# Package 'temStaR'

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Title Ter	mpered Stable Distribution
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_	<b>ion</b> This package provides useful tools to use the multivariate normal tempered stable distrition and process
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chf\_NTS

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# Description

chf\_NTS calculates Ch.F of the NTS distribution with parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ . If a time parameter value is given, it calculates Ch.F of the NTS profess  $\phi(u) = E[\exp(iu(X(t+s)-X(s)))] = \exp(t\log(E[\exp(iuX(1))]))$ , where X is the NTS process generated by the NTS distribution with parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ .

# Usage

```
chf_NTS(u, param)
```

# **Arguments**

u An array of u  $\mbox{A vector of the NTS parameters } (\alpha,\theta,\beta,\gamma,\mu). \mbox{ For NTS process case it is a vector of parameters } (\alpha,\theta,\beta,\gamma,\mu,t).$ 

# Value

Characteristic function of the NTS distribution

```
alpha <- 1.2
theta <- 1
beta <- -0.2
gamma <- 0.3
mu <- 0.1
ntsparam <- c(alpha, theta, beta, gamma, mu)
u <- seq(from = -2*pi, to = 2*pi, length.out = 101)
phi <- chf_NTS(u, ntsparam)

#Annual based parameters
alpha <- 1.2
theta <- 1
beta <- -0.2</pre>
```

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```
gamma <- 0.3
mu <- 0.1
#scaling annual parameters to one day
dt <- 1/250 #one day
ntsparam <- c(alpha, theta, beta, gamma, mu, dt)</pre>
u \leftarrow seq(from = -2*pi, to = 2*pi, length.out = 101)
phi <- chf_NTS(u, ntsparam)</pre>
```

chf\_stdNTS

chf\_stdNTS

### **Description**

chf\_stdNTS calculates Ch.F of the standard NTS distribution with parameters  $(\alpha, \theta, \beta)$ . If a time parameter value is given, it calculates Ch.F of the standard NTS profess  $\phi(u) = E[\exp(iu(X(t +$  $(s) - X(s) = \exp(t \log(E[\exp(iuX(1))]))$ , where X is the standard NTS process generated by the standard NTS distribution with parameters  $(\alpha, \theta, \beta)$ .

# Usage

```
chf_stdNTS(u, param)
```

# **Arguments**

An array of u

ntsparam

A vector of the standard NTS parameters  $(\alpha, \theta, \beta)$ . For the standard NTS process case it is a vector of parameters  $(\alpha, \theta, \beta, t)$ .

### Value

Characteristic function of the standard NTS distribution

```
alpha <- 1.2
theta <- 1
beta <- -0.2
ntsparam <- c(alpha, theta, beta)</pre>
u \leftarrow seq(from = -2*pi, to = 2*pi, length.out = 101)
phi <- chf_stdNTS(u, ntsparam)</pre>
#Annual based parameters
alpha <- 1.2
theta <- 1
beta <- -0.2
#scaling annual parameters to one day
dt <- 1/250 #one day
ntsparam \leftarrow c(alpha, theta, beta, gamma, mu, dt)
u \leftarrow seq(from = -2*pi, to = 2*pi, length.out = 101)
phi <- chf_stdNTS(u, ntsparam)</pre>
```

4 cvarnts

cvarnts

cvarnts

# **Description**

cvarnts calculates Conditional Value at Risk (CVaR, or expected shortfall ES) of the NTS market model with parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ . If only three parameters are given, it calculates CVaR of the standard NTS distribution with parameter  $(\alpha, \theta, \beta)$ 

# Usage

```
cvarnts(eps, ntsparam)
```

### Arguments

ntsparam A vector of the NTS parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ . A vector of the standard NTS

parameters  $(\alpha, \theta, \beta)$ .

u Real value between 0 and 1

#### Value

CVaR of the NTS distribution.

#### References

- Y. S. Kim, S. T. Rachev, M. L. Bianchi, and F. J. Fabozzi (2010), Computing VaR and AVaR in infinitely divisible distributions, Probability and Mathematical Statistics, 30 (2), 223-245.
- S. T. Rachev, Y. S. Kim, M. L. Bianchi, and F. J. Fabozzi (2011), Financial Models with Levy Processes and Volatility Clustering, John Wiley & Sons

