Q2-Q3 HW2

Sunday, October 6, 2024 5:17 PM

2a) $\mathbb{E}(Y_i)=\mu \quad \mathbb{V}(Y_i)=\sigma_i^2 \; (i=1,2) \quad \bar{y}_w = w_1 Y_1 + w_2 Y_2$

$w_1 = w \quad w_2 = 1-w$

Under what circumstance will $\bar{y}_w$ = unbiased estimator of μ

=E

$E(\bar{y}_w) = E(wY_1 + (1-w)Y_2)$ con con

$= w\,E(Y_1) + (1-w)\,E(Y_2)$

$= w\mu + (1-w)\mu$

$= \mu(w+1-w) \rightarrow \mu$ = expected value of $\bar{y}_w$.

$\boxed{\text{Because } E(\bar{y}_w)=\mu,\ \bar{y}_w \text{ is an unbiased estimator of } \mu}$

b) Variance of $\bar{y}_w$

$V(\bar{y}_w) = V(wY_1 + (1-w)Y_2)$

$= w^2 V(Y_1) + (1-w)^2 V(Y_2)$

$= \boxed{w^2\sigma_1^2 + (1-w)^2\sigma_2^2}$

c) $\mathbb{V}(\bar{y}_w)$ minimized $w_i \propto 1/\sigma_i$
=der

1. $\mathbb{V}(\bar{y}_w) = w^2\sigma_1^2 + (1-w)^2\sigma_2^2$

$\frac{d}{dw}\mathbb{V}(\bar{y}_w) = 2w\sigma_1^2 - 2(1-w)\sigma_2^2 = 0$

$2w\sigma_1^2 - 2\sigma_2^2 + 2w\sigma_2^2 = 0$

$\cancel{2}w\sigma_1^2 + \cancel{2}w\sigma_2^2 = \cancel{2}\sigma_2^2$

$w(\sigma_1^2 + \sigma_2^2) = \sigma_2^2$

$$\boxed{w_1 = \frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2} \rightarrow \underset{w_2}{1-w} = \frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2} \qquad \frac{1}{\sigma_i^2} \text{ proportional to}}$$

3a) uncentered model: $y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + e_i \qquad \tilde{x}_i = x_i - \bar{x}$

centered: $y_i = \gamma_0 + \gamma_1 \tilde{x}_i + \gamma_2 \tilde{x}_i^2 + e_i =$

① distribute γ
② β_2 = coef x^2 β_1 = cot x $\beta_0 \neq x$

$y_i = \gamma_0 + \gamma_1(x_i - \bar{x}) + \gamma_2(x_i - \bar{x})^2 + e_i$

$y_i = \gamma_0 + \gamma_1(x_i - \bar{x}) + \gamma_2(x_i^2 - 2x_i\bar{x} + \bar{x}^2) + e_i$

$y_i = \gamma_0 + \gamma_1 x_i - \gamma_1\bar{x} + \gamma_2 x_i^2 - 2\gamma_2 x_i\bar{x} + \gamma_2\bar{x}^2 + e_i$

$y_i = \gamma_2 x_i^2 + (\gamma_1 - 2\gamma_2\bar{x})x_i + (\gamma_0 - \gamma_1\bar{x} + \gamma_2\bar{x}^2) + e_i$

$y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + e_i$

$$\boxed{\beta_2 = \gamma_2 \quad \beta_1 = \gamma_1 - 2\gamma_2\bar{x} \quad \beta_0 = \gamma_0 - \gamma_1\bar{x} + \gamma_2\bar{x}^2}$$