Vector Operations with SCILAB

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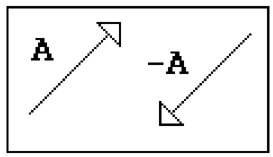
Vectors with SCILAB

A vector in two- or three-dimensions represents a *directed segment*, i.e., a mathematical object characterized by a magnitude and a direction in the plane or in space. Vectors in the plane or in space can be used to represent physical quantities such as the position, displacement, velocity, and acceleration of a particle, angular velocity and acceleration or a rotating body, forces and moments.

Operations with vectors

Vectors in two- or three-dimensions are represented by arrows. The length of the arrow represents the *magnitude* of the vector, and the arrowhead indicates the *direction* of the vector. The figure below shows a vector A, and its negative –A. As you can see, *the negative of a vector* is a vector of the same magnitude, but with opposite sense.



Vectors can be added and subtracted. To illustrate vector addition and subtraction refer to the figure below. Consider two vectors in the plane or in space, **A** and **B**, as shown in the figure below, item (a). There are two ways that you can construct *the vector sum*, **A**+**B**:

(1) By attaching the origin of the vector **B** to the tip of vector **A**, as shown in the figure, item (b). In this case, the vector sum is the vector extending from the origin of vector **A** to the tip of vector **B**.

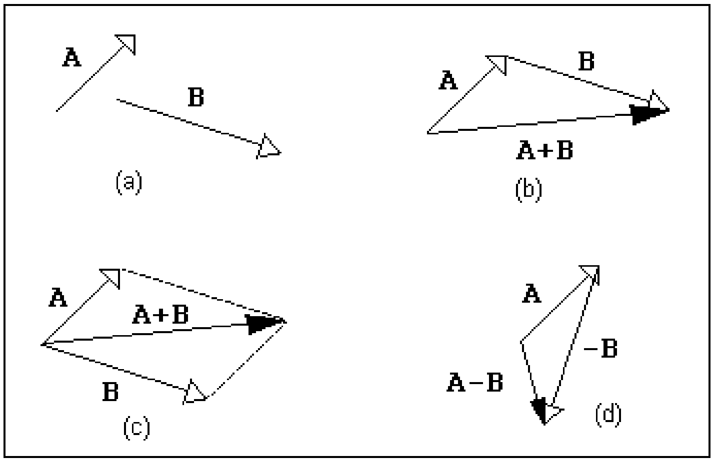
(2) By showing the vectors **A** and **B** with a common origin, and completing the parallelogram resulting from drawing lines parallel to vectors **A** and **B** at the tips of vectors **B** and **A**, respectively, as shown in the figure below, item (c).

The vector sum, also known as the *resultant*, is the diagonal of the parallelogram that starts at the common origin of vectors **A** and **B**.

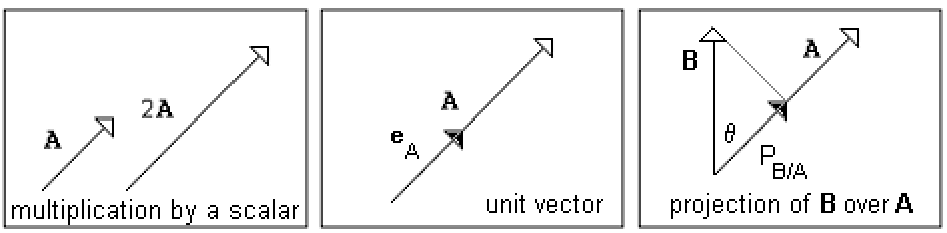
Subtraction is accomplished by using the definition:

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B}).$$

In other words, subtracting vector **B** from vector **A** is equivalent to adding vectors **A** and (–**B**),. This operation is illustrated in the figure below, item (d).



A vector **A** can be *multiplied by a scalar* **c**, resulting in a vector $c \cdot \mathbf{A}$, parallel and oriented in the same direction as **A**, and whose magnitude is *c* times that of **A**. The figure below illustrates the case in which $c = 2$. If the scalar *c* is negative, then the orientation of the vector $c \cdot \mathbf{A}$ will be opposite that of **A**.



The *magnitude* of a vector **A** is represented by $|\mathbf{A}|$. A *unit vector* in the direction of **A** is a vector of magnitude 1 parallel to **A**. The unit vector corresponding to a vector **A** is shown in the figure below, item (a). The unit vector along the direction of **A** is defined by

$$\mathbf{e}_A = \frac{\mathbf{A}}{|\mathbf{A}|}.$$

By definition, $|\mathbf{e}_A| = 1.0$.

The *projection of vector B onto vector A*, $P_{B/A}$, is shown in the figure below, item (b). If θ represents the angle between the two vectors, we see from the figure that $P_{B/A} = |\mathbf{B}| \cdot \cos\theta = |\mathbf{e}_A| \cdot |\mathbf{B}| \cdot \cos\theta$, or

$$P_{B/A} = \frac{|\mathbf{A}| \cdot |\mathbf{B}| \cdot \cos\theta}{|\mathbf{A}|}.$$

We will define the *dot product, or internal product*, of two vectors A and B as the scalar quantity

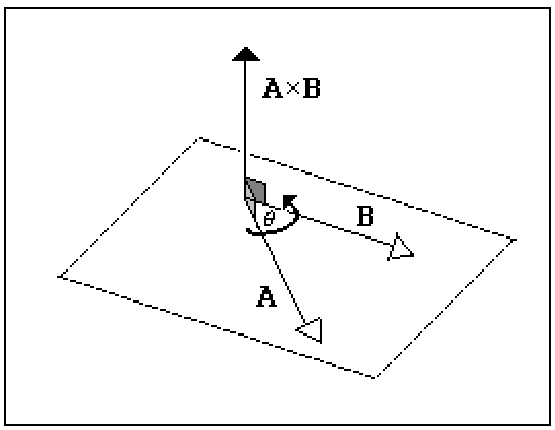
$$\mathbf{A}\bullet\mathbf{B} = |\mathbf{A}|\cdot|\mathbf{B}|\cdot\cos\theta,$$

where θ is the angle between the vectors when they have a common origin. Notice that $\mathbf{A}\bullet\mathbf{B} = \mathbf{B}\bullet\mathbf{A}$.

From the definition of the dot product it follows that the *angle between two vectors* can be found from

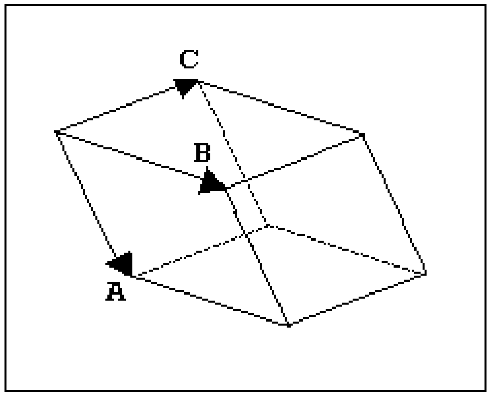
$$\theta = \arccos\left(\frac{\mathbf{A}\bullet\mathbf{B}}{|\mathbf{A}|\cdot|\mathbf{B}|}\right).$$

From the same definition it follows that if the angle between the vectors **A** and **B** is $\theta = 90^{o} = \pi/2\ ^{rad}$, i.e., if **A** is perpendicular (or normal) to **B** ($\mathbf{A}\perp\mathbf{B}$), $\cos\theta = 0$, and $\mathbf{A}\bullet\mathbf{B} = 0$. The reverse statement is also true, i.e., if $\mathbf{A}\bullet\mathbf{B} = 0$, then $\mathbf{A}\perp\mathbf{B}$.



The *cross product, or vector product*, of two vectors in space is defined as the vector $\mathbf{C} = \mathbf{A}\times\mathbf{B}$, such that $|\mathbf{C}| = |\mathbf{A}|\cdot|\mathbf{B}|\cdot\sin\theta$, where θ is the angle between the vectors, and $\mathbf{A}\bullet\mathbf{C} = 0$ and $\mathbf{B}\bullet\mathbf{C} = 0$ (i.e., **C** is perpendicular to both **A** and **B**). The cross product is illustrated in the figure below. Since there could be two orientations for a vector **C** perpendicular to both **A** and **B**, we need to refine the definition of $\mathbf{C} = \mathbf{A}\times\mathbf{B}$ by indicating its orientation. The so-called right-hand rule indicates that if we were to curl the fingers of the right hand in the direction shown by the curved arrow in the figure (i.e., from **A** to **B**), the right hand thumb will point towards the orientation of **C**. Obviously, the order of the factors in a cross product affects the sign of the result, for the right-hand rule indicates that $\mathbf{B}\times\mathbf{A} = -\mathbf{C}$.

Three vectors A, B, and C, in space, having a common origin, determine a solid figure called a parallelepiped, as shown in the figure below. It can be proven that the *volume of the parallelepiped* is obtained through the expression $\mathbf{A}\bullet(\mathbf{B}\times\mathbf{C})$. This expression is known as the *vector triple product*.