```
alpha <- 1.2
theta <- 1
beta <- -0.2
ntsparam <- c(alpha, theta, beta)</pre>
u \leftarrow c(0.01, 0.05)
q <- cvarnts(u, ntsparam)</pre>
alpha <- 1.2
theta <- 1
beta <- -0.2
gamma <- 0.3
mu <- 0.1
ntsparam <- c(alpha, theta, beta, gamma, mu)</pre>
u \leftarrow c(0.01, 0.05)
q \leftarrow cvarnts(u, ntsparam)
#Annual based parameters
alpha <- 1.2
theta <- 1
beta <- -0.2
```

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```
gamma <- 0.3 mu <- 0.1 #scaling annual parameters to one day dt <- 1/250 #one day ntsparam <- c(alpha, theta, beta, gamma, mu, dt) u <- c(0.01,0.05) q <- cvarnts(u, ntsparam)
```

dnts

dnts

### **Description**

dnts calculates pdf of the NTS distribution with parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ . If only three parameters are given, it calculates pdf of the standard NTS distribution with parameter  $(\alpha, \theta, \beta)$  If a time parameter value is given, it calculates pdf of the NTS profess f(x)dx = d(P((X(t+s) - X(s)) < x)), where X is the NTS process generated by the NTS distribution with parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ .

### Usage

```
dnts(xdata, ntsparam)
```

# **Arguments**

xdata An array of x

ntsparam A vector of the NTS parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ . For the NTS process case it is a

vector of parameters  $(\alpha, \theta, \beta, \gamma, \mu, t)$ . A vector of the standard NTS parameters

 $(\alpha, \theta, \beta)$ .

#### Value

Density of NTS distribution

### References

Kim, Y. S. (2020) Portfolio Optimization on the Dispersion Risk and the Asymmetric Tail Risk https://arxiv.org/pdf/2007.13972.pdf

```
alpha <- 1.2
theta <- 1
beta <- -0.2
ntsparam <- c(alpha, theta, beta)
x <- seq(from = -6, to = 6, length.out = 101)
d <- dnts(x, ntsparam)
plot(x,d,type = 'l')

alpha <- 1.2
theta <- 1
beta <- -0.2
gamma <- 0.3</pre>
```

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```
mu <- 0.1
ntsparam <- c(alpha, theta, beta, gamma, mu)</pre>
x < - seq(from = -2, to = 2, by = 0.01)
d <- dnts(x, ntsparam)</pre>
plot(x,d,type = 'l')
#Annual based parameters
alpha <- 1.2
theta <- 1
beta <- -0.2
gamma <- 0.3
mu <- 0.1
#scaling annual parameters to one day
dt <- 1/250 #one day
ntsparam <- c(alpha, theta, beta, gamma, mu, dt)</pre>
x <- seq(from = -0.02, to = 0.02, length.out = 101)
d <- dnts(x, ntsparam)</pre>
plot(x,d,type = 'l')
```

fitmnts

fitmnts

# Description

fitmnts fit parameters of the n-dimensional NTS distribution.

```
r=\mu+diag(\sigma)X where X \mbox{ follows } stdNTS_n(\alpha,\theta,\beta,\Sigma)
```

# Usage

```
\code{res <- fitmnts(returndata, n)}
\code{res <- fitmnts(returndata, n, alphaNtheta = c(alpha, theta))}
\code{res <- fitmnts(returndata, n, stdflag = TRUE ) }
\code{res <- fitmnts(returndata, n, alphaNtheta = c(alpha, theta), stdflag = TRUE)}</pre>
```

# Arguments

returndata Raw data to fit the parameters. The data must be given as a matrix form. Each

column of the matrix contains a sequence of asset returns. The number of row

of the matrix is the number of assets.

n Dimension of the data. That is the number of assets.

alphaNtheta If  $\alpha$  and  $\theta$  are given, then put those numbers in this parameter. The func-

tion fixes those parameters and fits other remaining parameters. If you set alphaNtheta = NULL, then the function fits all parameters including  $\alpha$  and

 $\theta$ .

stdflag If you want only standard NTS parameter fit, set this value be TRUE.

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#### Value

#### References

Kim, Y. S. (2020) Portfolio Optimization on the Dispersion Risk and the Asymmetric Tail Risk https://arxiv.org/pdf/2007.13972.pdf

