Piecewise polynomial interpolation

Consider the function

$$f(x) = \frac{1}{1+x^2},\ x \in [-10, 10]$$

and suppose you want to find a piecewise polynomial approximation.

(**a**) Read the documentation for `scipy.interpolate.CubicSpline` on `docs.scipy.org`. Make sure you understand the inputs and outputs. In particular, the output is a function handle, not the list of spline coefficients or the interpolant evaluated at some set of points.

(**b**) Write a script that generates N equally spaced knots on the domain and computes the corresponding cubic spline interpolant C_N.

(**c**) Plot the cubic spline interpolant together with f for $N = 4, 8, 16, 32$. Also, plot the error

$$\max_{x \in [-10,10]} |f(x) - C_N(x)|$$

Is cubic spline interpolation on these nodes useful for approximating f on this domain?

(**d**) You have approximated the same function with high-order polynomials (tutorial 7) and with cubic splines. Which approach is better for this test function? Why?

Discussion: For equally spaced interpolation nodes, the polynomial interpolation did not work well. The interpolation error did not decrease as the number of interpolation nodes increased. The interpolation error for cubic splines does decrease with the number of knots. What is the trade-off? What is a disadvantage of using splines over polynomials?