Exercise 13.3

Question NO: 3

$$z = 9x^2y - 3x^5y$$

$\frac{\partial z}{\partial x} = ?$ $\frac{\partial z}{\partial y} = ?$

Solution:

$$z = 9x^2y - 3x^5y$$

Apply partial derivative w.r.t. x

$$\frac{\partial}{\partial x} \cdot z = \frac{\partial}{\partial x}(9x^2y - 3x^5y)$$

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}(9x^2y) - \frac{\partial}{\partial x}(3x^5y)$$

$$\frac{\partial z}{\partial x} = 9y \cdot \frac{\partial}{\partial x}(x^2) - 3y\frac{\partial}{\partial x}(x^5)$$

$$\frac{\partial z}{\partial x} = 9y \cdot 2x(1) - 3y \cdot 5x^4 \cdot (1)$$

$$\boxed{\frac{\partial z}{\partial x} = 18xy - 15x^4y}$$

$$z = 9x^2y - 3x^5y$$

$$\frac{\partial}{\partial y} \cdot z = \frac{\partial}{\partial y}(9x^2y - 3x^5y)$$

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}(9x^2y) - \frac{\partial}{\partial y}(3x^5y)$$

$$\frac{\partial z}{\partial y} = 9x^2\frac{\partial}{\partial y}(y) - 3x^5\frac{\partial}{\partial y}(y)$$

$$\frac{\partial z}{\partial y} = 9x^2.(1) - 3x^5.(1)$$

$$\boxed{\frac{\partial z}{\partial y} = 9x^2 - 3x^5}$$ Ans

Question NO: 4

$$f(x,y) = 10x^2y^4 - 6xy^2 + 10x^2$$

$f_x(x,y) = ?$, $f_y(x,y) = ?$

Solution:

$$f(x,y) = 10x^2y^4 - 6xy^2 + 10x^2$$

Apply partial derivative w.r.t. x

$$\frac{\partial}{\partial x}.f(x,y) = \frac{\partial}{\partial x}(10x^2y^4 - 6xy^2 + 10x^2)$$

$$f_x(x,y) = \frac{\partial}{\partial x}(10x^2y^4) - \frac{\partial}{\partial x}(6xy^2) + \frac{\partial}{\partial x}(10x^2)$$

$$f_x(x,y) = 10y^4.\frac{\partial}{\partial x}(x^2) - 6y^2\frac{\partial}{\partial x}(x) + 10\frac{\partial}{\partial x}(x^2)$$

$$f_x(x,y) = 10y^4.(2x).(1) - 6y^2(1) + 10.2x.(1)$$

$$\boxed{f_x(x,y) = 20xy^4 - 6y^2 + 20x}$$ Ans

$$f(x,y) = 10x^2y^4 - 6xy^2 + 10x^2$$

Apply partial derivative w.r.t. y

$$\frac{\partial}{\partial y} \cdot f(x,y) = \frac{\partial}{\partial y}(10x^2y^4 - 6xy^2 + 10x^2)$$

$$f_y(x,y) = \frac{\partial}{\partial y}(10x^2y^4) - \frac{\partial}{\partial y}(6xy^2) + \frac{\partial}{\partial y}(10x^2)$$

$$f_y(x,y) = 10x^2\frac{\partial}{\partial y}(y^4) - 6x\frac{\partial}{\partial y}(y^2) + \frac{\partial}{\partial y}(10x^2)$$

$$f_y(x,y) = 10x^2 \cdot 4y^3(1) - 6x \cdot 2y \cdot (1) - 0$$

$$\boxed{f_y(x,y) = 40x^2y^3 - 12xy}$$ Ans

Question No: 5

$$z = (x^2+5x-2y)^8$$

$$\frac{\partial z}{\partial x} = ? \qquad \frac{\partial z}{\partial y} = ?$$

Solution:

$$z = (x^2+5x-2y)^8$$

Apply partial derivative w.r.t. x on both sides

$$\frac{\partial}{\partial x} \cdot z = \frac{\partial}{\partial x}(x^2+5x-2y)^8$$

$$\frac{\partial z}{\partial x} = 8 \cdot (x^2+5x-2y)^7 \cdot \frac{\partial}{\partial x}(x^2+5x-2y)$$