```
library(functional)
library(nloptr)
library(pracma)
library(spatstat)
library(Matrix)
library(quantmod)
getSymbols("^GSPC", src="yahoo", from = "2016-1-1", to = "2020-08-31")
pr1 <- as.numeric(GSPC$GSPC.Adjusted)</pre>
getSymbols("^DJI", src="yahoo", from = "2016-1-1", to = "2020-08-31")
pr2 <- as.numeric(DJI$DJI.Adjusted)</pre>
returndata <- matrix(data = c(diff(log(pr1)), diff(log(pr2))),</pre>
                      ncol = 2, nrow = (length(pr1)-1))
res <- fitmnts( returndata = returndata, n=2 )</pre>
#Fix alpha and theta.
#Estimate alpha dna theta from DJIA and use those parameter for IBM, INTL parameter fit.
getSymbols("^DJI", src="yahoo", from = "2020-8-25", to = "2020-08-31")
prDJ <- as.numeric(DJI$DJI.Adjusted)</pre>
ret <- diff(log(prDJ))</pre>
ntsparam <- fitnts(ret)</pre>
getSymbols("IBM", src="yahoo", from = "2016-1-1", to = "2020-08-31")
pr1 <- as.numeric(IBM$IBM.Adjusted)</pre>
getSymbols("INTL", src="vahoo", from = "2016-1-1", to = "2020-08-31")
pr2 <- as.numeric(INTL$INTL.Adjusted)</pre>
returndata <- matrix(data = c(diff(log(pr1)), diff(log(pr2))),</pre>
                      ncol = 2, nrow = (length(pr1)-1))
res <- fitmnts( returndata = returndata,</pre>
                 n = 2.
                 alphaNtheta = c(ntsparam["alpha"], ntsparam["theta"]) )
```

8 fitnts

### **Description**

fitnts fit parameters  $(\alpha, \theta, \beta, \gamma, \mu)$  of the NTS distribution. This function using the curvefit method between the empirical cdf and the NTS cdf.

### Usage

```
\code{fitnts(rawdat)}
\code{fitnts(rawdat), ksdensityflag = 1}
\code{fitnts(rawdat, initialparam = c(alpha, theta, beta, gamma, mu))}
\code{fitnts(rawdat, initialparam = c(alpha, theta, beta, gamma, mu)), ksdensityflag = 1}
\code{fitnts(rawdat, initialparam = c(alpha, theta, beta, gamma, mu)), maxeval = 100, ksdensityflag
```

### **Arguments**

rawdat Raw data to fit the parameters.

initialparam A vector of initial NTS parameters. This function uses the nloptr package. If

it has a good initial parameter then estimation performs better. If users do not know a good initial parameters, then just set it as initialparam=NaN, that is default. The function cffitnts() may be helpful to find the initial parameters.

maxeval Maximum evaluation number for nloptr. The iteration stops on this many func-

tion evaluations.

ksdensityflag This function fit the parameters using the curvefit method between the empirical

cdf and the NTS cdf. If ksdensityflag = 1 (default), then the empirical cdf is calculated by the kernel density estimation. If ksdensityflag = 0, then the

empirical cdf is calculated by the empirical cdf.

# Value

Estimated parameters

### References

Kim, Y. S. (2020) Portfolio Optimization on the Dispersion Risk and the Asymmetric Tail Risk https://arxiv.org/pdf/2007.13972.pdf

```
library("quantmod")
getSymbols("^GSPC", src="yahoo", from = "2010-1-1", to = "2020-12-31")
pr <- as.numeric(GSPC$GSPC.Adjusted)
ret <- diff(log(pr))
ntsparam <- fitnts(ret)

Femp = ecdf(ret)
x = seq(from=min(ret), to = max(ret), length.out = 100)
cemp = Femp(x)
ncdf = pnts(x, c(ntsparam))</pre>
```

fitstdnts

```
plot(x,ncdf,type = 'l', col = "red")
points(x,cemp, type = 'l', col = "blue")
a = density(ret)
p = dnts(x,ntsparam)
plot(x,p,type = 'l', col = "red")
lines(a,type = 'l', col = "blue")
```

fitstdnts

fitstdnts

#### **Description**

fitstdnts fit parameters  $(\alpha, \theta, \beta)$  of the standard NTS distribution. This function using the curvefit method between the empirical cdf and the standard NTS cdf.

# Usage

