PART IV — EXTENDED BLACK HOLE FAMILIES

Chapter 9 — CVP Reissner–Nordström Black Holes

Executive Premium / IEEE-Style Technical Chapter (Draft)

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Scope and Reader Contract

This chapter extends Curvature Variable Physics (CVP) black-hole dynamics to charged, nonrotating Reissner–Nordström (RN) geometries. The objective is not to claim astrophysical black holes are typically electrically charged; rather, RN serves as a mathematically controlled testbed for (i) curvature-scalar sensitivity, (ii) extremal limits, and (iii) discriminating observable signatures under a declared CVP propagation law $c(\mathcal{R})$. All quantitative claims in this chapter are conditional on the frozen model specification and numerical settings used in the author's simulation pipeline.

At-a-Glance Definitions

Symbol	Meaning (geometric units unless stated)
M	Black-hole mass parameter.
Q	Charge parameter (electric or effective conserved charge).
$r_\pm$	Outer/inner horizons: $r_\pm = M \pm \sqrt{M^2 - Q^2}$.
κ	Surface gravity; $\kappa \to 0$ in the extremal limit $Q \to M$.
T_H	Hawking temperature; in GR, T_H $\kappa/(2\pi)$.∝
$\mathcal{R}$	Declared curvature functional used by CVP (e.g., $\sqrt{K}$, K, or horizon curvature).

$c(\mathcal{R})$	Curvature-conditioned propagation speed specification.
D	Dillon Operator: $D \equiv \partial c/\partial \mathcal{R}$ (or $\partial c/\partial R$ where $\mathcal{R}=R$).

9.1 RN Geometry

The Reissner–Nordström solution describes a static, spherically symmetric spacetime sourced by a central mass and an electromagnetic field. In standard geometric units (G=c=1), the line element is:

$$ds^2 = -f(r)\,dt^2 + f(r)^{-1}\,dr^2 + r^2(d\vartheta^2 + \sin^2\vartheta\,d\varphi^2), \quad f(r) = 1 - 2M/r + Q^2/r^2.$$

The horizon radii are the roots of f(r)=0, yielding the familiar outer and inner horizons:

$$r_\pm = M \pm \sqrt{M^2 - Q^2}.$$

CVP uses RN as a controlled environment to study how curvature-conditioned propagation interacts with: (i) a two-horizon causal structure, (ii) a charge-dependent near-horizon curvature profile, and (iii) the extremal limit Q→M where κ→0 in GR.

9.2 Curvature Scalars and Declared Curvature Functionals

For RN in classical GR with an electromagnetic stress-energy tensor, the Ricci scalar is R=0 in the exterior region; therefore CVP typically uses a curvature functional that remains informative in RN, such as the Kretschmann scalar K ≡ R_{abcd}R^{abcd}, or an effective curvature $\mathcal{R}$_eff built from K and horizon data.

A commonly used scalar is the Kretschmann invariant (illustrative form):

$$K(r;M,Q) = 48M^2/r^6 - 96MQ^2/r^7 + 56Q^4/r^8 \quad \text{(geometric units)}.$$

In CVP, the curvature-conditioning is written generically as c = c($\mathcal{R}$), with a declared choice such as $\mathcal{R}$ $\mathcal{R}$=√K, =K, or $\mathcal{R}$=horizon-curvature. The Dillon Operator is then D ≡ ∂c/∂$\mathcal{R}$. The practical requirement is that $\mathcal{R}$ must be:

- well-defined for RN (including near-horizon and near-extremal regimes),
- numerically stable in the simulation grid, and
- connected to a measurable or observationally discriminating effect.

9.3 Charge-Dependent Suppression and Induced K(M,Q)

In Parts II–III, CVP introduces a curvature-conditioned Hawking-like coefficient K(M) induced by c($\mathcal{R}$). For RN, the natural extension is K(M,Q), where charge alters both the curvature profile and the horizon structure.

A convenient executive form is to factor the suppression into (i) a curvature term and (ii) a charge term:

$$K(M,Q) = K_0 \cdot S(\mathcal{R}\_h(M,Q)) \cdot G(q), \quad \text{where } q \equiv |Q|/M \in [0,1].$$

Here S(·) is the declared curvature-suppression law derived from c(ℛ), evaluated at a chosen horizon curvature proxy ℛ_h, and G(q) is a charge-dependent modifier that enforces correct limits. For example, G(q) may be chosen to satisfy:

- Neutral limit: G(0)=1 so RN reduces to the Schwarzschild baseline.
- Extremal limit: as q→1, κ→0; the effective emission coefficient should not diverge. ⍰
  Monotonicity/regularity: G(q) should remain bounded and smooth on [0,1].

This chapter does not impose a single mandatory G(q) form; instead, it defines pass/fail constraints that any chosen charge modifier must satisfy for physical and numerical stability. The author's pipeline uses a frozen choice per run, reported alongside the parameter ledger.

