881713 1.3164527 1.1844078
1.2898196 1.5071208 1.1372976 1.0578785
 [958] 1.2937026 1.0934788 1.7625975 1.4012230 1.0825887 1.2834583 1.5860446
0.8880887 1.3065783 0.9766463 1.5088792
 [969] 1.7156496 1.5692309 1.2120213 1.5957480 1.1062495 0.9705363 1.1350202
1.1372556 0.8781491 1.0534440 0.7739596
 [980] 1.3496500 1.7585879 1.7346150 1.1703072 1.4124715 1.4913921 1.1710857
2.0389477 1.1232172 1.4602068 0.6277074
 [991] 1.5479157 2.2243141 0.7902318 1.5129470 1.2413882 1.2425497 1.6718969
1.0174736 1.6125461 1.5744446
 [ reached getOption("max.print") -- omitted 39000 entries ]

$theta2
   [1] 0.7574787 0.9019248 1.6140401 0.6826475 1.2987035 1.6846625 0.8846178
1.1273901 0.6052151 0.8175929 1.0370796
  [12] 0.7250115 0.6201143 0.9638341 1.1946831 1.0120154 1.1742109 0.4308292
0.6758281 1.4631490 0.7942742 0.7079303
  [23] 0.9605838 0.4475407 0.9650541 0.8666344 0.8068350 0.7572347 1.1597645
1.3537567 0.7948142 1.2100811 0.8539531
  [34] 0.7775325 0.7825828 1.1911814 0.8775203 1.0790176 1.4782855 1.0056067
1.0274780 0.5641805 1.2708610 0.8315795
  [45] 0.9767902 0.9428179 0.8959226 0.9770281 0.8488108 0.9624689 0.6639200
0.8121500 1.2215395 1.3862647 1.3123339
  [56] 0.9385239 1.0806124 1.2845639 1.1051567 1.0717089 0.7444732 0.8721463
1.3502628 0.4713307 0.8220370 1.3247379
  [67] 0.7188717 1.5364054 1.2041528 1.5057417 0.7836189 0.8542025 1.1251696
0.9219722 1.2629677 0.6969987 0.7573880
  [78] 0.5975002 1.1544130 1.0628363 1.1815019 1.3297666 1.0726308 0.7537161
1.1649796 0.7154180 1.1738117 1.0524360
```

```
 [89]  0.6638515  1.1034402  0.8427689  1.2782339  1.7736273  0.8195676  0.6615385
1.4624158 1.1609976 0.9728148 1.2498981
 [100]  0.4922689  0.4112311  0.5938118  0.7820977  1.1698861  0.9601481  0.5909874
0.8994630 1.1607487 0.7377509 1.1504828
 [111]  0.9860574  1.0503782  1.1679786  1.4069732  1.0021612  1.6573116  0.7875432
0.7111623 0.7970460 1.0450461 0.8894574
 [122]  1.7314695  0.9565740  0.6637309  0.7038871  1.5061711  1.1220603  1.8112142
0.9331078 0.7163602 0.7774984 0.4372365
 [133]  0.8350729  0.5571861  0.8422096  0.5571324  0.6813465  1.1013662  0.7018966
0.4617098 1.2320633 0.5111048 1.0373041
 [144]  1.5322660  1.0932938  0.8722172  0.8296500  1.1559771  0.8787556  0.8614645
1.1189317 0.7908080 0.9041094 1.3986221
 [155]  0.8573720  1.4257153  0.7875680  1.2681798  0.7496663  1.3455198  1.0887893
0.7257922 1.2068327 1.2015279 0.8217356
 [166]  0.8290216  1.3411925  0.4086797  1.3247615  0.8333434  0.7241168  0.5882754
0.6972979 1.5844830 1.4134141 0.3331145
 [177]  1.0299308  0.6856888  1.1481808  0.7825275  1.0341452  0.9107293  1.1117210
0.9831921 0.9078841 1.1077216 0.7899310
 [188]  0.9225305  1.0840681  1.0310554  0.6927719  0.7232490  0.9421857  1.0409933
0.8029538 0.5665657 1.4260619 0.9379270
 [199]  0.6790784  0.6880532  1.4116339  1.2780206  0.8469062  1.2165364  1.0393059
1.0631434 1.4362071 0.7826490 1.3201966
 [210]  1.2554386  1.1402102  0.7864886  0.8240168  1.2221216  0.8181182  0.8235685
0.9620658 1.9509194 0.5714547 0.8907602
 [221]  0.8792798  0.7133296  0.8305852  0.5931210  0.8908382  0.9012233  0.6031832
1.4841477 0.5309247 0.7147877 0.6057721
 [232]  0.8613806  1.2797182  0.7538132  1.2976549  0.6812009  1.1775548  0.9412777
0.8001408 0.6148156 0.8485840 1.4181049
 [243]  1.1816138  1.2790535  2.2309221  0.4921677  1.3914010  0.8538829  0.9436604
1.1570994 0.8604975 0.9931980 0.7773669
 [254]  0.8529079  1.0090593  0.9030576  0.5822983  0.4012310  1.1280135  0.8076251
0.7893342 1.3267993 1.0897775 0.8488181
 [265]  1.2033412  0.7504390  0.6799867  1.1463842  0.8501967  1.3639220  0.8242759
0.9200746 0.8683445 1.0343782 0.5764895
 [276]  1.4278928  1.1782266  1.3010773  1.3606194  1.1211476  0.9605352  0.9871633
0.6566259 1.2445573 0.7120787 0.9587038
 [287]  0.8328785  0.8679164  1.2623573  0.6069127  0.4256288  0.9699244  0.5762878
1.5443254 1.3090895 0.9591989 0.9169753
```

```
 [298]  1.6052952  0.8868748  1.0907989  1.1995587  1.5095092  0.9287329  0.9034919
1.0007738 1.2100361 1.3572985 0.7871845
 [309]  0.7437668  0.8379776  0.6519377  1.2909306  1.2834255  0.6695293  0.7572301
1.1005358 0.4966486 0.8642215 0.9199521
 [320]  0.9786385  0.7763854  1.9076469  0.9739157  1.3454086  1.2666173  1.6320060
0.9775656 1.2345841 1.1728573 0.8449967
 [331]  1.3820908  1.3581344  0.9425675  0.9850124  1.6538331  0.6903439  1.6567313
0.6901684 1.0669635 1.5872728 0.6810576
 [342]  1.2274498  0.8489510  1.2534039  0.9382665  0.9632684  1.0223978  0.8472564
1.2158271 2.0197309 0.7084777 1.1130041
 [353]  1.0247246  0.8478247  0.8567176  0.7581916  0.9806792  0.6365452  0.9809446
0.8592031 0.9447538 1.2692542 1.1330644
 [364]  1.0700780  1.2859050  0.8924308  0.8415927  1.0600069  0.8252778  0.9968204
1.0525839 1.0181025 0.9823561 1.2050078
 [375]  0.5989211  0.8284369  1.3496768  0.5150648  0.7598310  0.6341412  0.5492639
0.6001263 1.3788047 0.5596973 1.1379471
 [386]  0.7826726  0.5853959  0.7465428  1.1022378  0.9800250  1.2458178  1.3321958
0.9724945 0.7342986 1.2588558 1.1195780
 [397]  1.2439976  0.7006914  1.0104557  0.2085463  0.6122375  0.6866174  0.9648419
0.7038393 0.7159149 0.6828027 0.7221544
 [408]  0.7180432  0.6376754  0.5340406  0.9625766  0.9084627  0.9140459  1.0628738
1.0278834 0.7223730 0.8699549 0.4101203
 [419]  0.8442294  1.4576950  1.2133025  1.2454927  0.7356208  0.9348872  0.9975924
0.7780214 0.7949195 1.6922407 1.0340110
 [430]  0.8929925  1.3433828  0.6464437  0.8433385  0.8987972  0.5730106  1.2900808
1.3509865 1.0456786 1.2179165 0.9595390
 [441]  0.9623834  1.8302631  1.3925087  1.0608854  0.7811606  1.2471388  0.4056084
0.6344413 1.2971643 1.6634641 0.8128204
 [452]  0.6734920  1.4045076  0.9724315  0.7431019  1.1042069  0.7438061  0.6650308
0.6772153 0.5956232 0.9071525 1.2884187
 [463]  0.9424961  0.8402049  0.5793695  1.5917015  1.1793999  0.9850653  1.0168848
1.3532151 0.6038980 1.1501718 1.4260488
 [474]  0.8621912  1.9044964  1.1761978  0.4718150  0.9344458  0.8936354  0.7889023
0.9725500 0.8881087 1.2036681 1.0487138
 [485]  1.0928220  0.9219689  0.8928506  0.7485250  0.8638046  1.0634318  0.9732955
1.3483488 1.2659012 0.4532882 0.7096507
 [496]  0.8407860  1.1048223  1.7187808  1.1063664  0.6483287  0.8906235  0.7540137
1.3714099 0.7643260 0.8538829 0.8851590
```