Vectors in Cartesian coordinates

The mathematical representation of a vector typically requires it to be referred to a specific coordinate system. Using a Cartesian coordinate system, we introduce the *unit vectors* **i**, **j**, and **k**, corresponding to the x-, y-, and z-directions, respectively. The unit vectors **i**, **j**, and **k**, are shown in the figure below. These unit vectors are such that

$$\mathbf{i}\bullet\mathbf{j} = \mathbf{i}\bullet\mathbf{k} = \mathbf{j}\bullet\mathbf{k} = 0,$$

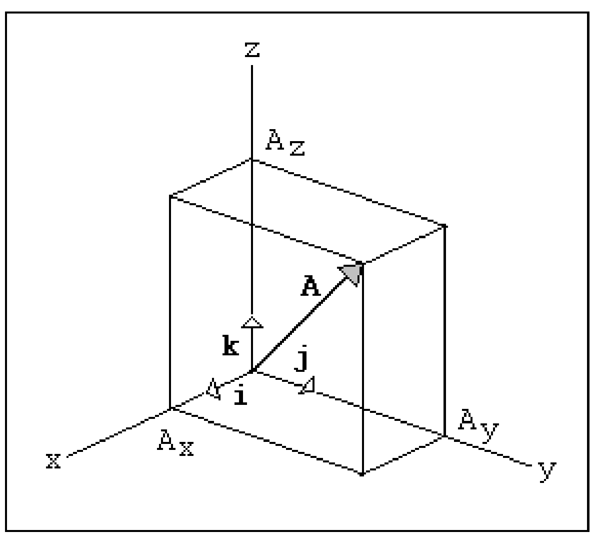
and

$$\mathbf{i}\times\mathbf{j} = \mathbf{k},\quad \mathbf{j}\times\mathbf{k} = \mathbf{i},\quad \mathbf{k}\times\mathbf{i} = \mathbf{j},\quad \mathbf{i}\times\mathbf{k} = -\mathbf{j},\quad \mathbf{k}\times\mathbf{j} = -\mathbf{i},\quad \mathbf{j}\times\mathbf{i} = -\mathbf{k}.$$

n terms of these unit vectors, any vector **A** can be written as

$$\mathbf{A} = A_x\cdot\mathbf{i} + A_y\cdot\mathbf{j} + A_z\cdot\mathbf{k},$$

where the values A_x, A_y, and A_z, are called the *Cartesian components* of the vector **A**.



Using Cartesian components, therefore, we can also write $\mathbf{B} = B_x \cdot \mathbf{i} + B_y \cdot \mathbf{j} + B_z \cdot \mathbf{k}$, and define the following *vector operations:*

- negative of vector: $-\mathbf{A} = -(A_x \cdot \mathbf{i} + A_y \cdot \mathbf{j} + A_z \cdot \mathbf{k}) = -A_x \cdot \mathbf{i} - A_y \cdot \mathbf{j} - A_z \cdot \mathbf{k}$.
- addition: $\mathbf{A}+\mathbf{B} = (A_x + B_x) \cdot \mathbf{i} + (A_y + B_y) \cdot \mathbf{j} + (A_z + B_z) \cdot \mathbf{k}$.
- subtraction: $\mathbf{A}-\mathbf{B} = (A_x - B_x) \cdot \mathbf{i} + (A_y - B_y) \cdot \mathbf{j} + (A_z - B_z) \cdot \mathbf{k}$.
- multiplication by a scalar, c: $c \cdot \mathbf{A} = c \cdot (A_x \cdot \mathbf{i} + A_y \cdot \mathbf{j} + A_z \cdot \mathbf{k}) = c \cdot A_x \cdot \mathbf{i} + c \cdot A_y \cdot \mathbf{j} + c \cdot A_z \cdot \mathbf{k}$.
- dot product: $\mathbf{A} \bullet \mathbf{B} = (A_x \cdot \mathbf{i} + A_y \cdot \mathbf{j} + A_z \cdot \mathbf{k}) \bullet (B_x \cdot \mathbf{i} + B_y \cdot \mathbf{j} + B_z \cdot \mathbf{k}) = A_x \cdot B_x + A_y \cdot B_y + A_z \cdot B_z$.
- magnitude: $|\mathbf{A}|^2 = \mathbf{A} \bullet \mathbf{A} = A_x^2 + A_y^2 + A_z^2$; $|\mathbf{A}| = \sqrt{(\mathbf{A} \bullet \mathbf{A})} = (A_x^2 + A_y^2 + A_z^2)^{1/2}$.
- unit vector: $\mathbf{e}_A = \mathbf{A}/|\mathbf{A}| = (A_x \cdot \mathbf{i} + A_y \cdot \mathbf{j} + A_z \cdot \mathbf{k})/(A_x^2 + A_y^2 + A_z^2)^{1/2}$.
- cross product: $\mathbf{A} \times \mathbf{B} = (A_x \cdot \mathbf{i} + A_y \cdot \mathbf{j} + A_z \cdot \mathbf{k}) \times (B_x \cdot \mathbf{i} + B_y \cdot \mathbf{j} + B_z \cdot \mathbf{k}) =$
  $(B_z \cdot A_y - B_y \cdot A_z) \cdot \mathbf{i} + (A_z \cdot B_x - B_z \cdot A_x) \cdot \mathbf{j} + (B_y \cdot A_x - A_y \cdot B_x) \cdot \mathbf{k}$.

Vector operations in SCILAB

Consider the vectors $\mathbf{A} = 3\mathbf{i}+5\mathbf{j}-\mathbf{k}$ and $\mathbf{B} = 2\mathbf{i}+2\mathbf{j}+3\mathbf{k}$. You can enter these three-dimensional vectors as row vectors in SCILAB, i.e.,

```
-->A = [3, 5, -1], B = [2, 2, 3]
 A  =

!   3.    5.  - 1. !
 B  =

!   2.    2.    3. !
```

Addition and subtraction of vectors is straightforward:

```
-->A+B
 ans  =

!   5.    7.    2. !

-->A-B
 ans  =

!   1.    3.  - 4. !
```

The negative of these vectors are calculated as:

```
--> -A
 A  =

!   -3.    -5.   1. !

--> -B
```

```
 B  =

!   -2.    -2.    -3. !
```

The product of the vectors by a constant is also straightforward:

```
-->2*A, -5*B
 ans  =

!   6.    10.  - 2. !
 ans  =

! - 10.  - 10.  - 15. !
```

The following is a linear combination of the two vectors:

```
-->5*A-3*B
 ans  =

!   9.    19.  - 14. !
```

The magnitude of a vector is obtained by using the function *norm*. For example, the magnitudes of vectors **A** and **B** are calculated as:

```
-->norm(A)
 ans  =

   5.9160798

-->norm(B)
 ans  =

   4.1231056
```

Unit vectors are calculated by dividing a vector by its magnitude. For example, unit vectors parallel to **A** and **B** (i.e., $\mathbf{e}_A$ and $\mathbf{e}_B$) are given by:

```
-->eA = A/norm(A)
 eA  =

!    .5070926     .8451543  -  .1690309 !

-->eB = B/norm(B)
 eB  =

!    .4850713     .4850713     .7276069 !
```

You can check that these are indeed unit vectors by using *norm*:

```
-->norm(eA)
 ans  =

    1.

-->norm(eB)
 ans  =

    1.
```


The matrix multiplication of a row vector times the transpose of a second row vector (i.e., a column vector) which will produce a dot product, i.e.,

```
-->A*B'
 ans  =

    13.

-->B*A'
 ans  =

    13.
```

Following the rules of matrix multiplication (see Chapter…), the product of a column vector times a row vector produces a matrix, e.g.,

```
-->A'*B
 ans  =

!   6.      6.      9.  !
!   10.     10.     15. !
! - 2.   - 2.    - 3.   !

-->B'*A
 ans  =

!   6.     10.   - 2. !
!   6.     10.   - 2. !
!   9.     15.   - 3. !
```

When the multiplication symbol between two row (or column) vectors is preceded by a dot, the result is a vector whose components are the product of the corresponding components of the factors. This is referred to as a term-by-term multiplication. For example, try:

```
-->A.*B
 ans  =

!   6.     10.   - 3. !

-->B.*A
 ans  =

!   6.     10.   - 3. !
```

Term-by-term multiplication requires that the factors have the same dimensions. Thus, the term-by-term multiplication of a row vector and a column vector is not defined. The following term-by-term multiplications, for example, will produce error messages:

```
-->A.*B'
      !--error  9999
inconsistent element-wise operation


-->A'.*B
      !--error  9999
inconsistent element-wise operation
```

The dot product of two row or column vectors can be obtained by combining a term-by-term multiplication with the function *sum*. The function *sum*, when applied to a vector, produces the sum of the vector components, for example:


```
-->sum(A)
 ans  =

    7.
```

In the following examples, the function *sum* is used to calculate the dot product of two vectors:

```
-->sum(A.*B)
 ans  =

    13.
```

The cosine of the angle between two vectors is obtained from the dot product of the vectors divided by the product of their magnitudes. For example, the cosine of the angle between vectors **A** and **B** is given by

```
-->costheta = A*B'/(norm(A)*norm(B))
 costheta  =

     .5329480
```

The corresponding angle is obtained by using the function *acos*.

```
-->acos(costheta)
 ans  =

    1.0087155
```

Of course, the result is in radians, the natural angular units. To convert to degrees use:

```
-->theta = 180/%pi*acos(costheta)
 theta  =

    57.795141
```

The projection of vector B over vector A is calculated by dividing the dot product of the two vectors by the magnitude of A, i.e.,

```
-->PB_A = A*B'/norm(A)
 PB_A  =

    2.1974011
```

The calculation of cross products requires the use of determinants, as described in the following section.

Calculating 2×2 and 3×3 determinants

The calculation of a cross product can be simplified if the cross product is written as a determinant of a matrix of 3 rows and 3 columns (also referred to as a 3×3 matrix). A *determinant* is a number associated with a square matrix, i.e., a matrix with the same number of rows and columns. While there is a general rule to obtain the determinant of any square matrix, we concentrate our attention on 2×2 and 3×3 determinants. Next, we present a simple way to calculate the determinant for 2×2 and 3×3 matrices.

