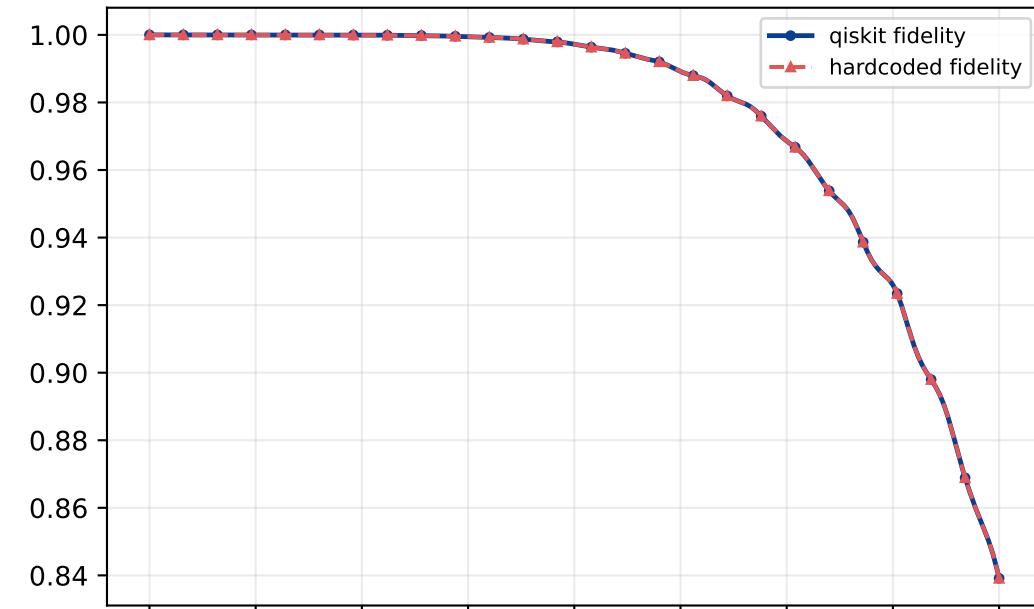
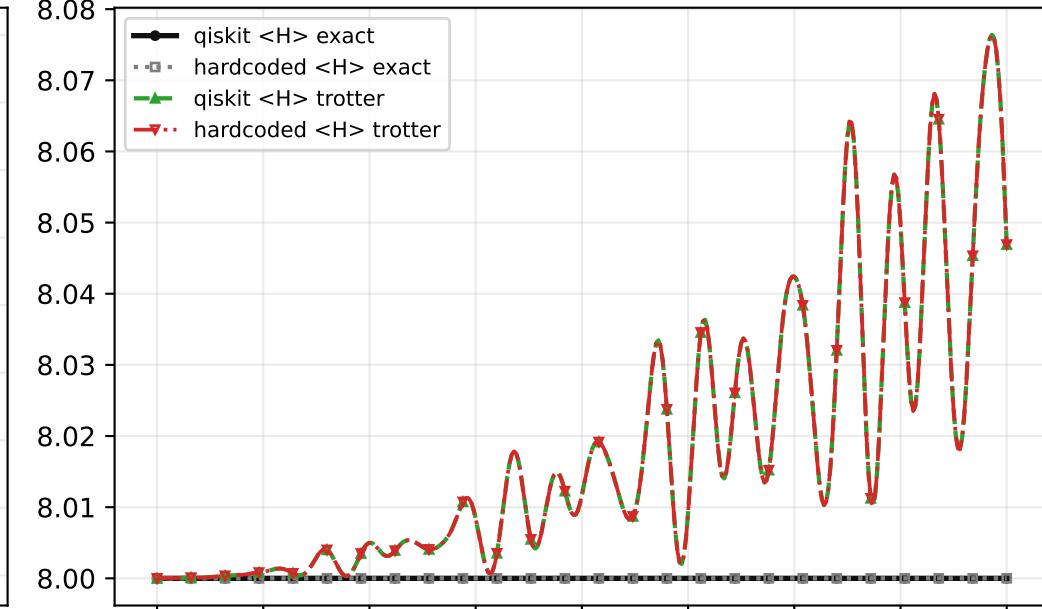


