Hubbard Dimer: Exact vs Suzuki-Trotter Dynamics (Qiskit Built-ins)

Interpretation of State Fidelity vs Time:

$F(t) = |\langle \psi_{\text{exact}}(t) | \psi_{\text{trotter}}(t) \rangle|^2$,

which measures similarity between exact and trotterized time-evolved states.

Exact version in the standalone script:

- The mapped qubit Hamiltonian (H) is evolved with Qiskit's MatrixExponential,
  i.e. full exp(-i H t), not product-formula splitting.
- Setup: hubbard_dimer_suzuki_trotter_standalone.py:334
- exact evolution call: hubbard_dimer_suzuki_trotter_standalone.py:349

The Trotter version:

- Splits (H) into single-Pauli terms: hubbard_dimer_suzuki_trotter_standalone.py:204
- Evolves with Lie/Suzuki product formula: hubbard_dimer_suzuki_trotter_standalone.py:336,
  hubbard_dimer_suzuki_trotter_standalone.py:338
- Uses decomposed gates so simulation applies the approximation
  (not the exact parent gate matrix): hubbard_dimer_suzuki_trotter_standalone.py:200

Relevant symbolic math (LaTeX):

Exact: $|\psi_{\text{exact}}(t)\rangle = e^{-iHt}|\psi(0)\rangle$

Hamiltonian split: $H = \sum_{j=1}^{m} h_j P_j$

Lie-Trotter: $U_{\text{LT}}(t;r) = \left(\prod_{j=1}^{m} e^{-i\frac{t}{r} h_j P_j} \right)^r$

2nd-order Suzuki:

$U_{\text{S2}}(t;r) = \left(\prod_{j=1}^{m} e^{-i\frac{t}{2r} h_j P_j} \prod_{j=m}^{1} e^{-i\frac{t}{2r} h_j P_j} \right)^r$

Fidelity: $F(t) = |\langle \psi_{\text{exact}}(t) | \psi_{\text{trotter}}(t) \rangle|^2$

Included figures:

1) hubbard_dimer_time_dynamics_plots.png

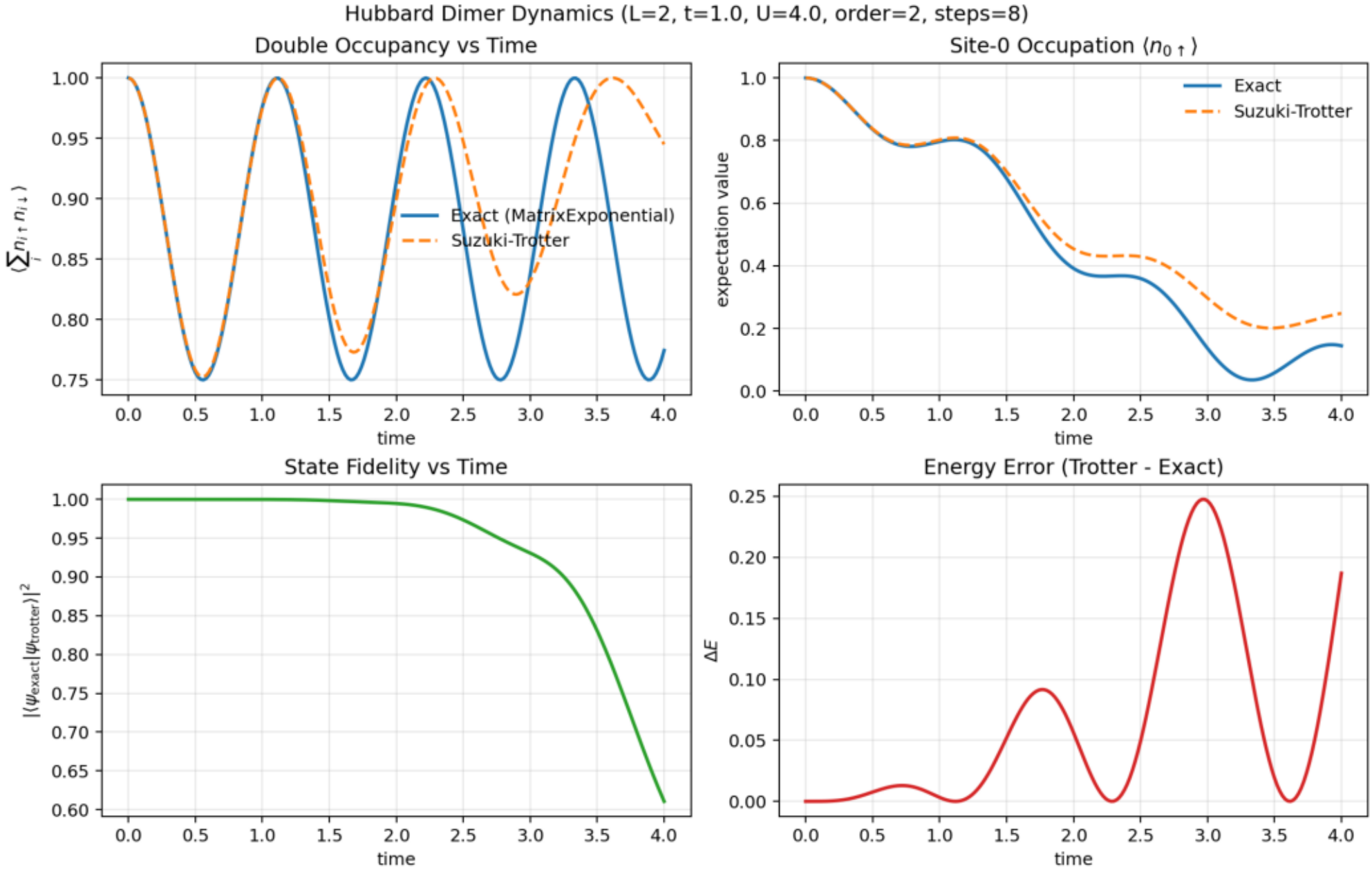
2) vqe_hardcoded_vs_qiskit_vs_exact_L2_L4_barplot.png

