1. Assume that a new light bulb will burn out after t hours, where t is chosen from $[0;\infty)$ with an exponential density $f(t) = \lambda e^{-\lambda t}$

   In this context, λ is often called the *failure rate* of the bulb.

   (a) Assume that $\lambda = 0.01$, and

   find the probability that the bulb will *not* burn out before T hours. This probability is often called the *reliability* of the bulb.

   (b) For what T is the reliability of the bulb $= \frac{1}{2}$.

2. The psychologist Tversky and his colleagues say that about four out of

   five people will answer (a) to the following question:

   A certain town is served by two hospitals. In the larger hospital about 45 babies are born each day, and in the smaller hospital 15 babies are born each day. Although the overall proportion of boys is about 50 percent, the actual proportion at either hospital may be more or less than 50 percent on any day.

   At the end of a year, which hospital will have the greater number of days on which more than 60 percent of the babies born were boys?

   (a) the large hospital
   (b) the small hospital
   (c) neither—the number of days will be about the same.

   Assume that the probability that a baby is a boy is .5 (actual estimates make this more like .513). Find the right answer to the question. Can you suggest why so many people go wrong?

3. Assume that the probability of a "success" on a single experiment with n outcomes is $\frac{1}{n}$. Let m be the number of experiments necessary to make it a favorable bet that at least one success will occur.

   (a) Show that the probability that, in m trials, there are no successes is $(1 - \frac{1}{n})^m$.

   (b) (de Moivre) Show that if $m = nlog2$ then

   $$\lim_{n\to\infty} \left(1 - \frac{1}{n}\right)^m = \frac{1}{2}$$

   Hint:

   $$\lim_{n\to\infty} \left(1 - \frac{1}{n}\right)^n = e^{-1}$$

   Hence for large n we should choose m to be about $nlog2$.

4. Tversky and Kahneman asked a group of subjects to carry out the following task. They are told that:

   > Linda is 31, single, outspoken, and very bright. She majored in philosophy in college. As a student, she was deeply concerned with racial discrimination and other social issues, and participated in anti-nuclear demonstrations.

   The subjects are then asked to rank the likelihood of various alternatives, such as:

   (1) Linda is active in the feminist movement. (2) Linda is a bank teller. (3) Linda is a bank teller and active in the feminist movement.

   Tversky and Kahneman found that between 85 and 90 percent of the subjects rated alternative (1) most likely, but alternative (3) more likely than alternative (2). Is it?

   They call this phenomenon the *conjunction fallacy*, and note that it appears to be unaffected by prior training in probability or statistics. Is this phenomenon a fallacy? If so, why? Can you give a possible explanation for the subjects' choices?

5. Choose independently two numbers B and C at random from the interval $[0, 1]$ with uniform density. Note that the point (B, C) is then chosen at *random* in the unit square. Find the probability that

   (a) $B + C < 1/2$

   (b) $BC < 1/2$

   (c) $|B - C| < 1/2$

   (d) $max B, C < 1/2$

   (e) $min B, C < 1/2$

   (f) $B < 1/2$ and $1 - C < 1/2$

   (g) conditions (c) and (f) both hold

   (h) $B^2 + C^2 \leq 1/2$

   (i) $(B - 1/2)^2 + (C - 1/2)^2 \leq 1/4$

6. Take a stick of unit length and break it into two pieces, choosing the break point at random. Now break the longer of the two pieces at a random point. What is the probability that the three pieces can be used to form a triangle?

7. Three points are chosen at random on a circle of *unit circumference*. What is the probability that the triangle defined by these points as vertices has three acute angles? Hint: One of the angles is obtuse if and only if all three points lie in the same semicircle. Take the circumference as the interval $[0, 1]$. Take one point at 0 and the others at B and C.

8. From a deck of five cards numbered 2, 4, 6, 8, and 10, respectively, a card is drawn at random and replaced. This is done three times. What is the probability that the card numbered 2 was drawn exactly two times, given that the sum of the numbers on the three draws is 12?

9. How many times must we roil a single die in order to get the same score

    (a) at least twice?

    (b) at least three times?

    (c) at least n times, for $n \geq 4$?

10. Prove that if A and B are independent so are
(a) A and $\bar(B)$
(b) $\bar(A)$ and $\bar(B)$

11. A student is applying to Penn and Cornell. He estimates that he has a probability of .5 of being accepted at Cornell and .3 of being accepted at Penn. He further estimates the probability that he will be accepted by both is .2. What is the probability that he is accepted by Cornell if he is accepted by Penn? Is the event "accepted at Penn" independent of the event "accepted at Cornell?"?

12. The 50 members of Nardine's aerobics class line up to get their equipment. Assuming that no two of these people have the same height, as the line is equipped from first to last, how many of them have successive heights that either decrease or increase?

13. If a set has $2n$ elements, show that it has more subsets with n elements than with any other number of elements.

14. A baseball player, Smith, has a batting average of .300 and in a typical game comes to bat three times. Assume that Smith's hits in a game can be considered to be a Bernoulli trials process with probability .3 for success. Find the probability that Smith gets 0, 1, 2, and 3 hits.

15. Find x if $\displaystyle\sum_{t=0}^{50} \binom{50}{t} 8^t = x^{100}$

16. Determine the coefficient of x^9y^3 in the expansions of

    (a) $(x+y)^{12}$

    (b) $(x+2y)^{12}$

    (c) $(2x-3y)^{12}$

17. In how many ways can l5 (identical) candy bars be distributed among five children so that the youngest gets only one or two of them?

18. How many ways are there to place 25 different flags on 10 numbered Flagpoles if the order of the flags on a flagpole is

    (a) not relevant?

    (b) relevant?

    (c) relevant and every flagpole flies at least one flag?

19. In how many ways can the letters in WONDERING be arranged with exactly two consecutive vowels?

20. In how many ways can the 11 identical horses on a carousel be painted so that three are brown, three are white, and five are black?