# L=4 Time Dynamics: hardcoded vs Qiskit

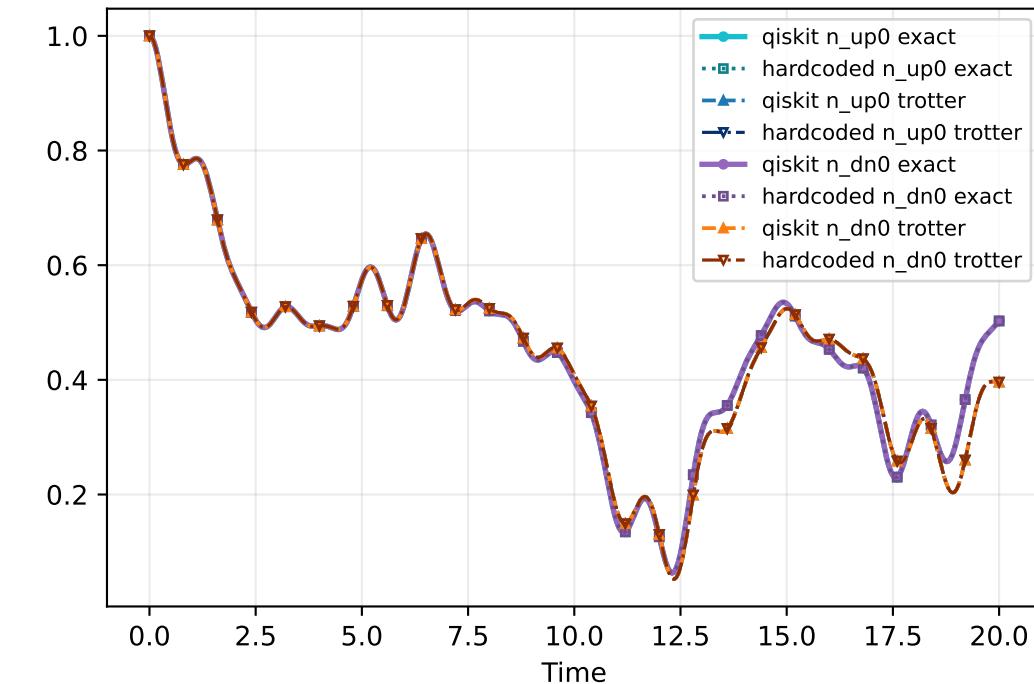
Fidelity(t)



