5.(a)

$$x^{(i+1)} = x^{(i)} - \frac{f(x^{(i)})}{f'(x^{(i)})}$$

$$z^{(i+1)} = z^{(i)} - \frac{g(z^{(i)})}{g'(z^{(i)})} = z^{(i)} - \frac{f(Az^{(i)})}{Af'(Az^{(i)})}$$

$$z^{(0)} = A^{-1} x^{(0)}$$

suppose $z^{(k)} = A^{-1} x^{(k)}$

$$z^{(k+1)} = z^{(k)} - \frac{f(Az^{(k)})}{Af'(Az^{(k)})} = A^{-1}x^{(k)} - \frac{f(x^{(k)})}{Af'(x^{(k)})}$$

$$= A^{-1}x^{(k)} - \frac{1}{A}(x^{(k)} - x^{(k+1)})$$

$$= A^{-1}x^{(k)} - A^{-1}x^{(k)} + A^{-1}x^{(k+1)}$$

$$= A^{-1}x^{(k+1)}$$

so for $\forall i$, $z^{(i)} = A^{-1}x^{(i)}$

so Newton's method is invariant to linear reparameterization

(b) $x^{(i+1)} = x^{(i)} - \alpha f'(x^{(i)})$

$$z^{(i+1)} = z^{(i)} - \alpha A f'(Az^{(i)})$$

suppose $z^{(i)} = A^{-1}x^{(i)}$

$$Az^{(i+1)} = x^{(i)} - \alpha A^2 f'(x^{(i)})$$

$$Az^{(i+1)} - x^{(i+1)} = \alpha f'(x^{(i)})(I - A^2)$$

we can't ensure if $A^2 = I$, or $f'(x^{(i)}) = 0$

So gradient descent is not invariant to linear reparameterization