

PROBLEM 4 of ML-1, Homework-1

GIVEN \Rightarrow

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ and } A = \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix}$$

\Rightarrow EVALUATING THE EXPRESSION \Rightarrow

$$x^T \cdot A \cdot x$$

$$\Rightarrow \begin{bmatrix} x_1 & x_2 \end{bmatrix}_{1 \times 2} \cdot \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix}_{2 \times 2} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{2 \times 1}$$

$$= \begin{bmatrix} a_{11} \times x_1 + a_{12} \times x_2 & a_{12} \times x_1 + a_{22} \times x_2 \end{bmatrix}_{1 \times 2} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{2 \times 1}$$

$$= (a_{11} \times x_1^2 + a_{12} \times x_2 \times x_1) + (a_{12} \times x_1 \times x_2 + a_{22} \times x_2^2)$$

$$= \cancel{2 \times a_{11} \times x_1} + \cancel{2 \times a_{12} \times x_1 \times x_2} + \cancel{2 \times a_{22} \times x_2}$$

$$+ \cancel{a_{12} \times x_1} + \cancel{a_{12} \times x_2}$$

Taking partial derivative of the above expression with respect to 'x'

$$\Rightarrow \frac{\partial (x^T \cdot A \cdot x)}{\partial x}$$

\Rightarrow Taking partial derivative w.r.t. x_1

$$\Rightarrow \frac{\partial (a_{11} \times x_1^2 + 2 \times a_{12} \times x_1 \times x_2 + a_{22} \times x_2^2)}{\partial x_1}$$

$$= 2 \times a_{11} \times x_1 + 2 \times a_{12} \times x_2 + 0 = 2 \times$$

\Rightarrow w.r.t. x_2 ,

$$\frac{\partial (a_{11} \times x_1^2 + 2 \times a_{12} \times x_1 \times x_2 + a_{22} \times x_2^2)}{\partial x_2}$$

$$= 0 + 2 \times a_{12} \times x_1 + 2 \times a_{22} \times x_2$$

\Rightarrow Evaluating the expression $2 \times A \cdot x$

$$\Rightarrow 2 \times \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$= 2 \times \begin{bmatrix} a_{11} \times x_1 + a_{12} \times x_2 \\ a_{12} \times x_1 + a_{22} \times x_2 \end{bmatrix}, \text{ which are the same as the expressions derived via the partial derivatives.}$$

$$\Rightarrow \begin{matrix} 2 \times (a_{11} \times x_1 + a_{12} \times x_2) \\ 2 \times (a_{12} \times x_1 + a_{22} \times x_2) \end{matrix}$$