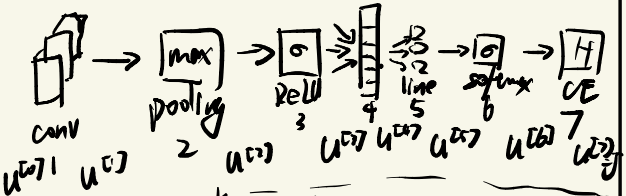
1. $\frac{\partial J}{\partial u^{[i-1]}} = B(n_i, u^{[i-1]}) \frac{\partial J}{\partial u^{[i]}}$, $\frac{\partial J}{\partial \theta^{[i]}} = B[n_i, \theta^{[i]}] \frac{\partial J}{\partial u^{[i]}}$



$J = CE(y, \hat{y}) = -\sum_{k=1}^{K} y_k \log \hat{y}_k$

$\frac{\partial J}{\partial u^{[7]}} = 1$

$\frac{\partial J}{\partial u^{[6]}} = B[CE, u^{[6]}] \frac{\partial J}{\partial u^{[7]}}$

$$B[CE, u^{[6]}] = \begin{bmatrix} \vdots \\ \frac{\partial - \sum_{k=1}^{K} y_k \log u_k^{[6]}}{\partial u_i^{[6]}} \\ \vdots \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -\frac{1}{u_T^{[6]}} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where $y_T = 1$, $u_T^{[6]} = \text{softmax}_T(u^{[5]})$

so $\frac{\partial J}{\partial u^{[6]}} = \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{u_T^{[6]}} \\ 0 \\ 0 \end{bmatrix} = -\text{lable/probability}$

$\frac{\partial J}{\partial u^{[5]}} = B[\text{softmax}, u^{[5]}] \frac{\partial J}{\partial u^{[6]}}$

$$B[\text{softmax}, u^{[5]}] = \begin{bmatrix} \frac{\partial \text{softmax}(u^{[5]})}{\partial u_1^{[5]}} & \cdots \\ \vdots & \end{bmatrix}$$

$$\frac{\partial J}{\partial u^{[5]}} = \begin{bmatrix} \frac{\partial \text{softmax}(u^{[5]})}{\partial u_1^{[5]}} \\ \vdots \\ \frac{\partial \text{softmax}(u^{[5]})}{\partial u_n^{[5]}} \end{bmatrix} \left(-\frac{1}{u_T}\right)$$

$$\frac{\partial \frac{\exp(x_j)}{\sum \exp(x_k)}}{\partial x_i} = -\frac{\exp(x_j)\exp(x_i)}{[\sum \exp(x_k)]^2}$$

$= -\text{softmax}_j(x)\ \text{softmax}_i(x)$

$$\frac{\partial \frac{\exp(x_j)}{\sum \exp(x_k)}}{\partial x_j} = \frac{\exp(x_j) \sum_{k \neq j} \exp(x_k)}{(\sum \exp(x_k))^2}$$

$= \text{softmax}_j(x) - \text{softmax}_j^2(x)$

$$\frac{\partial J}{\partial u^{[5]}} = \begin{bmatrix} \text{softmax}_1(u^{[5]}) \\ \vdots \\ \text{softmax}_T(u^{[5]}) - 1 \\ \vdots \\ \text{softmax}_n(u^{[5]}) \end{bmatrix}$$

$\frac{\partial J}{\partial u^{[4]}} = B[MM, u^{[4]}] \frac{\partial J}{\partial u^{[5]}}$

$B[MM, u^{[4]}] = W^T$

$\frac{\partial J}{\partial W^{[5]}} = B[MM, W^{[5]}] \frac{\partial J}{\partial u^{[5]}}$

$= \left(\frac{\partial J}{\partial u^{[5]}}\right) u^{[4]T}$

$\frac{\partial J}{\partial b^{[5]}} = B[MM, b^{[5]}] \frac{\partial J}{\partial u^{[5]}} = \frac{\partial J}{\partial u^{[5]}}$

$u^{[3]} = u^{[4]}$ (reshaped)

