$\log \pi_{ik} = \alpha_k^T \text{ dot } x_i - \log Z$

$\pi_{ik} \rightarrow$ i^{th} observat^n's probabili
score for the k^{th} class.
The target variable 'Y' is ~~the~~
a 'multinomial' variable which ta
class values 1 to K.

classmate

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To find Z's value using the given expression ⇒

$$\log \pi_{ik} = \alpha_k^T \text{ dot } x_i - \log Z$$

$$\log Z = \alpha_k^T \text{ dot } x_i - \log \pi_{ik}$$

$$Z = e^{(\alpha_k^T \text{ dot } x_i - \log \pi_{ik})}$$

$$= e^{\alpha_k^T \text{ dot } x_i} \times e^{(-\log \pi_{ik})}$$

$$= \left(e^{\alpha_k^T \text{ dot } x_i}\right) \times \left(\frac{1}{\pi_{ik}}\right)$$

$$\Rightarrow Z = \frac{e^{\alpha_k^T \text{ dot } x_i}}{\pi_{ik}}$$

$$\Rightarrow \pi_{ik} = \frac{e^{\alpha_k^T \text{ dot } x_i}}{Z} \Rightarrow$$

$$\sum_{k=1}^{K} \pi_{ik} = 1 \Rightarrow$$

$$1 = \sum \left(\frac{e^{\alpha_k^T \text{ dot } x_i}}{Z}\right)$$

$$\Rightarrow Z = \sum_{k=1}^{K} e^{\alpha_k^T \text{ dot } x_i}$$

The usual formulat^n of the multinomial logit picks a base category ⇒

$$\log\left(\frac{\pi_{ik}}{\pi_{i1}}\right) = (\beta_k^T) \text{ dot } (x_i)$$

$$k = 2, \ldots, K$$

This can be written as :⇒

$$\log(\pi_{ik}) - \log(\pi_{i1}) = \alpha_k^T \text{ dot } x_i - \alpha_1^T \text{ dot } x_i$$

$$= (\alpha_k^T - \alpha_1^T) \text{ dot } x_i$$