```
 [507]  1.9738426  0.7002265  1.1179550  0.5425592  0.9604809  0.8677430  1.3333574
1.2365593 1.5804333 0.6667509 0.7927137
 [518]  1.6474225  1.0261326  1.1300495  0.8968951  0.4284877  1.2704316  1.1582614
0.5486571 0.7174317 0.6821836 0.9996142
 [529]  0.5906496  0.6661056  0.8450735  0.8643440  0.9713551  0.9554051  1.2536501
0.6688874 1.2947461 0.7748627 0.8596122
 [540]  1.0424438  0.5788841  0.7914328  0.5223330  0.6915860  1.0563955  1.6826336
0.6230417 0.9540793 1.0121481 0.8699472
 [551]  1.5000260  0.9530249  0.9562739  1.1105026  1.0929138  0.4585768  0.6794035
1.3528846 1.6654153 0.9905680 1.3990740
 [562]  1.0588086  1.6383632  1.4119166  0.6693445  0.9187048  0.8848264  0.9985973
0.9107144 0.6947915 0.7130675 0.6474874
 [573]  0.4952394  0.8591609  0.6820655  1.0702178  0.6012253  0.8854683  0.8865626
0.8841901 0.6124014 1.0959698 1.3558776
 [584]  1.2725853  1.0074670  0.6452121  1.1555419  0.9403180  1.3901309  1.0601754
1.0810775 0.8784367 0.7675466 1.2159006
 [595]  0.8934292  0.8832584  0.4712532  0.6764847  1.4521061  0.6908702  0.5549289
0.9715454 0.7241427 1.4137822 0.5166278
 [606]  0.6081739  1.3352355  1.5227969  1.3376000  0.9187879  0.9723133  0.6483048
1.6699077 1.0098504 1.3472800 0.7803034
 [617]  1.4363646  1.7297263  1.8008976  1.0761621  1.1596460  1.1529787  1.1695249
0.8117184 0.9404558 1.4648943 0.9518659
 [628]  0.6180958  0.6204187  0.7739375  1.6847778  0.8421694  1.1467090  0.7637878
0.7504009 1.3964178 0.6408050 0.5931757
 [639]  1.0467183  0.6419261  0.5631956  1.0605870  0.9819580  0.8887193  0.8705930
1.0316676 0.9960116 1.2956994 1.0308586
 [650]  0.9336754  0.6904995  0.9112143  1.2497313  0.8377794  1.0019147  1.0879907
1.2638555 0.8901459 1.1867701 1.0267976
 [661]  0.8191767  1.0518165  0.8503729  0.9695825  1.1694404  0.9709655  1.1281952
1.1668638 1.1393703 1.7361772 0.8490960
 [672]  1.2397832  1.2610953  0.7790329  0.7102346  0.9985580  1.0445381  0.6406962
1.0600189 0.8179037 0.6807137 1.4253876
 [683]  1.3876157  0.7656162  1.1458040  1.1899073  1.1308855  0.7385961  1.0384865
0.8347130 0.5545028 1.0349232 0.7948715
 [694]  0.8931362  0.8066531  1.0474732  0.5957173  1.0696692  0.7666271  0.7002582
1.0208031 1.0974938 0.9071058 1.0642453
 [705]  0.8535607  1.7203032  1.0226821  0.9125418  1.2688094  0.8580910  1.1266829
1.5293348 0.5672212 1.1384473 1.3700969
```

```
 [716]  1.2093632  0.8395947  0.7916025  0.6543755  1.3680438  0.7419229  1.4709352
1.2402606 0.7449807 1.1335096 0.9360418
 [727]  0.7468293  0.7280449  0.9385239  1.3068051  0.7922201  1.5395856  0.7955877
0.9424961 0.8134874 0.7243651 0.7852629
 [738]  1.5129752  0.7547110  0.7985240  0.7342830  0.8863333  1.3057782  0.7438169
1.0073231 0.8799334 0.7568092 0.5672913
 [749]  0.8463708  0.6357544  1.1331992  1.2307404  0.6827363  0.8494120  1.0251371
1.0104609 0.7222247 1.0896602 0.5645844
 [760]  0.8032002  0.9977863  1.3921776  0.7435080  1.5577057  0.4931151  1.2269456
1.1961158 0.9306419 1.0188943 1.3349374
 [771]  1.1102745  0.9316341  0.8980450  0.7061033  0.9867651  0.7767794  0.7071876
0.6377543 0.7932560 0.9385308 0.7756970
 [782]  0.5812488  1.0209150  0.6514426  1.3293743  0.9540136  1.2229092  1.0726235
0.9454845 1.0350503 1.2325871 0.4100339
 [793]  1.6768950  1.0805586  1.3931553  1.0904636  1.3567092  0.6434153  0.8912321
0.5600142 0.9693168 1.3309525 0.9850647
 [804]  1.0299435  1.0020824  0.9282633  1.0113589  0.8234706  1.0628195  1.2520898
0.9101141 0.9628947 0.7046079 0.8219471
 [815]  0.6501450  1.4391686  0.7270753  1.0810154  0.7395239  0.7239067  0.6441750
1.3446063 0.8209725 1.1174399 1.2651716
 [826]  1.3709909  1.2662050  1.4643168  0.5302043  0.9484342  0.7845919  1.1925792
0.8045835 1.1156829 1.1946389 0.6704064
 [837]  0.8811277  0.8865820  1.3185038  0.8796388  0.9579819  0.7699191  0.9671802
0.8339726 0.6174497 1.2097031 1.0690659
 [848]  1.3229898  0.6243319  1.5180776  0.5179627  1.0692053  1.2427248  0.5574404
1.1104755 1.3704560 0.8584544 1.3026836
 [859]  0.7508828  1.2613865  0.9466501  1.0800143  1.0587082  1.1131976  1.5357806
1.0758674 0.7507169 0.7142753 1.4256563
 [870]  1.2885310  1.5727480  0.9470852  0.8476975  1.0246428  0.8481088  0.5723347
0.6567481 0.7843903 1.4249326 0.6052405
 [881]  0.8248510  0.7825559  1.2216951  1.1687616  1.2207839  0.9801218  1.5878678
0.9168197 0.9984367 0.9215908 0.9926214
 [892]  1.0998257  0.9525845  0.5798491  1.2046971  0.9108890  0.8558354  1.1949258
1.1864044 0.8189346 0.7383314 0.9745821
 [903]  1.2138336  0.9429906  1.1104592  0.6923653  0.8211921  0.6961996  1.3751153
0.8382070 1.0911780 0.4601651 1.6208234
 [914]  1.4060881  0.9318438  0.8066237  0.8229752  1.6892175  0.4385316  0.4713094
0.7370583 1.0034523 1.0697543 0.9404307
```