$$\boxed{\frac{\partial z}{\partial x} = 8(x^2+5x-2y)^7 (2x+5)}$$ Ans

$$z = (x^2+5x-2y)^8$$

$$\frac{\partial}{\partial y} \cdot z = \frac{\partial}{\partial y}(x^2+5x-2y)^8$$

$$\frac{\partial z}{\partial y} = 8(x^2+5x-2y)^7 \frac{\partial}{\partial y}(x^2+5x-2y)$$

$$\frac{\partial z}{\partial y} = 8(x^2+5x-2y)^7(0+0-2)$$

$$\frac{\partial z}{\partial y} = 8(x^2+5x-2y)^7(-2)$$

$$\boxed{\frac{\partial z}{\partial y} = -16(x^2+5x-2y)^7}$$ Ans

Question No: 6

$$f(x,y) = \frac{1}{xy^2 - x^2y}$$

$f_x(x,y) = ?$, $f_y(x,y) = ?$

Solution:

$$f(x,y) = \frac{1}{xy^2 - x^2y}$$

$$f(x,y) = (xy^2 - x^2y)^{-1}$$

Apply partial derivative w.r.t. x on both sides

$$\frac{\partial}{\partial x} \cdot f(x,y) = \frac{\partial}{\partial x} (xy^2 - x^2y)^{-1}$$

$$f_x(x,y) = -1\,(xy^2 - x^2y)^{-2} \cdot \frac{\partial}{\partial x}(xy^2 - x^2y)$$

$$f_x(x,y) = -(xy^2 - x^2y)^{-2}\left[\frac{\partial}{\partial x}(xy^2) - \frac{\partial}{\partial x}(x^2y)\right]$$

$$f_x(x,y) = -(xy^2 - x^2y)^{-2}\left[y^2\frac{\partial}{\partial x}(x) - y\frac{\partial}{\partial x}(x^2)\right]$$

$$f_x(x,y) = -(xy^2 - x^2y)^{-2}\left[y^2(1) - y(2x)\right]$$

$$f_x(x,y) = -(xy^2 - x^2y)^{-2}(y^2 - 2xy)$$

$$f_x(x,y) = (xy^2 - x^2y)^{-2}(2xy - y^2)$$

$$\boxed{f_x(x,y) = \frac{(2xy - y^2)}{(xy^2 - x^2y)^{+2}}}$$ Ans

$$f(x,y) = \frac{1}{xy^2 - x^2y}$$

$$f(x,y) = (xy^2 - x^2y)^{-1}$$

Apply partial derivative w.r.t. y on both sides.

$$\frac{\partial}{\partial y} f(x,y) = \frac{\partial}{\partial y} (xy^2 - x^2y)^{-1}$$

$$f_y(x,y) = -1\,(xy^2 - x^2y)^{-2} \frac{\partial}{\partial y}(xy^2 - x^2y)$$

$$f_y(x,y) = -(xy^2 - x^2y)^{-2}\left[\frac{\partial}{\partial y}(xy^2) - \frac{\partial}{\partial y}(x^2y)\right]$$

$$f_y(x,y) = -(xy^2 - x^2y)^{-2}\left[x\frac{\partial}{\partial y}(y^2) - x^2\frac{\partial}{\partial y}(y)\right]$$

$$f_y(x,y) = -(xy^2 - x^2y)^{-2}\left[x \cdot 2y - x^2(1)\right]$$

$$f_y(x,y) = -(xy^2 - x^2y)^{-2}(2xy - x^2)$$

$$\boxed{f_y(x,y) = \frac{-(2xy - x^2)}{(xy^2 - x^2y)^2}}$$ Ans

Question No: 7

$$\frac{\partial}{\partial p}\left(e^{-7p/q}\right), \quad \frac{\partial}{\partial q}\left(e^{-7p/q}\right)$$

