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Milestone Report 1

(this is also sort of my notes doc, i am so sorry, I wanted to do so much and accomplished so little observable work)

- I am modeling the inside of the earth with prem, and I am going to calculate the density of each layer of the planet. I am going to extract the temperature data from an online source. With PREM data, I will then use researched thermal conductivities and diffusivities to find the average heat flow across the layers in the Earth.

In this project, I'm delving into modeling the Earth's interior by utilizing the Preliminary Reference Earth Model (PREM). The primary objective is to calculate the density of various layers within the planet. Additionally, I aim to extract temperature data from available online sources. With the data derived from PREM and information on thermal conductivities and diffusivities, my goal is to determine the average heat flow across the layers of the Earth.

In github, you will see two documents: piecewise.py, and earth_model.py. The functions of these files are to provide the data and equations for the seismic velocity calculations I will be doing in later code.

Prem class: (earth_model.py)

- The Prem class supports PREM-like 1D Earth models and contains various methods to evaluate properties at different radii within the Earth. It features methods to evaluate density, seismic velocities (S-wave and P-wave), quality factors, bulk modulus, shear modulus, mass, moment of inertia, gravity, pressure, and tabulate models both inwards and outwards.

Piecewise class:

- This class defines a piecewise polynomial function that's used as an alternative representation for the PREM (Preliminary Reference Earth Model) data.
- Methods:
- __init__ Method: Initializes the PiecewisePolynomial class with coefficients c and breakpoints x. It handles one-dimensional breakpoints and ensures the correct shape of coefficients.
- __call__ Method: Allows the class instance to be used as a callable function. It evaluates the piecewise polynomial at a given xp (breaks down the evaluation if required).
- _evaluate_at_point: Evaluates the piecewise polynomial at a specific point.
- _evaluate_polynomial: Evaluates the polynomial at a given point based on coefficients.
- _get_coefs: Retrieves coefficients at a specific point, handling different scenarios based on breakpoints and break-down options.
- derivative: Computes the derivative of the piecewise polynomial.
- antiderivative: Computes the antiderivative of the piecewise polynomial.

- integrate: Calculates the definite integral of the piecewise polynomial within a specified range.
- mult: Performs multiplication between two PiecewisePolynomial instances.

- The Deep Earth Research Group (DERG) has an equation for the piecewise representation of PREM.

$$\$V_P(r) = \begin{cases} p_{0,0} + p_{0,1}r + p_{0,2}r^2 + p_{0,3}r^3 & r \leq 1221.5 \text{ km} \\ p_{1,0} + p_{1,1}r + p_{1,2}r^2 + p_{1,3}r^3 & 1221.5 \leq r \leq 3480.0 \text{ km} \\ \vdots & \vdots \\ p_{12,0} + p_{12,1}r + p_{12,2}r^2 + p_{12,3}r^3 & 6368.0 \leq r \leq 6371.0 \text{ km} \end{cases}$$

- Density can be modeled in a similar way. The code integrates the density parameterization into the Earth model, enabling the calculation and visualization of density variation with depth. It employs numerical techniques, particularly numerical integration, to compute the total mass and moment of inertia based on the density distribution.

$$\rho(r) = \begin{cases} \rho_{0,0} + \rho_{0,1}r + \rho_{0,2}r^2 + \rho_{0,3}r^3 & r \leq 1221.5 \text{ km} \\ \rho_{1,0} + \rho_{1,1}r + \rho_{1,2}r^2 + \rho_{1,3}r^3 & 1221.5 \leq r \leq 3480.0 \text{ km} \\ \vdots & \vdots \\ \rho_{12,0} + \rho_{12,1}r + \rho_{12,2}r^2 + \rho_{12,3}r^3 & 6368.0 \leq r \leq 6371.0 \text{ km} \end{cases}$$

- This is the Vs equation for PREM, which uses this model with a period of 1 second.

$$V_S(r,T) = V_S(r,1)\left(1 - \frac{\ln T}{\pi} q_\mu(r)\right)$$

- Similarly, same format for Vp:

$$V_P(r,T) = V_P(r,1)\left(1 - \frac{\ln T}{\pi}[(1-E)q_\kappa(r) + Eq_\mu(r)]\right)$$

- Here are the approximations of Vp and Vs in terms of the bulk and shear moduli as well as the density! These can be used to find the values we need to use later on to calculate heat flow. Here we know that period (T) = 1 s

$$V_P = \sqrt{\frac{\kappa+\frac{4}{3}\mu}{\rho}}$$

$$V_S = \sqrt{\frac{\mu}{\rho}}$$

- A familiar classic, here is the mass function of the earth.