```
 [925]  1.0261807  1.6226833  1.2137951  1.2100588  1.7890775  1.1151031  0.6861135
1.1775602 1.0159166 1.0989668 0.9940323
 [936]  1.0502819  0.7788330  1.0811953  0.9468167  1.4799464  1.3297466  1.0297706
1.1235603 0.7715556 1.0369693 1.3091282
 [947]  0.8635260  0.8753085  1.0978174  0.7478644  0.6336379  1.2098409  1.1495614
0.9988079 0.5751173 0.8429591 1.0735952
 [958]  0.7209097  0.8025949  0.7725997  1.1546485  0.7383600  0.5618211  1.0382731
1.7648077 1.5220654 1.4418338 0.6434422
 [969]  1.0500036  1.0917082  0.6133952  0.8318700  1.7215216  1.1411049  0.7335624
0.7581218 1.1214615 1.0805279 0.7321482
 [980]  1.0804661  1.1896013  0.9286009  1.0775488  1.1235967  0.8887133  0.7926442
1.4323048 0.9045048 0.6495374 0.7117628
 [991]  1.4726247  0.9967177  0.8490409  0.8540966  1.4454481  0.8282162  0.8927566
0.8429840 1.3737715 0.7831965
 [ reached getOption("max.print") -- omitted 39000 entries ]

$diff
   [1]   0.509384394   0.071252425   0.259821236   0.931907226   0.536783517
-0.870185241  0.708475795 -0.389501892
   [9]   0.752369406   0.523412702   0.131864420   0.508668916   0.500194448
0.694054767 -0.149677954  0.115142089
  [17]  -0.265277934   0.723830377   0.571974711  -0.079046724   0.503501487
1.333794063  0.054651058  0.678981428
  [25]   0.123014927   0.161138205   0.722234512   0.278988213   0.118048807
-0.617659403  0.901121093  0.431245706
  [33]   0.201323609   0.263242316   0.925399827  -0.366730619   0.279433930
0.308829289 -0.257640797  0.231859306
  [41]   0.693158503   0.733556291  -0.189659430   0.738544123  -0.213452787
0.905338320  0.037848236  0.509701970
  [49]   0.745068491  -0.140822956   0.352325560   0.500991419   0.572941133
0.217631335 -0.068910375  0.282052912
  [57]  -0.043781139  -0.310733184   0.678382555  -0.006802936   0.229760894
0.423038131  0.625034323  1.582260922
  [65]   0.650504050  -0.488035978   0.183184794  -0.155373304  -0.512336656
-0.403590675  0.586332521  0.424152829
  [73]   0.192107607  -0.067365252  -0.493742564   0.285266997   0.977063231
0.378128290 -0.370625285  0.229833156
  [81]  -0.189320270  -0.344416554   0.131861693   0.104767299   0.002401728
```

```
0.260729441   0.351601299   0.587802145
  [89]      0.150085438   -0.339105273      0.861956429      0.187598700   -0.706203026
0.556131243   0.265407612  -0.657642297
  [97]   -0.051517664   -0.108784386      0.048935679      1.190921330      1.060667145
0.793525580   0.161751493   0.377228845
 [105]    -0.329615130       0.584099645    -0.043853405    -0.159094544    -0.009100924
0.503382644   0.186471383   0.264399523
 [113]      0.592368973   -0.050379432      0.550239515   -0.760149043      0.356407965
0.787802244   0.790881843   0.780624843
 [121]      0.002617608   -0.443473922      0.043664946      0.211594612      0.695053987
0.135031948   0.685328298  -0.650245515
 [129]      0.152231599      0.673551845      0.358005945      0.594521586      0.011422552
0.592578348   0.668008518   0.596973520
 [137]      0.477028700      0.243587888      0.831493856      0.431789888      0.200219511
0.507113354  -0.369580444  -0.391102550
 [145]   -0.212912984      0.824488564      0.393038297   -0.194893660      0.163250385
0.494411659  -0.544544600   1.058463595
 [153]      0.129842271   -0.538348008      0.608096074   -0.048677250      0.826312012
-0.469772269   0.552871013  -0.073956263
 [161]      0.737621863      0.670307427      0.283545199      0.195784755      0.236597505
0.673345547   0.142194284   1.326017338
 [169]   -0.107154735      0.165254795      0.968730812      0.409215917      0.403078763
-0.196255213   0.032686895   0.691794548
 [177]      0.119725153      0.283656967      0.316847172      1.124204066   -0.025769324
0.969082978  -0.083594013   0.325331632
 [185]      0.595377255   -0.020775840      0.279085858   -0.331656287      0.172412598
0.226264115   0.280749550   0.696521577
 [193]      0.543098911   -0.135437597      0.050374201      0.347821061   -0.044949974
0.627987471   0.831516930   0.627964511
 [201]   -0.350865439      0.375335710      0.235416283      0.683807726   -0.012859976
0.649707548   0.139336738   0.442858620
 [209]      0.097235202   -0.124509409      0.387742605      0.432851005      0.168317483
-0.212153234   0.249097145   0.219894885
 [217]      0.396613280   -1.194649516      0.917786910      0.704659898      0.263521836
0.429746328   0.815004748   1.313555527
 [225]   -0.061090504      0.414261551      0.740167655   -0.164784437      0.388897235
0.395638032   0.674129032   0.650297069
 [233]      0.314551425      0.037170820   -0.234514362      0.623757120      0.829183104
```

```
0.576922850   0.431203169   0.661615975
 [241]        0.617857093    -0.018755749    -0.023835328    -0.417833570    -0.840422513
0.738578184   0.144766658   0.495978316
 [249]      0.649365354      0.545813944   -0.123743527      0.354127150      0.863809497
0.124337318  -0.182353513  -0.344793537
 [257]      0.626076769      0.892779086   -0.419248412      0.434435088      0.471080012
-0.369917421  -0.330007803   0.436593925
 [265]      0.147686965      0.230772033      0.600606032      0.116974241      0.278593917
-0.739190220   0.276651215   0.564079343
 [273]       0.095785352   -0.217482401      1.364196224      0.018135791    -0.204835116
0.319631763  -0.481655552  -0.382028698
 [281]      0.474832158      0.034097896      0.610075574   -0.323813829      1.066069598
0.207657998   0.829745278   0.352931393
 [289]      0.236416733      0.894989448      0.807042549   -0.253852813      0.723635549
-0.797115299   0.240960720   0.527798900
 [297]       0.096115268   -0.135893443      0.374765457   -0.079991350      0.282600385
-0.386238186   0.842007152   0.198037000
 [305]       0.414498198   -0.024890290   -0.004254731      0.339823486      0.588310376
0.114313544   0.593952236  -0.017410651
 [313]      0.391325167      0.772541119      0.584186904      0.557185769      0.556357901
0.617270339   1.126921762   0.646389499
 [321]        0.202343290   -1.046948514      0.438240896    -0.280593745    -0.481123876
-0.918255415   0.245459108  -0.222808768
 [329]    -0.034279421       0.596874801    -0.131540677    -0.175097029    -0.153891694
0.398627441  -0.054863532   0.361354048
 [337]   -0.113136328      1.095657545      0.438498473   -0.619349144      0.322228607
0.085139099   0.097054409   0.128912415
 [345]      0.577491126      0.216371617      0.393058526      0.839710389      0.336171954
-0.963673579   0.739314664   0.026252415
 [353]      0.318831719      0.317347011      0.918238119      0.683368232      1.263686309
0.664344142   0.567902424   0.493908480
 [361]      0.853937847      0.172228133      0.103187493      0.272650570      0.112200183
0.214365832   0.338035486  -0.107334621
 [369]      0.566557542      0.018870641      0.844640156   -0.024948097      0.008462270
-0.137701393   0.664254967   0.948080961
 [377]   -0.005791148      0.663977446      0.367422080      0.281361058      1.107332279
0.444287441   0.420404295   0.824360255
 [385]      0.035860284      0.595957646      1.068730810      0.093243220      0.447471094
```

