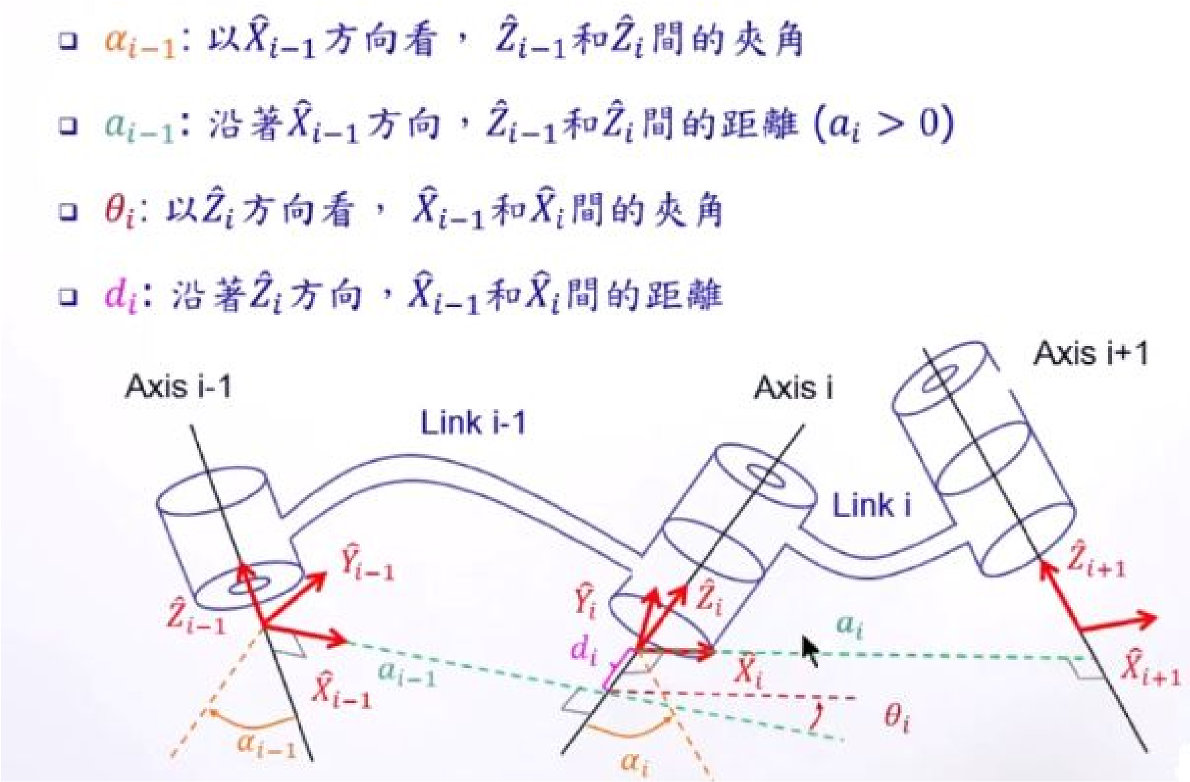
机器人学——学习笔记8(DH表达法总结)

1. Review: Denavit-Hartenberg表达法(Crag Version)

- α_{i-1}：以$\hat{X}_{i-1}$方向看，$\hat{Z}_{i-1}$和$\hat{Z}_i$之间的夹角
- a_{i-1}：沿着$\hat{X}_{i-1}$方向，$\hat{Z}_{i-1}$和$\hat{Z}_i$之间的距离$(a_i > 0)$
- θ_i：以$\hat{Z}_i$方向看，$\hat{X}_{i-1}$和$\hat{X}_i$之间的夹角
- d_i：沿着$\hat{Z}_i$方向，$\hat{X}_{i-1}$和$\hat{X}_i$之间的距离



