

Pseudo-code for LU decomposition with partial pivoting:

LU decomposition with partial pivoting

Input: $A \in \mathbb{R}^{n \times n}$

$U \leftarrow A, L \leftarrow I, P \leftarrow I$ (initialise matrices)

for $k = 1 : n - 1$ (loop through pivot columns)

 Select $i \geq k$ to maximise $|U_{i,k}|$ (choose pivot element)

$U_{k,k:n} \leftrightarrow U_{i,k:n}$ (swap rows of U)

$L_{k,1:k-1} \leftrightarrow L_{i,1:k-1}$ (swap rows of L up to pivot)

$P_{k,:} \leftrightarrow P_{i,:}$ (swap rows of P)

for $j = k + 1 : n$ (loop through rows under pivot)

$L_{j,k} \leftarrow U_{j,k} / U_{k,k}$ (store multiplier in L matrix)


$U_{j,k:n} \leftarrow U_{j,k:n} - L_{j,k} U_{k,k:n}$ (update row j of U matrix)

end for

end for

Output: Matrices L, U and P

Iterate over columns to find pivot elements



$$\begin{pmatrix}
 a_{00} & a_{01} & \cdots & \cdots & \cdots & \cdots & a_{0(n-1)} \\
 a_{10} & a_{11} & \cdots & \cdots & \cdots & \cdots & a_{1(n-1)} \\
 \vdots & & \ddots & & & & \vdots \\
 \vdots & & & a_{kk} & & & \vdots \\
 \vdots & & & & \ddots & & \vdots \\
 a_{(n-2)0} & a_{(n-2)1} & \cdots & a_{(n-2)k} & \cdots & a_{(n-2)(n-2)} & a_{(n-2)(n-1)} \\
 a_{(n-1)0} & a_{(n-1)1} & \cdots & a_{(n-1)k} & \cdots & \cdots & a_{(n-1)(n-1)}
 \end{pmatrix}$$

$$\begin{array}{c}
 \textcolor{red}{k} \\
 \textcolor{red}{\downarrow} \\
 \left(\begin{array}{ccccccc}
 a_{00} & a_{01} & \cdots & \cdots & \cdots & \cdots & a_{0(n-1)} \\
 a_{10} & a_{11} & \cdots & \cdots & \cdots & \cdots & a_{1(n-1)} \\
 \vdots & & \ddots & & & & \vdots \\
 \vdots & & & \textcolor{red}{a_{kk}} & & & \vdots \\
 \vdots & & & & \ddots & & \vdots \\
 a_{(n-2)0} & a_{(n-2)1} & \cdots & a_{(n-2)k} & \cdots & a_{(n-2)(n-2)} & a_{(n-2)(n-1)} \\
 a_{(n-1)0} & a_{(n-1)1} & \cdots & a_{(n-1)k} & \cdots & \cdots & a_{(n-1)(n-1)}
 \end{array} \right) \textcolor{red}{\leftarrow k}
 \end{array}$$

Find pivot element

k
↓

$$\begin{pmatrix}
 a_{00} & a_{01} & \cdots & \cdots & \cdots & \cdots & a_{0(n-1)} \\
 a_{10} & a_{11} & \cdots & \cdots & \cdots & \cdots & a_{1(n-1)} \\
 \vdots & & \ddots & & & & \vdots \\
 \vdots & & & & & & \vdots \\
 \vdots & & & & \ddots & & \vdots \\
 a_{(n-2)0} & a_{(n-2)1} & \cdots & a_{(n-2)k} & \cdots & a_{(n-2)(n-2)} & a_{(n-2)(n-1)} \\
 a_{(n-1)0} & a_{(n-1)1} & \cdots & a_{(n-1)k} & \cdots & \cdots & a_{(n-1)(n-1)}
 \end{pmatrix}
 \quad \leftarrow k$$

arg max

a_{kk}	0	k
$a_{(k+1)k}$	1	k + 1
\vdots	\vdots	\vdots
\vdots	\vdots	\vdots
\vdots	\vdots	\vdots
\vdots	\vdots	\vdots
$a_{(n-2)k}$	(n - 2) - k	(n - 2)
$a_{(n-1)k}$	(n - 1) - k	(n - 1)

Swap Rows

$$\left(\begin{array}{ccccccc} a_{00} & a_{01} & \cdots & \cdots & \cdots & \cdots & a_{0(n-1)} \\ a_{10} & a_{11} & \cdots & \cdots & \cdots & \cdots & a_{1(n-1)} \\ \vdots & & \ddots & & & & \vdots \\ \vdots & & & & & & \vdots \\ \vdots & & & & \ddots & & \vdots \\ a_{(n-2)0} & a_{(n-2)1} & \cdots & a_{(n-2)k} & \cdots & a_{(n-2)(n-2)} & a_{(n-2)(n-1)} \\ a_{(n-1)0} & a_{(n-1)1} & \cdots & a_{(n-1)k} & \cdots & \cdots & a_{(n-1)(n-1)} \end{array} \right) \begin{array}{l} \mathbf{U, P} \\ \\ \\ \\ \\ \end{array}$$

Diagram illustrating the initial state of a matrix $\mathbf{U, P}$ during row swapping. The matrix is partitioned into two main sections. The top section (rows 0 to $k-1$) is highlighted in red, and the bottom section (rows k to $n-1$) is highlighted in green. An orange arrow labeled "Swap" indicates the operation of swapping the row containing a_{kk} with the row containing $a_{(n-2)k}$.

$$\left(\begin{array}{ccccccc} a_{00} & a_{01} & \cdots & \cdots & \cdots & \cdots & a_{0(n-1)} \\ a_{10} & a_{11} & \cdots & \cdots & \cdots & \cdots & a_{1(n-1)} \\ \vdots & & \ddots & & & & \vdots \\ \vdots & & & & & & \vdots \\ \vdots & & & & & & \vdots \\ \vdots & & & & & & \vdots \\ a_{(n-2)0} & a_{(n-2)1} & \cdots & a_{(n-2)k} & \cdots & a_{(n-2)(n-2)} & a_{(n-2)(n-1)} \\ a_{(n-1)0} & a_{(n-1)1} & \cdots & a_{(n-1)k} & \cdots & \cdots & a_{(n-1)(n-1)} \end{array} \right) \mathbf{L}$$

Diagram illustrating the state of the matrix \mathbf{L} after the row swap operation. The matrix is partitioned into two main sections. The top section (rows 0 to $k-1$) is highlighted in red, and the bottom section (rows k to $n-1$) is highlighted in green. An orange arrow labeled "Swap" indicates the operation of swapping the row containing a_{kk} with the row containing $a_{(n-2)k}$.