Projects of

Monte Carlo and Finite Difference Methods

2023/2024

Here is a list of project proposals for the course Monte Carlo and Finite Difference Methods. You have to choose and study one of these subjects in groups of two students. You are asked to write a report and to send it, together with your codes, to

`claisse@ceremade.dauphine.fr`

before **Monday, January 15 2024**. This report should not exceed **fifteen pages** and must include:

- a mathematical description of the problem and the methods proposed by the article(s),
- numerical results that you get (using a programming language of your choice) with tests on the influence of simulation and discretization parameters,
- comments and propositions to improve the methods.

I draw your attention to the fact that the report and the codes are personal productions that should reflect your own understanding of the topic. **It is strictly forbidden to copy text or codes written by others.** You should also mention clearly in the report all the references you used to complete your project.

You have to register by sending an email including the names of the students in the group and the chosen subject at the email address above before **Friday, December 15 2023**.

Defenses with 15 minutes of presentation and 5 minutes of questions will take place on **Friday, January 19 2024**. Questions on the course can be asked during the defense.

1 Computation of Sensitivities

Compute the sensitivities (delta, gamma, vega) of a European Call option in the Black-Scholes model using: 1/ finite difference approach, 2/ pathwise differentiation, 3/ Malliavin differentiation. See [6, Chapter 5] and [4, Chapter 4] for a presentation. Use first an exact simulation of the price process and then consider the Euler scheme. Compute also the sensitivities using a finite difference method for PDEs. See, *e.g.*, [12, Section 5.2].
References: [6], [4], [12]

2 Asian Options

Study, implement and compare the different methods proposed by [13] for the valuation of Asian options. See also [4, Section 2.3, Example 3.3.2]. Test these three procedures in the Black-Scholes model. Additionally, propose and implement a finite difference method to solve the PDE for the valuation of Asian option by using the transformation introduced by [16] or the splitting method presented in [3].
References: [13], [4], [16], [3]

3 Lookback Options

Consider a lookback put option with floating strike whose payoff is of the following form:

$$\left(\max_{t\in[0,T]}\{X_t\} - X_T\right)^+ .$$

Using a Brownian bridge approach, propose and implement an algorithm for the valuation of such options by Monte-Carlo. See [14, Section 8.2.3] and [4, Section 2.5]. Test this procedure in the Black-Scholes model and compare it to a naive approach. Compute also the price by using a finite difference method for PDEs after using the transformation proposed in [17, Section 12.4] or the splitting method presented in [3].
References: [14], [4], [17], [3]

4 Bermudean Options

Study the nested Monte Carlo presented in [9, Section 8.3] for the valuation of Bermudean options. This approach was originally developed by [5]. Make a numerical study in the case of a Bermudean put option in the Black–Scholes model. Compute also the price of this option by using a finite difference method for PDEs. See, *e.g.,* [10, Section 6.7].
References: [5],[9], [10]

5 American Options (hard)

Study the approach presented in [9, Section 8.7] for the valuation of American options. It is based on an original result by [15] and developments introduced in [2]. See also [10, Section 6.8.4] for a quick overview of the corresponding algorithm. Make a numerical study in the case of an American put option in the Black–Scholes model. Compute also the price of the American put option by using a finite difference method for PDEs. See, *e.g.,* [10, Section 6.7] and [12, Section 5.3].
References: [15], [2], [9], [10], [12]

6 Unbiased Simulation (hard)

Study and implement the approach introduced in [11] to construct unbiased Monte Carlo estimator of $\mathbb{E}[g(X)]$ where X is a diffusion process. Reproduce some of the numerical results presented in the paper. Compare them with the results obtained by using a classical Euler discretization scheme. Implement also a finite difference scheme for PDEs whenever possible.
References: [11]

7 Multilevel Monte Carlo (hard)

Study and implement the approach introduced in [8] for the computation of $\mathbb{E}[g(X)]$ where X is a diffusion process. Use this method to price some of the options considered in Section 6 therein. Compare them with the results obtained by using a classical Euler discretization scheme. Implement also a finite difference scheme for PDEs whenever possible.
References: [8]

References

[1] A. Alfonsi. *Affine diffusions and related processes: simulation, theory and applications*, volume 6 of *Bocconi & Springer Series*. Springer, Cham; Bocconi University Press, Milan, 2015.

[2] L. Andersen and M. Broadie. Primal-dual simulation algorithm for pricing multidimensional american options. *Management Science*, 50(9):1222–1234, 2004.

[3] G. Barles. Convergence of numerical schemes for degenerate parabolic equations arising in finance theory. In *Numerical methods in finance*, volume 13 of *Publ. Newton Inst.*, pages 1–21. Cambridge Univ. Press, Cambridge, 1997.

[4] B. Bouchard. Méthodes de Monte Carlo en finance, 2007.

[5] M. Broadie and P. Glasserman. Pricing American-style securities using simulation. *J. Econom. Dynam. Control*, 21(8-9):1323–1352, 1997. Computational financial modelling.

[6] J. Claisse. Monte Carlo and finite differences methods with applications in finance, 2020.

[7] E. Ekström, P. Lötstedt, and J. Tysk. Boundary values and finite difference methods for the single factor term structure equation. *Appl. Math. Finance*, 16(3-4):253–259, 2009.

[8] M. B. Giles. Multilevel Monte Carlo path simulation. *Oper. Res.*, 56(3):607–617, 2008.

[9] P. Glasserman. *Monte Carlo methods in financial engineering*, volume 53 of *Applications of Mathematics (New York)*. Springer-Verlag, New York, 2004. Stochastic Modelling and Applied Probability.

[10] J. Guyon and P. Henry-Labordère. *Nonlinear option pricing*. Chapman & Hall/CRC Financial Mathematics Series. CRC Press, Boca Raton, FL, 2014.

[11] P. Henry-Labordère, X. Tan, and N. Touzi. Unbiased simulation of stochastic differential equations. *Ann. Appl. Probab.*, 27(6):3305–3341, 2017.

[12] D. Lamberton and B. Lapeyre. *Introduction to stochastic calculus applied to finance*. Chapman & Hall/CRC Financial Mathematics Series. Chapman & Hall/CRC, Boca Raton, FL, second edition, 2008.

[13] B. Lapeyre and E. Temam. Competitive Monte Carlo methods for the pricing of Asian options. *The Journal of Computational Finance*, 5(1):39 – 57, 2001.

[14] G. Pagès. *Numerical probability*. Universitext. Springer, Cham, 2018. An introduction with applications to finance.

[15] L. C. G. Rogers. Monte Carlo valuation of American options. *Math. Finance*, 12(3):271–286, 2002.

[16] L. C. G. Rogers and Z. Shi. The value of an Asian option. *J. Appl. Probab.*, 32(4):1077–1088, 1995.

[17] P. Wilmott, J. Dewynne, and S. Howison. *Option Pricing: Mathematical Models and Computation*. Oxford Financial Press, 1993.