HW1

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##Q1: If $A \in R^{m\times n}$, show that $||A||_F \leq \sqrt{Rank(A)}||A||_2$

##Ans1: Since A has SVD which is $A = U\Sigma V' = \sum_{i=1}^{n} \sigma_i u_i v_i'$, so $||A||_F = ||\sum_{i=1}^{n} \sigma_i u_i v_i'||_F = \sqrt{\sum_{i=1}^{n} \sigma_i^2}$, and $\sqrt{Rank(A)} = \sqrt{max\{r \geq 1, \sigma_r > 0\}}$, $||A||_2 = \sigma_1$. Times $\sqrt{Rank(A)}$ and $||A||_2$ together, we get $\sqrt{r}\sigma_1$, we want to show that $\sqrt{\sum_{i=1}^{r} \sigma_i^2} \leq \sqrt{r}\sigma_1$. Since both sides are positive, square both sides, we want to show that $\sum_{i=1}^{r} \sigma_i^2 \leq r\sigma_1^2$, which can be written in the form $\sigma_1^2 + \sigma_2^2 + ... + \sigma_r^2 \leq r\sigma_1^2$. Since in definition, $\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq ... \geq \sigma_n \geq 0$, the formula is true. So we can say $||A||_F \leq \sqrt{Rank(A)}||A||_2$ is true.