```
0.529536162   0.048896098   0.034739719
 [393]   0.302179451   0.280438080  -0.654836602   0.353907132   0.013019754
0.562809634   0.741900473   1.256303900
 [401]   0.472539678   0.490737826  -0.081743791   1.017583814   1.029661906
0.047334015   0.775636836   1.078742846
 [409]   0.731223124   0.528654958   0.138200916   0.502971739   0.200710752
-0.264530803   0.191515788   0.778529680
 [417]   0.297082836   0.898045066   0.135558866  -0.469036574   0.377948559
0.157873821   0.370225589  -0.132132543
 [425]  -0.442618815   0.311093867   0.559587386  -0.757441533   0.271753657
0.158800369  -0.157677445   0.763369527
 [433]   1.338465689   0.534831986   0.632144931  -0.353418420  -0.548921272
0.497064952   0.138219220   0.463017884
 [441]   0.114952860  -0.512702990   0.458946614  -0.114558550  -0.237929055
-0.405610768   0.492108325   0.424651887
 [449]  -0.321058848  -0.899903664   0.841487419   0.641623474  -0.551470580
0.839679374   0.053542265  -0.308372558
 [457]   0.575781891   0.758888455   0.276509014   0.136028897   0.596268288
-0.672073825   0.597904472   0.041894007
 [465]   0.254652605  -0.949290546  -0.205580317   0.255396938   0.179716792
0.212586218   0.366058961   0.730523984
 [473]   0.591767688   0.692186771  -0.378976687   0.001365066   1.214458817
0.645475510   0.160406910   0.198224115
 [481]   0.582505635  -0.005815834  -0.157059601   0.225944063   0.488415853
-0.020483911   0.617022589   0.394024912
 [489]   0.265857442   0.432780526   0.479505304  -0.423494861   0.171985846
0.805592586   0.290518770   0.125031154
 [497]   0.549310547  -0.651676570   0.415169830   0.916664820  -0.005910020
0.471909133  -0.335523658   0.430698200
 [505]   0.495978316   0.264101853  -0.990616243   0.591272344  -0.318785732
0.984705810   0.322845371   0.809304910
 [513]   0.038197769   0.076251397   0.478697431   0.837770731   0.247768914
-0.969908818   0.117658265  -0.088733655
 [521]  -0.056948969   0.370035669  -0.405491113  -0.205162328   0.617701147
0.317055437   0.802258538  -0.106607742
 [529]   0.587666923   0.900026599   0.282219284  -0.109189536   0.229980936
0.105474886  -0.267410824   0.661755087
 [537]   0.100239442   1.174858301   0.593459067   0.434966982   0.364079699
```

```
0.565115178  1.152196714  1.318306630
 [545]  0.088209331 -0.799510612  1.190065540 -0.006444497  0.064202038
0.426301106 -0.292985699 -0.265466874
 [553]  0.059217254 -0.103289758  0.137987939  0.353157398  0.992729022
0.279999699 -0.365870195  0.191566442
 [561] -0.347804287  0.998207947 -0.550559476  0.064680558  0.806485792
0.448102941  0.130644212  0.685224197
 [569]  0.073658490  0.086849659  0.287658388  0.332632006  0.524343292
0.669328026  0.743445141  0.183769850
 [577]  0.503857540  0.282511986  0.611101492  0.797096169  0.448257256
0.970173684  0.101338477 -0.286233294
 [585]  0.977821345  0.588936833  0.550218135  0.495737506 -0.399624127
-0.178188110 -0.135327909 -0.024739084
 [593]  0.807910403 -0.108645643  0.459648185  0.348748266  0.912448322
0.701305881 -0.299887379  0.297921061
 [601]  0.533856019 -0.005356541  0.505476006  0.087103804  1.222861566
0.646179313 -0.339486700 -0.468780056
 [609]  0.036216265  0.419869789  0.230315548  1.067516635 -0.901466570
0.943842079  0.147106576  0.466819202
 [617] -0.149972174 -0.523815290 -0.574888969 -0.065710090  0.508513009
-0.423741951  0.307275732  0.583043464
 [625]  0.191504006 -0.607800773  0.461431778  0.316999581  0.209560634
0.315069159 -0.018387653  0.581580408
 [633]  0.010262414  0.890529205  0.436321735  0.190033723  0.551240976
1.290218153  0.018943905  0.889167471
 [641]  0.597124541  0.261680067  0.074515487  0.189753327  0.541564654
-0.246632367  0.025902702 -0.363104137
 [649]  0.129517294  0.457776380  0.870172879 -0.240655868 -0.231547701
0.742658330 -0.146104641  0.331659245
 [657] -0.437210823  0.575872674  0.234705120  0.039435487  0.481873236
0.217925882  0.271363253  0.156256199
 [665]  0.279066156  0.275233038 -0.013159416 -0.229976890  0.068275400
-0.755211480  0.474391924  0.062541153
 [673] -0.157509720  0.098666451  0.485597076 -0.052373880  0.572658311
0.984732880 -0.144914518  0.252346643
 [681]  0.506786552 -0.057952011  0.075376563  0.786197713  0.966688990
0.322285359  0.443105421  0.709575379
 [689]  0.817739922  0.514526485  0.544731647  0.159777827  0.129977916
```

```
1.009103905  -0.077965890  -0.399270745
 [697]   0.458297940  -0.406252097   0.822406934   0.872082035  -0.463692413
-0.024470956   0.177056003   0.134902510
 [705]   0.547801437  -0.051125238   0.322519621   0.885898334  -0.476139783
0.820344670  -0.080541484   0.057827730
 [713]   0.906232163  -0.067711143   0.568857047   0.397258479   1.182231227
1.074639155   0.348035143  -0.183827761
 [721]   0.302402845   0.033214494   0.335896885   0.249402773   0.199080636
0.752484323   0.530413820   0.343579874
 [729]   0.282052912  -0.107647578   0.509407454   0.490675028   1.230863488
0.433111866   1.148184821   0.750461980
 [737]   0.191566527  -0.167115148   0.580231492   0.474923991   0.847841811
0.367079779   0.394628748   1.054252269
 [745]  -0.252274968   0.547811646   0.361597757   0.907217377   0.667081957
0.162256960   0.124231353  -0.060283440
 [753]   1.989399089   0.583409060   0.383867591   0.611747862   1.159450241
0.065531678   0.515526579   0.433381695
 [761]  -0.024288152  -0.293372773   0.582090412   0.122669790   0.937119880
0.042154633  -0.001151496   0.286036174
 [769]   0.085099310  -0.218576418  -0.002789293   0.737022193   0.581552156
0.894087902   0.472227168   0.560365453
 [777]   0.504302831   1.480808203   0.217829302   0.395409340   0.152548156
0.999000781   0.206116021   0.697718936
 [785]  -0.281676267   0.279066203  -0.453384824  -0.367742592   0.188277238
-0.397788895   0.368404123   0.973642682
 [793]  -0.677583752  -0.233394206  -0.182705950   0.349257223  -0.002484638
0.995253411   0.566278294   0.847833476
 [801]   0.405049971  -0.324727235   0.102255687   0.226741943   0.533737272
0.269540998   0.284714142   0.506871834
 [809]   0.934275631   0.594592768   0.466884242   0.941941663   0.351603754
0.072768744   0.802447360  -0.219298195
 [817]  -0.201836326  -0.209018086   0.110088791   0.357533403   0.789240648
-0.522573577   0.610015763   0.466643942
 [825]  -0.480662644   0.350727851   0.633179601   0.080743868   0.996878222
0.192845100   0.794616499   0.263481023
 [833]   1.172980866  -0.403563954  -0.236452071   0.488199664   0.829539749
0.220537308  -0.469230309   0.499248206
 [841]  -0.006687702   1.193734077   0.126578900   0.882154198   1.487602140
```

