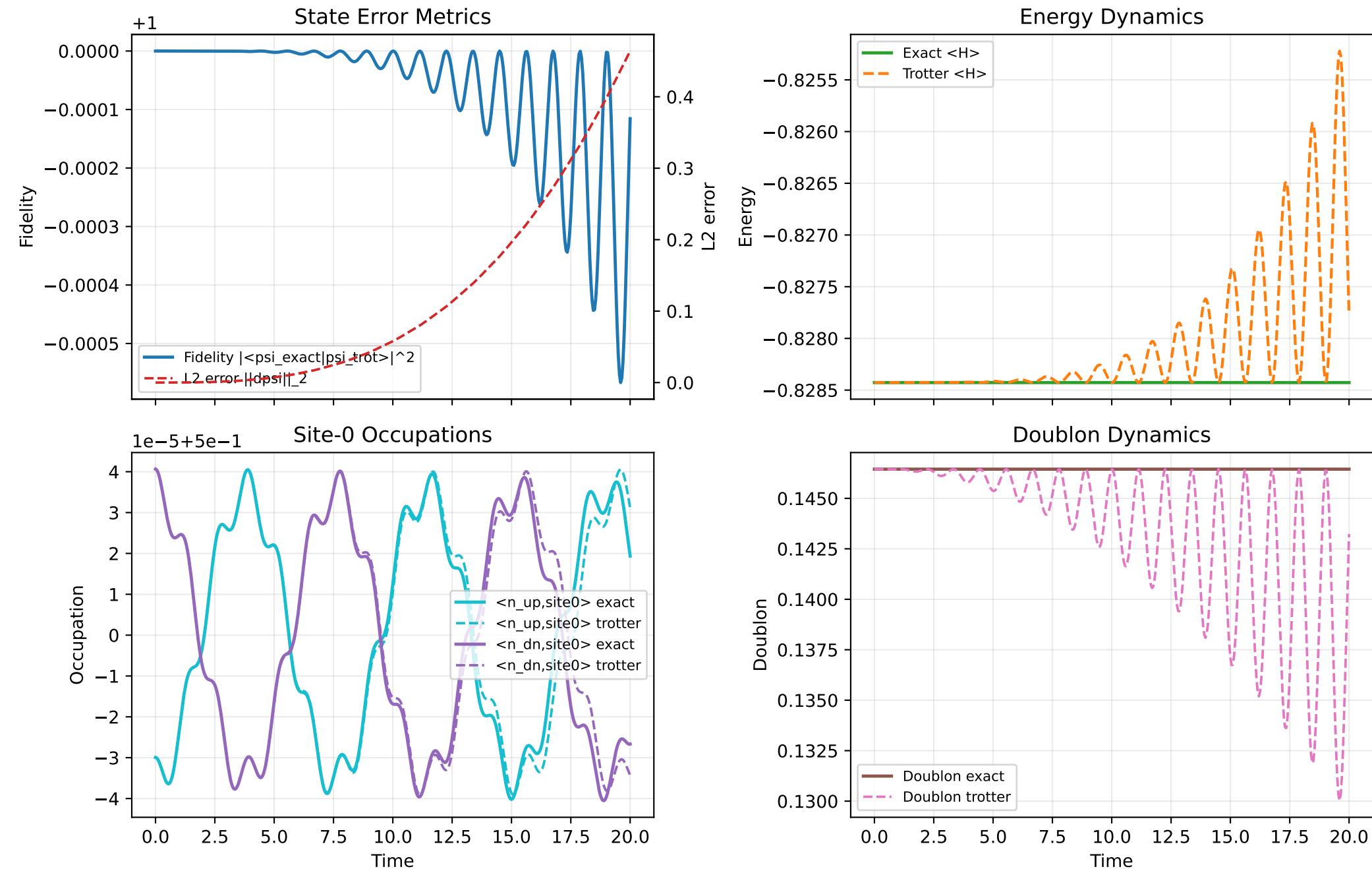


Hubbard Dimer Time Dynamics (Qiskit Built-ins)



```

initial_state=vqe_uccsd (superposition)
L=2, t=1.0, U=4.0, boundary=periodic, ordering=blocked
Suzuki order 2, trotter_steps=128, t_final=20.0, num_times=401
Summary: min_fidelity=0.9994332123, final_fidelity=0.9998843364, max_|dE|=3.206e-03
VQE(UCCSD): energy=-0.828427122588, reps=2, params=6, best_restart=1, F(vqe,hf)=0.0732282487
  
```

Detailed Explanation of Dynamics Results

Key quantitative outcomes

- min fidelity: 0.9994332123
- mean fidelity: 0.9999369216
- final fidelity at t=20.000: 0.9998843364
- max state L2 error: 4.640133e-01
- final state L2 error: 4.640133e-01
- max |energy_exact - energy_trotter|: 3.206043e-03
- mean |energy_exact - energy_trotter|: 3.567799e-04
- max |n_up exact - trotter|: 1.195971e-05
- max |n_dn exact - trotter|: 1.224707e-05
- max |doublon exact - trotter|: 1.642577e-02
- exact-trajectory span in n_up: 8.070119e-05
- exact-trajectory span in n_dn: 8.115834e-05
- exact-trajectory span in doublon: 1.611709e-06

Interpretation and meaning

- The exact curve uses MatrixExponential synthesis and is the reference trajectory.
- The approximate curve uses Suzuki order 2 with reps=128; all deviations from exact come from product-form Trotterization.
- Fidelity close to 1 across the full interval means the Trotterized state remains close to the exact state.
- The energy drift between exact and Trotter trajectories quantifies accumulated approximation error.
- For a time-independent Hamiltonian, exact $\langle H \rangle$ is conserved; any drift in $\langle H \rangle_{\text{trotter}}$ is numerical/Trotterization error.
- Occupation and doublon agreement indicates that physically relevant observables are preserved well by this Trotterization.
- Because the initial state is the VQE(UCCSD) ground-state candidate, small exact observable span indicates the VQE preparation quality is high.
- VQE preparation quality marker: $F(|\psi_{\text{vqe}}\rangle, |\psi_{\text{hf}}\rangle) = 0.0732282487$ (lower means VQE moved away from HF).

How to use this baseline

- This run is a Qiskit built-in reference for later hardcoded-algorithm validation.
- Compare hardcoded trajectories against the exact and Trotter curves with identical Hamiltonian, ordering, and parameters.
- Matching fidelity/observable error scales indicates the hardcoded implementation reproduces the same dynamics.