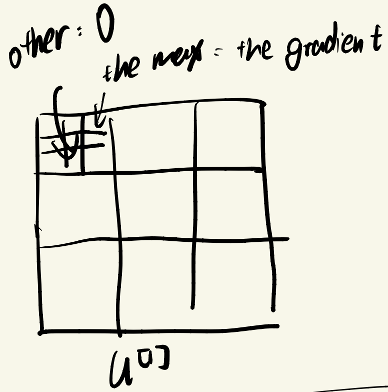
$\frac{\partial J}{\partial u^{[3]}} = \frac{\partial J}{\partial u^{[4]}}$ (reshaped)

$\frac{\partial J}{\partial u^{[2]}} = B[\text{ReLU}, u^{[2]}] \frac{\partial J}{\partial u^{[3]}}$

$= \text{ReLU}'(u^{[2]}) \odot \frac{\partial J}{\partial u^{[2]}}$

$\text{ReLU}'(x) = \begin{cases} 1 & (x > 0) \\ 0 & (x < 0) \end{cases}$

$$\frac{\partial J}{\partial u^{[l]}} = B[\text{pooling}, u^{[l]}]\left(\frac{\partial J}{\partial u^{[l]}}\right)$$



$$\frac{\partial J}{\partial u^{[0]}} = B[\text{conv}, u^{[0]}]\left(\frac{\partial J}{\partial u^{[1]}}\right)$$

$$\begin{bmatrix} O_{11} & O_{12} \\ O_{21} & O_{22} \end{bmatrix} = \text{conv}\left(\underbrace{\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix}}_{X}, \underbrace{\begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}}_{F} \right) + \begin{bmatrix} b & b \\ b & b \end{bmatrix}$$

$O_{11} = X_{11}F_{11} + X_{12}F_{12} + X_{21}F_{21} + X_{22}F_{22} + b$

$$\frac{\partial J}{\partial F_{11}} = \frac{\partial J}{\partial O_{11}}X_{11} + \frac{\partial J}{\partial O_{12}}X_{12} + \frac{\partial J}{\partial O_{21}}X_{21} + \frac{\partial J}{\partial O_{22}}X_{22}$$

so $\frac{\partial J}{\partial F} = \text{conv}\left(X, \frac{\partial J}{\partial O}\right)$, with $\frac{\partial J}{\partial O_{ij}} \cdot X_{i \to i+\text{conv}-1,\ j \to j+\text{conv}-1}$

$\frac{\partial J}{\partial O_{ij}}$ contributes $\frac{\partial J}{\partial F}$ (all)

$$\frac{\partial J}{\partial b} = \frac{\partial J}{\partial O_{11}}\frac{\partial O_{11}}{\partial b} + \cdots \frac{\partial J}{\partial O_{22}}\frac{\partial O_{22}}{\partial b} = \sum_{ij}\frac{\partial J}{\partial O_{ij}}$$

$$\frac{\partial O_{11}}{\partial X_{11}} = F_{11}$$

$$\frac{\partial J}{\partial X_{11}} = \frac{\partial J}{\partial O_{11}} \cdot \frac{\partial O_{11}}{\partial X_{11}} + \frac{\partial J}{\partial O_{12}}\frac{\partial O_{12}}{\partial X_{11}} + \frac{\partial J}{\partial O_{21}}\frac{\partial O_{21}}{\partial X_{11}} + \frac{\partial J}{\partial O_{22}}\frac{\partial O_{22}}{\partial X_{11}}$$
$$= F_{11} \cdot \frac{\partial J}{\partial O_{11}}$$

$$\frac{\partial J}{\partial X_{12}} = F_{12}\frac{\partial J}{\partial O_{11}} + F_{11}\frac{\partial J}{\partial O_{12}}$$

$\vdots$

$$\frac{\partial J}{\partial X} = \text{conv}\left(\widehat{\frac{\partial J}{\partial O}}, \hat{F}\right)$$

where $\hat{F}$ is F being rotated $180°$

$\widehat{\frac{\partial J}{\partial O}}$ is:

$$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & \multicolumn{2}{c}{\frac{\partial J}{\partial O}} & 0 \\ 0 & & & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

and ensure the width of added 0 is length of $F - 1$.

