The Greeks

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In the world of the financial derivatives, the Greeks play a crucial role to measure different types of risk and sensitivity. When used to trade options, the Greeks are calculated as first partial derivative of the option pricing model, such as the Black-Scholes Model. The primary greeks are Delta (Δ), Gamma (Γ), Vega, Theta (Θ) and Rho (ρ).
First of all, I recall the Black-Scholes pricing formula for a call option:

$$C = S(t) \cdot N(d_1) - K \cdot e^{-r(T-t)} \cdot N(d_2)$$

Where:

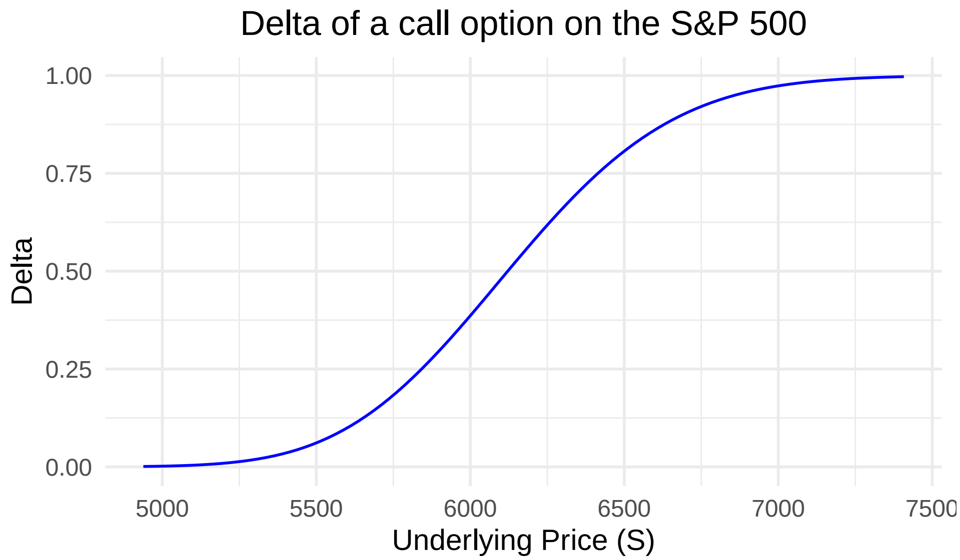
- $$d_1 = \frac{\ln\left(\frac{S(t)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) \cdot (T-t)}{\sigma \cdot \sqrt{T-t}},$$
- $$d_2 = d_1 - \sigma \cdot \sqrt{T-t}$$
- N(x) represents the cumulative density of a standard normal distribution.
- S(t) is the price of the underlying at time t
- K is the strike price
- r is the risk-free rate

1 Delta

As I said in the introduction, Delta is the first partial derivative with respect to S(t) By computing it, we obtain:

$$\frac{\partial C(t)}{\partial S(t)} = N(d_1)$$

It measures the sensitivity of the option's price to small changes in the price of the underlying asset. If the underlying prices increases by €1, how much does the option price change?



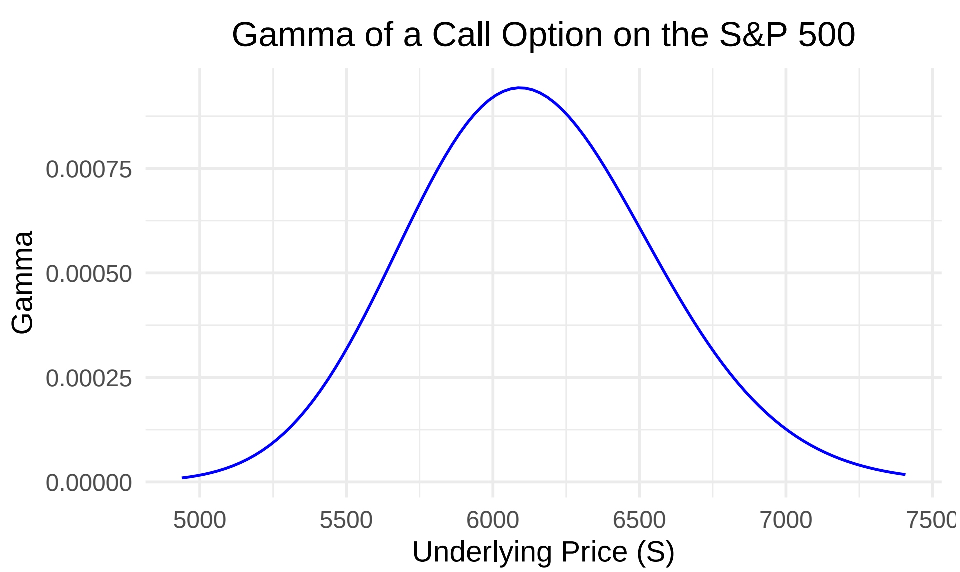
The S-shaped curve shows Delta smoothly increases from 0 to 1 as the underlying price moves from below to above the strike price.

