

Question NO: 2

Let $z = e^{2x} \sin y$ Find

(a) $\frac{\partial z}{\partial x}$

(b) $\frac{\partial z}{\partial y}$

(c) $\frac{\partial z}{\partial x} \big|_{(0,y)}$

(d) $\frac{\partial z}{\partial x} \big|_{(x,0)}$

(e) $\frac{\partial z}{\partial y} \big|_{(0,y)}$

(f) $\frac{\partial z}{\partial y} \big|_{(x,0)}$

(g) $\frac{\partial z}{\partial x} \big|_{(\ln 2, 0)}$

(h) $\frac{\partial z}{\partial y} \big|_{(\ln 2, 0)}$

Solution:

(a) $\frac{\partial z}{\partial x} = ? \quad z = e^{2x} \sin y$

Apply $\frac{\partial}{\partial x}$ on both side

$$\frac{\partial}{\partial x} z = \frac{\partial}{\partial x} (e^{2x} \sin y)$$

$$\frac{\partial z}{\partial x} = \sin y \cdot \frac{\partial}{\partial x} (e^{2x})$$

$$\frac{\partial z}{\partial x} = \sin y \cdot e^{2x} \cdot 2$$

$$\frac{\partial z}{\partial x} = \sin y \cdot 2e^{2x}$$

$$\frac{\partial z}{\partial x} = 2e^{2x} \sin y$$

(b) $\frac{\partial z}{\partial y} = ? \quad z = e^{2x} \sin y$

Apply partial derivative w.r.t 'y' on both sides

$$z = e^{2x} \sin y$$

$$\frac{\partial}{\partial y} \cdot z = \frac{\partial}{\partial y} [e^{2x} \sin y]$$

$$\frac{\partial z}{\partial y} = e^{2x} \frac{\partial}{\partial y} (\sin y)$$

$$\frac{\partial z}{\partial y} = e^{2x} \cdot \cos y$$

$$\frac{\partial z}{\partial y} = e^{2x} \cos y$$

(c) $\frac{\partial z}{\partial x} \big|_{(0,y)} = ? \quad z = e^{2x} \sin y$

Apply partial derivative w.r.t. x on both side.

$$z = e^{2x} \sin y$$

$$\frac{\partial}{\partial x} z = \frac{\partial}{\partial x} [e^{2x} \sin y]$$

$$\frac{\partial z}{\partial x} = \sin y \left[\frac{\partial}{\partial x} e^{2x} \right]$$

$$\frac{\partial z}{\partial x} = \sin y (2e^{2x})$$

$$\frac{\partial z}{\partial x} = 2e^{2x} \sin y$$

$$\frac{\partial z}{\partial x} \Big|_{(0,y)} = ?$$

$$\frac{\partial z}{\partial x} \Big|_{(0,y)} = 2e^{2(0)} \sin y$$

$$\frac{\partial z}{\partial x} \Big|_{(0,y)} = 2(1) \sin y$$

$$\frac{\partial z}{\partial x} \Big|_{(0,y)} = 2 \sin y \text{ Ans.}$$

$$(d) \frac{\partial z}{\partial x} \Big|_{(x,0)} = ? \quad z = e^{2x} \sin y$$

$$\left[\because \frac{\partial z}{\partial x} = 2e^{2x} \sin y \right]$$

$$\frac{\partial z}{\partial x} \Big|_{(x,0)} = 2e^{2x} \sin(0)$$

$$\frac{\partial z}{\partial x} \Big|_{(x,0)} = 2e^{2x} (0)$$

$$= 0 \text{ Ans}$$

$$(e) \frac{\partial z}{\partial y} \Big|_{(0,y)} = ? \quad z = e^{2x} \sin y$$

$$\because \frac{\partial z}{\partial y} = e^{2x} \cos y$$

$$\frac{\partial z}{\partial y} \Big|_{(0,y)} = \cancel{e^{2(0)}} e^{2(0)} \cos y$$

$$= (1) \cos y$$

$$= \cos y \text{ Ans}$$

$$(f) \quad \frac{\partial z}{\partial y} \Big|_{(x,0)} = ? \quad z = e^{2x} \sin y$$

$$\because \frac{\partial z}{\partial y} = e^{2x} \cos y$$

$$\frac{\partial z}{\partial y} \Big|_{(x,0)} = e^{2x} \cos 0^\circ \quad [\cos 0^\circ = 1]$$

$$= e^{2x} (1)$$

$$= e^{2x} \text{ Ans}$$

$$(g) \quad \frac{\partial z}{\partial x} \Big|_{(\ln 2, 0)} = ? \quad z = e^{2x} \sin y$$

$$\left[\because \frac{\partial z}{\partial x} = 2e^{2x} \sin y \right]$$

$$\frac{\partial z}{\partial x} \Big|_{(\ln 2, 0)} = 2e^{2(\ln 2)} \cdot \sin 0$$

$$= 2e^{2(\ln 2)} (0) \quad \left[\because \sin 0 = 0 \right]$$

$$= 0$$

$$\frac{\partial z}{\partial y} \Big|_{(\ln 2, 0)} = ? \quad z = e^{2x} \sin y$$

$$\left[\because \frac{\partial z}{\partial y} = e^{2x} \cos y \right]$$

$$\frac{\partial z}{\partial y} \Big|_{(\ln 2, 0)} = e^{2(\ln 2)} \cos 0$$

$$= e^{\ln(2)^2} \cos 0$$

$$(\log a^b = b \log a)$$

$$= (2)^2 (1)$$

$$= 4 \text{ Ans}$$