```
\code{fitstdnts(rawdat)}
\code{fitstdnts(rawdat), ksdensityflag = 1}
\code{fitstdnts(rawdat, initialparam = c(alpha, theta, beta))}
\code{fitstdnts(rawdat, initialparam = c(alpha, theta, beta)), ksdensityflag = 1}
\code{fitstdnts(rawdat, initialparam = c(alpha, theta, beta)), maxeval = 100, ksdensityflag = 1}
```

### **Arguments**

rawdat Raw data to fit the parameters.

initialparam A vector of initial standard NTS parameters. This function uses the nloptr

package. If it has a good initial parameter then estimation performs better. If users do not know a good initial parameters, then just set it as initialparam=NaN,

that is default.

maxeval Maximum evaluation number for nloptr. The iteration stops on this many func-

tion evaluations.

ksdensityflag This function fit the parameters using the curvefit method between the empirical

cdf and the standard NTS cdf. If ksdensityflag = 1 (default), then the empirical cdf is calculated by the kernel density estimation. If ksdensityflag = 0,

then the empirical cdf is calculated by the empirical cdf.

# Value

Estimated parameters

# References

Kim, Y. S. (2020) Portfolio Optimization on the Dispersion Risk and the Asymmetric Tail Risk https://arxiv.org/pdf/2007.13972.pdf

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#### **Examples**

```
library("quantmod")
getSymbols("^GSPC", src="yahoo", from = "2010-1-1", to = "2020-12-31")
pr <- as.numeric(GSPC$GSPC.Adjusted)</pre>
ret <- diff(log(pr))</pre>
stdret <- (ret-mean(ret))/sd(ret)</pre>
stdntsparam <- fitstdnts(stdret)</pre>
Femp = ecdf(stdret)
x = seq(from=min(stdret), to = max(stdret), length.out = 100)
cemp = Femp(x)
ncdf = pnts(x, c(stdntsparam))
plot(x,ncdf,type = 'l', col = "red")
lines(x,cemp, type = '1', col = "blue")
a = density(stdret)
p = dnts(x,stdntsparam)
plot(x,p,type = 'l', col = "red", ylim = c(0, max(a$y, p)))
lines(a,type = 'l', col = "blue")
```

gensamplepathnts

gensamplepathnts

# **Description**

gensamplepathnts generate sample paths of the NTS process with parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ . If only three parameters are given, it generate sample paths of the standard NTS process with parameters  $(\alpha, \theta, \beta)$ .

### Usage

```
gensamplepathnts(npath, ntimestep, ntsparam, dt)
```

### **Arguments**

npath Number of sample paths  $\begin{array}{ll} \text{number of sample paths} \\ \text{ntimestep} \\ \text{ntsparam} & \text{A vector of the NTS parameters } (\alpha, \theta, \beta, \gamma, \mu). \text{ A vector of the standard NTS } \\ \text{parameters } (\alpha, \theta, \beta). \\ \text{dt} & \text{the time length of one time step by the year fraction. "dt=1" means 1-year.} \\ \end{array}$ 

### Value

Structure of the sample path. Matrix of sample path. Column index is time.

```
#standard NTS process sample path
alpha <- 1.2
theta <- 1
beta <- -0.2
ntsparam <- c(alpha, theta, beta)</pre>
```

getPortNTSParam 11

```
npath <- 5
ntimestep <- 250
dt <- 1/250
simulation <- gensamplepathnts(npath, ntimestep, ntsparam, dt)</pre>
matplot(colnames(simulation), t(simulation), type = 'l')
#NTS process sample path
alpha <- 1.2
theta <- 1
beta <- -0.2
gamma <- 0.3
mu <- 0.1
ntsparam <- c(alpha, theta, beta, gamma, mu)</pre>
npath <- 5
ntimestep <- 250
dt <- 1/250
simulation <- gensamplepathnts(npath, ntimestep, ntsparam, dt)</pre>
matplot(colnames(simulation), t(simulation), type = 'l')
```

getPortNTSParam

getPortNTSParam

# Description

Portfolio return with capital allocation weight is  $R_p = \langle w, r \rangle$ , which is a weighted sum of of elements in the N-dimensional NTS random vector.  $R_p$  becomes an 1-dimensional NTS random variable. getPortNTSParam find the parameters of  $R_p$ .

# Usage

