To merge a domain assoc space with the background space, we use an adaptation of the algorithm from Brand 2006[1]. Here I try to explain the adaptation in (relatively) plain English.

We are given two matrices X and Y in the form of eigenvalue decompositions $X = U_X S_X {U_X}^T$ and $Y = U_Y S_Y {U_Y}^T$; we want the eigenvalue decomposition of $X+Y$. In practice $X+Y$ has large dimension, but U_X and U_Y are "tall and thin" (that is, have few columns). The key insight is that the eigenvalue decomposition of $X+Y$ can be computed in the low-dimensional subspace spanned by U_X and U_Y.

We proceed as follows.

1. Obtain an orthogonal basis for the combined column spaces of U_X and U_Y. In practice we do this by computing $U_Y - U_X {U_X}^T U_Y$, which is the component of U_Y acting in the subspace orthogonal to U_X, and applying QR decomposition:
$$U_Y - U_X {U_X}^T U_Y = QR \tag{1}$$
Q is guaranteed to be orthogonal. U_X and Q together then span the column spaces of U_X and U_Y.

2. Express U_X and U_Y in this new orthogonal basis. The components of U_X are easy; they are the identity in the U_X part of the basis and zero in the Q part. The components of U_Y are ${U_X}^T U_Y$ and R, as seen by a trivial rearrangement of the above:
$$U_Y = U_X {U_X}^T U_Y + QR$$
The combined basis can be written using a single matrix
$$\begin{bmatrix} U_X & Q \end{bmatrix}$$
with the above relations summarized as
$$U_X = \begin{bmatrix} U_X & Q \end{bmatrix} \begin{bmatrix} I \\ 0 \end{bmatrix}$$
$$U_Y = \begin{bmatrix} U_X & Q \end{bmatrix} \begin{bmatrix} {U_X}^T U_Y \\ R \end{bmatrix}$$

3. Express the desired matrix $X+Y$ in the basis $\begin{bmatrix} U_X & Q \end{bmatrix}$. Inserting the expressions for U_X and U_Y into the given decompositions,
$$X = \begin{bmatrix} U_X & Q \end{bmatrix} \begin{bmatrix} S_X & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} U_X & Q \end{bmatrix}^T$$
$$Y = \begin{bmatrix} U_X & Q \end{bmatrix} \begin{bmatrix} {U_X}^T U_Y \\ R \end{bmatrix} S_Y \begin{bmatrix} {U_X}^T U_Y \\ R \end{bmatrix}^T \begin{bmatrix} U_X & Q \end{bmatrix}^T$$
So
$$X + Y = \begin{bmatrix} U_X & Q \end{bmatrix} K \begin{bmatrix} U_X & Q \end{bmatrix}^T$$
where K, the expression of $X+Y$ in the newly constructed basis, is
$$K = \begin{bmatrix} S_X & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} {U_X}^T U_Y \\ R \end{bmatrix} S_Y \begin{bmatrix} {U_X}^T U_Y \\ R \end{bmatrix}^T \tag{2}$$

4. Compute the eigenvalue decomposition of K. (Note that K is symmetric by construction.) This is easy because K has dimension equal to the total number of columns in U_X and U_Y, which is small.
$$K = U' S' U'^T \tag{3}$$
S' is, by construction, the matrix of eigenvalues of $X+Y$.

5. Find the new eigenbasis, which is simply a matter of inserting the decomposition of K into the previous formula:
$$X + Y = \left(\begin{bmatrix} U_X & Q \end{bmatrix} U' \right) S' \left(\begin{bmatrix} U_X & Q \end{bmatrix} U' \right)^T \tag{4}$$
The eigenbasis is properly column-normalized so long as U_X and Q are. (Remember that U_X and Q are orthogonal by construction.)

[1] http://www.merl.com/publications/docs/TR2006-059.pdf