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Branch :- CSE-I

subject :- M1

No. of sheets :- 6

Assignment-2

(1) $(D^2-8D+9)y=0$

It is in the form of $F(D)y=\Theta(x)$

Auxilary Equation is $F(m)=0$

$m^2-8m+9=0$

$\cancel{m^2}$

$= \frac{8 \pm \sqrt{64-4(1)9}}{2(1)}$

$= \frac{8 \pm \sqrt{28}}{2}$

$= \frac{8 \pm 2\sqrt{7}}{2}$

$= 4 \pm \sqrt{7}$

$y_c = e^{4x}(c_1 \cosh\sqrt{7}x + c_2 \sinh\sqrt{7}x)$

$y_P = 0$

General solution is $y = y_c + y_p$

$y = e^{4x}(c_1 \cosh\sqrt{7}x + c_2 \sinh\sqrt{7}x)$

(2) $(D^3+4D)y = 5$

$F(D)y = \Theta(x)$

$F(m)=0$

$m^3+4m=0$

$m=0 \quad m=\pm 2i$

$y_P = \frac{1}{F(D)} \Theta(x)$

$= \frac{1}{D^3+4D} 5e^{0x}$

$D=a=0$

$= \frac{5}{\ldots} e^{0x}$ (failure)

$$= \frac{5x}{3D^2+4} e^{0x}$$

$$y_p = \frac{5x}{4} e^{0x}$$

General solution is $y = y_c + y_p$

$$y = c_1 e^{0x} + e^{0x} (c_1 \cos 2x + c_2 \sin 2x) + \frac{5x}{4}$$

(3) $(D^2+9)y = \cos 3x$

It is in the form of $f(D)y = Q(x)$

$$y_p = \frac{1}{f(D)} Q(x)$$

$$= \frac{1}{D^2+9} \cos 3x$$

$$D^2 = -b = -9$$

$$= \frac{1}{-9+9} \cos 3x \text{(failure)}$$

$$= \frac{x}{2D} \cos 3x$$

$$y_p = \frac{x}{2} \left[\frac{\sin 3x}{3}\right]$$

(4) $(D^2+3D+2)\, y = x$

It is in the form of $f(D)y = Q(x)$

$$y_p = \frac{1}{f(D)} Q(x)$$

$$= \frac{1}{D^2+3D+2} x$$

$$= \frac{1}{2\left(1+\frac{D^2}{2}+\frac{3D}{2}\right)} x$$

$$= \frac{1}{2}\left[1+\left(\frac{D^2}{2}+\frac{3D}{2}\right)\right]^{-1} x$$

$$= \frac{1}{2}\left[1-\left(\frac{D^2}{2}+\frac{3D}{2}\right)\right] x$$

$$= \frac{1}{2}\left[x-\left(\frac{0}{2}+\frac{3}{2}\right)\right]$$

$$y_p = \frac{1}{2}\left[x-\frac{3}{2}\right]$$

(5) $u = \frac{y}{x}, \quad v = xy$

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$$

$$= \begin{vmatrix} -\frac{y}{x^2} & \frac{1}{x} \\ y & x \end{vmatrix}$$

$$= -\frac{y}{x} - \frac{y}{x}$$

$$= -\frac{2y}{x} \neq 0$$

$\therefore$ Functionally Independent

(6) $u = x^2 - 2y, \quad v = x+y+z, \quad w = x-2y+3z$

$$J\left[\frac{u,v,w}{x,y,z}\right] = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$$

$$= \begin{vmatrix} 2x & -2 & 0 \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix}$$

$= 2x\,[3+2] + 2(3-1] + 0$

$= 10x + 4$

$\neq 0$