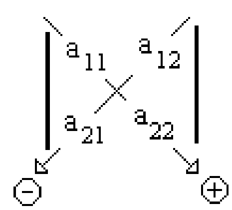
The elements of a matrix are identified with two sub-indices, the first representing the row and the second the column. Therefore, a 2×2 and a 3×3 matrix will be represented as:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}.$$

The determinants corresponding to these matrices are represented by the same arrangement of elements enclosed between vertical lines, i.e.,

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}, \quad \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}.$$

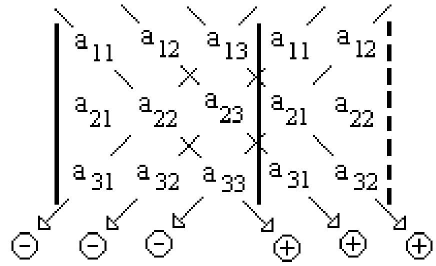
A 2×2 determinant is calculated by multiplying the elements in its diagonal and adding those products accompanied by the positive or negative sign as indicated in the diagram shown below.



The 2×2 determinant is, therefore,

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} \cdot a_{22} - a_{12} \cdot a_{21}.$$

A 3×3 determinant is calculated by *augmenting* the determinant, an operation that consists on copying the first two columns of the determinant, and placing them to the right of column 3, as shown in the diagram below. The diagram also shows the elements to be multiplied with the corresponding sign to attach to their product, in a similar fashion as done earlier for a 2×2 determinant. After multiplication the results are added together to obtain the determinant.



Therefore, a 3×3 determinant produces the following result:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\cdot a_{22}\cdot a_{33} + a_{12}\cdot a_{23}\cdot a_{31} + a_{13}\cdot a_{21}\cdot a_{32}$$

$$-(a_{13}\cdot a_{22}\cdot a_{31} + a_{11}\cdot a_{23}\cdot a_{31} + a_{12}\cdot a_{21}\cdot a_{33}).$$

Determinants can be calculated with SCILAB by using the function *det*. The following commands shown examples of 2x2 and 3x3 determinant calculations in SCILAB.

```
-->A2x2 = [3, -1; 2, 5]
 A2x2  =

!   3.  - 1. !
!   2.    5. !

-->det(A2x2)
 ans  =

    17.

-->A3x3 = [2,3, -1
-->        5,5, -2
-->        4, -3, 1]
 A3x3  =

!   2.    3.  - 1. !
!   5.    5.  - 2. !
!   4.  - 3.    1. !

-->det(A3x3)
 ans  =

  - 6.

-->det([2,-2; 4, 5])
 ans  =

    18.
```


Cross product as a determinant

The cross product **A**×**B** in Cartesian coordinates can be expressed as a determinant if the first row of the determinant consists of the unit vectors **i**, **j**, and **k.** The components of vector A and B constitute the second and third rows of the determinant, i.e.,

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}.$$

Evaluation of this determinant, as indicated above, will produce the result

$$\mathbf{A}\times\mathbf{B} = (B_z\cdot A_y - B_y\cdot A_z)\cdot\mathbf{i} + (A_z\cdot B_x - B_z\cdot A_x)\cdot\mathbf{j} + (B_y\cdot A_x - A_y\cdot B_x)\cdot\mathbf{k}.$$

SCILAB does not provide a cross-product function of its own. The following function, *CrossProd*, calculates the cross product of two three-dimensional vectors:

```
function [p] = CrossProd(u,v)
//Calculates the cross-product of two vectors u and v.
//Vectors u and v can be columns or row vectors, but
//they must have only three elements each.

[nu,mu] = size(u);
[nv,mv] = size(v);

if nu*mu <> 3 | nv*mv <> 3 then
       error('Vectors must be three-dimensional only')
       abort;
end

A1 = [ u(2), u(3); v(2), v(3)];
A2 = [ u(3), u(1); v(3), v(1)];
A3 = [ u(1), u(2); v(1), v(2)];

px = det(A1); py = det(A2); pz = det(A3);

p = [px, py, pz]

//end function
```

An application of the function *CrossProd* follows:

```
-->getf('CrossProd')

-->u = [2, 3, -1], v = [-3, 1, 4]
 u  =

!   2.    3.  - 1. !
 v  =

! - 3.    1.    4. !

-->CrossProd(u,v)
 ans  =
```



```
!   13.  - 5.    11. !
```

These operations are interpreted as **u** = 2**i**+3**j**-**k**, **v** = -3**i**+**j**+4**k**, and **u**×**v** = 13**i**+**j**+4**k**.

Earlier in this chapter we indicated that the volume of the parallelepiped defined by vectors **A**, **B**, and **C**, is given by the magnitude of the vector triple product **A•**(**B**×**C**). As an example in SCILAB, try the following calculation:

```
-->A = [2, 2, -1], B = [5, 3, 1], C = [1, 2, 3]
 A  =

!   2.    2.  - 1. !
 B  =

!   5.    3.    1. !
 C  =

!   1.    2.    3. !

-->abs(A*CrossProd(B,C)')
 ans  =

    21.
```

Polynomials as vector components

Although SCILAB is not a symbolic environment, some pseudo-symbolic calculations are possible using polynomials. Polynomials are a special type of SCILAB objects useful in linear system analysis. They are presented in more detail elsewhere in this book. In this section we present simple applications of polynomials as vector components.

Simple "symbolic" variables can be defined using the following call to the SCILAB function *poly*. For example, the following statement creates the symbolic variable *c*:

```
-->c=poly(0,'c')
 c  =

    c
```

With the symbolic variable c defined above, you can produce the following vector operations:

```
 -->c*A, c*B, c*(A+B), c*(A-B)
 ans  =

!   3c      5c    - c   !
 ans  =

!   2c      2c      3c   !
 ans  =

!   5c      7c      2c   !
 ans  =
```

```
!    c       3c     - 4c   !
```

It should be kept in mind that these are not symbolic results in the most general sense of the word (i.e., as those obtained in a symbolic mathematical environment such as *Maple* or *Mathematica*). The results obtained above, in terms of the polynomial *c*, are three-dimensional vectors whose components are polynomials. Other examples of vectors with polynomial components would be:

```
-->u = [ 1+c, (c+1)^2, c^3+2*c^2 ]
 u  =

!                        2       2   3 !
!   1 + c     1 + 2c + c      2c + c   !

-->u+c*A+3*c*B
 ans  =

!                            2            2   3 !
!   1 + 10c     1 + 13c + c      8c + 2c + c   !
```

Operations such as term-by-term multiplication, or linear combinations, are permitted with vectors whose components are polynomials. For example,

```
-->A.*u
 ans  =

!                         2      2   3 !
!   3 + 3c     5 + 10c + 5c   - 2c - c  !

-->u.*B
 ans  =

!                       2      2    3 !
!   2 + 2c     2 + 4c + 2c     6c + 3c  !

-->A-B.*u
 ans  =

!                       2          2    3 !
!   1 - 2c     3 - 4c - 2c   - 1 - 6c - 3c  !

-->u +3*A.*u
 ans  =

!                            2      2    3 !
!   10 + 10c     16 + 32c + 16c   - 4c - 2c  !
```

Some operations, such as *norm*, are not defined when a vector has polynomial components:

```
-->norm(u)
        !--error     4
undefined variable : %p_norm


-->norm(c*A)
          !--error     4
undefined variable : %p_norm
```

Also, in keeping with SCILAB's numerical nature, operations that attempt to combine two polynomial variables (i.e., symbolic operations) are not allowed. For example, the following command defines a new polynomial "symbolic" variable *r*:

```
-->r=poly(0,'r')
 r  =

    r
```

The following linear combination of the polynomial variables *c* and *r* fails to produce a result:

```
-->c*A+r*B
        !--error     4
undefined variable : %p_a_p
```

Assignment statements can replace the values of polynomial variables. In the next example, the value of *c*, which so far has been used as a polynomial variable, gets redefined as a constant:

```
-->c=2
 c =

   2.
```

With the new value of *c* the following operation produces a constant vector:

```
-->c*A
 ans =

!  6.   10.  - 2. !
```

As an application of polynomials as components of vectors, suppose that you want to determine the value of c such that vector **A** = 2**i**+c**j**-**k** is normal to vector **B** = -5**i**+3**j**-2**k**. We will use the fact that if vectors **A** and **B** are perpendicular then their dot product is zero, i.e., **A•B** = 0. Using SCILAB:

```
-->c = poly(0,'c')
 c  =

    c

-->A = [2, c, -1], B = [-5, 3, -2]
 A  =

!   2     c   - 1  !
 B  =

! - 5.    3.  - 2. !

-->A*B'
 ans  =

  - 8 + 3c
```

This translates into the equation $-8 + 3c = 0$. From which we can find $c = 8/3$.

Applications of vector algebra using SCILAB

In this section we present examples of operations with 2- and 3-dimensional vectors using SCILAB functions. The examples are taken from applications in different physical sciences.

