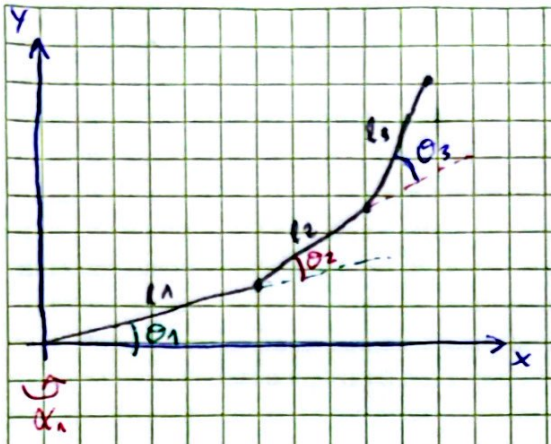
Examen Parcial 1



T_1 → Rotación base α_1 en el eje y → $R_y(\alpha_1)$

T_2 → Rotación articulación 1 θ_1 eje z → $R_z(\theta_1)$

T_3 → Traslación l_1 ⟶ $T_x(l_1)$

T_4 → Rotación articulación 2 θ_2 eje z → $R_z(\theta_2)$

T_5 → Traslación l_2 ⟶ $T_x(l_2)$

T_6 → Rotación articulación 3 θ_3 eje z → $R_z(\theta_3)$

T_7 → Traslación l_3 ⟶ $T_x(l_3)$

$$T_1 = R_y(\alpha_1) = \begin{bmatrix} \cos\alpha_1 & 0 & \sin\alpha_1 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\alpha_1 & 0 & \cos\alpha_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_1 \quad T_2 = R_z(\theta_1) = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3 = T_x(l_1) = \begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T_4 = R_z(\theta_2) = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_5 = T_x(l_2) = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T_6 = R_z(\theta_3) = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_7 = T_x(l_3) = \begin{bmatrix} 1 & 0 & 0 & l_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Pasos que voy a seguir: ① T_2T_3 (T4) ② T_4T_5, ③ T_6T_7, ④ $T_{45}T_{67}$, ⑤ $T_{23}T_{4567}$, ⑥ T_1T_{2345}

① $$T_2T_3 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & l_1\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & l_1\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

② $$T_4T_5 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & l_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & l_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

③ $$T_6T_7 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & l_3\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & l_3\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(4) $T_{45}\,T_{67} = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & l_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & l_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & l_3\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & l_3\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$T_{4567} = \begin{bmatrix} \cos\theta_2\cos\theta_3 - \sin\theta_2\sin\theta_3 & -\cos\theta_2\sin\theta_3 - \sin\theta_2\cos\theta_3 & 0 & \\ \sin\theta_2\cos\theta_3 + \cos\theta_2\sin\theta_3 & +\sin\theta_2\sin\theta_3 + \cos\theta_2\cos\theta_3 & 0 & \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & \end{bmatrix} \rightarrow$

$\rightarrow \begin{bmatrix} \cos\theta_2 l_3\cos\theta_3 - \sin\theta_2 l_3\sin\theta_3 + l_2\cos\theta_2 \\ \sin\theta_2 l_3\cos\theta_3 + \cos\theta_2 l_3\sin\theta_3 + l_2\sin\theta_2 \\ 0 \\ 1 \end{bmatrix}$ $R_z(\theta_2+\theta_3)$

$T_{4567} = \begin{bmatrix} \cos(\theta_2+\theta_3) & -\sin(\theta_2+\theta_3) & 0 & l_3\cos(\theta_2+\theta_3) + l_2\cos\theta_2 \\ \sin(\theta_2+\theta_3) & \cos(\theta_2+\theta_3) & 0 & l_3\sin(\theta_2+\theta_3) + l_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(5) $T_{23}\,T_{4567} = \begin{bmatrix} \cos\theta_1\cdot\cos(\theta_2+\theta_3) - \sin\theta_1\cdot\sin(\theta_2+\theta_3) & -\cos\theta_1\cdot\sin(\theta_2+\theta_3) - \sin\theta_1\cdot\cos(\theta_2+\theta_3) & 0 & \\ \sin\theta_1\cdot\cos(\theta_2+\theta_3) + \cos\theta_1\cdot\sin(\theta_2+\theta_3) & -\sin\theta_1\cdot\sin(\theta_2+\theta_3) + \cos\theta_1\cdot\cos(\theta_2+\theta_3) & 0 & \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & \end{bmatrix}$

$\begin{bmatrix} 0 \cdot \dots - \cos\theta_1\left[l_3\cos(\theta_2+\theta_3) + l_2\cos(\theta_2)\right] - \sin\theta_1\left[l_3\sin(\theta_2+\theta_3) + l_2\sin\theta_2\right] + l_1\cos\theta_1 \\ 0 \quad \sin\theta_1\left[l_3\cdot\cos(\theta_2+\theta_3) + l_2\cdot\cos\theta_2\right] + \cos\theta_1\left[l_3\sin(\theta_2+\theta_3) + l_2\sin\theta_2\right] + l_1\sin\theta_1 \\ 1 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 0 \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 1 \end{bmatrix}$ $R_z(\theta_1+\theta_2+\theta_3)$

$T_{23457} = \begin{bmatrix} \cos(\theta_1+\theta_2+\theta_3) & -\sin(\theta_1+\theta_2+\theta_3) & 0 & \\ \sin(\theta_1+\theta_2+\theta_3) & \cos(\theta_1+\theta_2+\theta_3) & 0 & \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & \end{bmatrix} \rightarrow$

$\begin{bmatrix} l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2) + l_3\cos(\theta_1+\theta_2+\theta_3) \\ l_1\sin\theta_1 + l_2\sin(\theta_1+\theta_2) + l_3\sin(\theta_1+\theta_2+\theta_3) \\ 0 \\ 1 \end{bmatrix}$ $R_y(\alpha_1)\cdot R_z(\theta_1+\theta_2+\theta_3)$

(6) $T_E = T_0\cdot T_{234567} = \begin{bmatrix} \cos\alpha_1\cdot\cos(\theta_1+\theta_2+\theta_3) & -\cos\alpha_1\cdot\sin(\theta_1+\theta_2+\theta_3) & \sin\alpha_1 & \\ \sin(\theta_1+\theta_2+\theta_3) & \cos(\theta_1+\theta_2+\theta_3) & 0 & \\ -\sin\alpha_1\cdot\cos(\theta_1+\theta_2+\theta_3) & \sin\alpha_1\cdot\sin(\theta_1+\theta_2+\theta_3) & \cos\alpha_1 & \\ 0 & 0 & 0 & \end{bmatrix} \rightarrow$

$\begin{bmatrix} \cos\alpha_1\left[l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2) + l_3\cdot\cos(\theta_1+\theta_2+\theta_3)\right] \\ l_1\sin\theta_1 + l_2\sin(\theta_1+\theta_2) + l_3\cdot\sin(\theta_1+\theta_2+\theta_3) \\ -\sin\alpha_1\left[l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2) + l_3\cdot\cos(\theta_1+\theta_2+\theta_3)\right] \\ 1 \end{bmatrix}$