Solution:

$$\frac{\partial}{\partial p}\left(e^{-7p/q}\right) = e^{-7p/q} \cdot \frac{\partial}{\partial p}\left(\frac{-7p}{q}\right)$$

$$\frac{\partial}{\partial p}\left(e^{-7p/q}\right) = e^{-7p/q} \cdot \frac{-7}{q} \frac{\partial}{\partial p}(p)$$

$$\frac{\partial}{\partial p}\left(e^{-7p/q}\right) = e^{-7p/q} \cdot \frac{-7}{q}(1)$$

$$\boxed{\frac{\partial}{\partial p}\left(e^{-7p/q}\right) = \frac{-7\, e^{-7p/q}}{q}}$$ Ans

$$\frac{\partial}{\partial q}\left(e^{-7P/q}\right)$$

$$\frac{\partial}{\partial q}\left(e^{-7P/q}\right) = e^{-7P/q} \cdot \frac{\partial}{\partial q}\left(-\frac{7P}{q}\right)$$

$$\frac{\partial}{\partial q}\left(e^{-7P/q}\right) = e^{-7P/q} \cdot -7P\,\frac{\partial}{\partial q}\left(\frac{1}{q}\right)$$

$$\frac{\partial}{\partial q}\left(e^{-7P/q}\right) = e^{-7P/q} \cdot -7P\,\frac{\partial}{\partial q}\left(q^{-1}\right)$$

$$\frac{\partial}{\partial q}\left(e^{-7P/q}\right) = e^{-7P/q} \cdot -7P \cdot -1\, q^{-2}(1)$$

$$\frac{\partial}{\partial q}\left(e^{-7P/q}\right) = e^{-7P/q} \cdot 7P \cdot q^{-2}$$

$$\frac{\partial}{\partial q}\left(e^{-7P/q}\right) = e^{-7P/q} \cdot \frac{7P}{q^2}$$

$$\frac{\partial}{\partial q}\left(e^{-7P/q}\right) = \frac{7P\,e^{-7P/q}}{q^2}$$ Ans.

Q:8 $\dfrac{\partial}{\partial x}\left(x\, e^{\sqrt{15xy}}\right)$

Sol:

$$= x \cdot \frac{\partial}{\partial x} e^{\sqrt{15xy}} + e^{\sqrt{15xy}} \cdot \frac{\partial}{\partial x}(x)$$

$$= x \cdot e^{\sqrt{15xy}} \cdot \frac{\partial}{\partial x}(15xy)^{1/2} + e^{\sqrt{15xy}}$$

$$= x \cdot e^{\sqrt{15xy}} \cdot \frac{1}{2\sqrt{15xy}} \cdot 15y + e^{\sqrt{15xy}}$$

$$= \frac{x \cdot e^{\sqrt{15xy}} \cdot 15y}{2\sqrt{15xy}} + e^{\sqrt{15xy}}$$

$$= e^{\sqrt{15xy}} \boxed{\left(\frac{15xy}{2\sqrt{15xy}} + 1\right)}$$

$$\frac{\partial}{\partial y}\left(x e^{\sqrt{15xy}}\right) = x \cdot \frac{\partial}{\partial y}\left(e^{\sqrt{15xy}}\right)$$

$$= x \cdot e^{\sqrt{15xy}} \cdot \frac{\partial}{\partial y}\left(\sqrt{15xy}\right)$$

$$= x e^{\sqrt{15xy}} \cdot \frac{1}{2\sqrt{15xy}} \frac{\partial}{\partial y}(15xy)$$

$$= \frac{15 x^2 \cdot e^{\sqrt{15xy}}}{2\sqrt{15xy}} \text{ Ans}$$

Question NO: 10

$$f(x,y) = \cos(2xy^2 - 3x^2y^2)$$

$f_x(x,y) = ?$, $f_y(x,y) = ?$

Solution:

$$f(x,y) = \cos(2xy^2 - 3x^2y^2)$$

Apply partial derivative w.r.t. x

$$\frac{\partial}{\partial x} f(x,y) = \frac{\partial}{\partial x} \cos(2xy^2 - 3x^2y^2)$$

$$\boxed{f_x(x,y) = -\sin(2xy^2 - 3x^2y^2)(2y^2 - 6xy^2)}$$ Ans

$$f(x,y) = \cos(2xy^2 - 3x^2y^2)$$

Apply partial derivative w.r.t. y

$$\frac{\partial}{\partial y} f(x,y) = \frac{\partial}{\partial y} \cos(2xy^2 - 3x^2y^2)$$

$$f_y(x,y) = -\sin(2xy^2 - 3x^2y^2)\frac{\partial}{\partial y}(2xy^2 - 3x^2y^2)$$

$$\boxed{f_y(x,y) = -\sin(2xy^2 - 3x^2y^2)(4xy - 6x^2y)}$$ Ans