1.几何关系

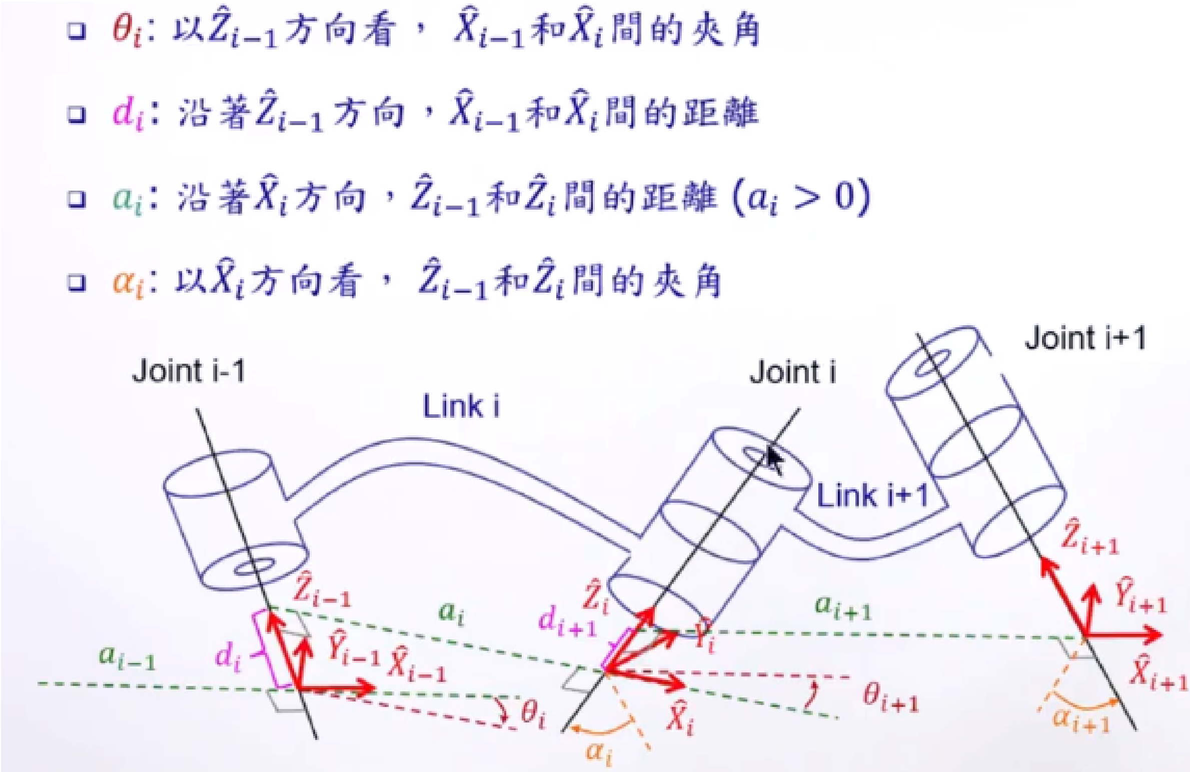
在这个操作顺序下面：

$$
\begin{aligned}
{}^{i-1}_{i}T &= {}^{i-1}_{R}T\,{}^{R}_{Q}T\,{}^{Q}_{P}T\,{}^{P}_{i}T \\
&= T_{\hat{X}_{i-1}(\alpha_{i-1})} T_{\hat{X}_R}(a_{i-1}) T_{\hat{Z}_Q}(\theta_i) T_{\hat{Z}_P}(d_i) \\
&= \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

这个表达法不是那么直观，因为只有 θ_i, d_i 是在第 i 个link下面，而 α_{i-1}, a_{i-1} 都是在第 i - 1 个Link下面。

2. Denavit-Hartenberg表达法(Standard)

- θ_i：以$\hat{Z}_{i-1}$方向看，$\hat{X}_{i-1}$和$\hat{X}_i$之间的夹角
- d_i：沿着$\hat{Z}_{i-1}$方向，$\hat{X}_{i-1}$和$\hat{X}_i$之间的距离
- a_i：沿着$\hat{X}_i$方向，$\hat{Z}_{i-1}$和$\hat{Z}_i$之间的距离$(a_i > 0)$
- α_i：以$\hat{X}_i$方向看，$\hat{Z}_{i-1}$和$\hat{Z}_i$之间的夹角



