The error of polynomial interpolation

Consider the functions

$$f(x) = \frac{1}{1 + x^2} \qquad g(x) = \exp(-x/10) \text{ for } x \in [-10, 10]$$

and suppose you want to find a polynomial approximation to both.

(**a**) Write a script that generates N equally spaced interpolation nodes on the domain and computes the corresponding interpolating polynomials P_N and Q_N. Use the code we wrote in lecture, which you can find in the course codes repository.

(**b**) Plot the interpolants together with f and g for $N = 4, 8, 16, 32$. Also, plot the approximate errors $|f(x) - P_N(x)|$ and $|g(x) - Q_N(x)|$ (find the maximum on a fine grid). Is interpolation on these nodes useful for approximating f on this domain? What about g?

(**c**) In lecture 11 we looked at a formula for the error of polynomial interpolation. Taking this formula into consideration, can you explain why the error is much larger for one function than for the other?

Discussion: The error of interpolation depends on three things: the location of the interpolation nodes, the order of the interpolant and the magnitude of the high-order derivatives of the function that generates the interpolation data. What about the derivatives of the current test functions? Share your code, ideas and conclusions on the dedicated Slack channel!