# Package 'temStaR'

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<b>Fitle</b> Tempered Stalbe Distribution
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<b>Description</b> This package provides useful tools to use the multivariate normal tempered stable distribution and process
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Imports functional, nloptr, pracma, spatstat, Matrix  Suggests functional, nloptr, pracma, spatstat, Matrix
R topics documented:
chf_NTS chf_stdNTS cvarnts dnts fitnts fitstdnts gensamplepathnts ipnts moments_NTS moments_stdNTS  cvarnts 1

chf\_NTS

Index																							16
	pnts qnts rnts																						13

chf\_NTS

 $chf\_NTS$ 

## **Description**

chf\_NTS calculates Ch.F of the NTS distribution with parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ . If a time parameter value is given, it calculates Ch.F of the NTS profess  $\phi(u) = E[exp(iu(X(t+s)-X(s)))] = exp(tlog(E[exp(iuX(1))]))$ , where X is the NTS process generated by the NTS distribution with parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ .

# Usage

```
chf_NTS(u, param)
```

## **Arguments**

u An array of u  $\text{A vector of the NTS parameters } (\alpha, \theta, \beta, \gamma, \mu). \text{ For NTS process case it is a vector of parameters } (\alpha, \theta, \beta, \gamma, \mu, t).$ 

#### Value

Characteristic function of the NTS distribution

```
alpha <- 1.2
theta <- 1
beta <- -0.2
gamma <- 0.3
mu <- 0.1
ntsparam <- c(alpha, theta, beta, gamma, mu)</pre>
u \leftarrow seq(from = -2*pi, to = 2*pi, length.out = 101)
phi <- chf_NTS(u, ntsparam)</pre>
#Annual based parameters
alpha <- 1.2
theta <- 1
beta <- -0.2
gamma <- 0.3
mu <- 0.1
#scaling annual parameters to one day
dt <- 1/250 #one day
ntsparam <- c(alpha, theta, beta, gamma, mu, dt)</pre>
u \leftarrow seq(from = -2*pi, to = 2*pi, length.out = 101)
phi <- chf_NTS(u, ntsparam)</pre>
```

chf\_stdNTS 3

chf\_stdNTS

chf\_stdNTS

# Description

chf\_stdNTS calculates Ch.F of the standard NTS distribution with parameters  $(\alpha, \theta, \beta)$ . If a time parameter value is given, it calculates Ch.F of the standard NTS profess  $\phi(u) = E[exp(iu(X(t+s)-X(s))] = exp(tlog(E[exp(iuX(1))]))$ , where X is the standard NTS process generated by the standard NTS distribution with parameters  $(\alpha, \theta, \beta)$ .

## Usage

```
chf_stdNTS(u, param)
```

#### **Arguments**

u An array of u

ntsparam

A vector of the standard NTS parameters  $(\alpha, \theta, \beta)$ . For the standard NTS process case it is a vector of parameters  $(\alpha, \theta, \beta, t)$ .

#### Value

Characteristic function of the standard NTS distribution

```
alpha <- 1.2
theta <- 1
beta <- -0.2
ntsparam <- c(alpha, theta, beta)
u <- seq(from = -2*pi, to = 2*pi, length.out = 101)
phi <- chf_stdNTS(u, ntsparam)

#Annual based parameters
alpha <- 1.2
theta <- 1
beta <- -0.2
#scaling annual parameters to one day
dt <- 1/250 #one day
ntsparam <- c(alpha, theta, beta, gamma, mu, dt)
u <- seq(from = -2*pi, to = 2*pi, length.out = 101)
phi <- chf_stdNTS(u, ntsparam)</pre>
```

4 cvarnts

cvarnts

cvarnts

## **Description**

cvarnts calculates Conditional Value at Risk (CVaR, or expected shortfall ES) of the NTS market model with parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ . If only three parameters are given, it calculates CVaR of the standard NTS distribution with parameter  $(\alpha, \theta, \beta)$ 

## Usage

```
cvarnts(eps, ntsparam)
```

#### Arguments

ntsparam A vector of the NTS parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ . A vector of the standard NTS

parameters  $(\alpha, \theta, \beta)$ .

u Real value between 0 and 1

#### Value

CVaR of the NTS distribution.