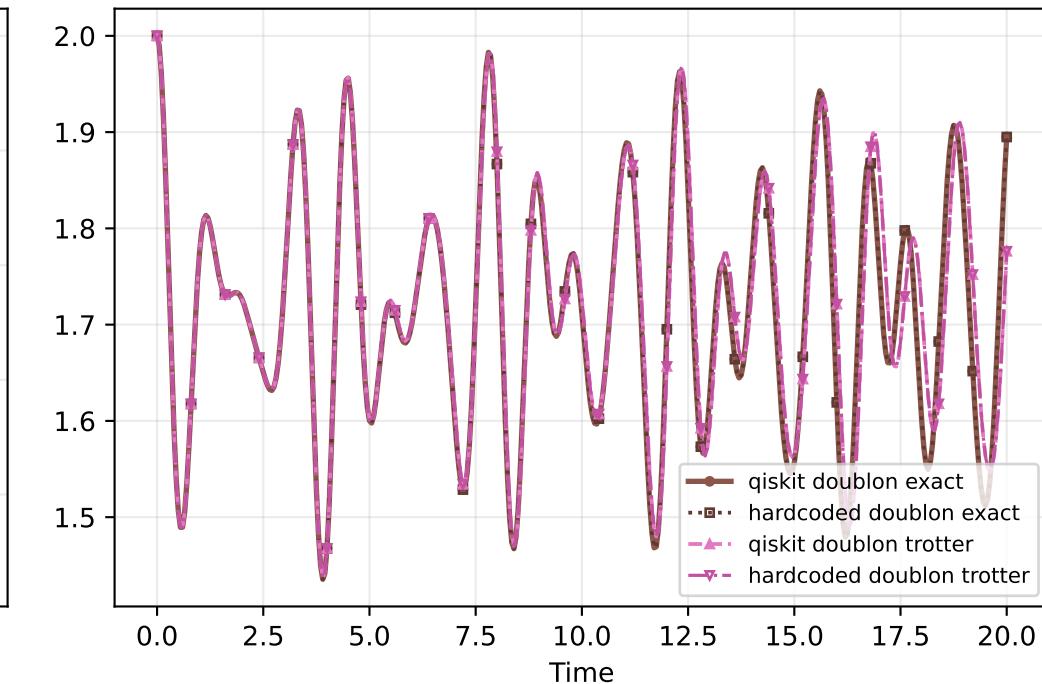
Energy with Exact Overlays



Site-0 Occupations with Exact Overlays

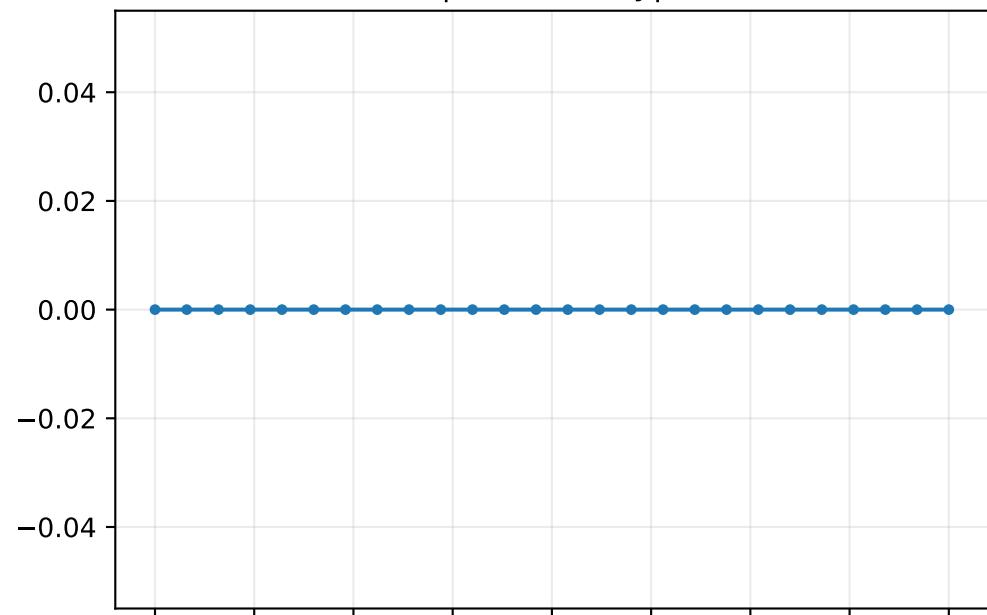


Total Doublon with Exact Overlays

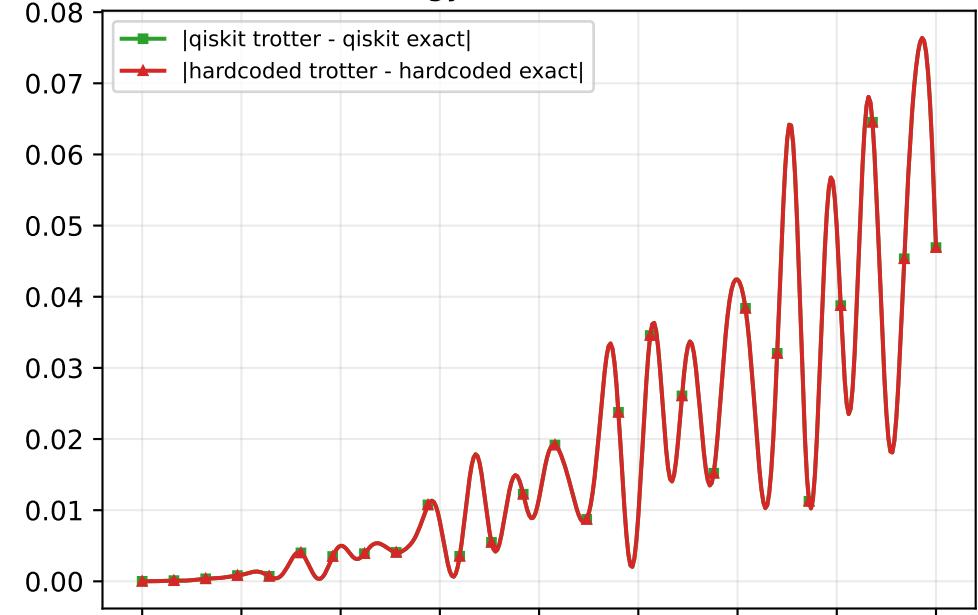


# L=4 Delta Diagnostics

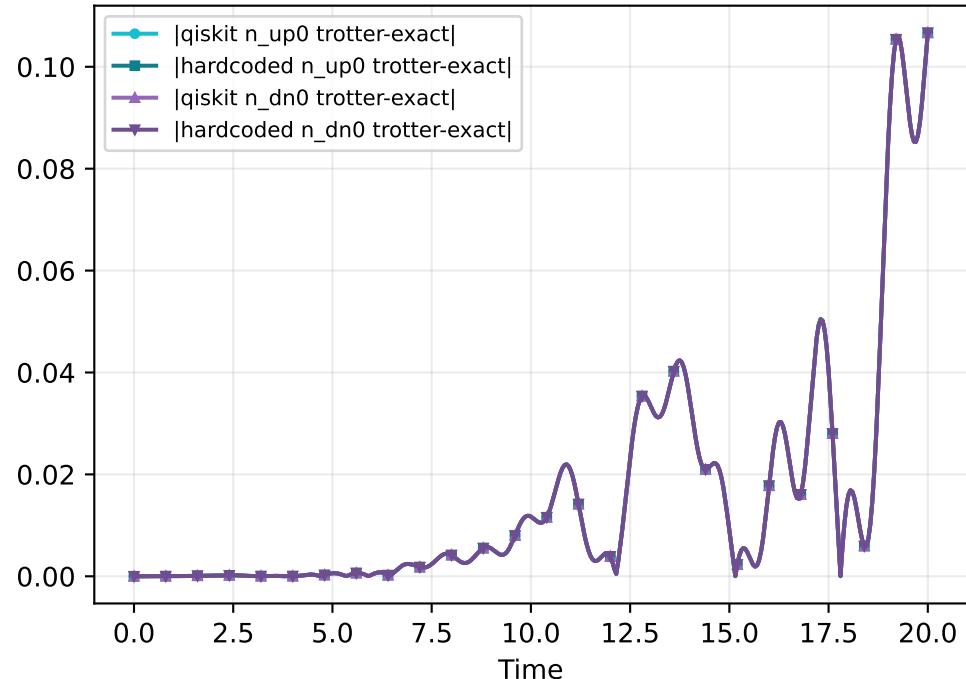
$|\delta \text{ fidelity}|$



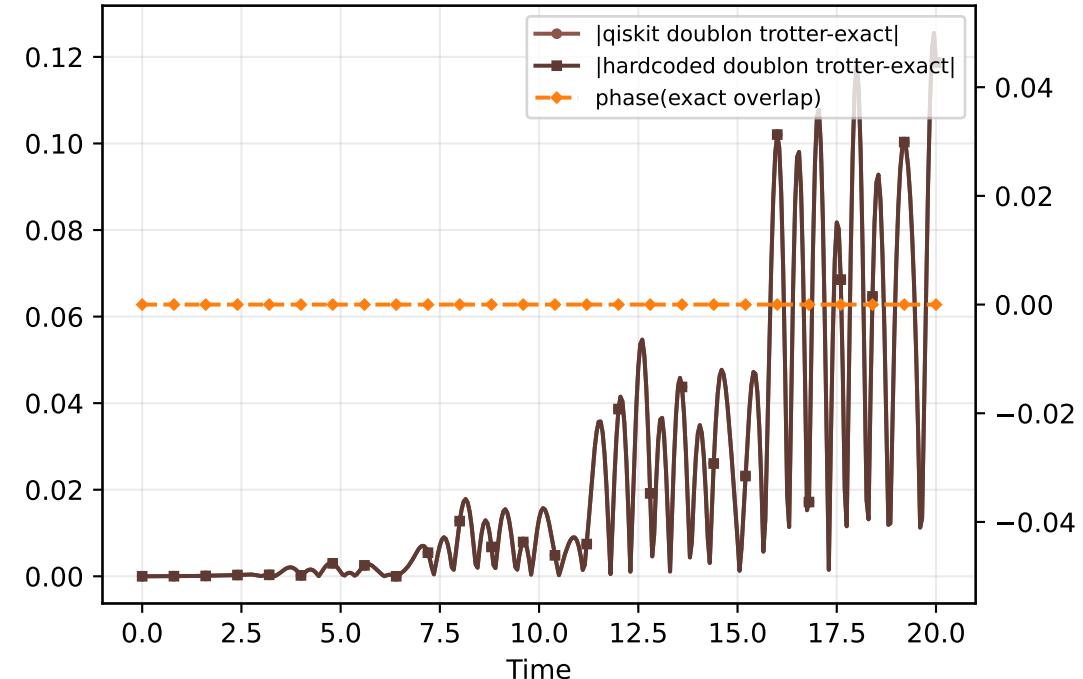
Energy Trotter-vs-Exact



Occupation Trotter-vs-Exact



Doublon Trotter-vs-Exact + Phase



L=4 validation summary

```
ground_state_energy_abs_delta = 0.000000000000e+00
fidelity max/mean/final delta = 0.000000000000e+00 / 0.000000000000e+00 / 0.000000000000e+00
energy_trotter max/mean/final delta = 0.000000000000e+00 / 0.000000000000e+00 / 0.000000000000e+00