每个Link所对应的Joint是放在这个Link的后方

与Crag Version的几个区别：

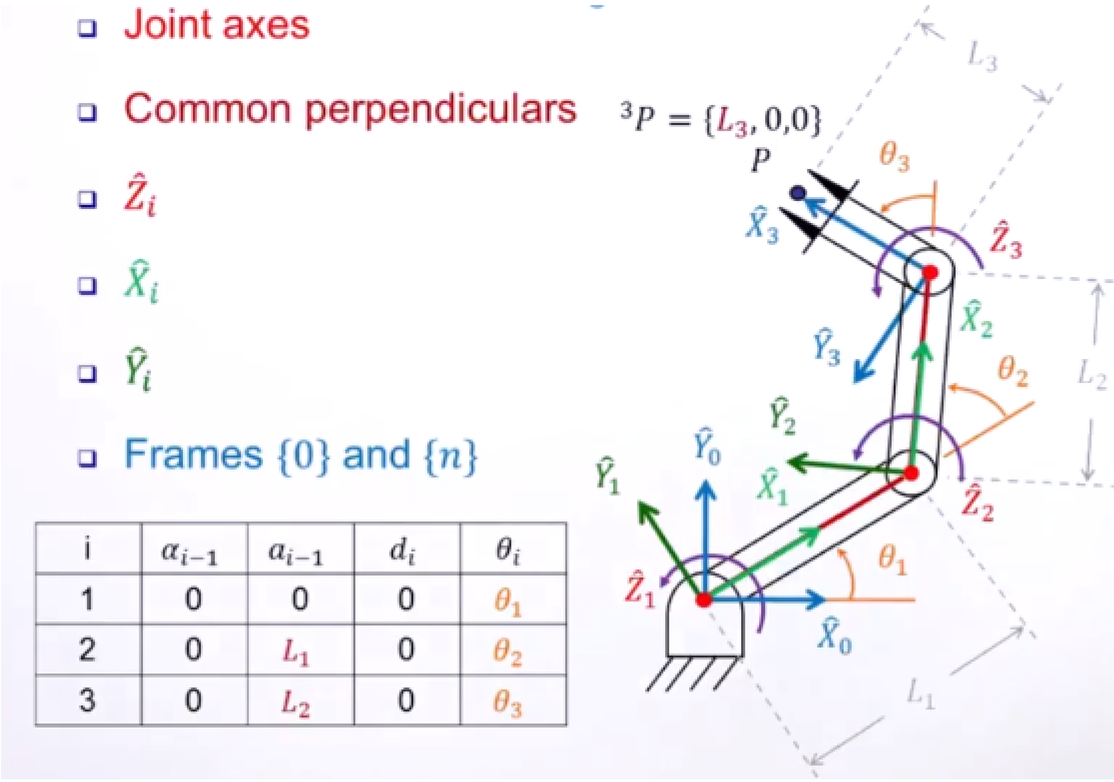
- Standard Ver更习惯用Joint而不是Axis;
- 每个Link所对应的Joint是放在这个Link的后方;
- X_i的定义不同， Std Ver下，X_i的方向是$Joint_{i-1} \rightarrow Joint_i$的方向;
- 符号定义不同，$Joint_i$下，X_i与X_{i+1}之间的夹角是θ_{i+1};

采用Std Ver的好处，在进行Trans时，其Trans Matrix求法如下：

$$
\begin{aligned}
{}^{i-1}_{i}T &= {}^{i-1}_{R}T\,{}^{R}_{Q}T\,{}^{Q}_{P}T\,{}^{P}_{i}T \\
&= T_{\hat{Z}_{i-1}}(\theta_i)T_{\hat{Z}_R}(d_i)T_{\hat{X}_Q}(a_i)T_{\hat{X}_P}(\alpha_i) \\
&= \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

- step1: 绕Z_{i-1}轴旋转θ_i使X_{i-1}, X_i平行;
- step2: 沿Z_R移动d_i使Z_R, Z_i平行;
- step3: 沿X_Q移动a_i, 使移动后{P}frame与{i}frame重合;
- step4: 绕X_P轴旋转α_i使Z_i, Z_P重合，完成变换;

Example-1 Craig DH&Std DH方式表达差异(A RRR Manipulator)



i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	L_1	0	θ_2
3	0	L_2	0	θ_3

Craig DH 情况下的Transformation Matrix分别是：（θ 用 t 代替），**可以发现，Crag DH的一个优点就是关于Translation的描述会更加干净（与下面Std Ver对比）。**

$$
{}_1^0T \quad \begin{pmatrix} \cos t_1 & -\sin t_1 & 0 & 0 \\ \sin t_1 & \cos t_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}_2^1T \quad \begin{pmatrix} \cos t_2 & -\sin t_2 & 0 & L_1 \\ \sin t_2 & \cos t_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}_3^2T \quad \begin{pmatrix} \cos t_3 & -\sin t_3 & 0 & L_2 \\ \sin t_3 & \cos t_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

