6. We want to show that if we fit $Y = B_0 + B_1 + \varepsilon$ when true model is $Y = B_0 + B_1 X_1 + B_2 X_2 + \varepsilon$.

$$E(\hat{B}_1) = B_1 + B_2 r \frac{s_2}{s_1}$$

Our model becomes $Y = B_0 + B_1 X_1 + (B_2 x_2 + \varepsilon)$

Then $\hat{B}_1 = \dfrac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$ $\qquad y_i - \bar{y} = B_1 (x_{i1} - \bar{x}_1) + B_2 (x_{i2} - \bar{x}_2)$

$$= \frac{\sum (x_i - \bar{x})(B_1 X_1 + B_2 x_2 - \bar{y})}{\sum (x_i - \bar{x})^2}$$

$$= \frac{\sum (x_{i1} - \bar{x}_1) \cdot B_1 (x_{i1} - \bar{x}_1) + \sum (x_{i1} - \bar{x}_1) \cdot B_2 (x_{i2} - \bar{x}_2)}{\sum (x_{i1} - \bar{x}_1)^2}$$

$$= B_1 + \frac{B_2 \sum (x_{i1} - \bar{x}_1)(x_{i2} - \bar{x}_2)}{S_{11}}$$

$$= B_1 + B_2 r \frac{s_2}{s_1}$$

The Bias is 0 when r, the sample correlation coefficient between X_1 and X_2, is 0. Or $B_2 = 0$

$S_2 = 0$

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$$V(b) = \begin{bmatrix} z_{11} & z_{21} & z_{31} & \cdots & z_{n1} \\ z_{12} & z_{22} & z_{32} & \cdots & z_{n2} \end{bmatrix} \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \\ z_{31} & \vdots \\ \vdots & z_{n2} \\ z_{n1} & \end{bmatrix}$$

$$= \begin{bmatrix} \sum_{i=1}^{n} z_{i1}^2 & \sum_{i=1}^{n} z_{i1} z_{i2} \\ \sum_{i=1}^{n} z_{i1} z_{i2} & \sum_{i=1}^{n} z_{i2}^2 \end{bmatrix}$$

$$= \begin{bmatrix} n-1 & r \cdot n-1 \\ r \cdot n-1 & n-1 \end{bmatrix}^{-1}$$

$$= \frac{1}{(n-1)^2 - r^2 (n-1)^2}$$

$$= \frac{1}{(n-1)^2 \cdot (1-r^2)}$$

$$= \frac{1}{(n-1)^2} \cdot VIF$$