Location and length of the stub using Reflection coefficient.

The i/p impedence is given by.

$$Z_s = Z_0 \left[\frac{1 + K e^{-2\gamma l}}{1 - K e^{-2\gamma l}} \right]$$

For lossless line $\alpha = 0$, $\gamma = j\beta$, $K = |K| e^{j\phi}$

$$Z_s = Z_0 \left[\frac{1 + |K| e^{j\phi} e^{-j2\beta l}}{1 - |K| e^{j\phi} e^{-j2\beta l}} \right]$$

$$= Z_0 \left[\frac{1 + |K| e^{j(\phi - 2\beta l)}}{1 - |K| e^{j(\phi - 2\beta l)}} \right]$$

$Z_s = 1/Y_s$, $Z_0 = 1/Z_0$.

30/3/23 Location & length of the stub using reflection coefficient.

The i/p Impedance is given by

$$Z_s = Z_0\left[\frac{1+ke^{-2\gamma l}}{1-ke^{-2\gamma l}}\right]$$

For lossless:

$$Y_s = \frac{1}{Z_s}, \quad G_0 = \frac{1}{Z_0} = \frac{1}{R_0}$$

$$Y_s = G_0\left[\frac{1-|k|e^{j(\phi-2\beta l)}}{1+|k|e^{j(\phi-2\beta l)}}\right]$$

$$= G_0\left[\frac{1-|k|[\cos(\phi-2\beta l)+j\sin(\phi-2\beta l)]}{1+|k|[\cos(\phi-2\beta l)+j\sin(\phi-2\beta l)]}\right]$$

Take complex conjugate of the denominator.

$$= G_0\left[\frac{1-|k|\cos(\phi-2\beta l)-j|k|\sin(\phi-2\beta l)}{1+|k|[\cos(\phi-2\beta l)]-j\sin(\phi-2\beta l)}\right] \times$$

$$\left[\frac{1+|k|\cos(\phi-2\beta l)-j|k|\sin(\phi-2\beta l)}{1+|k|\cos(\phi-2\beta l)-j|k|\sin(\phi-2\beta l)}\right]$$

$$[\because (a-ib)(a+ib) = a^2+b^2]$$

$$= G_0\left\{\frac{[1-|k|\cos(\phi-2\beta l)-j\sin(\phi-2\beta l)|k|]\,[1+|k|\cos(\phi-2\beta l)-j|k|\sin(\phi-2\beta l)]}{[1+|k|\cos(\phi-2\beta l)]^2+[|k|]^2\sin^2(\phi-2\beta l)}\right\}$$

$$= G_0\left[\frac{1+\cancel{|k|\cos(\phi-2\beta l)}-j|k|\sin(\phi-2\beta l)-\cancel{|k|\cos(\phi-2\beta l)}-|k|^2\cos^2(\phi-2\beta l)+\cancel{j|k|^2\cos(\phi-2\beta l)\sin(\phi-2\beta l)}-j|k|\sin(\phi-2\beta l)-\cancel{j|k|^2\sin(\phi-2\beta l)\cos(\phi-2\beta l)}-|k|^2\sin^2(\phi-2\beta l)}{[1+|k|\cos(\phi-2\beta l)]^2+|k|^2\sin^2(\phi-2\beta l)}\right]$$

$$= G_0 \frac{\left[1 - \cancel{2|k|\cos(\phi-2\beta l)} - 2j|k|\sin(\phi-2\beta l) - |k|^2[\cos^2(\phi-2\beta l) + \sin^2(\phi-2\beta l)]\right]}{[1+|k|\cos(\phi-2\beta l)]^2 + |k|^2\sin^2(\phi-2\beta l)}$$

$$= G_0\left\{\frac{1-2|k|[\cancel{\cos(\phi-2\beta l)} + j\sin(\phi-2\beta l)] - |k|^2}{1+|k|^2\cos^2(\phi-2\beta l)+2|k|\cos(\phi-2\beta l)+|k|^2\sin^2(\phi-2\beta l)}\right\}$$

$$= G_0\left[\frac{1-|k|^2 - 2j\sin(\phi-2\beta l)|k|}{1+2|k|\cos(\phi-2\beta l)+|k|^2}\right]$$

$$\frac{Y_S}{G_0} = \left[\frac{1-|k|^2-2|k|j\sin(\phi-2\beta)}{1+|k|^2+2|k|\cos(\phi-2\beta l)}\right]$$