该例题下的3个Trans Matrix

现在把Trans Matrix乘开，进行进一步地分析

- Craig

$$
{}_3^0T \quad \begin{pmatrix} \mathrm{Cos}[t1+t2+t3] & -\mathrm{Sin}[t1+t2+t3] & 0 & \mathrm{L1\,Cos}[t1]+\mathrm{L2\,Cos}[t1+t2] \\ \mathrm{Sin}[t1+t2+t3] & \mathrm{Cos}[t1+t2+t3] & 0 & \mathrm{L1\,Sin}[t1]+\mathrm{L2\,Sin}[t1+t2] \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$${}^0_3T.T_{\hat{X}_3}([L_3,0,0])$$

$$\begin{pmatrix} \boxed{\begin{matrix} \text{Cos}[t1+t2+t3] & -\text{Sin}[t1+t2+t3] & 0 \\ \text{Sin}[t1+t2+t3] & \text{Cos}[t1+t2+t3] & 0 \\ 0 & 0 & 1 \end{matrix}} & \begin{matrix} \text{L1 Cos}[t1]+\text{L2 Cos}[t1+t2]+\text{L3 Cos}[t1+t2+t3] \\ \text{L1 Sin}[t1]+\text{L2 Sin}[t1+t2]+\text{L3 Sin}[t1+t2+t3] \\ 0 \end{matrix} \\ \begin{matrix} 0 & \quad 0 & \quad 0 \end{matrix} & 1 \end{pmatrix}$$

最终Craig的形式，做一个L3的平移，才会跑到Std Ver 末端的原点

◆ Standard

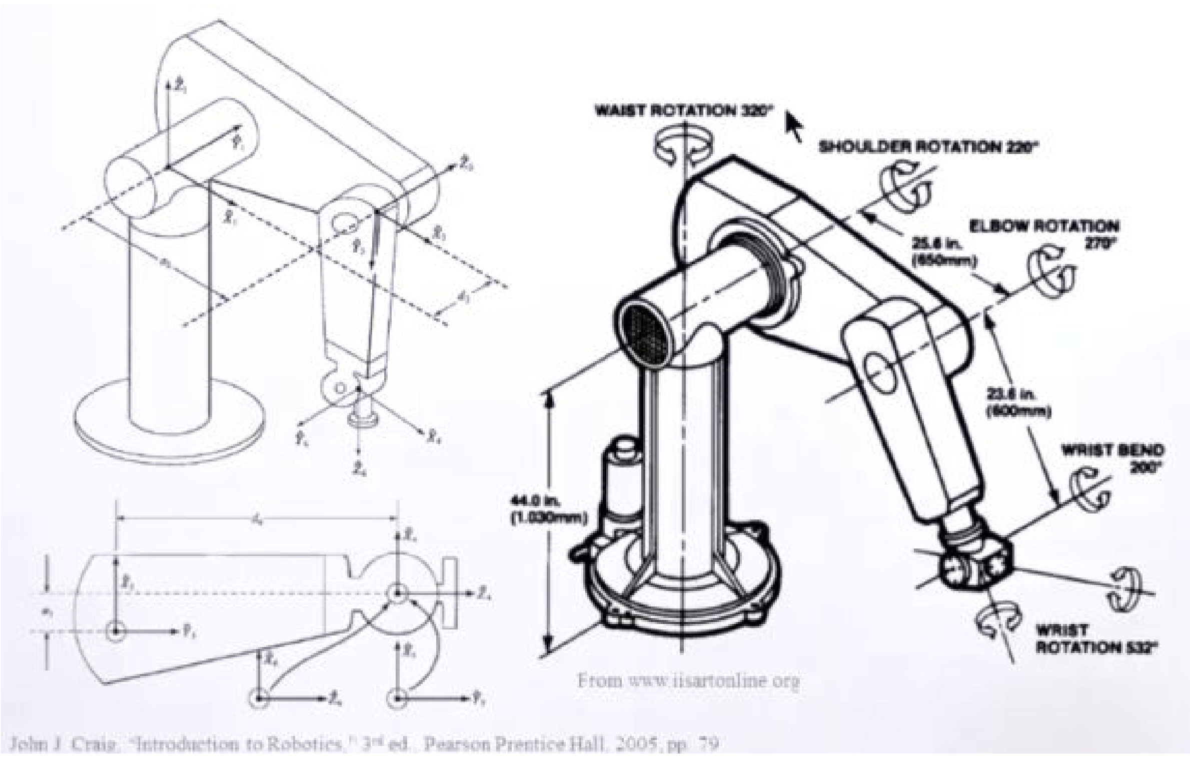
$${}^0_3T$$

$$\begin{pmatrix} \text{Cos}[t1+t2+t3] & -\text{Sin}[t1+t2+t3] & 0 & \text{L1 Cos}[t1]+\text{L2 Cos}[t1+t2]+\text{L3 Cos}[t1+t2+t3] \\ \text{Sin}[t1+t2+t3] & \text{Cos}[t1+t2+t3] & 0 & \text{L1 Sin}[t1]+\text{L2 Sin}[t1+t2]+\text{L3 Sin}[t1+t2+t3] \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

