5.(a) $n \times 2$ matrix has the form $\begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix}$

(b) $X^T X = \begin{bmatrix} 1 & \cdots & 1 \\ x_1 & \cdots & x_n \end{bmatrix} \begin{bmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} = \begin{bmatrix} n & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 \end{bmatrix}$

(c) Yes, it is symmetric

(d) $A = \begin{bmatrix} 1 & \cdots & 1 \\ x_{11} & \cdots & x_{1n} \\ x_{21} & \cdots & x_{2n} \\ \vdots & & \vdots \\ x_{p1} & \cdots & x_{pn} \end{bmatrix} \begin{bmatrix} 1 & x_{11} & x_{21} & \cdots & x_{p1} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_{1n} & x_{2n} & \cdots & x_{pn} \end{bmatrix}$

$A = \begin{bmatrix} n & \Sigma_i x_{1i} & & \\ \Sigma_i x_{1i} & \sum_{i=1}^{n} x_{1i}^2 & & \\ & & \ddots & \\ & & & \sum_{i=1}^{n} x_{pi}^2 \end{bmatrix}$

we wish to show $A_{ij} = A_{ji}$, for all $i \neq j$

$A_{ij} = X^T(i,:) \cdot X(:,j)$, $A_{ji} = X^T(j,:)\, X(:,i)$

Notice that $X^T(i,:) = X(:,i)$. Row i of X^T = Column i of X

By definition of a Transpose of a matrix

$X(:,j) = X^T(j:,)$ by same reason. Since it is a dot product, where we sum the multiple of the vector, $A_{ij} = A_{ji}$ for all $i \neq j$