9.4 Extremal RN Remnants and CVP Endpoint Behavior

In GR, RN black holes approach the extremal limit (|Q|→M) with vanishing surface gravity κ and thus vanishing Hawking temperature T_H. This makes RN a compelling case study for CVP endpoint narratives: the classical geometry already suggests suppressed emission near extremality, while CVP adds curvature-conditioned propagation that can induce additional fixed points and "lock-in" behavior.

The GR surface gravity for RN (illustrative) is:

$$\kappa\_{RN} = (r_+ - r_-)/(2r_+^2) \to 0 \text{ as } |Q| \to M.$$

In CVP, a practical remnant condition can be stated as the existence of a stable fixed point in the mass-loss law:

$$dM/dt = -K(M,Q)/M^2 \quad \text{with} \quad K(M,Q) \to 0 \text{ sufficiently fast as } M \to M\_{rem}(Q).$$

A CVP extremal RN remnant is then defined operationally as a regime where (i) the effective emission coefficient is suppressed by curvature-conditioning and/or extremality, (ii) the fixed point is stable under perturbations in M and Q, and (iii) the result remains compatible with thermodynamic accounting (no perpetual-motion claims).

Operational Failure Modes (RN)

- D=0 within uncertainty across controlled curvature gradients: curvature-conditioning is unsupported in the tested regime.
- K(M,Q) becomes negative or non-physical under declared parameters: the chosen suppression law is invalid.

- Endpoint behavior depends sensitively on numerics (step size, tolerance) with no convergence: simulation claims are not robust.

9.5 Observational Signatures and Discriminants

Astrophysical charge is typically expected to neutralize rapidly; however, RN-like phenomenology can arise as (i) true electric charge in exotic environments, (ii) an effective conserved charge from a hidden-sector U(1), or (iii) a proxy for additional structure that modifies near-horizon curvature. CVP treats RN primarily as a discriminant generator: charge changes curvature scalars and horizon structure, which can shift lensing and emission signatures under the same curvature-conditioned propagation law.

Primary Discriminants

Extended-structure microlensing residuals: Deviations from point-mass lensing, including centroid-trajectory asymmetries and caustic morphology changes in astrometric catalogs.

Absence of endpoint bursts: If CVP suppression and/or extremality yields remnants, the model predicts suppressed terminal flash signatures relative to naive Hawking extrapolations.

Population-level constraints: Event-rate and timescale distributions for candidate compact objects can be compared against RN/CVP parameter families; failure to match rejects those regimes.

Pass/Fail Summary (Chapter-Level)

		Fail Condition	Implication
Curvatureconditioning (D)	Measured ∂c/c above noise under controlled curvature gradients	Consistent with D=0 within uncertainty	CVP propagation response disfavored in tested regime
RN endpoint behavior	K(M,Q) remains bounded and yields stable fixed point under convergence checks	Non-physical K(M,Q) or nonconvergent endpoint dependence	Chosen suppression law invalid / simulation not robust
Lensing discriminants	Extended-structure residuals detected above declared thresholds	>99% events fit point-mass after controls	Remnant/extendedstructure narrative disfavored

Thermodynamics	$dS_{total} \ge 0$ under full accounting	$dS_{total} < 0$ after uncertainty budget	Extraction/endpoint mechanism falsified
Test	**Pass Condition**		

Key Takeaways

- RN provides a controlled two-horizon testbed to validate that curvature-conditioning is not an artifact of the Schwarzschild case.
- A CVP RN run is fully specified by (i) $\mathcal{R}$ choice, (ii) $c(\mathcal{R})$ parameterization, and (iii) a bounded charge modifier G(q) that respects neutral and extremal limits.
- The extremal limit ($q \to 1$) is a hard stress test: any divergence, sign flip, or non-convergence constitutes a failure of the chosen specification.
- Observable discriminants should prioritize astrometric microlensing residuals (centroid asymmetries / caustic morphology) and population-level event statistics under declared selection functions.

Run Disclosure Checklist (Minimum)

Item to Disclose	Why it matters (audit / replication)
Chosen curvature functional $\mathcal{R}$ (e.g., √K, K, horizon proxy)	Defines what $D \equiv \partial c/\partial\mathcal{R}$ means; RN needs $\mathcal{R}$ that is non-trivial even when R=0.
$c(\mathcal{R})$ functional form + coefficients	Allows independent re-runs of induced suppression behavior and stability checks.
Charge modifier G(q) and bounds	Ensures correct limits ($q \to 0$, $q \to 1$) and prevents non-physical endpoint behavior.
Numerical settings (grid, tolerances, step sizes)	Prevents "result-by-discretization"; supports convergence reporting.
Selection functions / thresholds for lensing tests	Prevents post-hoc tuning; enables clean pass/fail discriminants on catalogs.

Transition to Chapter 10 (Kerr): RN isolates charge effects; Kerr generalizes to angular momentum and anisotropic curvature. The same CVP design discipline applies: declare $\mathcal{R}$, declare $c(\mathcal{R})$, induce $K(\cdot)$, and publish hard discriminants.