n_up_site0_trotter max/mean/final delta = 0.000000000000e+00 / 0.000000000000e+00 / 0.000000000000e+00
n_dn_site0_trotter max/mean/final delta = 0.000000000000e+00 / 0.000000000000e+00 / 0.000000000000e+00
doublon_trotter max/mean/final delta = 0.000000000000e+00 / 0.000000000000e+00 / 0.000000000000e+00
```

```
Checks: {'ground_state_energy_abs_delta': True, 'fidelity_max_abs_delta': True, 'energy_trotter_max_abs_delta': True, 'n_up_si
PASS: True
```

Locked settings:

```
{
    "hopping_t": 1.0,
    "onsite_u": 4.0,
    "dv": 0.0,
    "boundary": "periodic",
    "ordering": "blocked",
    "t_final": 20.0,
    "num_times": 401,
    "suzuki_order": 2,
    "trotter_steps": 128
}
```

## L=4 interpretation notes

- 1) 'exact' means exact state propagation  $\exp(-i H t)$  from diagonalization.  
'qiskit exact' and 'hardcoded exact' use the same definition but each path's own H coefficients.
- 2) Site-0 occupations show  $n_{\text{up}}(\text{site0})$  and  $n_{\text{dn}}(\text{site0})$ .  
Each has four curves: qiskit exact, qiskit trotter, hardcoded exact, hardcoded trotter.
- 3) Fidelity is  $F(t)=|\langle \psi_{\text{exact}}(t) | \psi_{\text{trotter}}(t) \rangle|^2$  within each path.
- 4) Delta diagnostics are absolute errors:  
 $|\text{qiskit trotter} - \text{qiskit exact}|$  and  $|\text{hardcoded trotter} - \text{hardcoded exact}|$ .
- 5) The phase overlay is an extra diagnostic:  $\text{angle}(\langle \psi_{\text{exact_qiskit}} | \psi_{\text{exact_hardcoded}} \rangle)$ .  
It does not redefine doublon; it is shown on a secondary y-axis.