殊途同归！！！

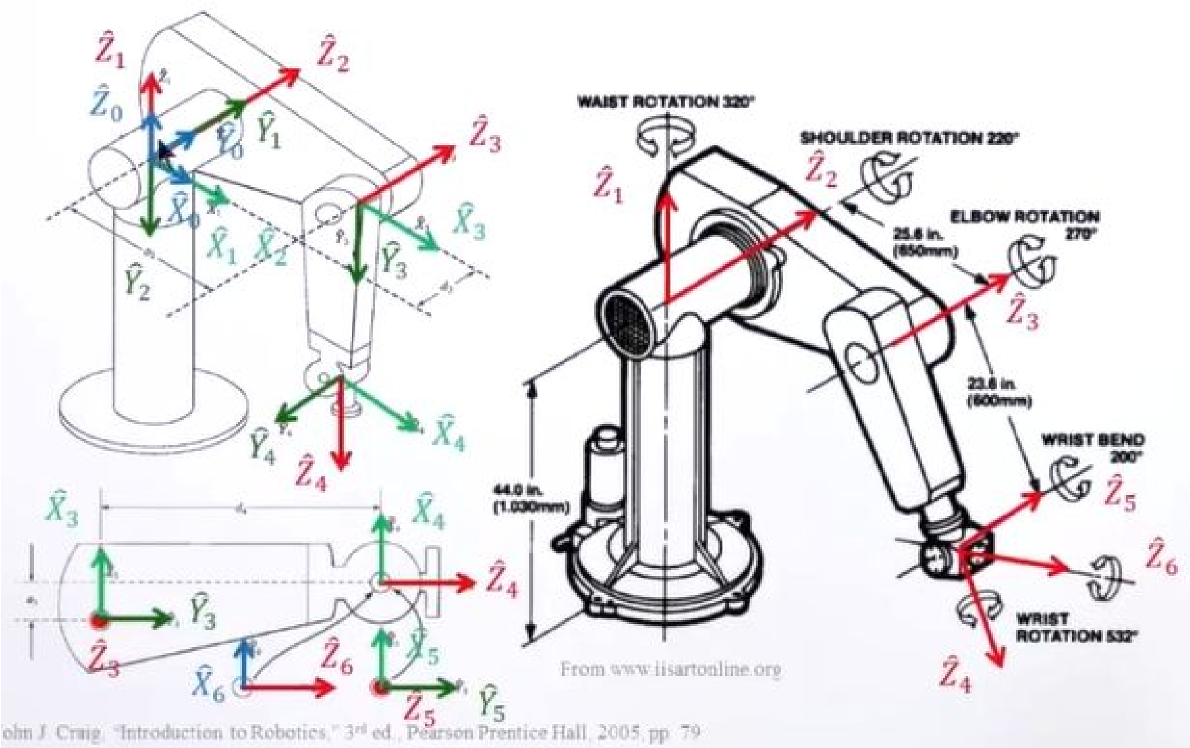
可以发现，因为是取Link3相对于地的Trans Matrix，所以其<u>**旋转坐标部分一模一样**</u>；

CLASSICAL EXAMPLE: PUMA 560



6轴手臂，可以达到任意一点(6 DOF)

1. **Craig 法： 先找Axis→Z→X(**Z_1, Z_2<u>相交， X选择和两者都垂直的方向</u>）**→Y→<u>补上frame{0}与frame{n}</u>→确定几何关系→目视法确定Craig DH Table：**



i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0°	0	0	θ_1
2	-90°	0	0	θ_2
3	0°	a_2	d_3	θ_3
4	-90°	a_3	d_4	θ_4
5	90°	0	0	θ_5
6	-90°	0	0	θ_6

Craig DH Table

- $Transformation \quad Matrices$

$$
{}^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c_{\theta_1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^3_4T = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & a_3 \\ 0 & 0 & 1 & d_4 \\ -s\theta_4 & -c\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^1_2T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_2 & -c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^4_5T = \begin{bmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^2_3T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_2 \\ s\theta_3 & c_{\theta_3} & 0 & 0 \\ 0 & 0 & 1 & d3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^5_6T = \begin{bmatrix} c\theta_6 & -s\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_6 & -c\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