$$M = 4\pi \int_0^{R_e} \rho(r) r^2 \,\mathrm{d}r.$$

- We can also calculate the moment of inertia of the Earth. Utilizing the density distribution, the code calculates the total mass of the Earth and its moment of inertia. These calculations offer insights into the Earth's bulk properties and mass distribution throughout its interior.

$$I = \frac{2}{3} 4\pi \int_0^{R_e} \rho(r) r^4 \,\mathrm{d}r.$$

- Another very familiar equation!

$$g(r) = \frac{GM(r)}{r^2}$$

- The code further determines the gravitational field g and pressure P at different depths within the Earth. These metrics provide an understanding of the gravitational force and pressure variations experienced at different layers of the planet.
- P(r) is the integral of g(r).

$$P(r) = \int_{R_e}^{r} -g(r)\rho(r) \,\mathrm{d}r$$

Next steps:
I am writing code to incorporate the temperature profile of the Earth (we need thermal gradient) and calculating heat flow

What I need:

- Parameterize temperature profile
    - To include the temperature profile against Earth's depth in the analysis, we use a parameterized representation of temperature variation with depth T(r)
- Obtain thermal conductivity, calculate heat flow
    - Different layers within the Earth exhibit varying thermal conductivities K(r). These values will be related to the temperature profile at respective depths to assess the rate of heat transfer through the materials, and I will change them for the layers based on the average mineral compositions of each layer.
    - Utilizing Fourier's law of heat conduction, the heat flux q(r) can be determined as the product of the temperature gradient (dT/dr) and the thermal conductivity k(r):
    - The temperature profile T(r) against depth enables the determination of the temperature gradient with respect to depth (dT/dr). Utilizing Fourier's law, the heat flux (q(r)) as a function of depth is derived. Integrating the heat flux function q(r) over depth for each time interval t results in the total heat flow Q(t) over time.
    - $$Q(t) = \int_{R_{\text{core}}}^{R_{\text{surface}}} q(r,t)\,dr$$

Insights derived from analyzing the temporal evolution of integrated heat flow provide valuable information on the dynamic nature of Earth's thermal processes. Identification of periods exhibiting higher or lower heat flow aids in understanding temporal variations in Earth's thermal behavior.

- Geothermal gradient is a function of Fourier's theorem

- Understanding Mars's heat flow helps unveil its geological evolution, interior structure, and potential for geothermal activity, offering insights into past habitability and aiding future missions. By comparing it with Earth's data, it contributes to understanding planetary diversity and the broader context of the solar system's evolution.

Neat Insight blurb that i will 100% be referencing in my presentation:

The goals of the mission are to provide insight on formation and evolution of terrestrial planets by investigating the interior structure and processes of Mars. It will determine thickness, structure and composition of crust, mantle and core, determine the thermal state of the interior, and measure the rate and distribution of internal seismic activity and the rate of meteorite impacts. To achieve this, InSight will make use of advanced single-seismometer analysis techniques that are currently applied on Earth, along with extremely precise measurements of variations in the spin axis of the planet, and the subsurface thermal gradient, to provide the first direct measurements of the internal structure of Mars.

Mars represents a near-ideal laboratory, because Mars is large enough, in terms of interior pressure and temperature, to have shared most of the early processes that shaped all terrestrial planets, but small enough to have retained the fingerprints of those processes for more than four billion years. While InSight will land on Mars, its scientific implications reach across and beyond our Solar System, providing knowledge about the fundamental processes of terrestrial-planet formation and evolution, of critical importance for ongoing discoveries of extra-solar planets and our ability to characterize their interior structures.
https://seg.ethz.ch/research/terrestial-planets.html

Here is the prem data I am using!