Example 1 - Position vector

The position vector of a particle is a vector that starts at the origin of a system of coordinates and ends at the particle's position. If the current position of the particle is P(x,y,z), then the *position vector* is

$$\mathbf{r} = x\cdot\mathbf{i}+y\cdot\mathbf{j}+z\cdot\mathbf{k},$$

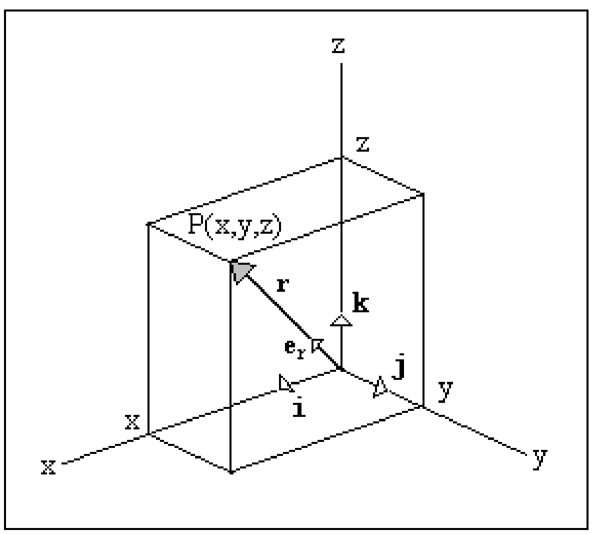
as illustrated in the figure below.

If a particle is located at point P(3, -2, 5), the position vector for this particle is $\mathbf{r} = 3\mathbf{i} -2\mathbf{j}+5\mathbf{k}$. In SCILAB this position vector is written as [3 –2 5].

```
-->r = [3,-2,5]
 r  =

!   3.  - 2.    5. !
```

To determine the *magnitude* of the position vector the function *norm*.

```
-->abs_r = norm(r)
 abs_r  =

    6.164414
```



In paper, you can write $|\mathbf{r}|$ = 6.164414. To determine the unit vector, use:


```
-->e_r = r/abs_r
 e_r  =  !    .4866643  -  .3244428     .8111071 !
```

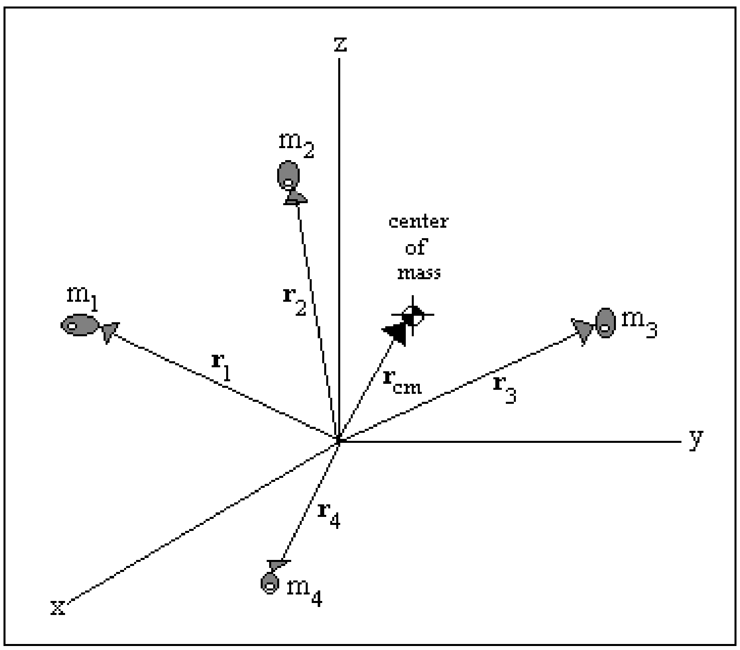
In paper, we can write: $\mathbf{e}_r = \mathbf{r}/|\mathbf{r}|$ = [0.4866   -0.3244 0.81111].

Example 2 - Center of mass of a system of discrete particles

Consider a system of discrete particles of mass m_i, located at position P_i (x_i, y_i, z_i), with $i = 1, 2, 3, \dots, n$. We can write position vectors for each particle as $\mathbf{r}_i = x_i\cdot\mathbf{i}+y_i\cdot\mathbf{j}+z_i\cdot\mathbf{k}$. The center of mass of the system of particles will be located at a position $\mathbf{r}_{cm}$ defined by

$$\mathbf{r}_{cm} = \frac{\sum_{i=1}^{n} m_i \cdot \mathbf{r}_i}{\sum_{i=1}^{n} m_i}.$$

A system of four particles is illustrated in the figure below.



The following table shows the coordinates and masses of 5 particles. Determine their center of mass.

i	x_i	y_i	z_i	m_i
1	2	3	5	12
2	-1	6	4	15
3	3	-1	2	25
4	5	4	-7	10
5	5	3	2	30

First, we enter vectors containing the coordinates x_i, y_i, and z_i, as well as the masses m_i:

```
-->x = [2,-1,3,5,5]
 x  =

!   2.  - 1.    3.    5.    5. !

-->y = [3,6,-1,4,3]
 y  =

!   3.    6.  - 1.    4.    3. !

-->z = [5,4,2,-7,2]
 z  =

!   5.    4.    2.  - 7.    2. !

-->m = [12,15,25,10,30]
 m  =

!   12.    15.    25.    10.    30. !
```

Then, we proceed to calculate the coordinates of the center of mass, (x_{cm}, y_{cm}, z_{cm}) using the formulas:

$$x_{cm} = \frac{\sum_{i=1}^{n} x_i \cdot m_i}{\sum_{i=1}^{n} m_i},\ y_{cm} = \frac{\sum_{i=1}^{n} y_i \cdot m_i}{\sum_{i=1}^{n} m_i},\ x_{cm} = \frac{\sum_{i=1}^{n} z_i \cdot m_i}{\sum_{i=1}^{n} m_i}.$$

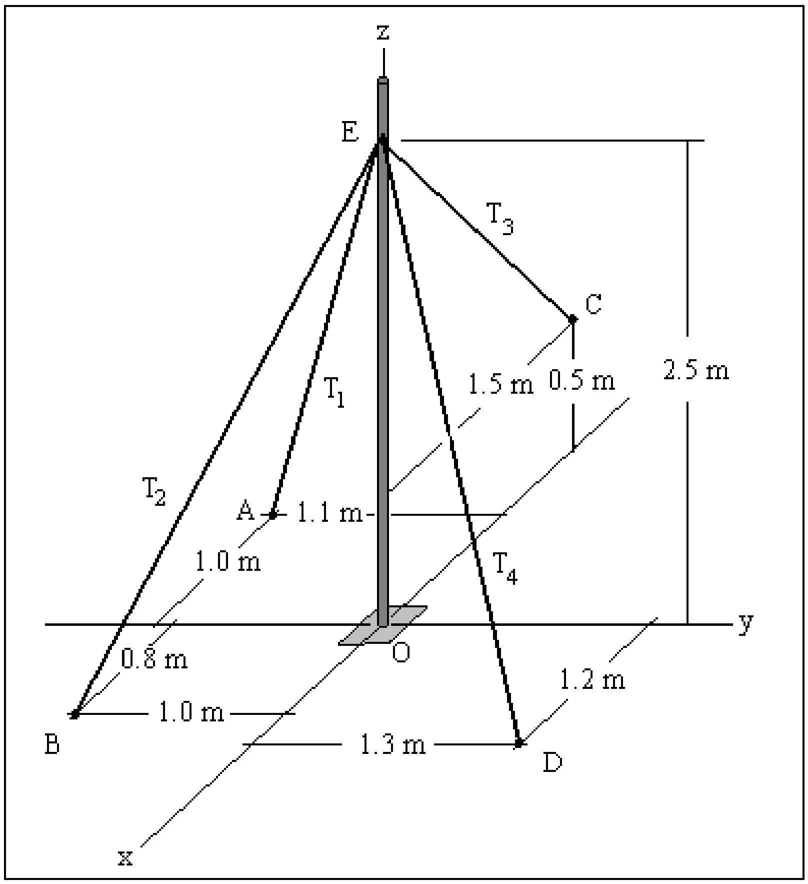
Using SCILAB, these formulas are calculated using the function *sum*:

```
-->x_cm = sum(x.*m)/sum(m)
 x_cm  =

    3.0869565

-->y_cm = sum(y.*m)/sum(m)
 y_cm  =

    2.5108696

-->z_cm = sum(z.*m)/sum(m)
 z_cm  =

    1.7391304
```

Example 3 - Resultant of forces

The figure below shows a vertical pole buried in the ground and supported by four cables EA, EB, EC, and ED. The magnitude of the tensions in each of those cables, as shown in the figure, are T_1 = 150 N, T_2 = 300 N, T_3 = 200 N, and T_4 = 150 N.

To find the vector resultant of all those forces you need to add the four tensions written out as vectors. The tension in cable *i* will be given by $\mathbf{T}_i = T_i \cdot \mathbf{e}_i$, where $\mathbf{e}_i$ is a unit vector in the direction of the cable where the tension acts. (The tension vectors act so that the cables pull away from point E, where the cables are attached to the pole.)



Writing out tension $\boldsymbol{T_1}$

Tension $\mathbf{T}_1$ acts along cable EA. To determine a unit vector along cable EA, you need to write the vector $\mathbf{r}_{EA} = \mathbf{r}_A - \mathbf{r}_E$, where $\mathbf{r}_A$ is the position vector of point A (the tip of the vector $\mathbf{r}_{EA}$), and $\mathbf{r}_A$ is the position vector of point A (the origin of the vector $\mathbf{r}_{EA}$). The coordinates of these points, obtained from the figure, are A(-1.0 m, -1.1 m, 0) and E(0, 0, 2.5 m). Therefore, we can write $\mathbf{r}_E = (2.5\mathbf{k})$ m, and $\mathbf{r}_A = (-\mathbf{i} - 1.1\mathbf{j})$ m.

