To merge a domain assoc space with the background space, we use an adaptation of the algorithm from Brand 2006[1]. Here I try to explain the adaptation in (relatively) plain English.

We are given two matrices X and Y in the form of eigenvalue decompositions $X = U_X S_X {U_X}^T$ and $Y = U_Y S_Y {U_Y}^T$; we want the eigenvalue decomposition of $X+Y$. In practice $X+Y$ has large dimension, but U_X and U_Y are "tall and thin" (that is, have few columns). The key insight is that the eigenvalue decomposition of $X+Y$ can be computed in the low-dimensional subspace spanned by U_X and U_Y.

We proceed as follows:

1. Obtain an orthogonal basis for the combined column spaces of U_X and U_Y. In practice we do this by computing $U_Y - U_X {U_X}^T U_Y$, which is the component of U_Y acting in the subspace orthogonal to U_X, and applying QR decomposition:
$$U_Y - U_X {U_X}^T U_Y = QR \tag{1}$$
Q is guaranteed to be orthogonal. U_X and Q together then span the column spaces of U_X and U_Y.

2. Express U_X and U_Y in this new orthogonal basis. The components of U_X are easy; they are the identity in the U_X part of the basis and zero in the Q part. The components of U_Y are ${U_X}^T U_Y$ and R, as seen by a trivial rearrangement of the above:
$$U_Y = U_X {U_X}^T U_Y + QR$$
The combined basis can be written using a single matrix
$$\begin{bmatrix} U_X & Q \end{bmatrix}$$
with the above relations summarized as
$$U_X = \begin{bmatrix} U_X & Q \end{bmatrix} \begin{bmatrix} I \\ 0 \end{bmatrix}$$
$$U_Y = \begin{bmatrix} U_X & Q \end{bmatrix} \begin{bmatrix} {U_X}^T U_Y \\ R \end{bmatrix}$$

3. Express the desired matrix $X+Y$ in the basis $\begin{bmatrix} U_X & Q \end{bmatrix}$. Inserting the expressions for U_X and U_Y into the given decompositions,
$$X = \begin{bmatrix} U_X & Q \end{bmatrix} \begin{bmatrix} S_X & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} U_X & Q \end{bmatrix}^T$$
$$Y = \begin{bmatrix} U_X & Q \end{bmatrix} \begin{bmatrix} {U_X}^T U_Y \\ R \end{bmatrix} S_Y \begin{bmatrix} {U_X}^T U_Y \\ R \end{bmatrix}^T \begin{bmatrix} U_X & Q \end{bmatrix}^T$$
So
$$X + Y = \begin{bmatrix} U_X & Q \end{bmatrix} K \begin{bmatrix} U_X & Q \end{bmatrix}^T$$
where K, the expression of $X+Y$ in the newly constructed basis, is
$$K = \begin{bmatrix} S_X & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} {U_X}^T U_Y \\ R \end{bmatrix} S_Y \begin{bmatrix} {U_X}^T U_Y \\ R \end{bmatrix}^T \tag{2}$$

4. Compute the eigenvalue decomposition of K. (Note that K is symmetric by construction.) This is easy because K has dimension equal to the total number of columns in U_X and U_Y, which is small.
$$K = U' S' U'^T \tag{3}$$
S' is, by construction, the matrix of eigenvalues of $X+Y$.

[1] http://www.merl.com/publications/docs/TR2006-059.pdf

5. Find the new eigenbasis, which is simply a matter of inserting the decomposition of K into the previous formula:

$$X + Y = \left(\begin{bmatrix} U_X & Q \end{bmatrix} U' \right) S' \left(\begin{bmatrix} U_X & Q \end{bmatrix} U' \right)^T \tag{4}$$

The eigenbasis is properly column-normalized so long as U_X and Q are. (Remember that U_X and Q are orthogonal by construction.)

We can adapt this for any number of matrices $X_1 = U_1 S_1 U_1^T, X_2 = U_2 S_2 U_2^T, X_3 = U_3 S_3, U_3^T, \ldots, X_n = U_n S_n U_n^T$. Again, the eigenvalue decomposition of $\sum_{i=0}^{n} X_i$ can be computed in the low-dimensional subspace spanned by the U_i.

1. Obtain an orthogonal basis for the combined column spaces of the U_i. As before, U_1 will form the first part of the desired basis, and we can find the contribution from U_2 by calculating the QR decomposition.

$$U_2 - U_1 U_1^T U_2 = Q_2 R_2$$

That is, we project U_2 into the subspace spanned by U_1, subtract that from U_2, and orthogonalize the result via QR decomposition. This left side of this equation is the component of U_2 acting in the subspace orthogonal to U_1. U_1 and Q_2 together span the column spaces of U_1 and U_2. To obtain the contribution from U_3, then, we need to remove both of these:

$$U_3 - (U_1 U_1^T + Q_2 Q_2^T) U_3 = Q_3 R_3$$

Now U_1, Q_2, and Q_3 together span the column spaces of U_1, U_2, and U_3. Extending this, we can find the component of U_m for $m > 1$ acting in the subspace orthogonal to the subspace spanned by U_1, $\ldots$, U_{m-1} and then orthogonalize the result. For notational convenience, we define $Q_1 = U_1$. (We will define $R_1 = I$ for convenience as well.)

$$U_m - \left(\sum_{i=1}^{m-1} Q_i Q_i^T \right) U_m = Q_m R_m \tag{5}$$

Then $U_1, Q_2, \ldots, Q_n$ together form an orthonormal basis for the combined column spaces of the U_i.