Preliminary Reference Earth Model (PREM) (Dziewonski & Anderson, 1981)									
-	radius	Vp	Vs	Density	Ks	mu	lambda	P	g
Region	(km)	(m/s)	(m/s)	(kg/m3)	(Gpa)	(Gpa)		(Gpa)	(m/s2)
Inner Core	0	11266.2	3667.8	13088.48	1425.3	176.1	0.4407	363.85	0
	200	11255.93	3663.42	13079.77	1423.1	175.5	0.4408	362.9	0.7311
	400	11237.12	3650.27	13053.64	1416.4	173.9	0.441	360.03	1.4604
	600	11205.76	3628.35	13010.09	1405.3	171.3	0.4414	355.28	2.1862
	800	11161.86	3597.67	12949.12	1389.8	167.6	0.442	348.67	2.9068
	1000	11105.42	3558.23	12870.73	1370.1	163	0.4428	340.24	3.6203
	1200	11036.43	3510.02	12774.93	1346.2	157.4	0.4437	330.05	4.3251
	1221.5	11028.27	3504.32	12763.6	1343.4	156.7	0.4438	328.85	4.4002
Outer Core	1221.5	10355.68	0	12166.34	1304.7	0	0.5	328.85	4.4002
	1400	10249.59	0	12069.24	1267.9	0	0.5	318.75	4.9413
	1600	10122.91	0	11946.82	1224.2	0	0.5	306.15	5.5548
	1800	9985.54	0	11809	1177.5	0	0.5	292.22	6.1669
	2000	9834.96	0	11654.78	1127.3	0	0.5	277.04	6.7715
	2200	9668.65	0	11483.11	1073.5	0	0.5	260.68	7.3645
	2400	9484.09	0	11292.98	1015.8	0	0.5	243.25	7.9425
	2600	9278.76	0	11083.35	954.2	0	0.5	224.85	8.5023
	2800	9050.15	0	10853.21	888.9	0	0.5	205.6	9.0414
	3000	8795.73	0	10601.52	820.2	0	0.5	185.64	9.557
	3200	8512.98	0	10327.26	748.4	0	0.5	165.12	10.0464
	3400	8199.39	0	10029.4	674.3	0	0.5	144.19	10.5065
	3480	8064.82	0	9903.49	644.1	0	0.5	135.75	10.6823
D"	3480	13716.6	7264.66	5566.45	655.6	293.8	0.3051	135.75	10.6823
	3600	13687.53	7265.75	5506.42	644	290.7	0.3038	128.71	10.5204
	3630	13680.41	7265.97	5491.45	641.2	289.9	0.3035	126.97	10.4844
Lower mantle	3630	13680.41	7265.97	5491.45	641.2	289.9	0.3035	126.97	10.4844
	3800	13447.42	7188.92	5406.81	609.5	279.4	0.3012	117.35	10.3095
	4000	13245.32	7099.74	5307.24	574.4	267.5	0.2984	106.39	10.158
	4200	13015.79	7010.53	5207.13	540.9	255.9	0.2957	95.76	10.0535

	4400	12783.89	6919.57	5105.9	508.5	244.5	0.2928	85.43	9.9859
	4600	12544.66	6825.12	5002.99	476.6	233.1	0.2898	75.36	9.9474
	4800	12293.16	6725.48	4897.83	444.8	221.5	0.2864	65.52	9.9314
	5000	12024.45	6618.91	4789.83	412.8	209.8	0.2826	55.9	9.9326
	5200	11733.57	6563.7	4678.44	380.3	197.9	0.2783	46.49	9.9467
	5400	11415.6	6378.13	4563.07	347.1	185.6	0.2731	37.29	9.9698
	5600	11065.57	6240.46	4443.17	313.3	173	0.2668	28.29	9.9985
	5600	11065.57	6240.46	4443.17	313.3	173	0.2668	28.29	9.9985
	5701	10751.31	5945.08	4380.71	299.9	154.8	0.2798	23.83	10.0143
	5701	10266.22	5570.2	3992.14	255.6	123.9	0.2914	23.83	10.0143
	5771	10157.82	5516.01	3975.84	248.9	121	0.2909	21.04	10.0038
	5871	9645.88	5224.28	3849.8	218.1	105.1	0.2924	17.13	9.9883
	5971	9133.97	4932.59	3723.78	189.9	90.6	0.2942	13.35	9.9686
	5971	8905.22	4769.89	3543.25	173.5	80.6	0.2988	13.35	9.9686
	6061	8732.09	4706.9	3489.51	163	77.3	0.2952	10.2	9.9361
Transition Zone	6151	8558.96	4643.91	3435.78	152.9	74.1	0.2914	7.11	9.9048
	6151	7989.7	4418.85	3359.5	127	65.6	0.2797	7.11	9.9048
Low Velocity Zone	6221	8033.7	4443.61	3367.1	128.7	66.5	0.2796	4.78	9.8783
	6291	8076.88	4469.53	3374.71	130.3	67.4	0.2793	2.45	9.8553
	6291	8076.88	4469.53	3374.71	130.3	67.4	0.2793	2.45	9.8553
Lid	6346.6	8110.61	4490.94	3380.76	131.5	68.2	0.2789	0.604	9.8394
	6346.6	6800	3900	2900	75.3	44.1	0.2549	0.604	9.8394
	6356	6800	3900	2900	75.3	44.1	0.2549	0.337	9.8332
	6356	5800	3200	2600	52	26.6	0.2812	0.337	9.8332
Crust	6368	5800	3200	2600	52	26.6	0.2812	0.3	9.8222
	6368	1450	0	1020	2.1	0	0.5	0.3	9.8222
Ocean	6371	1450	0	1020	2.1	0	0.5	0	9.8156