Exotic Options

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Exotic options are a type of options contract that differ from standard options in terms of their payment methods, expiration dates, and strike prices.
The underlying asset or security can vary with exotic options, providing more diverse investment opportunities.
These options are hybrid financial instruments that can often be tailored to meet the specific needs of the investor and fall somewhere in between American and European options, with the first being more flexible than the second.
They are usually sold in the OTC market.

In this document I will analyze different types of Exotic options using expected value theory and indicator functions, following principles from stochastic processes and risk-neutral pricing.
The indicator function is defined as the following:

$$\mathbf{1}_{\{\varepsilon\}} = \begin{cases} 1 & \text{if the event } \varepsilon \text{occurs} \\ 0 & \text{otherwise} \end{cases}$$

Moreover, let's recall the Put-Call Parity: We can define the Call option as the following:

$$\begin{aligned} C(t) &= E_t\left[(S(T)-K)\mathbb{1}_{\{S(T)>K\}} \cdot \frac{G(t)}{G(T)}\right] \\ &= E_t\left[(S(T)-K)\left(1-\mathbb{1}_{\{K>S(T)\}}\right) \cdot \frac{G(t)}{G(T)}\right] \\ &= E_t\left[(S(T)-K)\cdot \frac{G(t)}{G(T)}\right] + E_t\left[(K-S(T))\cdot \mathbb{1}_{\{K>S(T)\}} \cdot \frac{G(t)}{G(T)}\right] \end{aligned}$$

As the last equation clearly states, the Call Option can be replicated with the purchase of the Underlying $S(t)$, a Put Option $P(t)$ and the sale of K zero-coupon bonds: $KB(t,T)$.
Therefore, every single Option in this document can be expressed as a Put Option with the right adjustements.

1 Down-and-Out Option

$$E_t\left[(S(T)-K)\mathbb{1}_{\{S(T)>K\}} \cdot \frac{G(t)}{G(T)} \cdot \mathbb{1}_{\{t_L>T\}}\right]$$

It's a type of knock-out option where the option buyer loses the right to exercise the option when the price of the underlying falls under a certain level L before the expiration date.

2 Down-and-In Option

$$E_t\left[(S(T)-K)\mathbb{1}_{\{S(T)>K\}}\cdot\frac{G(t)}{G(T)}\cdot\mathbb{1}_{\{t_L<T\}}\right]$$

It's a type of knock-in option that starts inactive, becomes active when the asset falls below the barrier L; otherwise, it expires worthless.

3 Up-and-Out Option

$$E_t\left[(S(T)-K)\mathbb{1}_{\{S(T)>K\}}\cdot\frac{G(t)}{G(T)}\cdot\mathbb{1}_{\{t_H>T\}}\right]$$

It's a second type of knock-out option where the option expires when the asset rises above the barrier level H

4 Up-and-In Option

$$E_t\left[(S(T)-K)\mathbb{1}_{\{S(T)>K\}}\cdot\frac{G(t)}{G(T)}\cdot\mathbb{1}_{\{t_H<T\}}\right]$$

It's a second type of knock-in option that only kicks in if the asset rises above the barrier level H; otherwise it expires worthless.

5 Double Barrier Option

$$E_t\left[(S(T)-K)\mathbb{1}_{\{S(T)>K\}}\cdot\frac{G(t)}{G(T)}\cdot\mathbb{1}_{\{t^*>T\}}\right]$$

This type of option expires (in case they are *out*) or remains valid (in case they are *in*) when the value of the underlying falls in between a lower bound L and a upper bound H.

6 Lookback Option

$$E_t\left[(S(T)-\hat{K})\cdot\frac{G(t)}{G(T)}\right]$$

These are path-dependent options whose payoff is based on the maximum or minimum price of the underlying during some period and not just its maturity price. It pays the difference between the value of the underlying at times T and the minimum value during its life. The indicator function is not used as the value of the underlying at the maturity will never be lower than the lowest value registered during the option's life

7 Asset-or-Nothing Option

$$E_t\left[(S(T)\mathbb{1}_{\{S(T)>K\}}\cdot\frac{G(t)}{G(T)}\right]$$

This type of option (when *in the money*) pays a single unit of the underlying when, otherwise it pays zero.

8 Digital Option

$$E_t\left[\mathbb{1}_{\{S(T)>K\}} \cdot \frac{G(t)}{G(T)}\right]$$

This type of option pays 0 or 1 depending on whether the underlying met specified condition/occurance of an event.

9 Parisian Option

$$E_t\left[((S(T)-K)\mathbb{1}_{\{S(T)>K\}} \cdot \frac{G(t)}{G(T)}\mathbb{1}_{t^*\geq T}\right]$$

Barrier option with a twist: the barrier condition only triggers if the underlying price stays beyond (above or below) the barrier for a predefined amount of time (e.g., a continuous period of T days), not just a one-time touch

10 Asian Option

$$E_t\left[(\hat{S}(t_0,T)-K)\mathbb{1}_{\{\hat{S}(t_0,T)>K)\}} \cdot \frac{G(t)}{G(T)}\right]$$

Type of path-dependent option where the payoff is based on the average price ($\hat{S}(t_0,T)$) of the underlying asset over a specified period, rather than its price at maturity. They are often used to reduce the impact of price manipulation or extreme volatility near expiration. Asian options are popular in commodities and energy markets.

11 Rainbow Option

$$E_t\left[max(S_1(T)-K, S_2(T)-K, 0) \cdot \frac{G(t)}{G(T)}\right]$$

These are multi-asset options whose payoff depends on the best or worst performer among two or more underlyings. The holder can decide whether exercise the option on the first underlying, on the second or not exercise it at all.