3) qpe_hardcoded_vs_prior_qiskit_L2_L3_barplot.png

Note: requested "...parplot.png" was not found; existing "...barplot.png" is included.

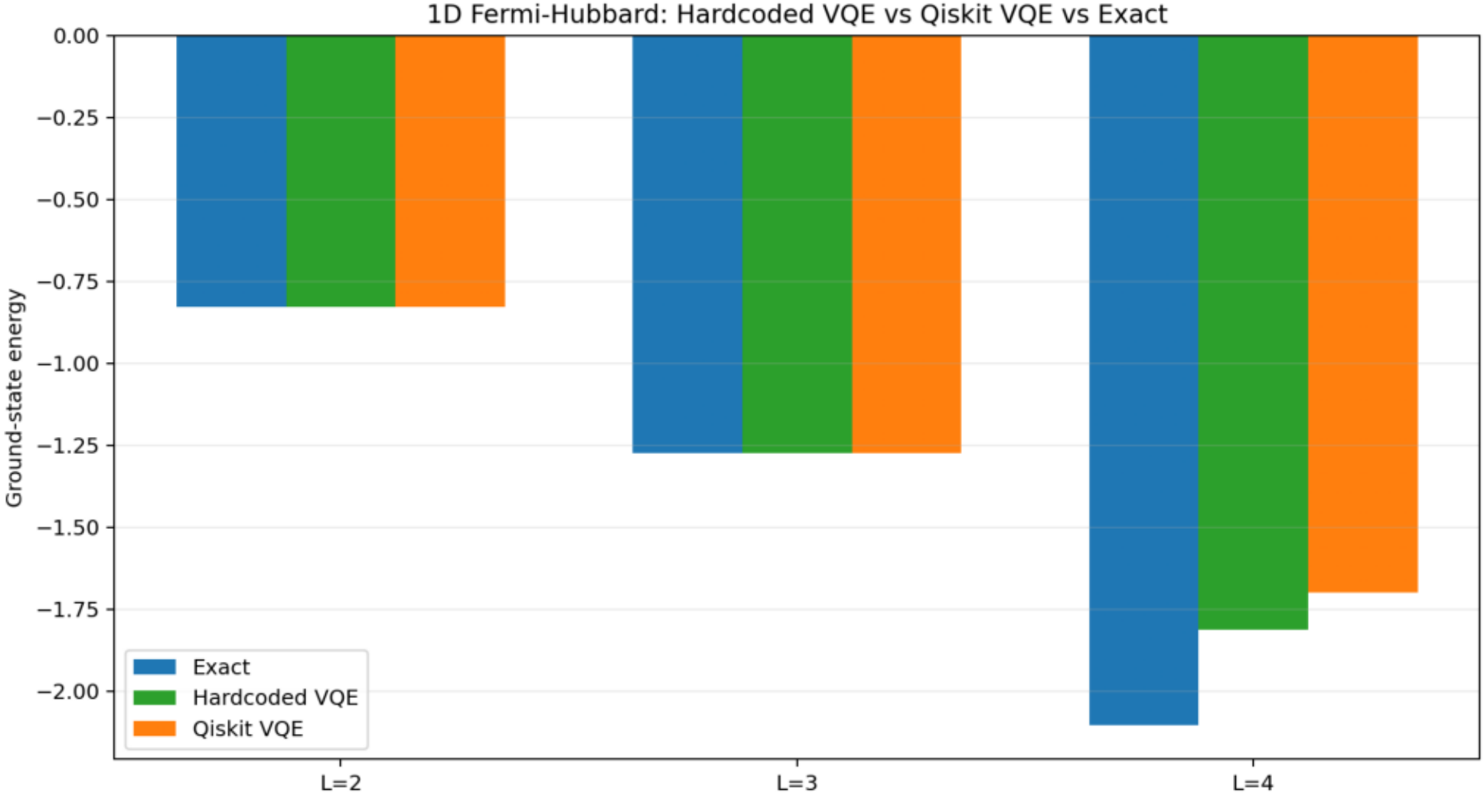
Hubbard Dimer Time Dynamics

hubbard_dimer_time_dynamics_plots.png



VQE Hardcoded vs Qiskit vs Exact (L2-L4)

vqe_hardcoded_vs_qiskit_vs_exact_L2_L4_barplot.png



QPE Hardcoded vs Prior Qiskit (L2-L3)

qpe_hardcoded_vs_prior_qiskit_L2_L3_barplot.png