#### References

- Y. S. Kim, S. T. Rachev, M. L. Bianchi, and F. J. Fabozzi (2010), Computing VaR and AVaR in infinitely divisible distributions, Probability and Mathematical Statistics, 30 (2), 223-245.
- S. T. Rachev, Y. S. Kim, M. L. Bianchi, and F. J. Fabozzi (2011), Financial Models with Levy Processes and Volatility Clustering, John Wiley & Sons

```
alpha <- 1.2
theta <- 1
beta <- -0.2
ntsparam <- c(alpha, theta, beta)</pre>
u \leftarrow c(0.01, 0.05)
q <- cvarnts(u, ntsparam)</pre>
alpha <- 1.2
theta <- 1
beta <- -0.2
gamma <- 0.3
mu <- 0.1
ntsparam <- c(alpha, theta, beta, gamma, mu)</pre>
u \leftarrow c(0.01, 0.05)
q \leftarrow cvarnts(u, ntsparam)
#Annual based parameters
alpha <- 1.2
theta <- 1
beta <- -0.2
```

dnts 5

```
gamma <- 0.3 mu <- 0.1 #scaling annual parameters to one day dt <- 1/250 #one day ntsparam <- c(alpha, theta, beta, gamma, mu, dt) u <- c(0.01,0.05) q <- cvarnts(u, ntsparam)
```

dnts

dnts

#### **Description**

dnts calculates pdf of the NTS distribution with parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ . If only three parameters are given, it calculates pdf of the standard NTS distribution with parameter  $(\alpha, \theta, \beta)$  If a time parameter value is given, it calculates pdf of the NTS profess f(x)dx = d(P((X(t+s) - X(s)) < x)), where X is the NTS process generated by the NTS distribution with parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ .

#### Usage

```
dnts(xdata, ntsparam)
```

## **Arguments**

xdata An array of x

ntsparam A vector of the NTS parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ . For the NTS process case it is a

vector of parameters  $(\alpha, \theta, \beta, \gamma, \mu, t)$ . A vector of the standard NTS parameters

 $(\alpha, \theta, \beta)$ .

#### Value

Density of NTS distribution

#### References

Kim, Y. S. (2020) Portfolio Optimization on the Dispersion Risk and the Asymmetric Tail Risk https://arxiv.org/pdf/2007.13972.pdf

```
alpha <- 1.2
theta <- 1
beta <- -0.2
ntsparam <- c(alpha, theta, beta)
x <- seq(from = -6, to = 6, length.out = 101)
d <- dnts(x, ntsparam)
plot(x,d,type = 'l')

alpha <- 1.2
theta <- 1
beta <- -0.2
gamma <- 0.3</pre>
```

6 fitnts

```
mu <- 0.1
ntsparam <- c(alpha, theta, beta, gamma, mu)
x < - seq(from = -2, to = 2, by = 0.01)
d <- dnts(x, ntsparam)</pre>
plot(x,d,type = 'l')
#Annual based parameters
alpha <- 1.2
theta <- 1
beta <- -0.2
gamma <- 0.3
mu <- 0.1
#scaling annual parameters to one day
dt <- 1/250 #one day
ntsparam <- c(alpha, theta, beta, gamma, mu, dt)</pre>
x <- seq(from = -0.02, to = 0.02, length.out = 101)
d <- dnts(x, ntsparam)</pre>
plot(x,d,type = 'l')
```

fitnts

fitnts

#### **Description**

fitnts fit parameters  $(\alpha, \theta, \beta, \gamma, \mu)$  of the NTS distribution. This function using the curvefit method between the empirical cdf and the NTS cdf.