```
\code{res <- setPortfolioParam(strPMNTS,w)}
\code{res <- setPortfolioParam(strPMNTS,w, FALSE)}</pre>
```

# **Arguments**

strPMNTS Structure of parameters for the n-dimensional NTS distribution. strPMNTS\$ndim: dimension strPMNTS\$mu :  $\mu$  mean vector (column vector) of the input data. strPMNTSsigma :  $\sigma$  standard deviation vector (column vector) of the input data. strPMNTS\$alpha:  $\alpha$  of the std NTS distribution (X). strPMNTS\$theta :  $\theta$  of the std NTS distribution (X). strPMNTS\$beta :  $\beta$  vector (column vector) of the std NTS distribution (X). res\$Rho :  $\rho$  matrix (Correlation) of the std NTS distribution (X). resSigma: Covariance  $\Sigma$  matrix of return data r. Capital allocation weight vector. If stdform is FALSE, then the return parameter has the following representation stdform  $R_p = \langle w, r \rangle = \mu + diag(\sigma)X,$ X follows  $stdNTS_1(\alpha, \theta, \beta, 1)$ . If stdform is TRUE, then the return parameter has the following representation

 $R_p = \langle w, r \rangle$  follows  $stdNTS_1(\alpha, \theta, \beta, \gamma, \mu)$ 

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#### Value

```
The weighted sum follows 1-dimensional NTS. R_p = < w, r >= \mu + diag(\sigma)X, where X \text{ follows } stdNTS_1(\alpha, \theta, \beta, 1). Hence we obtain \operatorname{res\$mu}: \mu \text{ mean of } R_p. \operatorname{res\$sigma}: \sigma \text{ standard deviation of } R_p. \operatorname{res\$alpha}: \alpha \text{ of } X. \operatorname{res\$theta}: \theta \text{ of } X. \operatorname{res\$theta}: \beta \text{ of } X.
```

### References

Proposition 2.1 of Kim, Y. S. (2020) Portfolio Optimization on the Dispersion Risk and the Asymmetric Tail Risk https://arxiv.org/pdf/2007.13972.pdf

# **Examples**

ipnts

ipnts

# Description

ipnts calculates inverse cdf of the NTS distribution with parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ . If only three parameters are given, it calculates inverse cdf of the standard NTS distribution with parameter  $(\alpha, \theta, \beta)$ 

# Usage

```
ipnts(u, ntsparam, maxmin = c(-10, 10), du = 0.01)
```

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# **Arguments**

u Real value between 0 and 1  $\text{A vector of the NTS parameters } (\alpha, \theta, \beta, \gamma, \mu). \text{ A vector of the standard NTS parameters } (\alpha, \theta, \beta).$ 

#### Value

Inverse cdf of the NTS distribution. It is the same as quts function.

### References

Kim, Y. S. (2020) Portfolio Optimization on the Dispersion Risk and the Asymmetric Tail Risk https://arxiv.org/pdf/2007.13972.pdf

```
alpha <- 1.2
theta <- 1
beta <- -0.2
ntsparam <- c(alpha, theta, beta)</pre>
u \leftarrow seq(from = 0.01, to = 0.99, length.out = 99)
q <- ipnts(u, ntsparam)</pre>
plot(u,q,type = 'l')
alpha <- 1.2
theta <- 1
beta <- -0.2
gamma <- 0.3
mu <- 0.1
ntsparam <- c(alpha, theta, beta, gamma, mu)</pre>
u \leftarrow seq(from = 0.01, to = 0.99, length.out = 99)
q <- ipnts(u, ntsparam)</pre>
plot(x,q,type = 'l')
#Annual based parameters
alpha <- 1.2
theta <- 1
beta <- -0.2
gamma <- 0.3
mu <- 0.1
#scaling annual parameters to one day
dt <- 1/250 #one day
u \leftarrow seq(from = 0.01, to = 0.99, length.out = 99)
q <- ipnts(u, ntsparam)</pre>
plot(x,q,type = 'l')
```

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 ${\tt moments\_NTS}$ 

moments\_NTS

# Description

moments\_NTS calculates mean, variance, skewness, and excess kurtosis of the NTS distribution with parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ .

# Usage

```
moments_NTS(param)
```

# **Arguments**

param

A vector of the NTS parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ .

# Value

First 4 moments (Mean, Variance, Skewness, Excess Kurtosis) of NTS distribution. The mean is always the same as the parameter  $\mu$ .

### References

Kim, Y.S, K-H Roh, R. Douady (2020) Tempered Stable Processes with Time Varying Exponential Tails https://arxiv.org/pdf/2006.07669.pdf

# **Examples**