$\frac{\partial J}{\partial O_{ij}}$ contributes to

$$\frac{\partial J}{\partial X_{i \to i+\text{conv}-1}\, y_{j \to j+\text{conv}-1}}$$

total $F \frac{\partial J}{\partial O_{ij}}$

2. (a)

$$E_{\substack{s\sim p(s)\\ a\sim\pi_0(s,a)}} \frac{\pi_1(s,a)}{\hat{\pi}_0(s,a)} R(s,a)$$

$$= E_{s\sim p(s)} \sum_{s,a} \pi_0(s,a) \frac{\pi_1(s,a)}{\hat{\pi}_0(s,a)} R(s,a)$$

$$\underline{\underline{\hat{\pi}_0 = \pi}} E_{s\sim p(s)} \sum_{s,a} \pi_1(s,a) R(s,a)$$

$$= E_{\substack{s\sim p(s)\\ a\sim\pi_1(s,a)}} R(s,a)$$

(b) $E_{\substack{s\sim p(s)\\ a\sim\pi_0(s,a)}} \frac{\pi_1(s,a)}{\hat{\pi}_0(s,a)}$

$$= E_{s\sim p(s)} \sum_{s,a} \pi_0(s,a) \frac{\pi_1(s,a)}{\hat{\pi}_0(s,a)}$$

$$\underline{\underline{\pi_0 = \hat{\pi}_0}} E_{s\sim p(s)} \sum_{s,a} \pi_1(s,a) = 1$$

so the answer is as same as (a):

$$E_{\substack{s\sim p(s)\\ a\sim\pi_1(s,a)}} R(s,a)$$

(c) when there is only one data element the estimator is $R(s,a)$, the expectation is $E_{\substack{s\sim p(s)\\ a\sim\pi_0(s,a)}} R(s,a) \neq E_{\substack{s\sim p(s)\\ a\sim\pi_1(s,a)}} R(s,a)$

so it's biased if $\pi_1 \neq \pi_0$

(d) (i) if $\hat{\pi}_0 = \pi_0$, it becomes that

$$E_{\substack{s\sim p(s)\\ a\sim\pi_0(s,a)}} E_{a\sim\pi_1(s,a)} \hat{R}(s,a) + E_{\substack{s\sim p(s)\\ a\sim\pi_1(s,a)}} \left(R(s,a) - \hat{R}(s,a)\right)$$

$$= E_{s\sim p(s)} \underbrace{\sum_{s,a} \pi_0(s,a)}_{=1} \sum_{s,a} \pi_1(s,a) \hat{R}(s,a) - \sum_{s,a} \pi_1 \hat{R}(s,a)$$

$$+ E_{\substack{s\sim p(s)\\ a\sim\pi_1(s,a)}} R(s,a) = E_{\substack{s\sim p(s)\\ a\sim\pi_1(s,a)}} R(s,a)$$

(ii) $R(s,a) = \hat{R}(s,a)$

$$E_{\substack{s\sim p(s)\\ a\sim\pi_0(s,a)}} E_{a\sim\pi_1(s,a)} \hat{R}(s,a) = E_{\substack{s\sim p(s)\\ a\sim\pi_1(s,a)}} R(s,a)$$

(e) (i)

Importance sampling. Because in this situation, $\hat{\pi}_0$ is easy to get.