Using SCILAB we would calculate the vector $\mathbf{r}_{EA}$ as follows. First, enter $\mathbf{r}_E$ and $\mathbf{r}_A$:

```
--> rE = [ 0 0 2.5]; rA = [-1 -1.1 0];
```

To obtain, $\mathbf{r}_{EA} = \mathbf{r}_A - \mathbf{r}_E$, we will use

```
--> rEA = rA - rE;
```



```
rEA =  ! -1.  -1.1  0. !
```

i.e., $\mathbf{r}_{EA}$ = **-i** –1.1**j**-2.5**k**.

To find the unit vector along which T_1, first we find the magnitude,

```
--> rEA_abs = norm(rEA)
```

i.e., ($|\mathbf{r}_{EA}|$ = 2.9086079).

Finally, the unit vector, is calculated as

```
--> eEA = rEA/rEA_abs
eEA = ! -0.344 - 0.378  -0.860 !
```

i.e., $\mathbf{e}_{EA} = \mathbf{r}_{EA}/|\mathbf{r}_{EA}|$ = [-0.344 - 0.378 -0.860] = -0.344**i** - 0.378**j** -0.860**k**. (To verify that this is indeed a unit vector, use the command `--> norm(eEA)` . You should get as a result 1.000).

The tension vector is calculated by multiplying the magnitude of the tension T_1 = 150, times the unit vector $\mathbf{e}_{EA}$, i.e.,

```
--> T1 = 150*eEA
T1 = !  -51.57 -56.73 -128.93  !
```

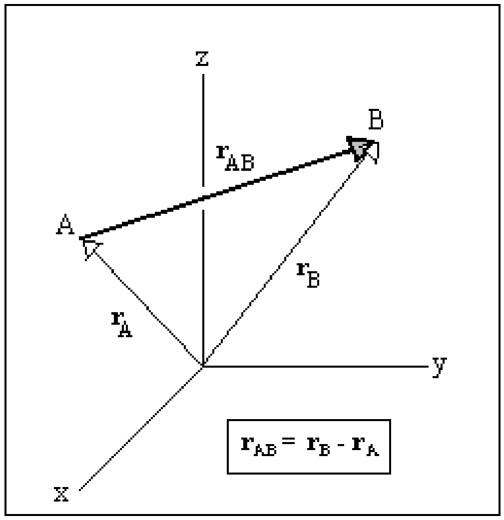
The result is the vector [-51.57 -56.73 -128.93], or $\mathbf{T}_1$ = (-51.57**i** -56.73**j** -128.93**k**) N.

Writing the vector that joins two points in space

The vector that joints points A and B, where $A(x_A, y_A, z_A)$ is the origin and point $B(x_B, y_B, z_B)$ the tip of the vector, is written as

$$\mathbf{r}_{AB} = \mathbf{r}_B - \mathbf{r}_A.= (x_B - x_A)\cdot\mathbf{i}+(y_B - y_A)\cdot\mathbf{j}+(z_B - z_A)\cdot\mathbf{k},$$

where $\mathbf{r}_A = x_A\cdot\mathbf{i}+y_A\cdot\mathbf{j}+z_A\cdot\mathbf{k}$ is the position vector of point A, and $\mathbf{r}_B = x_B\cdot\mathbf{i}+y_B\cdot\mathbf{j}+z_B\cdot\mathbf{k}$ is the position vector of point B. This operation is illustrated in the figure below.



Relative position vector

The previous operation can also be interpreted as obtaining the relative position vector of point B(x_B, y_B, z_B) with respect to point A(x_A, y_A, z_A), and written as

$$\mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A.= (x_B - x_A)\cdot\mathbf{i}+(y_B - y_A)\cdot\mathbf{j}+(z_B - z_A)\cdot\mathbf{k}.$$

Writing out tensions T_2, T_3, and T_4

Having developed a procedure to determine the unit vector along any of the cables, we can proceed to write out the vectors representing the tensions T_2, T_3, and T_4. First, we determine the coordinates of relevant points from the figure describing the problem. These points are: A(-1.0 m, -1.1m, 0) B(0.8 m, -1.0 m, 0.0), C(-1.5 m, 0.0, 0.5 m), D(1.2 m, 1.3m, 0.0), and E(0, 0, 2.5 m). The tensions, as vectors, will be written as $\mathbf{T}_2 = T_2\cdot\mathbf{e}_{EB}$, $\mathbf{T}_3 = T_3\cdot\mathbf{e}_{EC}$, and $\mathbf{T}_4 = T_4\cdot\mathbf{e}_{ED}$. (Recall that T_2 = 300 N, T_3 = 200 N, and T_4 = 150 N.). Thus, we need to write out the vectors $\mathbf{r}_{EB}$, $\mathbf{r}_{EC}$, and $\mathbf{r}_{ED}$, find their magnitudes, $|\mathbf{r}_{EB}|$, $|\mathbf{r}_{EC}|$, and $|\mathbf{r}_{ED}|$, and calculate the unit vectors $\mathbf{e}_{EB} = \mathbf{r}_{EB}/|\mathbf{r}_{EB}|$, $\mathbf{e}_{EC} = \mathbf{r}_{EC}/|\mathbf{r}_{EC}|$, and $\mathbf{e}_{ED} = \mathbf{r}_{ED}/|\mathbf{r}_{ED}|$, in order to obtain $\mathbf{T}_2$, $\mathbf{T}_3$, and $\mathbf{T}_4$.

The following SCILAB commands perform the required calculations. First, we enter the position vectors for points A, B, C, D, and E:

```
-->rA =[-1, -1.1, 0], rB = [0.8, -1, 0], rC = [-1.5, 0,0.5]
 rA  =

! - 1.  - 1.1    0. !
 rB  =

!    .8  - 1.    0. !
 rC  =

! - 1.5    0.     .5 !

-->rD = [1.2, 1.3, 0], rE = [0, 0, 2.5]
 rD  =
```



```
!   1.2    1.3    0. !

 rE  =

!   0.    0.    2.5 !
```

Next, we calculate the relative position vectors, $\mathbf{r}_{EB}$, $\mathbf{r}_{EC}$, and $\mathbf{r}_{ED}$:

```
-->rEB = rB - rE, rEC = rC - rE, rED = rD - rE
 rEB  =

!    .8  - 1.  - 2.5 !
 rEC  =

! - 1.5    0.  - 2. !
 rED  =

!   1.2    1.3  - 2.5 !
```

The magnitudes of the vectors, , $|\mathbf{r}_{EB}|$, $|\mathbf{r}_{EC}|$, and $|\mathbf{r}_{ED}|$, are calculated next:

```
-->abs_rEB = norm(rEB), abs_rEC = norm(rEC), abs_rED= norm(rED)
 abs_rEB  =

    2.8089144
 abs_rEC  =

    2.5
 abs_rED  =

    3.0626786
```

The unit vectors along the cables, $\mathbf{e}_{EB} = \mathbf{r}_{EB}/|\mathbf{r}_{EB}|$, $\mathbf{e}_{EC} = \mathbf{r}_{EC}/|\mathbf{r}_{EC}|$, and $\mathbf{e}_{ED} = \mathbf{r}_{ED}/|\mathbf{r}_{ED}|$, are calculated as:

```
-->eEB = rEB/abs_rEB, eEC = rEC/abs_rEC, eED = rED/abs_rED
 eEB  =

!    .2848075  -  .3560094  -  .8900236 !
 eEC  =

! -  .6    0.  -  .8 !
 eED  =

!    .3918139     .4244650  -  .8162789 !
```

Next, we enter the magnitudes of the tensions, T_2 = 300 N, T_3 = 200 N, and T_4 = 150 N. These are identified in SCILAB as `T1m`, `T2m`, and `T3m`, respectively:

```
-->T2m = 300, T3m = 200, T4m = 150
 T2m  =

    300.
 T3m  =

    200.
 T4m  =

    150.
```


The tension vectors, themselves, are calculated by using $\mathbf{T}_2 = T_2 \cdot \mathbf{e}_{EB}$, $\mathbf{T}_3 = T_3 \cdot \mathbf{e}_{EC}$, and $\mathbf{T}_4 = T_4 \cdot \mathbf{e}_{ED}$. In SCILAB we use:

```
-->T2 = T2m*eEB, T3 = T3m*eEC, T4 = T4m*eED
 T2  =

!   85.442263  - 106.80283  - 267.00707 !
 T3  =

! - 120.    0.  - 160. !
 T4  =

!   58.772083    63.669757  - 122.44184 !
```

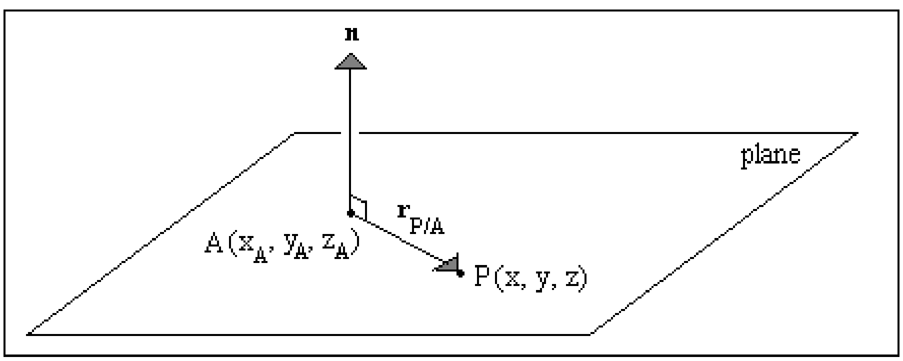
The results are: $\mathbf{T}_2 = (85.44\mathbf{i} -106.80\mathbf{j} -267.01\mathbf{k})$ N, $\mathbf{T}_3 = (-120\mathbf{i} -160\mathbf{k})$ N, and for cable ED, $\mathbf{T}_4 = (58.77\mathbf{i} +63.67\mathbf{j} -122.44\mathbf{k})$ N.