```
-0.226443898  0.171028251 -0.080082150
 [849]  0.434078319 -0.273638160  0.449086323 -0.253338084  0.382343390
0.839809925 -0.040906534 -0.366129214
 [857]  0.396515920 -0.148712461  0.653681943  0.173974031  0.917695419
-0.133529098  0.431914212  0.930861799
 [865]  0.563168378 -0.025570436 -0.102848328  0.328881838 -0.158002186
-0.121845778 -0.618605122  0.573697153
 [873]  0.133540512  0.953472362  0.527172705  0.356175177  0.006028467
0.573369992  0.211430841  0.761393009
 [881]  0.112491833  0.567255894  0.456235228 -0.110760784 -0.157002208
-0.043372174 -0.244109734  0.443029716
 [889]  0.267765540  0.358405220  0.537783797 -0.014190633  0.418563578
0.834881917 -0.551714279  0.688038905
 [897]  0.358543115  0.364594326 -0.035431094  0.580011220  0.534516806
0.294340372 -0.137573968  0.654867052
 [905] -0.022493339  0.687938957  0.940786972  0.541089286 -0.447211527
0.302961468  0.770280133  0.560111343
 [913] -0.431170269  0.359830568 -0.008436357  0.964421918 -0.051187833
-0.739934568  0.891773568  0.610799121
 [921]  0.184536208  0.475608774  0.251142652  0.834148253 -0.278887538
-0.546681698  0.183502620  0.540963994
 [929] -0.694229777 -0.284232964  0.656261622  0.012123903  0.234686345
0.149233643  0.077289069  0.015158891
 [937]  0.813135638  0.705754879  0.559697158  0.402680537  0.021804727
0.113362320  0.489751409  0.434526827
 [945]  0.313908363 -0.467609111  0.089016661  0.584418351  0.491155773
1.021762202  0.454533397  0.106611774
 [953]  0.034846387  0.291011730  0.932003526  0.294338482 -0.015716725
0.572792919  0.290883937  0.989997821
 [961]  0.246574480  0.344228697  0.721637260  0.547771472 -0.876719059
-0.215487034 -0.465187588  0.865437006
 [969]  0.665645970  0.477522687  0.598626052  0.763877965 -0.615272069
-0.170568597  0.401457821  0.379133805
 [977] -0.243312376 -0.027083921  0.041811444  0.269183918  0.568986574
0.806014044  0.092758459  0.288874858
 [985]  0.602678773  0.378441556  0.606642910  0.218712311  0.810669371
-0.084055406  0.075291010  1.227596456
 [993] -0.058809052  0.658850457 -0.204059920  0.414333563  0.779140310
```

```
0.174489612   0.238774682   0.791248131
 [ reached getOption("max.print") -- omitted 39000 entries ]

$lp__
    [1]  -22.01150  -22.14223  -24.30875  -22.76194  -23.09873  -24.62624  -22.08393
-23.45458 -22.83478 -21.85200 -21.68035
   [12]  -22.14283  -22.81056  -22.16652  -22.06357  -21.72373  -22.48480  -24.68958
-22.37054 -22.58254 -21.89702 -24.05258
   [23]  -21.96468  -24.49067  -21.79397  -22.02948  -22.11072  -22.28878  -21.75087
-23.94618 -22.52908 -22.33086 -21.98495
   [34]  -22.20979  -22.59855  -22.95008  -21.77535  -21.71040  -22.59721  -21.62938
-22.33336 -23.12858 -22.14303 -22.13129
   [45]  -23.18049  -22.74978  -22.29488  -21.80487  -22.14631  -22.79629  -22.76174
-21.85307 -22.78897 -22.68493 -22.07329
   [56]  -21.65749  -21.93270  -22.45898  -22.57540  -21.86055  -22.51709  -21.72427
-23.74780 -25.97403 -21.97817 -23.17850
   [67]  -22.90410  -22.84828  -23.97021  -22.82299  -21.96458  -21.75355  -21.71171
-22.63560 -23.45922 -22.69311 -22.75723
   [78]  -23.31485  -23.15133  -21.64628  -22.18576  -22.53786  -21.67228  -22.98056
-21.80955 -22.62897 -22.01694 -22.12945
   [89]  -23.64455  -23.22525  -22.43647  -22.13053  -24.03479  -21.87123  -23.08043
-23.79882 -21.87870 -22.55454 -21.92183
  [100]  -24.43641  -25.07728  -22.94235  -22.49799  -22.05155  -24.40149  -22.95359
-22.65450 -22.12357 -23.90791 -22.26222
  [111]  -21.67062  -21.64472  -22.58641  -22.37371  -21.91920  -24.06131  -21.99081
-22.39395 -22.25597 -22.66595 -22.48431
  [122]  -23.63222  -22.00997  -23.30252  -22.29360  -23.17982  -22.67074  -24.07047
-21.81800 -22.22892 -22.03236 -24.82133
  [133]  -22.81296  -23.25607  -21.99414  -23.25153  -22.39921  -21.69842  -22.49708
-24.98484 -21.98244 -23.95278 -24.00676
  [144]  -22.86482  -22.51289  -22.36040  -21.81526  -22.24898  -21.97404  -21.76881
-25.15271 -23.01549 -21.96009 -23.26455
  [155]  -21.89351  -22.44979  -22.34181  -23.27096  -22.04324  -22.15987  -22.69853
-22.19466 -22.01491 -21.88400 -22.04673
  [166]  -22.00929  -22.32063  -25.70854  -22.11607  -22.17640  -22.76767  -23.31476
-22.40125 -23.04459 -22.48438 -26.68576
  [177]  -21.69857  -22.79190  -21.87909  -23.24387  -21.98163  -22.88816  -21.98498
-21.63074 -21.87871 -21.84878 -22.10798
```

