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Q1 HW-7 ML-1

GIVEN → π → retention rate

Total cancellation = n
Cancellation periods = t_i

Total ongoing subscriptions = m
Ongoing durations = c_i

Likelihood of cancellation is given by

$$\prod_{i=1}^{n} \pi^{t_i-1} \times (1-\pi)$$

If π is probability of retention during a period 't', then '$1-\pi$' is the probability of cancellation. Hence, the final term in the product expression is '$1-\pi$' to denote cancellation, after 't_i-1' periods.

Likelihood for censored customers ⇒

$$\prod_{i=1}^{m} \pi^{c_i}$$

This does not have the '$1-\pi$' term since the cancellation never occurred.

The total likelihood will be the product of the 2 expressions ⇒

$$\Rightarrow \left\{\prod_{i=1}^{n} \pi^{t_i-1} \times (1-\pi)\right\} \times \left\{\prod_{i=1}^{m} \pi^{c_i}\right\}$$

Taking the logarithm of this expression to make further calculations simpler

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$$\log\left(\left\{\prod_{i=1}^{n} \pi^{t_i-1} \times (1-\pi)\right\} \times \left\{\prod_{i=1}^{m} \pi^{u_i}\right\}\right)$$

$$= \sum_{i=1}^{n} \left\{(t_i - 1) \times \log(\pi) + \log(1-\pi)\right\}$$

$$+ \sum_{i=1}^{m} u_i \times \log(\pi) = \log(\text{Likelihood}(\pi))$$

Taking differential of the above expression, and setting it to 0, to solve for π.
The value of π will be our maximum likelihood estimate.

$$\partial \{ \cancel{\text{Like}} \quad \frac{\partial(\log(\text{likelihood}(\pi)))}{\partial \pi} = 0$$

$$\Rightarrow \sum_{i=1}^{n} \left(\frac{t_i - 1}{\pi} - \frac{1}{1-\pi}\right)$$

$$+ \sum_{i=1}^{m} \left(\frac{u_i}{\pi}\right) = 0$$

$$\Rightarrow \sum_{i=1}^{n} \left(\frac{t_i - 1}{\pi}\right) - \left(\frac{n}{1-\pi}\right) + \sum_{i=1}^{m} \frac{u_i}{\pi} = 0$$

$$\frac{1}{\pi} \times \left(\sum_{i=1}^{n} (t_i - 1) + \sum_{i=1}^{m} u_i\right) = \frac{n}{1-\pi}$$

$$\frac{1-\pi}{\pi} = \frac{n}{\sum_{i=1}^{n} (t_i - 1) + \sum_{i=1}^{m} u_i}$$

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$$\Rightarrow \frac{1}{\pi} - 1 = \frac{n}{\sum_{i=1}^{n} t_i - 1 + \sum_{i=1}^{m} u_i}$$

$$\Rightarrow \frac{1}{\pi} = 1 + \frac{n}{\sum_{i=1}^{n} t_i - 1 + \sum_{i=1}^{m} u_i}$$

$$\Rightarrow \frac{1}{\pi} = \frac{\sum_{i=1}^{n} (t_i - 1) + \sum_{i=1}^{m} u_i + n}{\sum_{i=1}^{n} t_i - 1 + \sum_{i=1}^{m} u_i}$$

$$\Rightarrow \pi = \frac{\sum_{i=1}^{n} (t_i - 1) + \sum_{i=1}^{m} u_i}{\sum_{i=1}^{n} (t_i - 1) + \sum_{i=1}^{m} u_i + n}$$

$$= \frac{\left(\sum_{i=1}^{n} (t_i - 1) + \sum_{i=1}^{m} u_i\right)}{\left(\sum_{i=1}^{n} t_i + \sum_{i=1}^{m} u_i + n - n\right)}$$