The resultant of these four forces, namely, $\mathbf{R} = \mathbf{T}_1 + \mathbf{T}_2 + \mathbf{T}_3 + \mathbf{T}_4$, turns out to be

$$\mathbf{R} = (-27.36\mathbf{i} -99.86\mathbf{j} -678.38\mathbf{k}) \text{ N}.$$

Example 4 – Equation of a plane in space

The equation of a plane in space, in Cartesian coordinates, can be obtained given a point on the plane $A(x_A,y_A,z_A)$, and a vector normal to the plane $\mathbf{n} = n_x \mathbf{i}+n_y \mathbf{j}+n_z \mathbf{k}$. Let $P(x,y,z)$ be a generic point on the plane of interest. We can form a relative position vector $\mathbf{r}_{P/A} = \mathbf{r}_P - \mathbf{r}_A = (x-x_A)\cdot\mathbf{i}+(y-y_A)\cdot\mathbf{j}+(z-z_A)\cdot\mathbf{k}$, which is contained in the plane, as shown in the figure below.



Because the vectors $\mathbf{n}$ and $\mathbf{r}_{P/A}$ are perpendicular to each other, then, we can write $\mathbf{n}\bullet\mathbf{r}_{P/A} = 0$, or

$$(n_x \mathbf{i}+n_y \mathbf{j}+n_z \mathbf{k})\bullet((x-x_A)\cdot\mathbf{i}+(y-y_A)\cdot\mathbf{j}+(z-z_A)\cdot\mathbf{k}) = 0,$$

which results in the equation

$$n_x\cdot(x-x_A) + n_y\cdot(y-y_A) + n_z\cdot(z-z_A) = 0.$$

To implement a SCILAB function to produce the equation of a plane in space we use the fact that the equation can be re-written as $n_x\cdot x + n_y\cdot y + n_z\cdot z = \mathbf{n}\bullet\mathbf{r}_A$. The function *PlaneEquation*, shown below, calculates the right-hand side of the equation, $\mathbf{n}\bullet\mathbf{r}_A$, and uses the function *string* to convert numerical results to strings. The final result of the function is a string representing the equation.

```
function [eq] = PlaneEquation(n,p)

//This function produces the equation of a plane
//in space given the normal vector n and the point p.

[nn,mn] = size(n);
[np,mp] = size(p);

if nn*mn <> 3 | np*mp <> 3 then
       error('Vector n or point p in PlaneEquation must have 3 elements')
       abort;
end

c = n*p';
eq = string(n(1))+'*x+'+string(n(2))+'*y+'+string(n(3))+'*z='+string(c)

//end function
```

The function requires as input the normal vector **n** and the point A (referred to as *p* in the function), both entered as SCILAB vectors. For example, to find the equation of the plane with normal vector **n** = 3**i**-5**j**+6**k** passing through point A(3, 6, -2) use:

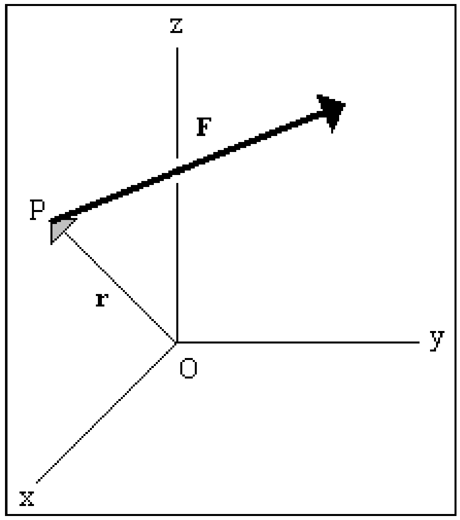
```
-->getf('PlaneEquation')

-->n = [3, -5, 6], A = [3, 6, -2]

-->myEquation = PlaneEquation(n,A)
 myEquation  =

 3*x+-5*y+6*z=-33
```

To verify that the equation is satisfied by point A use:

```
-->x = 3, y = 6, z = -2
 x  =

    3.
 y  =

    6.
 z  =

  - 2.

-->3*x-5*y+6*z
 ans  =

  - 33.
```

Example 5 – Moment of a force

The figure below shows a force **F** acting on a point P in a rigid body. Suppose that the body is allowed to rotate about point O. Let **r** be the position vector of point P with respect to the point of rotation O, which we make coincide with the origin of our Cartesian coordinate system. The moment of the force F about point O is defined as **M** = **r**×**F**.



Referring to the figure of Example 3, if we want to calculate the moment of tension $\mathbf{T}_1$ = (-51.57**i** -56.73**j** -128.93**k**) N, about point O, we will use as the position vector **r** of the force's line of action through the pole, the vector $\mathbf{r}_{OE}$ = (2.5**k**) m. The moment $\mathbf{M}_1 = \mathbf{r}_{OE}\times\mathbf{T}_1$, can be calculated as follows:

`-->rOE = [0 0 2.5]`	Enter $\mathbf{r}_{OE}$
`-->T1 = [-57.51 –56.73 –128.93]`	Enter $\mathbf{T}_1$
`-->M1 = CrossProd(r0E,T1)`	Calculate $\mathbf{M}_1 = \mathbf{r}_{OE}\times\mathbf{T}_1$

The result is [141.825 –143.775 0], or $\mathbf{M}_1$ = (141.825 **i** –143.775 **j**) m·N.

To find the magnitude of the moment, use [↰][ABS], thus, $|\mathbf{M}_1| = M_1$ = 201.954 m·N.

Note: Moments are vector quantities and obey all rules of vectors, i.e., they can be added, subtracted, multiplied by a scalar, undergo internal and external vector products. As an exercise, the reader may want to calculate the moments corresponding to the other tensions in Example 3, as well as the resultant moment from all four tensions.

Example 6 – Cartesian and polar representations of vectors in the x-y plane

A position vector in the x-y plane can be written simply as **r** = x·**i**+y·**j**. Let its magnitude be r = |**r**|. A unit vector along the direction of r is given by $\mathbf{e}_r$ = **r**/r = (x/r)·**i**+(y/r)·**j**. If we use polar coordinates, we recognize

$$x/r = \cos\theta, \text{ and } y/r = \sin\theta,$$

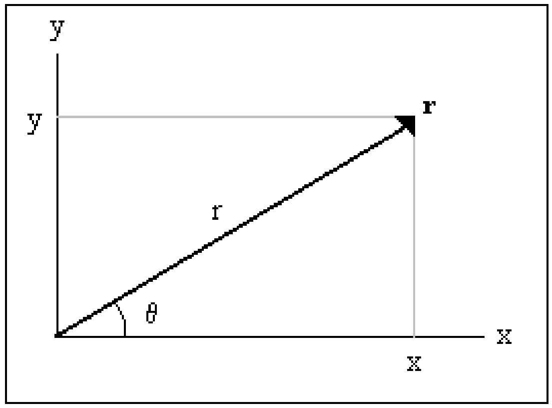
thus, we can write the unit vector as

$$\mathbf{e}_r = \mathbf{r}/r = \cos\theta\cdot\mathbf{i} + \sin\theta\cdot\mathbf{j}.$$

Thus, if we are given the magnitude, r, and the direction, θ, of a vector (i.e., its polar coordinates (r,θ)), we can easily put together the vector as

$$\mathbf{r} = r\cdot\mathbf{e}_r = r\cdot(\cos\theta\cdot\mathbf{i} + \sin\theta\cdot\mathbf{j}).$$

This result is illustrated in the figure below.



To enter a vector in SCILAB, given the magnitude *r* and the angle *θ,* you would use, for example:

```
-->r = 2.5*[cos(0.75), sin(0.75)]
 r  =

!   1.8292222    1.7040969 !
```

In this case the angle represents radians. If you want to use an angle in degrees, you need to transform it to radians in the trigonometric functions, for example:

```
-->r = 5.2*[cos(32*%pi/180), sin(32*%pi/180)]
 r  =

!   4.4098501    2.7555802 !
```

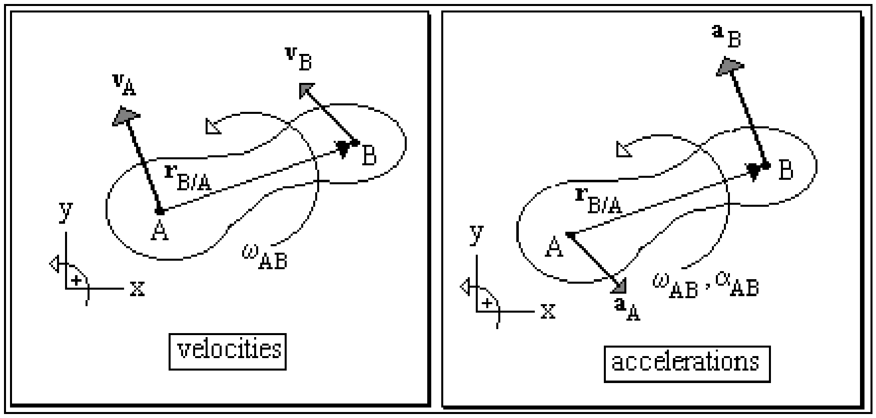
Example 7 – Planar motion of a rigid body