```
alpha <- 1.2
theta <- 1
beta <- -0.2
gamma <- 0.3
mu <- 0.1
ntsparam <- c(alpha, theta, beta, gamma, mu)
moments_NTS(param = ntsparam)</pre>
```

moments\_stdNTS

moments\_stdNTS

# Description

moments\_stdNTS calculates mean, variance, skewness, and excess kurtosis of the standard NTS distribution with parameters  $(\alpha, \theta, \beta)$ .

# Usage

```
moments_stdNTS(param)
```

### **Arguments**

param

A vector of the standard NTS parameters  $(\alpha, \theta, \beta)$ .

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#### Value

First 4 moments (Mean, Variance, Skewness, Excess Kurtosis) of NTS distribution. Of course, the mean and variance are always 0 and 1, respectively.

### References

Kim, Y.S, K-H Roh, R. Douady (2020) Tempered Stable Processes with Time Varying Exponential Tails https://arxiv.org/pdf/2006.07669.pdf

# **Examples**

```
alpha <- 1.2
theta <- 1
beta <- -0.2
ntsparam <- c(alpha, theta, beta)
moments_stdNTS(param = ntsparam)</pre>
```

pnts

pnts

# **Description**

pnts calculates cdf of the NTS distribution with parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ . If only three parameters are given, it calculates cdf of the standard NTS distribution with parameter  $(\alpha, \theta, \beta)$  If a time parameter value is given, it calculates cdf of the profess F(x) = P((X(t+s) - X(s)) < x), where X is the NTS process generated by the NTS distribution with parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ .

### Usage

```
pnts(xdata, ntsparam, dz = 2^-8, m = 2^12)
```

# **Arguments**

xdata An array of x

ntsparam A vector of the NTS parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ . For the NTS process case it is a

vector of parameters  $(\alpha, \theta, \beta, \gamma, \mu, t)$ . A vector of the standard NTS parameters

 $(\alpha, \theta, \beta)$ .

#### Value

Cumulative probability of the NTS distribution

### References

Kim, Y. S. (2020) Portfolio Optimization on the Dispersion Risk and the Asymmetric Tail Risk https://arxiv.org/pdf/2007.13972.pdf

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#### **Examples**

```
alpha <- 1.2
theta <- 1
beta <- -0.2
ntsparam <- c(alpha, theta, beta)</pre>
x \leftarrow seq(from = -6, to = 6, length.out = 101)
p <- pnts(x, ntsparam)</pre>
plot(x,p,type = 'l')
alpha <- 1.2
theta <- 1
beta <- -0.2
gamma <- 0.3
mu <- 0.1
ntsparam <- c(alpha, theta, beta, gamma, mu)</pre>
x \leftarrow seq(from = -2, to = 2, by = 0.01)
p <- pnts(x, ntsparam)</pre>
plot(x,p,type = 'l')
#Annual based parameters
alpha <- 1.2
theta <- 1
beta <- -0.2
gamma <- 0.3
mu <- 0.1
#scaling annual parameters to one day
dt <- 1/250 #one day
ntsparam <- c(alpha, theta, beta, gamma, mu, dt)</pre>
x <- seq(from = -0.02, to = 0.02, length.out = 101)
p <- pnts(x, ntsparam)</pre>
plot(x,p,type = 'l')
```

portfolioCVaRmnts

portfolioCVaRmnts

# **Description**

Calculate portfolio conditional value at risk (expected shortfall) on the NTS market model

#### Usage

```
portfolioCVaRmnts(strPMNTS, w, eta)
```

# **Arguments**

```
strPMNTS Structure of parameters for the n-dimensional NTS distribution.  \begin{split} & \mathsf{strPMNTS\$ndim}: \mathsf{dimension} \\ & \mathsf{strPMNTS\$mu}: \mu \; \mathsf{mean} \; \mathsf{vector} \; (\mathsf{column} \; \mathsf{vector}) \; \mathsf{of} \; \mathsf{the} \; \mathsf{input} \; \mathsf{data}. \\ & \mathsf{strPMNTS\$sigma}: \; \sigma \; \mathsf{standard} \; \mathsf{deviation} \; \mathsf{vector} \; (\mathsf{column} \; \mathsf{vector}) \; \mathsf{of} \; \mathsf{the} \; \mathsf{input} \; \mathsf{data}. \\ & \mathsf{strPMNTS\$alpha}: \; \alpha \; \mathsf{of} \; \mathsf{the} \; \mathsf{std} \; \mathsf{NTS} \; \mathsf{distribution} \; (\mathsf{X}). \end{split}
```

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strPMNTS\$theta :  $\theta$  of the std NTS distribution (X).

strPMNTS\$beta :  $\beta$  vector (column vector) of the std NTS distribution (X).

res\$Rho :  $\rho$  matrix (Correlation) of the std NTS distribution (X).

resSigma: Covariance  $\Sigma$  matrix of return data r.

w Capital allocation weight vector.

eta significanlt level

#### Value

portfolio value at risk on the NTS market model

portfolioVaRmnts

portfolioVaRmnts

# **Description**

Calculate portfolio value at risk on the NTS market model

# Usage