变换矩阵

$$ {}^4_6T = {}^4_5T\,{}^5_6T = \begin{bmatrix} c_5c_6 & -c_5s_6 & -s_5 & 0 \\ s_6 & c_6 & 0 & 0 \\ s_5c_6 & -s_5s_6 & c_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^3_6T = {}^3_4T\,{}^4_6T = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & -c_4s_5 & a_3 \\ s_5c_6 & -s_5s_6 & c_5 & d_4 \\ -s_4c_5c_6 - c_4s_6 & s_4c_5s_6 - c_4c_6 & s_4s_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^1_3T = {}^1_2T\,{}^2_3T = \begin{bmatrix} c_{23} & -s_{23} & 0 & a_2c_2 \\ 0 & 0 & 1 & d_3 \\ -s_{23} & -c_{23} & 0 & -a_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^1_6T = {}^1_3T\,{}^3_6T = \begin{bmatrix} {}^1r_{11} & {}^1r_{12} & {}^1r_{13} & {}^1p_x \\ {}^1r_{21} & {}^1r_{22} & {}^1r_{23} & {}^1p_y \\ {}^1r_{31} & {}^1r_{32} & {}^1r_{33} & {}^1p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^1r_{11} = c_{23}[c_4c_5c_6 - s_4s_6] - s_{23}s_5s_6 $$
$$ {}^1r_{21} = -s_4c_5c_6 - c_4s_6 $$
$$ {}^1r_{31} = -s_{23}[c_4c_5c_6 - s_4s_6] - c_{23}s_5c_6 $$
$$ {}^1r_{12} = -c_{23}[c_4c_5s_6 + s_4c_6] + s_{23}s_5s_6 $$
$$ {}^1r_{22} = s_4c_5s_6 - c_4c_6 $$
$$ {}^1r_{32} = s_{23}[c_4c_5s_6 + s_4c_6] + c_{23}s_5s_6 $$
$$ {}^1r_{13} = -c_{23}c_4s_5 - s_{23}c_5 $$
$$ {}^1r_{23} = s_4s_5 $$
$$ {}^1r_{33} = s_{23}c_4s_5 - c_{23}c_5 $$
$$ {}^1p_x = a_2c_2 + a_3c_{23} - d_4s_{23} $$
$$ {}^1p_y = d_3 $$
$$ {}^1p_z = -a_3s_{23} - a_2s_2 - d_4c_{23} $$

$$ {}^{0}_{6}T = {}^{0}_{1}T\,{}^{1}_{6}T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ r_{11} = c_1[c_{23}(c_4c_5c_6 - s_4s_5) - s_{23}s_5c_5] + s_1(s_4c_5c_6 + c_4s_6) $$
$$ r_{21} = s_1[c_{23}(c_4c_5c_6 - s_4s_6) - s_{23}s_5c_6] - c_1(s_4c_5c_6 + c_4s_6) $$
$$ r_{31} = -s_{23}(c_4c_5c_6 - s_4s_6) - c_{23}s_5c_6 $$
$$ r_{12} = c_1[c_{23}(-c_4c_5s_6 - s_4c_6) + s_{23}s_5s_6] + s_1(c_4c_6 - s_4c_5s_6) $$
$$ r_{22} = s_1[s_{23}(-c_4c_5s_6 - s_4c_6) + s_{23}s_5s_6] - c_1(c_4c_6 - s_4c_5s_6) $$
$$ r_{32} = -s_{23}(-c_4c_5s_6 - s_4c_6) + c_{23}s_5s_6 $$
$$ r_{13} = -c_1(c_{23}c_4s_5 + s_{23}c_5) - s_1s_4s_5 $$
$$ r_{23} = -s_1(c_{23}c_4s_5 + s_{23}c_5) + c_1s_4s_5 $$
$$ r_{33} = s_{23}c_4s_5 - c_{23}c_5 $$
$$ p_x = c_1[a_2c_2 + a_3c_{23} - d_4s_{23}] - d_3s_1 $$
$$ p_y = s_1[a_2c_2 + a_3c_{23} - d_4s_{23}] + d_3c_1 $$
$$ p_z = -a_3s_{23} - a_2s_2 - d_4c_{23} $$

Final Result