

1. (a) 7  
(b)  $\frac{25}{3}$

2.

$$\begin{aligned}
 & \sum_{k=7}^{1000} \frac{3^{2k+4}}{2^{3k+5}} \\
 &= \frac{3^4}{2^5} \sum_{k=7}^{1000} \left(\frac{9}{8}\right)^k \\
 &= \frac{3^4}{2^5} \frac{\left(\frac{9}{8}\right)^{1001} - \left(\frac{9}{8}\right)^7}{\frac{9}{8} - 1} \\
 &= \frac{3^4}{2^5} \frac{\left(\frac{9}{8}\right)^{1001} - \left(\frac{9}{8}\right)^7}{\frac{1}{8}} \\
 &= \frac{3^4 * 8}{2^5} \left(\left(\frac{9}{8}\right)^{1001} - \left(\frac{9}{8}\right)^7\right) \\
 &= \frac{3^4 * 8}{2^5} * \left(\frac{9}{8}\right)^7 \left(\left(\frac{9}{8}\right)^{994} - 1\right)
 \end{aligned}$$

3. Given,  $a_3 = 5$  and  $a_{11} = 87$  Hence,

$$\begin{aligned}
 5 &= a + 2d \\
 87 &= a + 10d \\
 \therefore 87 - 5 &= 10d - 2d \\
 \therefore 82 &= 8d \\
 \therefore d &= \frac{41}{4} \\
 \therefore a &= \frac{-31}{2}
 \end{aligned}$$

Hence,

$$\begin{aligned}
 \sum_{k=3}^{200} a_k &= \sum_{k=0}^{200} a_k - \sum_{k=0}^2 a_k \\
 &= \left(\frac{200}{2} \left(2\left(\frac{-31}{2}\right)\right) + (200-1)\frac{41}{4}\right) - \left(\frac{2}{2} \left(2\left(\frac{-31}{2}\right)\right) + (2-1)\frac{41}{4}\right)
 \end{aligned}$$

4. We know, for a geometric sequence, the  $n^{th}$  term is represented by  $a_n = ar^{n-1}$  where  $a$  is the first term of the sequence.

Hence,

$$\begin{aligned}
 a_3 &= ar^2 = 5 \\
 a_{11} &= ar^{10} = 87 \\
 \frac{87}{5} &= \frac{ar^{10}}{ar^2} \\
 \frac{87}{5} &= r^8 \\
 \left(\frac{87}{5}\right)^{\frac{1}{8}} &= r
 \end{aligned}$$

Hence, common ratio  $(r) = \left(\frac{87}{5}\right)^{\frac{1}{8}}$

5.

$$\begin{aligned}
 \sum_{j=5}^{45} (j+1)(j-3) &= \sum_{j=1}^{45} (j+1)(j-3) - \sum_{j=1}^4 (j+1)(j-3) \\
 \therefore \sum_{j=5}^{45} (j+1)(j-3) &= \sum_{j=1}^{45} (j^2 - 2j - 3) - \sum_{j=1}^4 (j^2 - 2j - 3) \\
 &= \left(\sum_{j=1}^{45} j^2 - 2\sum_{j=1}^{45} j - 3\sum_{j=1}^{45} 1\right) - \left(\sum_{j=1}^4 j^2 - 2\sum_{j=1}^4 j - 3\sum_{j=1}^4 1\right) \\
 &= \left(\frac{45 * 46 * 91}{6} - 2\frac{45 * 46}{2} - 3(45)\right) - \left(\frac{4 * 5 * 9}{6} - 2\frac{4 * 5}{2} - 12\right) \\
 &= \frac{(45 * 46 * 91) - (4 * 5 * 9)}{6} - 2\frac{(45 * 46) - (4 * 5)}{2} - (3(45) - 12)
 \end{aligned}$$

6. (a)  $a_n = -3 - 5n$   
 $\therefore a_{n+1} = -3 - 5(n+1)$   
 $\therefore a_{n+1} = -3 - 5n - 5$   
 $\therefore a_{n+1} = -8 - 5n$   
 $\therefore a_{n+1} = a_n - 5$

Hence, the recursive relation for the given sequence is  $a_{n+1} = a_n - 5$  with initial value  $a_1 = -13$ .

(b)  $a_n = (-5) * 3^n$   
 $\therefore a_{n+1} = (-5) * 3^{n+1}$   
 $\therefore a_{n+1} = (-15) * 3^n$   
 $\therefore a_{n+1} = 3 * a_n$

Hence, the recursive relation for the given sequence is  $a_{n+1} = 3 * a_n$  with initial value  $a_1 = -5$ .

(c)  $a_n = n! * 2^n$   
 $\therefore a_{n+1} = (n+1)! * 2^{n+1}$   
 $\therefore a_{n+1} = 2 * (n+1) * n! * 2^n$   
 $\therefore a_{n+1} = 2 * (n+1) a_n$

Hence, the recursive relation for the given sequence is  $a_{n+1} = 2 * (n+1) a_n$  with initial value  $a_1 = 2$ .