2. Express the U_i in this new orthogonal basis. The components of U_1 are easy; they are the identity on the U_1 part of the basis and zero on all of the $Q_2, \ldots, Q_n$ parts. The components of U_2 are $U_1^T U_2$ on the U_1 part of the basis, R_2 on the Q_2 part of the basis, and zero on all of the $Q_3, \ldots, Q_n$ parts. We can see this by rearranging our equation for $Q_2 R_2$.

$$U_2 = U_1 U_1^T U_2 + Q_2 R_2$$

Furthermore, the components of U_3 are $U_1^T U_3$, $Q_2^T U_3$, and R_3 on the first three parts of our orthogonal basis, respectively, and zero on all the other parts. This is once again seen by rearranging our equation for $Q_3 R_3$.

$$U_3 = U_1 U_1^T U_3 + Q_2 Q_2^T U_3 + Q_3 R_3$$

For U_m, $m > 1$, then, we can find the components of the appropriate part of the basis by rearranging the equation for U_m, once again naming $Q_1 = U_1$ for convenience.

$$U_m = \sum_{i=1}^{m-1} Q_i Q_i^T U_m + Q_m R_m$$

This tells us that the components of U_m on the Q_i part of the basis for $i < m$ are given by $Q_i^T U_m$, the components of U_m on the Q_m part of the basis are R_m, and the components on the Q_i part of the basis for $i > m$ are zero.

We can then express the U_i in terms of the basis $\begin{bmatrix} U_1 & Q_2 & Q_3 & Q_4 & \dots & Q_n \end{bmatrix}$ as follows:

$$U_1 = \begin{bmatrix} U_1 & Q_2 & Q_3 & Q_4 & \dots & Q_n \end{bmatrix} \begin{bmatrix} I \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$U_2 = \begin{bmatrix} U_1 & Q_2 & Q_3 & Q_4 & \dots & Q_n \end{bmatrix} \begin{bmatrix} U_1^T U_2 \\ R_2 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$U_3 = \begin{bmatrix} U_1 & Q_2 & Q_3 & Q_4 & \dots & Q_n \end{bmatrix} \begin{bmatrix} U_1^T U_3 \\ Q_2^T U_3 \\ R_3 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$U_m = \begin{bmatrix} U_1 & Q_2 & \dots & Q_{m-1} & Q_m & Q_{m+1} & \dots & Q_n \end{bmatrix} \begin{bmatrix} U_1^T U_m \\ Q_2^T U_m \\ \vdots \\ Q_{m-1}^T U_m \\ R_m \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

3. Express the desired sum $\sum_{i=1}^{n} X_i$ in the basis $B = \begin{bmatrix} U_1 & Q_2 & Q_3 & Q_4 & \dots & Q_n \end{bmatrix}$. Inserting the expressions for the U_i into the given decompositions:

$$X_1 = B \begin{bmatrix} I \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} S_1 \begin{bmatrix} I \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T B^T$$

$$X_2 = B \begin{bmatrix} U_1^T U_2 \\ R_2 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} S_2 \begin{bmatrix} U_1^T U_2 \\ R_2 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T B^T$$

$$X_3 = B \begin{bmatrix} U_1^T U_3 \\ Q_2^T U_3 \\ R_3 \\ 0 \\ \vdots \\ 0 \end{bmatrix} S_3 \begin{bmatrix} U_1^T U_3 \\ Q_2^T U_3 \\ R_3 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T B^T$$

$$X_m = B \begin{bmatrix} U_1^T U_m \\ Q_2^T U_m \\ \vdots \\ Q_{m-1}^T U_m \\ R_m \\ 0 \\ \vdots \\ 0 \end{bmatrix} S_m \begin{bmatrix} U_1^T U_m \\ Q_2^T U_m \\ \vdots \\ Q_{m-1}^T U_m \\ R_m \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T B^T$$

We can then factor out B and B^T to get the following:

$$\sum_{i=1}^{n} X_i = BKB^T$$

where K, the expression of $\sum_{i=1}^{n} X_i$ in B, is

$$K = \sum_{i=1}^{n} \begin{bmatrix} U_1^T U_i \\ Q_2^T U_i \\ \vdots \\ Q_{i-1}^T U_i \\ R_i \\ 0 \\ \vdots \\ 0 \end{bmatrix} S_i \begin{bmatrix} U_1^T U_i \\ Q_2^T U_i \\ \vdots \\ Q_{i-1}^T U_i \\ R_i \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T \tag{6}$$

Here we are once again naming $Q_1 = U_1$ and $R_1 = I$.

4. Compute the eigenvalue decomposition of K. (Note that K is symmetric by construction.) This is easy because K has dimension equal to the total number of columns in the U_i, which is relatively small.

$$K = U' S' U'^T \tag{7}$$

S' is, by construction, the matrix of eigenvalues of $\sum_{i=1}^{n} X_i$.

5. Find the new eigenbasis, which is simply a matter of inserting the decomposition of K into the previous formula.

$$\sum_{i=1}^{n} X_i = BU'S'(BU')^T \tag{8}$$

The eigenbasis is properly column-normalized so long as U_1 and the Q_i are. (Remember that U_1 and the Q_i are orthogonal by construction.)