```
portfolioVaRmnts(strPMNTS, w, eta)
```

# **Arguments**

strPMNTS Structure of parameters for the n-dimensional NTS distribution.

 $\verb|strPMNTS| sime : dimension|$ 

 $strPMNTS$mu: \mu mean vector (column vector) of the input data.$ 

 ${\tt strPMNTS\$sigma}: \sigma$  standard deviation vector (column vector) of the input

data.

strPMNTS\$alpha :  $\alpha$  of the std NTS distribution (X). strPMNTS\$theta :  $\theta$  of the std NTS distribution (X).

 $strPMNTS$beta: \beta vector (column vector) of the std NTS distribution (X).$ 

res\$Rho :  $\rho$  matrix (Correlation) of the std NTS distribution (X).

resSigma: Covariance  $\Sigma$  matrix of return data r.

w Capital allocation weight vector.

eta significanlt level

# Value

portfolio value at risk on the NTS market model

18 qnts

qnts qnts

# **Description**

qnts calculates quantile of the NTS distribution with parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ . If only three parameters are given, it calculates quantile of the standard NTS distribution with parameter  $(\alpha, \theta, \beta)$  If a time parameter value is given, it calculates quantile of NTS profess. That is it finds x such that u = P((X(t+s) - X(s)) < x), where X is the NTS process generated by the NTS distribution with parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ .

# Usage

```
qnts(u, ntsparam)
```

# **Arguments**

ntsparam A vector of the NTS parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ . For the NTS process case it is

a vector of parameters  $(\alpha, \theta, \beta, \gamma, \mu, t)$ . A vector of standard NTS parameters

 $(\alpha, \theta, \beta)$ .

vector of probabilities.

### Value

The quantile function of the NTS distribution

```
alpha <- 1.2
theta <- 1
beta <- -0.2
ntsparam <- c(alpha, theta, beta)</pre>
u \leftarrow c(0.01, 0.05, 0.25, 0.5, 0.75, 0.95, 0.99)
q <- qnts(u, ntsparam)</pre>
alpha <- 1.2
theta <- 1
beta <- -0.2
gamma <- 0.3
mu <- 0.1
ntsparam <- c(alpha, theta, beta, gamma, mu)</pre>
u \leftarrow c(0.01, 0.05, 0.25, 0.5, 0.75, 0.95, 0.99)
q <- qnts(u, ntsparam)</pre>
#Annual based parameters
alpha <- 1.2
theta <- 1
beta <- -0.2
gamma <- 0.3
mu <- 0.1
#scaling annual parameters to one day
dt <- 1/250 #one day
```

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```
ntsparam <- c(alpha, theta, beta, gamma, mu, dt) u <- c(0.01,0.05,0.25,0.5, 0.75, 0.95, 0.99) q <- qnts(u, ntsparam)
```

rmnts

rmnts

### **Description**

```
rmnts generates random vector following the n dimensional NTS distribution. r=\mu+diag(\sigma)X,
```

where

X follows  $stdNTS_n(\alpha, \theta, \beta, \Sigma)$ 

### Usage

```
rmnts(strPMNTS, numofsample, rW = NaN, rTau = NaN)
```

#### **Arguments**

Structure of parameters for the n-dimensional NTS distribution. 
$$\begin{split} & \mathsf{strPMNTS\$ndim}: \text{dimension} \\ & \mathsf{strPMNTS\$mu}: \mu \text{ mean vector (column vector) of the input data.} \\ & \mathsf{strPMNTS\$sigma}: \sigma \text{ standard deviation vector (column vector) of the input data.} \\ & \mathsf{strPMNTS\$slpha}: \alpha \text{ of the std NTS distribution (X).} \\ & \mathsf{strPMNTS\$theta}: \theta \text{ of the std NTS distribution (X).} \\ & \mathsf{strPMNTS\$theta}: \beta \text{ vector (column vector) of the std NTS distribution (X).} \\ & \mathsf{strPMNTS\$heta}: \beta \text{ vector (column vector) of the std NTS distribution (X).} \\ & \mathsf{number of samples.} \end{split}$$