In the study of the planar motion of a rigid body the following equations are used to determine the velocity and acceleration of a point B ($\mathbf{v}_B$, $\mathbf{a}_B$) given the velocity and acceleration of a reference point A ($\mathbf{v}_A$, $\mathbf{a}_A$):

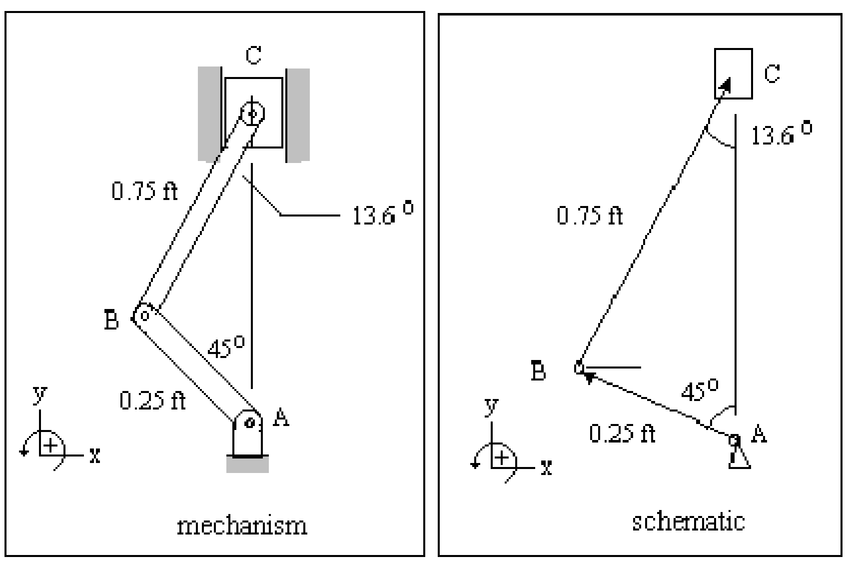
$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A},$$

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \cdot \mathbf{r}_{B/A}.$$

The equations also use the angular velocity of the body connecting A and B, $\boldsymbol{\omega}_{AB}$, whose magnitude is ω_{AB}; the angular acceleration of the body connecting A and B, $\boldsymbol{\alpha}_{AB}$, and the relative position vector of point B with respect to point A, $\mathbf{r}_{B/A}$. The following figure illustrates the calculation of relative velocity and acceleration in planar motion of a rigid body.



To present applications of this equation using the HP 49 G calculator, we use the data from the mechanism shown in the figure below.



In this mechanism there are two bars AB and AC pin-connected at B. Bar AB is pin supported at A, and bar BC is attached through a pin to piston C. Piston C is allowed to move in the vertical direction only. At the instant shown the angular velocity and acceleration of bar AB are 10 rad/s and 20 rad/s^2 clockwise. You are asked to determine the angular velocity and acceleration of bar BC and the linear velocity and acceleration of piston C.

Angular velocity and acceleration

Angular velocities and accelerations in the x-y plane are represented as vectors in the z-direction, i.e., normal to the x-y plane. These vectors are positive if the angular velocity or acceleration is counterclockwise. In general, thus we can write, $\boldsymbol{\omega}_{AB} = \pm\omega_{AB}\cdot\mathbf{k}$, and $\boldsymbol{\alpha}_{AB} = \pm\alpha_{AB}\cdot\mathbf{k}$. For the data in this problem we can write, therefore,

$$\boldsymbol{\omega}_{AB} = (-10\mathbf{k})\text{ rad/s},\ \boldsymbol{\alpha}_{AB} = (-20\mathbf{k})\text{rad/s}^2,\ \boldsymbol{\omega}_{BC} = (\omega_{BC}\cdot\mathbf{k})\text{ rad/s, and }\ \boldsymbol{\alpha}_{BC} = (\alpha_{BC}\cdot\mathbf{k}).$$

Using SCILAB, we will write:

```
-->wAB = [0, 0, -10], alphaAB = [0, 0, -20]
 wAB  =

!   0.    0.  - 10. !
 alphaAB  =

!   0.    0.  - 20. !
```

The next statements define the polynomial variables *wBCm* (standing for ω_{BC}) and *alBCm* (standing for α_{BC}). The *m* in the polynomial variable names stands for 'magnitude':

```
-->wBCm = poly(0,'wBCm'), alBCm = poly(0,'alBC')
 wBCm  =

    wBCm
 alBCm  =

    alBC
```

With the variables *wBCm* and *alBCm* we can define the vectors $\boldsymbol{\omega}_{BC}$ and $\boldsymbol{\alpha}_{BC}$ as:

```
-->wBC = [0, 0, wBCm], alBC = [0, 0, alBCm]
 wBC  =

!   0    0    wBCm  !
 alBC  =

!   0    0    alBC  !
```

Relative position vector

The relative position vectors of interest in this problem are $\mathbf{r}_{B/A}$ and $\mathbf{r}_{C/B}$, which are obtained as follows:

- To obtain $\mathbf{r}_{B/A}$, we use the fact that for the xy coordinate system shown, the angle corresponding to vector $\mathbf{r}_{B/A}$ is $\theta_{B/A} = 90^o + 45^o = 135^o$. Thus, we can write

```
-->rB_A = 0.25*[cos(135*%pi/180), sin(135*%pi/180), 0]
 rB_A  =

! -   .1767767     .1767767    0. !
```

  In paper, we would write, therefore, $\mathbf{r}_{B/A} = (-0.177\mathbf{i} + 0.177\mathbf{j})$ft.

- For $\mathbf{r}_{C/B}$, the angle $\theta_{C/B} = 90^o - 13.6^o = 76.4^o$. Thus, in SCILAB,


```
-->rC_B = 0.75*[ cos(76.4*%pi/180), sin(76.4*%pi/180), 0]
 rC_B  =

 !    .1763566     .7289708    0. !
```

In paper, we write, $\mathbf{r}_{C/B} = (0.175\mathbf{i} + 0.729\mathbf{j})$ft.

Velocity

Since point A is fixed point, $\mathbf{v}_A = 0$, and we can write $\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = (-10\mathbf{k})$ rad/s $\times$ $(-0.177\mathbf{i} + 0.177\mathbf{j})$ft. Using SCILAB, we would write:

```
-->getf('CrossProd')

-->vA = 0
 vA  =

    0.

-->vB = vA + CrossProd(wAB,rB_A)
 vB  =

!   1.767767    1.767767    0. !
```

Thus,

$$\boxed{\mathbf{v}_B = (1.77\mathbf{i}+1.77\mathbf{j}) \text{ ft/s.}}$$

Note: Radians are basically dimensionless units, thus rad·ft = ft, rad·m = m.

To find the velocity of point C, we use

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = (-1.77\mathbf{i}\text{-}1.77j) \text{ ft/s} + (\boldsymbol{\omega}_{BC}\cdot\mathbf{k}) \text{ rad/s} \times (0.175\mathbf{i} + 0.729\mathbf{j})\text{ft.}$$

With SCILAB, this operation is written as:

```
-->vC = vB + CrossProd(wBC,rC_B)
 vC  =

!   1.767767 -  .7289708wBCm     1.767767 +  .1763566wBCm     0 !
```

The result can be written in paper as:

$$\mathbf{v}_C = ((1.77\text{-}0.729\cdot\omega_{BC})\cdot\mathbf{i} + (1.77\text{+}0.176\cdot\omega_{BC})\cdot\mathbf{j}) \text{ ft/s.}$$

The figure above shows that the piston C is forced to move in the vertical direction, thus, the velocity of point C can be written as

$$\mathbf{v}_C = (v_C\cdot\mathbf{j}) \text{ ft/s.}$$

Equating the two results presented immediately above for $\mathbf{v}_C$ we get:

$$(1.77\text{-}0.729\cdot\omega_{BC})\cdot\mathbf{i} + (1.77\text{+}0.176\cdot\omega_{BC})\cdot\mathbf{j} = 0\cdot\mathbf{i} + v_C\cdot\mathbf{j}.$$

Since the x- and y-components of the two vectors in each side of the equal sign must be the same, we can write the system of equations:

$$1.77 - 0.729 \cdot \omega_{BC} = 0$$
$$1.77 + 0.176 \cdot \omega_{BC} = vC$$

Solution of equations – one at a time

Using SCILAB, we can solve for the two unknowns (ω_{BC} and v_C) as follows:

1. From the first equation, $\omega_{BC} = 1.77/0.729$, i.e.,

```
-->wBCm = 1.77/0.729
wBCm  = 2.4279835
```

2. From the second equation, $vC = 1.77+(0.176)\cdot(2.43)$, i.e.,

```
-->vCm = 1.77+0.176*2.43
vCm  = 2.19768
```

Thus, the solution of the system of equations is:

$$\omega_{BC} = 2.43 \text{ rad/s, and } v_C = 2.20 \text{ ft/s.}$$

The positive sign in ω_{BC} means that the angular velocity is counterclockwise. The positive sign in v_C means that point C is moving upwards in the vertical direction.