31/8/23. $\frac{Y_S}{G_0} = \frac{G_S}{G_0} + j\frac{S_S}{G_0}$

$$= \frac{1-|k|^2}{1+|k|^2+2|k|\cos(\phi-2\beta)} + \frac{(-2j|k|\sin(\phi-2\beta l))}{1+|k|^2+2|k|\cos(\phi-2\beta l)}.$$

$$\frac{G_S}{G_0} = \frac{1-|k|^2}{1+|k|^2+2|k|\cos(\phi-2\beta l)}$$

$$1 = \frac{1-|k|^2}{1+|k|^2+2|k|\cos(\phi-2\beta l)}$$

$$\cancel{1}-|k|^2 = \cancel{1}+|k|^2+2|k|\cos(\phi-2\beta l)$$

$$-\cancel{2}|k|^{\cancel{2}} = \cancel{2}|k|\cos(\phi-2\beta l)$$

$$-|k| = \cos(\phi-2\beta l)$$

$$\cos(\phi-2\beta l) = -|k|$$

$$\phi-2\beta l = \cos^{-1}(-|k|)$$

$l = l_s$ = location of the stub.

$$\cos(\phi-2\beta l_s) = -|k|$$

$$\phi-2\beta l_s = \cos^{-1}(-|k|)$$

$$2\beta l_s = \phi - \cos^{-1}(-|k|)$$

Since $\cos^{-1}(-\theta) = -\pi + \cos^{-1}\theta$

$$\cos^{-1}(-|K|) = -\pi + \cos^{-1}(|K|)$$

$$\phi - 2\beta l_s = -\pi + \cos^{-1}(|K|)$$

$$2\beta l_s = \phi + \pi - \cos^{-1}(|K|)$$

$$l_s = \frac{\phi + \pi - \cos^{-1}(|K|)}{2 \times \frac{2\pi}{\lambda}}$$

location of the stub.

$$l_s = \frac{\lambda}{4\pi}\left[\phi - \pi - \cos^{-1}(|K|)\right]$$

Length of the stub.

$$\frac{S_s}{G_0} = \frac{-2|K|\sin(\phi - 2\beta l)}{1 + |K|^2 + 2|K|\cos(\phi - 2\beta l)}$$

$[\phi - 2\beta l = \cos^{-1}(-|K|)$

$\cos(\phi - 2\beta l) = -|K|$

$$= \frac{-2|K|\sin(\cos^{-1}(-|K|))}{1 + |K|^2 + 2|K|(-|K|)}$$

$$= \frac{-2|K|\sin[-\pi + \cos^{-1}(|K|)]}{1 + |K|^2 - 2|K|^2}$$

Since $\sin(-\pi + \theta) = -\sin\theta$

$$= \frac{2|K|\sin(\cos^{-1}|K|)}{1 - |K|^2}$$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

$\sin = \sqrt{1 - |K|^2}$

Let $\cos^{-1}|K| = \theta$

$|K| = \cos\theta$

$\sin(\cos^{-1}(|K|)) = \sin\theta)$

$\sin(\cos^{-1}(K)) = \sqrt{1 - \cos^2\theta}$

$$= \sqrt{1 - |K|^2}$$

$$\frac{S_s}{G_0} = \frac{2|K|\sqrt{1 - |K|^2}}{1 - |K|^2}$$

$$S_s = G_0\left[\frac{2|K|}{\sqrt{1 - |K|^2}}\right]$$

$$G_0 \cot\beta l_t = G_0\left[\frac{2|K|}{\sqrt{1 - |K|^2}}\right]$$

$$\frac{1}{\tan\beta l_t} = \frac{2|K|}{\sqrt{1-|K|^2}}$$

($B_s = G_0 \cot \beta l_t$

$\beta = \frac{2\pi}{\lambda}$)

$$\tan\beta l_t = \frac{\sqrt{1-|K|^2}}{2|K|}$$

$$\beta l_t = \tan^{-1}\left(\frac{\sqrt{1-|K|^2}}{2|K|}\right)$$

$$\frac{2\pi}{\lambda} \times l_t = \tan^{-1}\left(\frac{\sqrt{1-|K|^2}}{2|K|}\right)$$

$$l_t = \frac{\lambda}{2\pi}\tan^{-1}\left(\frac{\sqrt{1-|K|^2}}{2|K|}\right)$$

The short circuited stub is normarrly preferred because of its simpler construction and inability of the stub to ~~rem~~ remain open circuited.

Short circuited stub is easily established with a large plate and it also has a lower radian loss of energy.

Drawbacks of Single Stub Matching.

It is Applicable for single frequency.