# Value

Simulated NTS random vectors

#### References

Kim, Y. S. (2020) Portfolio Optimization on the Dispersion Risk and the Asymmetric Tail Risk https://arxiv.org/pdf/2007.13972.pdf

20 rnts

rnts rnts

# **Description**

rnts generates random numbers following NTS distribution with parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ . If only three parameters are given, it generates random numbers of standard NTS distribution with parameter  $(\alpha, \theta, \beta)$  If a time parameter value is given, it generates random numbers of increments of NTS profess for time interval t.

# Usage

```
rnts(n, ntsparam)
```

# **Arguments**

n number of random numbers to be generated. A vector of NTS parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ . For NTS process case it is a vector of parameters  $(\alpha, \theta, \beta, \gamma, \mu, t)$ . A vector of standard NTS parameters  $(\alpha, \theta, \beta)$ .

### Value

NTS random numbers

### References

Kim, Y. S. (2020) Portfolio Optimization on the Dispersion Risk and the Asymmetric Tail Risk https://arxiv.org/pdf/2007.13972.pdf

```
alpha <- 1.2
theta <- 1
beta <- -0.2
ntsparam <- c(alpha, theta, beta)</pre>
r \leftarrow rnts(100, ntsparam) #generate 100 NTS random numbers
plot(r)
alpha <- 1.2
theta <- 1
beta <- -0.2
gamma <- 0.3
mu <- 0.1
ntsparam <- c(alpha, theta, beta, gamma, mu)</pre>
r <- rnts(100, ntsparam) #generate 100 NTS random numbers
plot(r)
#Annual based parameters
alpha <- 1.2
theta <- 1
beta <- -0.2
gamma <- 0.3
```

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```
mu <- 0.1
#scaling annual parameters to one day
dt <- 1/250 #one day
ntsparam <- c(alpha, theta, beta, gamma, mu, dt)
r <- rnts(100, ntsparam) #generate 100 NTS random numbers
plot(r)</pre>
```

setPortfolioParam

setPortfolioParam

# **Description**

Please use getPortNTSParam instead of setPortfolioParam.

Portfolio return with capital allocation weight is  $R_p = \langle w, r \rangle$ , which is a weighted sum of of elements in the N-dimensional NTS random vector.  $R_p$  becomes an 1-dimensional NTS random variable. setPortfolioParam find the parameters of  $R_p$ .

# Usage

```
\code{res <- setPortfolioParam(strPMNTS,w)}</pre>
```

# **Arguments**

strPMNTS Structure of parameters for the n-dimensional NTS distribution.

strPMNTS\$ndim: dimension

strPMNTS\$mu :  $\mu$  mean vector (column vector) of the input data.

 ${\tt strPMNTS\$sigma}: \sigma$  standard deviation vector (column vector) of the input

data.

strPMNTS\$alpha :  $\alpha$  of the std NTS distribution (X). strPMNTS\$theta :  $\theta$  of the std NTS distribution (X).

Strpmin 155 theta: 6 of the sid N 15 distribution (A).

 ${\tt strPMNTS\$beta}: \beta \ {\tt vector} \ ({\tt column} \ {\tt vector}) \ {\tt of} \ {\tt the} \ {\tt std} \ {\tt NTS} \ {\tt distribution} \ ({\tt X}).$ 

 ${\tt strPMNTS\$Rho}: \Sigma$  matrix of the std NTS distribution (X).

W

Capital allocation weight vector.

### Value

The weighted sum follows 1-dimensional NTS.

```
R_p = \langle w, r \rangle = \mu + diag(\sigma)X,
```

where

X follows  $stdNTS_1(\alpha, \theta, \beta, 1)$ .

Hence we obtain

res\$mu :  $\mu$  mean of  $R_p$ .

res\$sigma :  $\sigma$  standard deviation of  $R_p$ .

 $\label{eq:alpha:alpha:alpha:alpha:alpha:alpha:alpha:beta:alpha} \alpha \mbox{ of } X.$   $\mbox{res\$beta:} \beta \mbox{ } X.$ 

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### References

Kim, Y. S. (2020) Portfolio Optimization on the Dispersion Risk and the Asymmetric Tail Risk https://arxiv.org/pdf/2007.13972.pdf

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