(ii) Regression estimator. Because in this situation, estimating $\hat{R}(s,a)$ is easier

3.
$$\arg\min_{u:u^Tu=1} \sum_{i=1}^{m} \|x^{(i)} - f_u(x^{(i)})\|_2^2$$

$$= \arg\min_{u:u^Tu=1} \sum_{i=1}^{m} \left(\|x^{(i)}\|_2^2 - \|u^T x^{(i)}\|_2^2\right)$$

$$= \arg\max_{u:u^Tu=1} \sum_{i=1}^{m} (x^{(i)T} u)^T (x^{(i)T} u)$$

$$= \arg\max_{u:u^Tu=1} \sum_{i=1}^{m} u^T (x^{(i)} x^{(i)T}) u$$

4. (a)

$$\nabla_W \ell(W) = \sum_{i=1}^{n} \left((W^T)^{-1} + \nabla_W \sum_{j=1}^{d} \log \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(w_j^T x^{(i)})^2}{2}\right)\right)$$

$$= n(W^T)^{-1} - \frac{1}{2} \sum_{i=1}^{n} \nabla_W \sum_{j=1}^{d} (w_j^T x^{(i)})^2$$

$$= n(W^T)^{-1} - \frac{1}{2} \sum_{i=1}^{n} \nabla_W \sum_{j=1}^{d} w_j^T x^{(i)} x^{(i)T} w_j$$

$$= n(W^T)^{-1} - \frac{1}{2} \sum_{i=1}^{n} \begin{bmatrix} \vdots \\ \nabla_{w_j^T} w_j^T x^{(i)} x^{(i)T} w_j \\ \vdots \end{bmatrix}$$

$$= n(W^T)^{-1} - \sum_{i=1}^{n} \begin{bmatrix} \vdots \\ w_j^T x^{(i)} x^{(i)T} \\ \vdots \end{bmatrix}$$

$$= n(W^T)^{-1} - \sum_{i=1}^{n} W x^{(i)} x^{(i)T}$$

$$= n(W^T)^{-1} - W X^T X = 0$$

$$W^T W = n(X^T X)^{-1}$$

for $W' = RW$ with $R^T R = I$

$$W'^T W' = (RW)^T RW = W^T R^T R W$$

$$= W^T W = n(X^T X)^{-1}$$

so it cause ambiguity

(b) $\nabla_W \ell(W)$

$$= (W^T)^{-1} - \nabla_W \sum_{j=1}^{d} |w_j^T x^{(i)}|$$

$$= (W^T)^{-1} - \begin{bmatrix} \vdots \\ \nabla_{w_j^T} |w_j^T x^{(i)}| \\ \vdots \end{bmatrix}$$

$$= (W^T)^{-1} - \begin{bmatrix} \mathrm{sign}(w_1^T x^{(i)})\, x^{(i)T} \\ \vdots \\ x^{(i)T} \end{bmatrix} = (W^T)^{-1} - \mathrm{sign}(W x^{(i)})\, x^{(i)T}$$

$$W := W + \alpha\left((W^T)^{-1} - \mathrm{sign}(W x^{(i)})\, x^{(i)T}\right)$$

5. (a)

$$\|B(V_1) - B(V_2)\|_\infty = \gamma \left\| \max_{a \in A} \sum_{s' \in S} P_{sa}(s') V_1(s') - \max_{a \in A} \sum_{s' \in S} P_{sa}(s') V_2(s') \right\|_\infty$$

$$\overset{①}{\le} \gamma \left\| \max_{a \in A} \sum_{s' \in S} P_{sa}(s') (V_1(s') - V_2(s')) \right\|_\infty$$

$$\overset{②}{\le} \gamma \|V_1 - V_2\|_\infty$$

for ①, $\|\max f(x) - \max g(x)\| \le \|\max (f(x) - g(x))\|$

for ②, $\sum \alpha_i x_i < \max x_i$ if $\sum \alpha_i = 1,\ \alpha_i \ge 0$

(b) suppose $B(V_1) = V_1$

if $\exists\, V_2 \ne V_1,\ B(V_2) = V_2$

$$\|B(V_1) - B(V_2)\|_\infty \le \gamma \|V_1 - V_2\|_\infty$$

$$(\gamma - 1)\|V_1 - V_2\|_\infty \ge 0$$

$\gamma - 1 < 0$, so $\|V_1 - V_2\|_\infty = 0$

that is: $V_1 = V_2$

which leads a contradictory

so B has at most one fixed point