Acceleration

Again, because A is a fixed point, $\mathbf{a}_A = 0$. Thus, the acceleration of point B is given by

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \cdot \mathbf{r}_{B/A} = (-20\mathbf{k})\text{rad/s}^2 \times (-0.177\mathbf{i} + 0.177\mathbf{j})\text{ft} - (10 \text{ rad/s})^2 \times (-0.177\mathbf{i} + 0.177\mathbf{j})\text{ft}$$

Using SCILAB we can obtain $\mathbf{a}_B$ as follows:

```
-->aA = 0
 aA  =

    0.


-->wABm = -10
 wABm  =

  - 10.

-->aB = aA + CrossProd(alphaAB,rB_A)-wABm^2*rB_A
 aB  =

!   21.213203  - 14.142136    0. !
```

The result is [21.240 -14.142 0.000], or

$$\mathbf{a}_B = (21.24\mathbf{i} - 14.142\mathbf{j})\text{ft/s}^2.$$

To calculate the acceleration of point C we use:

$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}{}^2 \cdot \mathbf{r}_{C/B} =$

$$(21.24\mathbf{i} - 14.16\mathbf{j})\text{ft/s}^2 + (\alpha_{BC}\mathbf{k}) \times (0.175\mathbf{i} + 0.729\mathbf{j})\text{ft} - (-2.42 \text{ rad/s})^2 \cdot (0.175\mathbf{i} + 0.729\mathbf{j})\text{ft}$$

In Scilab:

```
-->aC = aB + CrossProd(alphaBC,rC_B) - wBCm^2*rC_B
 aC  =

!   20.173563 -  .7289708alBC   - 18.439494 +  .1763566alBC     0 !
```

The result is:

$$\mathbf{a}_C = ((20.17 - 0.729 \cdot \alpha_{BC}) \cdot \mathbf{i} + (-18.44 + 0.176 \cdot \alpha_{BC})) \cdot \mathbf{j}) \text{ ft/s}^2.$$

Also, because the motion of point C is in the vertical direction, we can write

$$\mathbf{a}_C = a_C \cdot \mathbf{j}.$$

Equating the two expressions for the vector aC shown above, we get the following equations:

$$20.17 - 0.729 \cdot \alpha_{BC} = 0,$$
$$-18.44 + 0.176 \cdot \alpha_{BC} = a_C.$$

Solution of a system of linear equations using matrices

The system of linear equations in two unknowns (α_{BC} and a_C) obtained above can be re-written in matricial form as follows:

1. The equations are first re-written as

$$a_c - 0.176 \cdot \alpha_{BC} = -18.44$$
$$0.729 \cdot \alpha_{BC} = 20.17$$

or, as a matrix equation:

$$\begin{bmatrix} 1 & -0.176 \\ 0 & 0.729 \end{bmatrix} \begin{bmatrix} a_C \\ \alpha_{BC} \end{bmatrix} = \begin{bmatrix} -18.44 \\ 20.17 \end{bmatrix}$$

2. This matrix equation can be solved by using the backward-slash operator (\), i.e., $\mathbf{x} = \mathbf{A} \backslash \mathbf{b}$. This result follows from the original matrix equation, $\mathbf{A} \cdot \mathbf{x} = \mathbf{b}$, by "dividing" both sides of the equation by **A**. However, since **A** is a matrix, this "division" is not a regular arithmetic division. Typically, this operation would be represented as $\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$, where $\mathbf{A}^{-1}$ is the inverse of matrix **A**. Modern matrix-based numerical environments, such as SCILAB, provide the user with the "backward-slash" operator instead of using the inverse matrix. The result, however, is exactly the same.

Using SCILAB, we would enter:

```
-->A = [1, -0.176; 0, 0.729], b = [-18.44;20.17]
 A  =

!   1.   -   .176 !
!   0.       .729 !
```



```
 b  =

! - 18.44 !
!   20.17 !

-->A\b
 ans  =

! - 13.570425 !
!   27.668038 !
```

The results are interpreted as

$$a_C = -13.57 \text{ ft/s}^2,\ \alpha_{BC} = 27.67 \text{ rad/s}^2.$$

The negative sign in a_c indicates that point C is decelerating. The positive sign in α_{BC} indicates that bar BC is accelerating angularly in the counterclockwise direction.

We have presented here two methods for solution of systems of linear equations. These methods, and others, are presented in more detail in a different chapter.

Exercises

For the following exercises use the Cartesian vectors:

$\mathbf{u}$ = 3$\mathbf{i}$+2$\mathbf{j}$-5$\mathbf{k}$, $\mathbf{v}$ = -$\mathbf{i}$-3$\mathbf{k}$, $\mathbf{w}$ = 5$\mathbf{i}$-10$\mathbf{j}$-3$\mathbf{k}$, $\mathbf{r}$ = -8$\mathbf{i}$+10$\mathbf{j}$-2.5$\mathbf{k}$, $\mathbf{s}$ = -3$\mathbf{i}$-2$\mathbf{j}$-5$\mathbf{k}$, $\mathbf{t}$ = 6$\mathbf{i}$-2$\mathbf{j}$+15$\mathbf{k}$,

[1]. Determine the result of the following operations:

(a) $|\mathbf{u}|$ (b) $|\mathbf{w}|$ (c) $|\mathbf{v}||\mathbf{r}|$
(d) $|\mathbf{s}|/|\mathbf{w}|$ (e) $\mathbf{a} = \mathbf{u} + \mathbf{v}$ (f) $\mathbf{b} = \mathbf{r}-\mathbf{t}$
(g) $\mathbf{c} = 3\mathbf{r}-2\mathbf{v}$ (h) $\mathbf{d} = -\mathbf{t} + 2\mathbf{s}$ (i) unit vectors: $\mathbf{e}_u$, $\mathbf{e}_v$, $\mathbf{e}_w$
(j) angles between vectors $\mathbf{u}$,$\mathbf{v}$; $\mathbf{r}$,$\mathbf{s}$; and $\mathbf{v}$,$\mathbf{w}$. (k) $\mathbf{u} \bullet \mathbf{v}$
(l) $\mathbf{w} \bullet \mathbf{r}$ (m) $\mathbf{s}\times\mathbf{t}$ (n) $\mathbf{u}\times\mathbf{v}$
(o) $\mathbf{r} \times \mathbf{w}$ (p) $\mathbf{u} \bullet(\mathbf{v}\times\mathbf{w})$ (q) $(\mathbf{s}\times\mathbf{t}) \bullet \mathbf{r}$
(r) $\mathbf{w}\times(\mathbf{r}\times\mathbf{u})$ (s) $(\mathbf{u}\times\mathbf{v})\times\mathbf{t}$ (t) $(\mathbf{u}\times\mathbf{v})\times(\mathbf{s}\times\mathbf{t})$

[2]. Four different cables are attached to point E(0,0,0) on a structure. The four cables are anchored to points A(-1,-1,-1), B(2,3,-5), C(-2,2,4), and D(2,3,-1). The tensions in the four cables are: AE = 150 lb, BE = 250 lb, CE = 100 lb, DE = 50 lb. Determine the resultant force from the four cables.

[3]. Determine the torque of the following forces $\mathbf{F}$ given the arm $\mathbf{r}$, i.e., $\mathbf{M} = \mathbf{r}\times\mathbf{F}$:

(a) $\mathbf{F}$ = (3$\mathbf{i}$+2$\mathbf{j}$-5$\mathbf{k}$) N, $\mathbf{r}$ = (-2$\mathbf{i}$+5$\mathbf{j}$-3$\mathbf{k}$) m (b) $\mathbf{F}$ = ($\mathbf{i}$-4$\mathbf{j}$-3$\mathbf{k}$) lb, $\mathbf{r}$ = ($\mathbf{i}$+8$\mathbf{j}$-13$\mathbf{k}$) ft
(a) $\mathbf{F}$ = 100($\mathbf{i}$+$\mathbf{j}$-$\mathbf{k}$) lb, $\mathbf{r}$ = (-2$\mathbf{i}$+5$\mathbf{j}$-3$\mathbf{k}$)/2 ft (b) $\mathbf{F}$ = 200($\mathbf{i}$-4$\mathbf{j}$-$\mathbf{k}$) N, $\mathbf{r}$ = 3($\mathbf{i}$+$\mathbf{j}$-10$\mathbf{k}$) m

[4]. Determine the equation of the plane through point A with normal vector $\mathbf{n}$. Sketch the plane:

(a) A(-2,3,5), $\mathbf{n}$ = [2,-2,3] (b) A(0, -1, 2), $\mathbf{n}$ = [5,5,-1]
(c) A(2,5,-1), $\mathbf{n}$ = [1, 1, 1] (d) A(-1,5,-2), $\mathbf{n}$ = [3,3,-1]

[5]. Two vectors n and m are said to be orthogonal if $\mathbf{n} \bullet \mathbf{m} = 0$. Determine the missing components in the following vectors m and n so that they are orthogonal:

(a) **n** = [2, y,-2], **m** = [5,5,-1]
(b) **n** = [x,5,4], **m** = [-1,0,2]
(c) **n** = [4,2,y], **m** = [3,3,2]
(d) **n** = [5,5,2], **m** = [x,-2,3]

Note: define the unknown variable as a SCILAB polynomial variable, e.g., for (a):

```
-->y = poly(0,'y')
```

Then, define the vectors as follows:

```
--> n = [2,y,-2]; m = [5,5,1];
```

Calculate the dot product as a polynomial:

```
--> p = n*m'
```

and solve for y using function *roots*:

```
--> roots(p)
```

[6]. For the mechanism presented in Example 7, if the velocity of point C is $v_c = 1.2\ ft/s$ and its acceleration is $a_c = -0.2\ ft/s^2$, determine the angular velocity and acceleration of bars AB and BC.

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