```
[188]  -24.90056  -21.66170  -21.62996  -22.74252  -22.23030  -21.81877  -22.35900
-22.85370  -23.79328  -22.45438  -21.96859
[199]  -22.56856  -22.31302  -22.57829  -22.50741  -21.94181  -23.14111  -21.93375
-22.32818  -22.78279  -21.93853  -22.17447
[210]  -22.02516  -21.97145  -21.92973  -22.21680  -22.20765  -22.03616  -22.07691
-21.66241  -26.33877  -23.24719  -22.08054
[221]  -21.78931  -22.26427  -22.30641  -24.20863  -22.81083  -21.68894  -22.84041
-22.61528  -24.09275  -22.30441  -22.80121
[232]  -21.95997  -22.37101  -23.38417  -22.24542  -22.34423  -23.47823  -21.87459
-21.88549  -22.73752  -21.91159  -22.44536
[243]  -21.84658  -22.92736  -26.39379  -23.84366  -22.55976  -21.77873  -22.02253
-22.40229  -23.49947  -21.64713  -22.43846
[254]  -22.20407  -22.75850  -25.35712  -22.99603  -25.08461  -23.69876  -21.86197
-21.90929  -22.62708  -23.24567  -21.76444
[265]  -21.84982  -22.47176  -22.34635  -21.73160  -21.85797  -25.04724  -21.95685
-21.83499  -22.22146  -22.82362  -24.46513
[276]  -22.53145  -22.24125  -22.48444  -23.05424  -23.43720  -21.73795  -21.94025
-22.47382  -22.56776  -23.06972  -21.68518
[287]  -22.34451  -21.74103  -22.14459  -22.99578  -24.70879  -23.55752  -23.02868
-24.50616  -22.35186  -21.81176  -22.00207
[298]  -23.22731  -21.70045  -22.01116  -21.98841  -22.80352  -22.50791  -21.81781
-21.70549  -21.87173  -22.22031  -22.01441
[309]  -22.07624  -22.32174  -22.50407  -22.01497  -22.57578  -22.51546  -22.03192
-22.21084  -24.02171  -21.90534  -23.52446
[320]  -22.08050  -22.39144  -25.44738  -21.70588  -22.36957  -23.35357  -25.13652
-21.63750  -22.22799  -21.85499  -21.88501
[331]  -22.26931  -22.23247  -23.01790  -21.67524  -23.67575  -22.53300  -23.57268
-23.19662  -21.85630  -23.48140  -22.70704
[342]  -21.87808  -22.32291  -21.97578  -21.87238  -21.67072  -21.70841  -22.37868
-22.14280  -25.33747  -22.32942  -21.77191
[353]  -21.64795  -21.81870  -22.61667  -22.12212  -24.37218  -22.59780  -21.91419
-21.77102  -22.58421  -22.07597  -21.71912
[364]  -21.66997  -22.06637  -21.82279  -21.81769  -22.17722  -21.87191  -21.95469
-22.91867  -22.02265  -22.02987  -22.03218
[375]  -22.85114  -22.67969  -22.19276  -23.62743  -22.10220  -23.29235  -23.78869
-23.09833  -23.20190  -23.21498  -21.76428
[386]  -21.97451  -23.47906  -23.10946  -21.96771  -21.84193  -21.91208  -22.15933
-21.62799  -22.43181  -25.01748  -21.85440
```

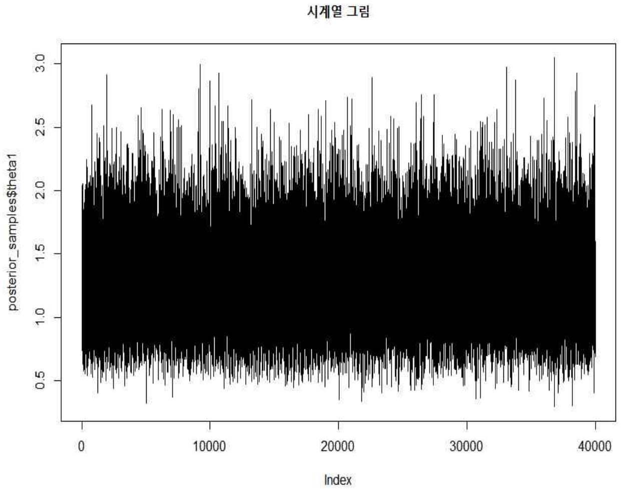
```
 [397]  -21.90715  -22.24427  -22.42306  -30.30636  -22.92490  -22.35367  -22.46151
-22.93703 -22.95335 -24.14271 -22.34420
 [408]  -23.10261  -22.62541  -23.61289  -21.77222  -21.75039  -21.78454  -22.95568
-21.64057 -22.34845 -21.77681 -24.94474
 [419]  -22.21270  -22.92416  -22.22000  -21.97837  -22.22503  -22.93057  -25.34843
-22.10440 -21.92022 -24.05478 -21.63459
 [430]  -21.93064  -22.18814  -22.60939  -24.22869  -21.78744  -23.07267  -22.60848
-23.46230 -21.91144 -21.88107 -21.72300
 [441]  -21.81580  -24.11606  -23.41782  -22.20153  -25.82811  -22.96147  -25.77322
-22.83021 -22.48257 -24.87603 -22.37075
 [452]  -22.38781  -23.32177  -22.61458  -23.38621  -23.00715  -22.07059  -22.51781
-22.89109 -24.67395 -21.87980 -24.94011
 [463]  -21.91473  -22.61686  -24.09221  -25.65778  -22.24176  -21.62955  -21.65137
-22.50430 -23.28730 -22.95753 -24.15031
 [474]  -22.03827  -24.73109  -21.81795  -24.68653  -22.00061  -21.92463  -22.32596
-21.92868 -22.53293 -22.07616 -21.63639
 [485]  -22.02335  -22.40364  -21.90830  -22.12231  -21.83177  -21.83846  -21.75590
-22.81121 -22.06360 -24.31424 -22.57846
 [496]  -22.26716  -22.20610  -23.77769  -21.91895  -22.84313  -22.51776  -22.03338
-22.51242 -22.01489 -21.77873 -21.77282
 [507]  -25.28478  -22.24760  -23.00041  -23.56541  -21.63304  -22.31467  -22.16610
-21.89702 -24.88854 -22.62407 -22.16480
 [518]  -25.53352  -21.70452  -21.97335  -22.74408  -25.96942  -22.88749  -22.28135
-23.31386 -22.44928 -22.50936 -22.40733
 [529]  -22.95327  -22.74347  -21.87021  -23.34977  -21.65122  -21.85357  -22.34612
-22.41910 -22.08521 -23.42858 -21.87202
 [540]  -21.79708  -23.57569  -21.93160  -24.09199  -23.99957  -21.71668  -24.23943
-23.67161 -22.18895 -21.81060 -21.72806
 [551]  -22.68302  -23.82020  -21.96602  -22.04131  -21.67619  -25.46948  -22.92144
-22.65396 -23.33554 -21.66147 -22.55988
 [562]  -23.54790  -23.37891  -22.52286  -22.56385  -21.69864  -22.03434  -22.22712
-22.09525 -23.69537 -22.56154 -22.96602
 [573]  -24.12139  -21.99402  -22.42884  -21.65118  -22.96715  -21.75210  -21.89624
-22.30033 -22.97324 -23.61564 -22.32370
 [584]  -22.38914  -23.23778  -22.54757  -22.40791  -21.75042  -22.69718  -22.47910
-22.21844 -22.69370 -22.31952 -21.97800
 [595]  -21.71780  -21.71241  -24.13379  -22.40959  -22.56076  -22.70192  -23.36805
-22.11416 -22.14795 -22.56762 -24.33176
```

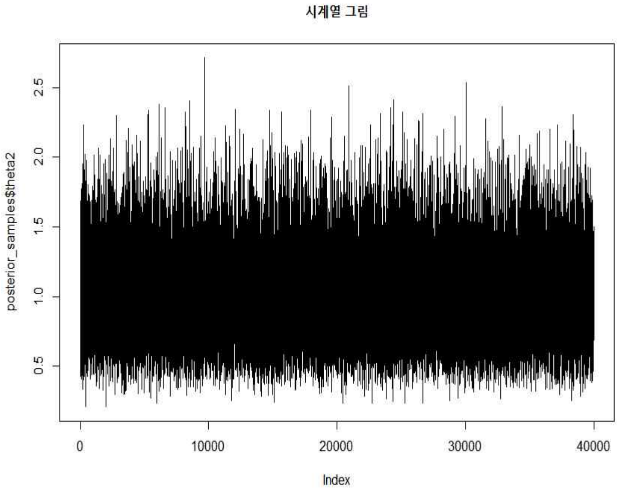
```
 [606]  -22.78536  -22.52029  -22.98240  -22.17964  -21.68026  -21.65007  -23.21424
-24.86886 -23.11505 -22.35535 -21.93874
 [617]  -22.44127  -23.64312  -23.97214  -21.99977  -22.31250  -23.55913  -21.92872
-21.90777 -21.73656 -23.50117 -21.71648
 [628]  -23.31683  -23.81431  -22.11757  -23.97564  -21.86851  -21.79335  -22.51584
-22.07025 -22.67755 -22.59730 -24.12412
 [639]  -21.84358  -22.81557  -23.19234  -21.65348  -21.85391  -21.87918  -21.80298
-23.02916 -21.93766 -22.63866 -21.68685
 [650]  -21.70642  -22.60500  -24.01727  -22.24257  -22.13957  -22.59383  -21.75064
-23.08499 -21.84217 -21.88338 -21.83460
 [661]  -21.83210  -21.63805  -21.86780  -21.73002  -21.88817  -21.63154  -21.82432
-22.35644 -21.74052 -24.08908 -21.77405
 [672]  -21.90077  -22.07980  -22.79709  -22.22670  -22.18249  -22.07004  -23.02377
-22.32724 -22.03032 -22.37520 -22.44105
 [683]  -22.42646  -22.27761  -23.87393  -22.02062  -22.05003  -22.20326  -22.76680
-21.81724 -23.35303 -21.65772 -22.53599
 [694]  -22.98987  -23.68337  -24.21197  -23.10844  -24.06802  -22.35181  -22.58204
-25.32044 -21.86565 -21.84510 -21.66984
 [705]  -21.82165  -24.14415  -21.64839  -22.61066  -23.31028  -22.33522  -21.95749
-23.14104 -23.25796 -21.91785 -23.66578
 [716]  -22.24693  -23.54281  -23.07197  -22.85575  -22.26052  -22.32514  -22.76586
-22.24220 -22.45023 -21.72885 -22.26331
 [727]  -22.05005  -22.31922  -21.65749  -22.07948  -21.90378  -24.60736  -23.66984
-21.68606 -23.36427 -22.29860 -22.37032
 [738]  -22.73386  -22.03738  -21.88239  -22.45519  -21.70231  -22.70110  -23.00232
-23.23946 -21.80611 -22.12579 -23.25896
 [749]  -21.99132  -23.91265  -21.71445  -21.92588  -27.34141  -21.86485  -21.70210
-22.07185 -23.37223 -21.72912 -23.30108
 [760]  -21.87535  -22.08454  -22.44212  -22.07421  -23.47730  -23.92425  -21.87038
-21.83838 -21.66547 -21.76079 -22.24551
 [771]  -21.81539  -22.21390  -21.85217  -22.61545  -21.76144  -21.96424  -22.23059
-24.75698 -22.24112 -21.66110 -22.57972
 [782]  -23.33866  -21.63535  -22.52810  -22.36367  -21.64267  -23.37097  -23.67754
-21.73077 -24.32453 -22.28037 -24.99107
 [793]  -23.76153  -22.67474  -22.31849  -21.77711  -22.21938  -23.03916  -21.82860
-23.23534 -21.67142 -22.47723 -21.78974
 [804]  -21.62969  -21.88714  -21.67872  -21.62676  -21.83234  -23.30517  -23.02607
-21.71559 -22.93942 -22.45463 -22.59765
```

