1. (a) Modus Tollens
   (b) Modus Pones
   (c) Resolution
   (d) Modeus Tollens
   (e) Hypothetical Syllogism
   (f) Existential Generalization

2. let $p =$ Your dog plays

   let $q =$ your dog is dirty

   let $r =$ your dog needs a bath

   $\therefore$ the given argument can be translated as:

   p

   $p \to q$

   $q \to r$

   $\therefore r$

   Using rules of inference:

$$p \to q \text{ (premise)} \quad (1)$$
$$q \to r \text{ (premise)} \quad (2)$$
$$p \to r \text{ (Hypothetical Syllogism 1 and 2)} \quad (3)$$
$$p \text{ (premise)} \quad (4)$$
$$\therefore r \text{ (Modus Pones 3 and 4)} \quad (5)$$

   Hence, argument is valid

3. let $p =$ It is Sunday.
   let $q =$ The store is closed

   Then, the given argument can be translated as:

$$p \to q \quad (1)$$
$$q \quad (2)$$
$$\therefore p \quad (3)$$

   This is the Fallacy of affirming the conclusion.

4. (a) let $S(x) = x$ is at least 16 years old.
   let $D(x) = x$ can get a driver's license
   Then the given argument can be written as:

$$\forall x(S(x) \to D(x))$$
$$\neg D(Azul)$$
$$\therefore \neg S(x)$$

   So, the argument can be re-written as:

$$\forall x(S(x) \to D(x)) \text{ (premise)} \quad (1)$$
$$S(Azul) \to D(x) \text{ (Universal Instantiation 1)} \quad (2)$$
$$\neg D(Azul) \text{ (premise)} \quad (3)$$
$$\therefore \neg S(Azul) \text{ (Modus Tollens 2, 3)} \quad (4)$$

   Hence, argument is valid.

   (b) let $B(x, y) = x$ barks at y
   let $D(x) = x$ is a Dog.
   let $C(x) = x$ is a Cat.
   Then the given argument can be written as:

$$\forall x \forall y((D(x) \wedge C(y)) \to B(x, y))$$
$$D(Max)$$
$$C(Moonbeam)$$
$$\therefore B(Max, Moonbeam)$$

So, the argument can be re-written as:

$$\forall x \forall y((D(x) \land C(y)) \to B(x, y)) \tag{1}$$
$$D(Max) \text{ (premise)} \tag{2}$$
$$C(Moonbeam) \text{ (premise)} \tag{3}$$
$$(D(Max) \land C(Moonbeam)) \to B(Max, Moonbeam) \text{ (Universal Instantiation 1)} \tag{4}$$
$$(D(Max) \land C(Moonbeam)) \text{ (Conjunction 2,3)} \tag{5}$$
$$\therefore B(Max, Moonbeam) \text{ (Modus Tollens 4,5)} \tag{6}$$

Hence, argument is valid

5. let $B(x) = x$ barks at cats
   let $D(x) = x$ is a Dog.
   let $L(x) = x$ is a Lion.
   Then the given argument can be written as:

$$\forall x(B(x) \to D(x))$$
$$\forall x(L(x) \to \neg D(x))$$
$$\text{Ł}(Ramsey)$$
$$\therefore \neg B(Ramsey)$$

Then the argument can be re-written as:

$$\forall x(B(x) \to D(x)) \text{ (premise)} \tag{1}$$
$$B(Ramsey) \to D(Ramsey) \text{ (Universal Instantiation 1)} \tag{2}$$
$$\forall x(L(x) \to \neg D(x)) \text{ (premise)} \tag{3}$$
$$L(Ramsey) \to \neg D(Ramsey) \text{ (Universal Instantiation 3)} \tag{4}$$
$$L(Ramsey) \text{ (premise)} \tag{5}$$
$$\neg D(Ramsey) \text{ (Modus Pones 4,5)} \tag{6}$$
$$\therefore \neg B(Ramsey) \text{ (Modus Tollens 2,6)} \tag{7}$$

Hence, argument is valid

6.

$$p \to \neg q \text{ (premise)} \tag{1}$$
$$\neg p \lor \neg q \text{ (conditional 1)} \tag{2}$$
$$p \lor u \text{ (premise)} \tag{3}$$
$$\neg q \lor u \text{ (resolution 2,3)} \tag{4}$$
$$((r \land t) \lor p) \lor \neg u \text{ (premise)} \tag{5}$$
$$q \text{ (premise)} \tag{6}$$
$$u \text{ (Disjunctive Syllogism 4,6)} \tag{7}$$
$$(r \land t) \lor p \text{ (Disjunctive Syllogism 5,7)} \tag{8}$$
$$\neg p \text{ (Modus Tollens 1,6)} \tag{9}$$
$$r \land t \text{ (Disjunctive Syllogism 8,9)} \tag{10}$$
$$\therefore r \text{ (Simplification 10)} \tag{11}$$

Hence, Proved.