2 Gamma

Gamma is the first partial derivative of Delta with respect to the price S(t). In other words is the second derivative of the BS formula with respect to the price S.

$$\frac{\partial^2 C(t)}{\partial S(t)^2} = \frac{\partial N(d_1)}{\partial S(t)} = N'(d_1) \cdot \frac{\partial d_1}{\partial S(t)} = \frac{1}{S(t)} \cdot \frac{1}{\sigma \cdot \sqrt{T-t}} \cdot N'(d_1)$$

Gamma represents the rate of change of Delta with respect to the underlying price. It shows how stable Delta is. How quickly does Delta itself change as the underlying moves?



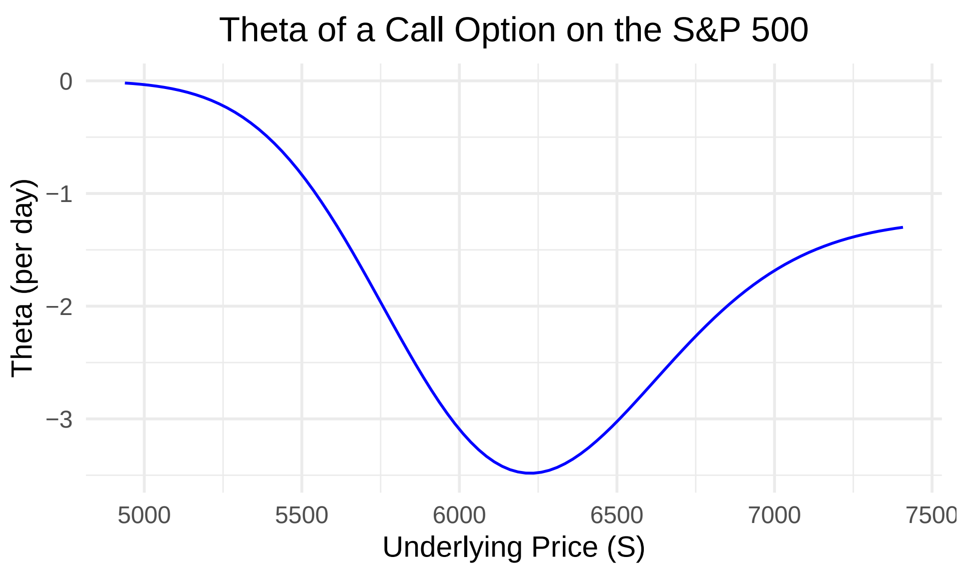
The bell-shaped curve peaks near the strike price, indicating Gamma is highest when the option is at-the-money and decreases as the underlying moves away.

3 Theta

Theta is the first partial derivative of the BS formula with respect to the Time $(T-t)$. This derivative equals to:

$$\frac{\partial C(t)}{\partial(T-t)} = K \cdot e^{-r(T-t)} \cdot (r \cdot N(d_2) + \frac{\sigma}{2 \cdot \sqrt{T-t}} \cdot N'(d_2))$$

Theta measures the sensitivity of the option price to the passage of time. It reflects time decay. How much value does the option lose with each passing day?



The graph is mostly negative and dips around the strike price, showing the option loses value faster near expiration, especially when near-the-money.