```
 [815]  -22.63937  -22.46228  -26.29606  -22.54380  -23.08138  -22.31673  -22.65029
-23.31957 -21.92438 -22.05523 -23.35696
 [826]  -22.94241  -23.24380  -22.81699  -23.68255  -21.71912  -22.27338  -21.93745
-23.44924 -23.65483 -22.32277 -22.45667
 [837]  -22.38546  -21.83025  -23.09131  -21.75659  -22.17327  -23.49817  -21.78272
-22.48722 -24.83049 -22.26482 -21.65350
 [848]  -22.10169  -22.89722  -22.73420  -24.04059  -22.84878  -22.35766  -23.24691
-21.88624 -22.59559 -21.74694 -22.10958
 [859]  -22.10530  -22.05007  -22.80329  -22.21478  -21.82638  -23.54110  -24.88248
-21.89639 -24.61538 -22.44159 -22.40511
 [870]  -22.06052  -23.47192  -21.87542  -22.20100  -23.21253  -21.80829  -23.68607
-24.89822 -21.95328 -22.88422 -22.84113
 [881]  -22.40810  -21.95362  -22.44739  -21.98966  -22.07017  -22.22120  -23.02499
-21.69541 -21.62393 -21.65965 -21.87730
 [892]  -21.84309  -21.67632  -23.07809  -24.35283  -22.06400  -21.76555  -22.11869
-21.86244 -21.89388 -22.08236 -21.62741
 [903]  -22.03052  -22.03373  -21.84983  -22.33117  -22.65174  -22.27076  -22.87688
-21.86578 -22.82012 -24.54170 -23.17170
 [914]  -23.18175  -22.29984  -22.71626  -23.31216  -23.98526  -24.52905  -24.25748
-22.74425 -21.79003 -21.65909 -22.51081
 [925]  -23.30210  -23.33511  -21.90719  -22.63174  -24.05586  -22.80124  -22.33504
-21.81017 -21.62725 -21.67692 -21.82023
 [936]  -21.84586  -22.31876  -22.56372  -21.85061  -23.81889  -22.14186  -21.70642
-22.12802 -21.98355 -21.65643 -23.10980
 [947]  -22.27148  -21.85487  -22.04490  -22.89751  -22.77597  -21.84468  -21.77108
-21.62499 -23.24680 -21.86086 -21.87735
 [958]  -22.15527  -22.02467  -22.79007  -21.80999  -22.25603  -23.14655  -21.99694
-24.59497 -22.75057 -22.90904 -22.76655
 [969]  -22.32837  -21.99695  -22.76373  -22.18587  -23.71653  -22.19478  -22.18957
-22.09452 -22.55390 -21.89284 -23.58398
 [980]  -21.68213  -22.61712  -22.39922  -21.70311  -21.78009  -21.88301  -21.94618
-24.25757 -21.78223 -22.65354 -24.99382
 [991]  -22.85170  -24.27523  -23.12813  -21.97547  -22.47536  -21.81140  -22.26362
-22.10140 -22.66693 -22.26750
 [ reached getOption("max.print") -- omitted 39000 entries ]
```

(b)