#### Usage

```
\code{fitnts(rawdat)}
\code{fitnts(rawdat), ksdensityflag = 1}
\code{fitnts(rawdat, initialparam = c(alpha, theta, beta, gamma, mu))}
\code{fitnts(rawdat, initialparam = c(alpha, theta, beta, gamma, mu)), ksdensityflag = 1}
\code{fitnts(rawdat, initialparam = c(alpha, theta, beta, gamma, mu)), maxeval = 100, ksdensityflag
```

#### **Arguments**

rawdat Raw data to fit the parameters.

initialparam A vector of initial NTS parameters. This function uses the nloptr package. If

it has a good initial parameter then estimation performs better. If users do not know a good initial parameters, then just set it as initialparam=NaN, that is default. The function cffitnts() may be helpful to find the initial parameters.

maxeval Maximum evaluation number for nloptr. The iteration stops on this many func-

tion evaluations.

ksdensityflag This function fit the parameters using the curvefit method between the empirical

cdf and the NTS cdf. If ksdensityflag = 1 (default), then the empirical cdf is calculated by the kernel density estimation. If ksdensityflag = 0, then the

empirical cdf is calculated by the empirical cdf.

#### Value

Estimated parameters

fitstdnts 7

#### References

Kim, Y. S. (2020) Portfolio Optimization on the Dispersion Risk and the Asymmetric Tail Risk https://arxiv.org/pdf/2007.13972.pdf

#### **Examples**

```
library("quantmod")
getSymbols("^GSPC", src="yahoo", from = "2010-1-1", to = "2020-12-31")
pr <- as.numeric(GSPC$GSPC.Adjusted)
ret <- diff(log(pr))
ntsparam <- fitnts(ret)

Femp = ecdf(ret)
x = seq(from=min(ret), to = max(ret), length.out = 100)
cemp = Femp(x)
ncdf = pnts(x, c(ntsparam))
plot(x,ncdf,type = 'l', col = "red")
points(x,cemp, type = 'l', col = "blue")
a = density(ret)
p = dnts(x,ntsparam)
plot(x,p,type = 'l', col = "red")
lines(a,type = 'l', col = "blue")</pre>
```

fitstdnts

fitstdnts

## Description

fitstdnts fit parameters  $(\alpha, \theta, \beta)$  of the standard NTS distribution. This function using the curvefit method between the empirical cdf and the standard NTS cdf.

# Usage

```
\code{fitstdnts(rawdat)}
\code{fitstdnts(rawdat), ksdensityflag = 1}
\code{fitstdnts(rawdat, initialparam = c(alpha, theta, beta))}
\code{fitstdnts(rawdat, initialparam = c(alpha, theta, beta)), ksdensityflag = 1}
\code{fitstdnts(rawdat, initialparam = c(alpha, theta, beta)), maxeval = 100, ksdensityflag = 1}
```

## **Arguments**

rawdat Raw data to fit the parameters.

initialparam A vector of initial standard NTS parameters. This function uses the nloptr

package. If it has a good initial parameter then estimation performs better. If users do not know a good initial parameters, then just set it as initialparam=NaN,

that is default.

maxeval Maximum evaluation number for nloptr. The iteration stops on this many func-

tion evaluations.

ksdensityflag This function fit the parameters using the curvefit method between the empirical

cdf and the standard NTS cdf. If ksdensityflag = 1 (default), then the empirical cdf is calculated by the kernel density estimation. If ksdensityflag = 0,

then the empirical cdf is calculated by the empirical cdf.

8 gensamplepathnts

#### Value

Estimated parameters

#### References

Kim, Y. S. (2020) Portfolio Optimization on the Dispersion Risk and the Asymmetric Tail Risk https://arxiv.org/pdf/2007.13972.pdf

