University of Glasgow

EPSRC Engineering and Physical Sciences Research Council

EPSRC Vacation Scholarship Summary Report

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Project Title	An extension of the theory of bigraphs and implementation of new algorithms.		

1. Original Aims and Objectives of the Project
   Bigraphical Reactive System (BRS) is a new formalism designed by Milner. It is intended for modelling systems with locality and connectivity. But there is a limitation in this formalism – there is no direct representation of locations that intersect or overlap. This is a significant limitation because these occur in many domains – mobile communication, wireless systems, social interactions and many others. Bigraphs with sharing is a redefined concept, where the underlying model of location is represented by DAGs (directed acyclic graphs), instead of forests (set of trees).
   The aim of the project was to extend the theory of bigraphs with sharing and implement new algorithms to enable powerful spatial analysis techniques for applications such as mixed reality systems, autonomous vehicles and wireless protocols for the IoT. The project was structured in three independent work packages. I focused on the third package: *RPOs for bigraphs with sharing.* This work package consisted in identifying classes of bigraphs with sharing that allow RPO (Relative Push Out) construction and implementing the RPO synthesis algorithm in the BigraphER tool.

2. Methodology
   The first part of this project was devoted to a literature review. The starting point was to read M. Sevegnani and M. Calder article (*Bigraphs with sharing*. Theoretical Computer Science, *577*, 43-73, 2015). This article provides an introduction to theory of bigraphs in condensed form. The next step was to study Sevegnani's PhD thesis (*Bigraphs with sharing and applications in wireless networks.* Doctoral dissertation, University of Glasgow, 2012). I also familiarised myself with Milner's introductory book on bigraphs (*The space and motion of communicating agents*. Cambridge University Press, 2009).
   The state of my prior knowledge on bigraphs was insufficient to understand the theory. Therefore, I had to further study the following related topics:
   - category theory
   - statistic and probabilistic (Markov Chain included)
   - graph theory
   - BiLog logic
   - functional programming (OCaml).

   I also extended my knowledge in following fields:

   - set theory and relational algebra
   - complexity concepts such as NP, NP-hard and NP-complete, soundness and completeness

   And finally, I familiarised myself with BigraphER - an implementation of both bigraphical reactive system (BRS) and stochastic BRS.

In the second part of my project, I paid a special attention to algorithms for the bigraphs matching problem: input size may be critical for the performance of an algorithm's implementation, because this problem is NP-hard.

Then I studied mechanics and properties of bigraphs' composition. I mostly focussed on how the structure of bigraphs is preserved (or changed) during composition. Then, I analysed two special kinds of concrete bigraphs – monomorphic and epimorphic. I was interested on how these properties influence bigraphs' composition.

Finally, I studied Relative Push-Outs (RPOs) together with labelled transition systems.

3. Key Findings

The ability of bigraphs to evolve in time is an important feature. We can describe this process by means of labelled transition systems. RPOs are essential to derive such systems. However, Sevegnani in his PhD thesis proves that RPOs do not exist in general for concrete bigraphs with sharing. He also conjectures that RPOs always exist for epimorphic bigraphs with sharing. I picked this problem and the aim of my study was to either provide a proof for Sevegnani conjecture or to disprove it. I have chosen this particular problem because the absence of RPOs seems to be a significant drawback of the theory.
As a result of my study the following proposition was formulated:
Proposition. *RPOs exist in the precategory of epimorphic concrete place graphs with sharing.*
I have also constructed a proof for this proposition. Sevegnani shows, that we can find more than one arrow between candidate RPO and another push-out, yet there is a requirement that this arrow is unique. I have shown in my proof that it is impossible to construct more than one arrow between candidate RPO and another push-out whenever this candidate is epimorphic. That means that the arrow is unique and the requirement for the candidate to be an RPO is fulfilled. The conclusion is that it is possible to construct RPOs for bigraphs with sharing as well. The proof is included in the full report of this project.

4. Areas for Future Research

Future work will be in two areas: algorithm for constructing RPOs and its implementation. A construction for RPOs for bigraphs without sharing already exists (Construction 5.5, Construction 5.9, Milner, R. (2009). *The space and motion of communicating agents*. Cambridge University Press). In my opinion, despite the fact that the underlying model of location in bigraphs with sharing is different, some parts of Milner's construction might be reused. Construction 5.5 is for link graphs. As link graphs are defined in the same manner in both kinds of bigraphs - this algorithm should deliver proper results also for bigraphs with sharing, note that Construction 5.9 for place graphs must be redefined. According to the proposition we need epimorphic place graphs. Therefore we need to restrict results, where roots of RPOs are idle or partners and in effect the place graph is not epimorphic. The roots are partners when they are parents of the same child.
The next step will be extending functionality of BigraphER by implementing the algorithm.

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