```
# 시계열 그림
plot(posterior_samples$theta1, type = "l", main = "시계열 그림")
plot(posterior_samples$theta2, type = "l", main = "시계열 그림")
```



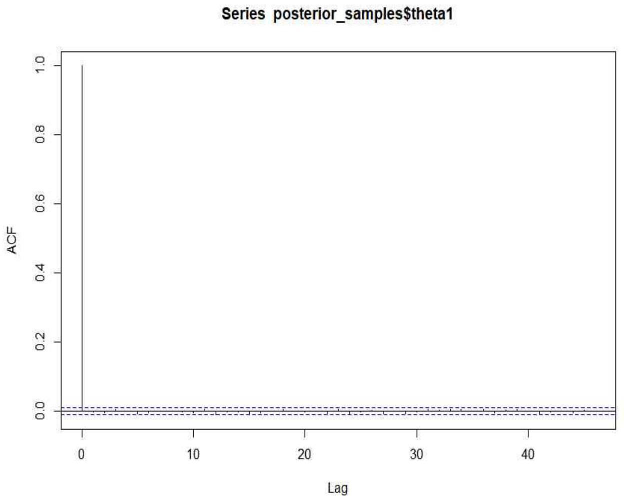


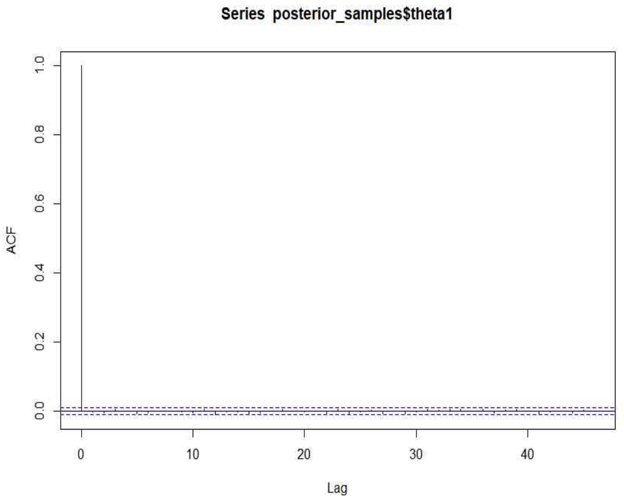
< 그림 1 theat1 시계열 >   < 그림 1 theat1 시계열 >

- (그림 1) 각 체인이 0.5~2 사이의 특정범위에서 집중적으로 변동

- (그림 2) 각 체인이 0.5~1.7 사이의 특정범위에서 집중적으로 변동

```
# 자기상관계수 그림
acf(posterior_samples$theta1)
acf(posterior_samples$theta2)
```





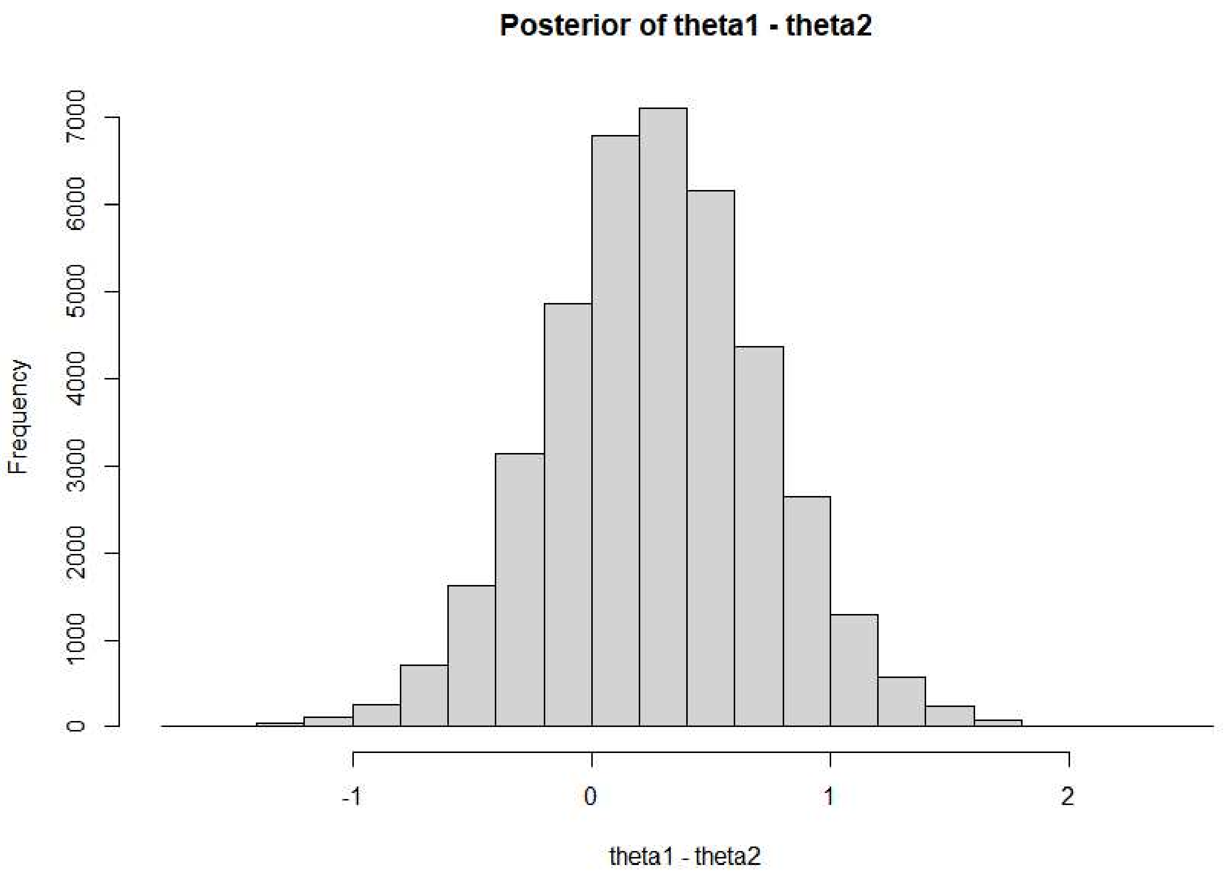
- 자기상관이 0에 가까워지면서 체인이 독립적으로 잘 섞이고 있음

```
# 각 체인을 개별 mcmc 객체로 변환
chains <- extract(fit, permuted = FALSE)  # 각 체인을 개별로 추출

# 각 체인을 mcmc 객체로 변환
mcmc_chains_theta1 <- lapply(1:dim(chains)[2], function(chain) {
  mcmc(as.matrix(chains[, chain, "theta1"]))
})
mcmc_chains_theta2 <- lapply(1:dim(chains)[2], function(chain) {
  mcmc(as.matrix(chains[, chain, "theta2"]))
})

# mcmc.list로 결합
mcmc_list_theta1 <- mcmc.list(mcmc_chains_theta1)
mcmc_list_theta2 <- mcmc.list(mcmc_chains_theta2)

# Gelman-Rubin 진단
gelman_diag_theta1 <- gelman.diag(mcmc_list_theta1)
gelman_diag_theta2 <- gelman.diag(mcmc_list_theta2)
print(gelman_diag_theta1)
Potential scale reduction factors:

     Point est. Upper C.I.
[1,]          1          1

print(gelman_diag_theta2)
Potential scale reduction factors:

     Point est. Upper C.I.
[1,]          1          1
```

=> Point estimate와 Upper C.I.가 모두 1: 이는 체인이 수렴했음을 나타냄.
여기서는 theta1과 theta2 두 변수 모두 1로 나타나므로 수렴한 것으로 판단할 수 있음.
따라서, 이 결과에 따르면 마르코프 체인이 수렴했다고 결론지을 수 있음.

(c)

```
# 히스토그램
hist(posterior_samples$diff, main = "Posterior of theta1 - theta2", xlab = "theta1 - theta2")
```



(d)

```
# 사후평균, 표준편차, 신용구간 계산
 mean_diff <- mean(posterior_samples$diff)
 sd_diff <- sd(posterior_samples$diff)
 ci_diff <- quantile(posterior_samples$diff, probs = c(0.025, 0.975))

 # 결과 출력
cat("Posterior mean of theta1 - theta2:", mean_diff, "₩n")
Posterior mean of theta1 - theta2: 0.2730707
cat("Posterior standard deviation of theta1 - theta2:", sd_diff, "₩n")
Posterior standard deviation of theta1 - theta2: 0.4562278
cat("95% credible interval of theta1 - theta2:", ci_diff, "₩n")
95% credible interval of theta1 - theta2: -0.6186622 1.179176
```