#### **Examples**

```
library("quantmod")
getSymbols("^GSPC", src="yahoo", from = "2010-1-1", to = "2020-12-31")
pr <- as.numeric(GSPC$GSPC.Adjusted)</pre>
ret <- diff(log(pr))</pre>
stdret <- (ret-mean(ret))/sd(ret)</pre>
stdntsparam <- fitstdnts(stdret)</pre>
Femp = ecdf(stdret)
x = seq(from=min(stdret), to = max(stdret), length.out = 100)
cemp = Femp(x)
ncdf = pnts(x, c(stdntsparam))
plot(x,ncdf,type = 'l', col = "red")
lines(x,cemp, type = '1', col = "blue")
a = density(stdret)
p = dnts(x,stdntsparam)
plot(x,p,type = 'l', col = "red", ylim = c(0, max(a$y, p)))
lines(a,type = 'l', col = "blue")
```

 ${\tt gensample} pathnts$ 

gensamplepathnts

# Description

gensamplepathnts generate sample paths of the NTS process with parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ . If only three parameters are given, it generate sample paths of the standard NTS process with parameters  $(\alpha, \theta, \beta)$ .

## Usage

```
gensamplepathnts(npath, ntimestep, ntsparam, dt)
```

#### **Arguments**

npath Number of sample paths  $\begin{array}{ll} \text{number of sample paths} \\ \text{ntimestep} \\ \text{ntsparam} & \text{A vector of the NTS parameters } (\alpha, \theta, \beta, \gamma, \mu). \text{ A vector of the standard NTS } \\ \text{parameters } (\alpha, \theta, \beta). \\ \text{dt} & \text{the time length of one time step by the year fraction. "dt=1" means 1-year.} \\ \end{array}$ 

#### Value

Structure of the sample path. Matrix of sample path. Column index is time.

ipnts 9

#### **Examples**

```
#standard NTS process sample path
alpha <- 1.2
theta <- 1
beta <- -0.2
ntsparam <- c(alpha, theta, beta)</pre>
npath <- 5
ntimestep <- 250
dt <- 1/250
simulation <- gensamplepathnts(npath, ntimestep, ntsparam, dt)</pre>
matplot(colnames(simulation), t(simulation), type = 'l')
#NTS process sample path
alpha <- 1.2
theta <- 1
beta <- -0.2
gamma <- 0.3
mu <- 0.1
ntsparam <- c(alpha, theta, beta, gamma, mu)</pre>
npath <- 5
ntimestep <- 250
dt <- 1/250
simulation <- gensamplepathnts(npath, ntimestep, ntsparam, dt)</pre>
matplot(colnames(simulation), t(simulation), type = 'l')
```

ipnts

ipnts

#### **Description**

ipnts calculates inverse cdf of the NTS distribution with parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ . If only three parameters are given, it calculates inverse cdf of the standard NTS distribution with parameter  $(\alpha, \theta, \beta)$ 

## Usage

```
ipnts(u, ntsparam, maxmin = c(-10, 10), du = 0.01)
```

## **Arguments**

u Real value between 0 and 1

ntsparam A vector of the NTS parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ . A vector of the standard NTS parameters  $(\alpha, \theta, \beta)$ .

## Value

Inverse cdf of the NTS distribution. It is the same as qnts function.

#### References

Kim, Y. S. (2020) Portfolio Optimization on the Dispersion Risk and the Asymmetric Tail Risk https://arxiv.org/pdf/2007.13972.pdf

10 moments\_NTS

#### **Examples**

```
alpha <- 1.2
theta <- 1
beta <- -0.2
ntsparam <- c(alpha, theta, beta)</pre>
u \leftarrow seq(from = 0.01, to = 0.99, length.out = 99)
q <- ipnts(u, ntsparam)</pre>
plot(u,q,type = 'l')
alpha <- 1.2
theta <- 1
beta <- -0.2
gamma <- 0.3
mu <- 0.1
ntsparam <- c(alpha, theta, beta, gamma, mu)</pre>
u \leftarrow seq(from = 0.01, to = 0.99, length.out = 99)
q <- ipnts(u, ntsparam)</pre>
plot(x,q,type = 'l')
#Annual based parameters
alpha <- 1.2
theta <- 1
beta <- -0.2
gamma <- 0.3
mu <- 0.1
#scaling annual parameters to one day
dt <- 1/250 #one day
ntsparam <- c(alpha, theta, beta, gamma, mu, dt)</pre>
u \leftarrow seq(from = 0.01, to = 0.99, length.out = 99)
q \leftarrow ipnts(u, ntsparam)
plot(x,q,type = 'l')
```

moments\_NTS

moments\_NTS

## **Description**

moments\_NTS calculates mean, variance, skewness, and excess kurtosis of the NTS distribution with parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ .

## Usage

```
moments_NTS(param)
```

## **Arguments**

param

A vector of the NTS parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ .

## Value

First 4 moments (Mean, Variance, Skewness, Excess Kurtosis) of NTS distribution. The mean is always the same as the parameter  $\mu$ .

moments\_stdNTS 11

#### References

Kim, Y.S, K-H Roh, R. Douady (2020) Tempered Stable Processes with Time Varying Exponential Tails https://arxiv.org/pdf/2006.07669.pdf

#### **Examples**

```
alpha <- 1.2
theta <- 1
beta <- -0.2
gamma <- 0.3
mu <- 0.1
ntsparam <- c(alpha, theta, beta, gamma, mu)
moments_NTS(param = ntsparam)</pre>
```

moments\_stdNTS

moments stdNTS

## **Description**

moments\_stdNTS calculates mean, variance, skewness, and excess kurtosis of the standard NTS distribution with parameters  $(\alpha, \theta, \beta)$ .

#### Usage

```
moments_stdNTS(param)
```

# Arguments

param

A vector of the standard NTS parameters  $(\alpha, \theta, \beta)$ .

## Value

First 4 moments (Mean, Variance, Skewness, Excess Kurtosis) of NTS distribution. Of course, the mean and variance are always 0 and 1, respectively.

#### References

Kim, Y.S, K-H Roh, R. Douady (2020) Tempered Stable Processes with Time Varying Exponential Tails https://arxiv.org/pdf/2006.07669.pdf

```
alpha <- 1.2
theta <- 1
beta <- -0.2
ntsparam <- c(alpha, theta, beta)
moments_stdNTS(param = ntsparam)</pre>
```

pnts

pnts pnts

## **Description**

pnts calculates cdf of the NTS distribution with parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ . If only three parameters are given, it calculates cdf of the standard NTS distribution with parameter  $(\alpha, \theta, \beta)$  If a time parameter value is given, it calculates cdf of the profess F(x) = P((X(t+s) - X(s)) < x), where X is the NTS process generated by the NTS distribution with parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ .

# Usage

```
pnts(xdata, ntsparam, dz = 2^-8, m = 2^12)
```

# **Arguments**

xdata An array of x

ntsparam A vector of the NTS parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ . For the NTS process case it is a

vector of parameters  $(\alpha, \theta, \beta, \gamma, \mu, t)$ . A vector of the standard NTS parameters

 $(\alpha, \theta, \beta)$ .

#### Value

Cumulative probability of the NTS distribution

#### References

Kim, Y. S. (2020) Portfolio Optimization on the Dispersion Risk and the Asymmetric Tail Risk https://arxiv.org/pdf/2007.13972.pdf

```
alpha <- 1.2
theta <- 1
beta <- -0.2
ntsparam <- c(alpha, theta, beta)</pre>
x \leftarrow seq(from = -6, to = 6, length.out = 101)
p <- pnts(x, ntsparam)</pre>
plot(x,p,type = 'l')
alpha <- 1.2
theta <- 1
beta <- -0.2
gamma <- 0.3
mu <- 0.1
ntsparam <- c(alpha, theta, beta, gamma, mu)</pre>
x < - seq(from = -2, to = 2, by = 0.01)
p <- pnts(x, ntsparam)</pre>
plot(x,p,type = 'l')
#Annual based parameters
alpha <- 1.2
```

qnts 13

```
theta <- 1 beta <- 0.2 gamma <- 0.3 mu <- 0.1 #scaling annual parameters to one day dt <- 1/250 #one day ntsparam <- 0.02, to = 0.02, length.out = 0.01 p <- 0.01 pnts(x, ntsparam) plot(x,p,type = 0.01 right)
```

qnts

qnts

## **Description**

qnts calculates quantile of the NTS distribution with parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ . If only three parameters are given, it calculates quantile of the standard NTS distribution with parameter  $(\alpha, \theta, \beta)$  If a time parameter value is given, it calculates quantile of NTS profess. That is it finds x such that u = P((X(t+s) - X(s)) < x), where X is the NTS process generated by the NTS distribution with parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ .

# Usage

```
qnts(u, ntsparam)
```

#### **Arguments**

ntsparam

A vector of the NTS parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ . For the NTS process case it is a vector of parameters  $(\alpha, \theta, \beta, \gamma, \mu, t)$ . A vector of standard NTS parameters  $(\alpha, \theta, \beta)$ .

vector

of probabilities.

## Value

The quantile function of the NTS distribution

```
alpha <- 1.2
theta <- 1
beta <- -0.2
ntsparam <- c(alpha, theta, beta)
u <- c(0.01,0.05,0.25,0.5, 0.75, 0.95, 0.99)
q <- qnts(u, ntsparam)

alpha <- 1.2
theta <- 1
beta <- -0.2
gamma <- 0.3
mu <- 0.1
ntsparam <- c(alpha, theta, beta, gamma, mu)
u <- c(0.01,0.05,0.25,0.5, 0.75, 0.95, 0.99)</pre>
```

14 rnts

```
q <- qnts(u, ntsparam)

#Annual based parameters
alpha <- 1.2
theta <- 1
beta <- -0.2
gamma <- 0.3
mu <- 0.1
#scaling annual parameters to one day
dt <- 1/250 #one day
ntsparam <- c(alpha, theta, beta, gamma, mu, dt)
u <- c(0.01,0.05,0.25,0.5, 0.75, 0.95, 0.99)
q <- qnts(u, ntsparam)
```

rnts

rnts

## **Description**

rnts generates random numbers following NTS distribution with parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ . If only three parameters are given, it generates random numbers of standard NTS distribution with parameter  $(\alpha, \theta, \beta)$  If a time parameter value is given, it generates random numbers of increments of NTS profess for time interval t.

#### Usage

```
rnts(n, ntsparam)
```

#### **Arguments**

n number of random numbers to be generated.

ntsparam A vector of NTS parameters  $(\alpha, \theta, \beta, \gamma, \mu)$ . For NTS process case it is a vector of parameters  $(\alpha, \theta, \beta, \gamma, \mu, t)$ . A vector of standard NTS parameters  $(\alpha, \theta, \beta)$ .

## Value

NTS randomnumbers

## References

Kim, Y. S. (2020) Portfolio Optimization on the Dispersion Risk and the Asymmetric Tail Risk https://arxiv.org/pdf/2007.13972.pdf

```
alpha <- 1.2
theta <- 1
beta <- -0.2
ntsparam <- c(alpha, theta, beta)
r <- rnts(100, ntsparam) #generate 100 NTS random numbers
plot(r)</pre>
```

rnts 15

```
alpha <- 1.2
theta <- 1
beta <- -0.2
gamma <- 0.3
mu <- 0.1
ntsparam <- c(alpha, theta, beta, gamma, mu)</pre>
r <- rnts(100, ntsparam) #generate 100 NTS random numbers
plot(r)
#Annual based parameters
alpha <- 1.2
theta <- 1
beta <- -0.2
gamma <- 0.3
mu <- 0.1
#scaling annual parameters to one day
dt <- 1/250 #one day
ntsparam <- \ c(alpha, \ theta, \ beta, \ gamma, \ mu, \ dt)
r <- rnts(100, ntsparam) #generate 100 NTS random numbers
plot(r)
```

# **Index**

```
chf_NTS, 2
chf_stdNTS, 3
cvarnts, 4

dnts, 5

fitnts, 6
fitstdnts, 7
gensamplepathnts, 8
ipnts, 9

moments_NTS, 10
moments_stdNTS, 11

pnts, 12
qnts, 13
rnts, 14
```