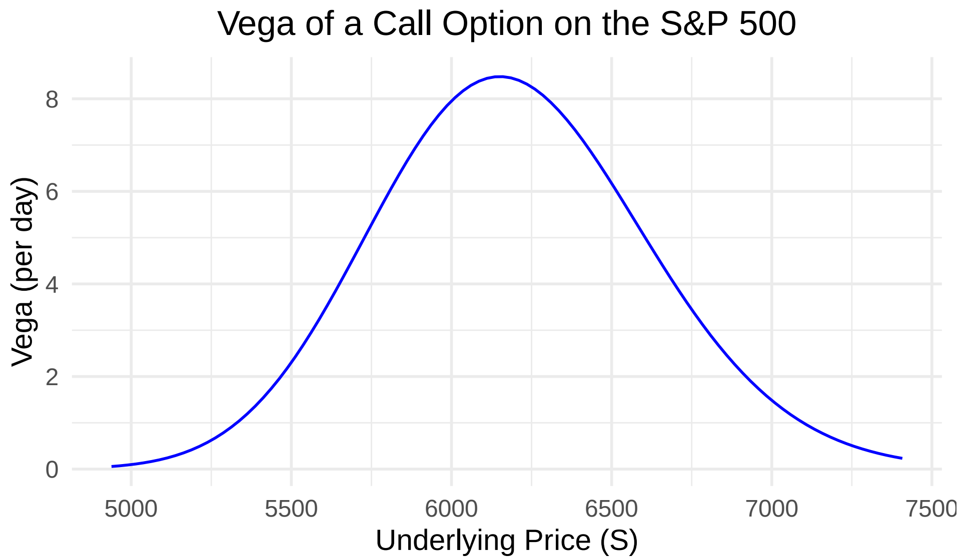
4 Vega

Vega is the first partial derivative of the BS formula with respect to the volatility σ. This derivative equals to:

$$\frac{\partial C(t)}{\partial \sigma} = \sqrt{T-t} \cdot S(t) \cdot N'(d_1)$$

Vega measures the sensitivity of the option price to changes in implied volatility.

How does the option value change when implied volatility increases by 1%?



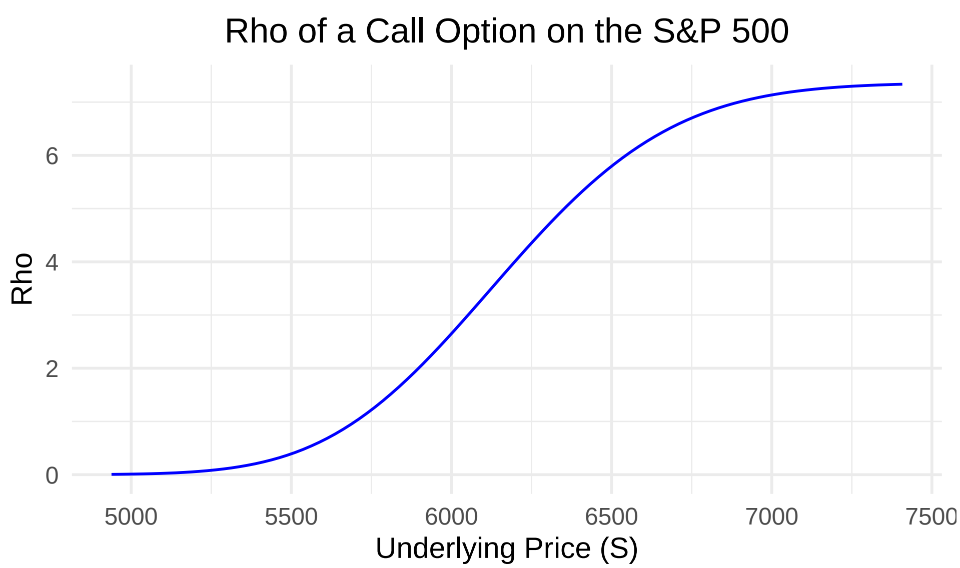
The bell-shaped curve peaks near the strike price, illustrating that Vega is highest for at-the-money options and decreases as the underlying moves away.

5 Rho

Rho is the first partial derivative of the BS formula with respect to r:

$$\frac{\partial C(t)}{\partial r} = (T-t) \cdot K \cdot e^{-r(T-t)} \cdot N(d_2)$$

Rho indicates the sensitivity of the option price to changes in the risk-free interest rate. How much does the option price change if the interest rate increases by 1%?



The curve generally increases with the underlying price, indicating that the option's sensitivity to interest rates grows as the option becomes more in-the-money.