regression

9/4/25

Announcements:

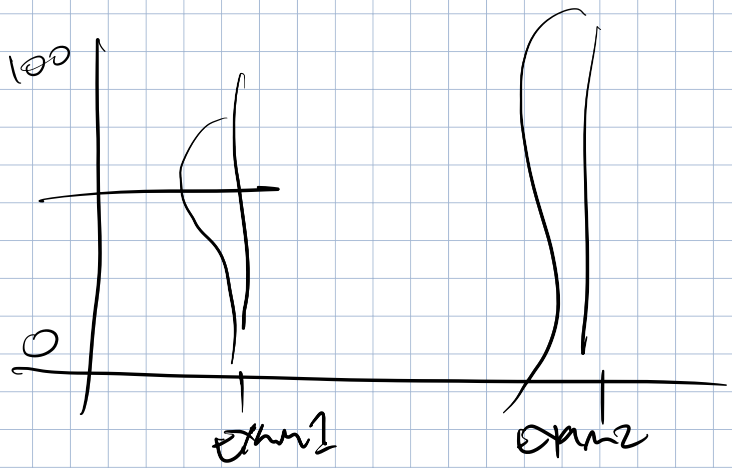
- Final Proj Sign Up on Slack Channel
  - ↳ you need my approval — DM me

- Final Proj:
  - "Can we use LLMs?"
    - → Yes for coding the app/graphic.
    - → No for writing the actual language on the app/blog!!

      I want <u>your</u> voice & your perspective.

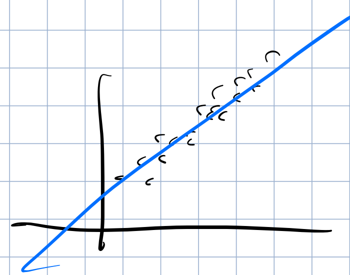
First Quiz Next Wk:

- Closed notes ← I provide the formula sheet
  - ↳ posted on Canvas/Slack
- 60 min
- no calculators



Recap

SLR model + assumptions



LS Estimators $\hat{\theta} \pm \text{crit} \cdot \hat{se}(\hat{\theta})$

	$E(\cdot)$	$Var(\cdot)$	CIs	Hyp Testing
$\hat{\beta}_0$	β_0	$\sigma^2\left(\frac{1}{n}+\frac{\bar{x}^2}{SSX}\right)$		
$\hat{\beta}_1$				
$\hat{y}_i$	$\beta_0+\beta_1 x_i$	$\sigma^2\left(\frac{1}{n}+\frac{(x_i-\bar{x})^2}{SSX}\right)$		
e_i	—	—	X	X

$$\hat{y}_i \pm t^*_{1-\frac{\alpha}{2},n-2} \cdot \hat{\sigma}\sqrt{\frac{1}{n}+\frac{(x_i-\bar{x})^2}{SSX}}$$

↳ $\frac{\sum_{i=1}^n e_i^2}{\sigma^2} \sim \chi^2_{n-p}$ $p=2$ for SLR

↳ unbiased est of σ^2:

$$MSE = \hat{\sigma}^2 = \frac{\sum_i e_i^2}{n-2} = \frac{SSE}{n-2}$$

Sum of Squares Decomp & F-test:

$$\sum_{i=1}^{n} (y_i - \bar{y})^2 = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 + \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2$$

$$SST = SSE + SSR$$

$$F = \frac{SSR/1}{SSE/n-2} \overset{H_0}{\sim} F_{1, n-2}$$

Diagnostics (?)

Pred Intervals

Target $(y_{new} \mid x=80) = \beta_0 + \beta_1(80) + \varepsilon_{new}$

$$y_{new} = \beta_0 + \beta_1(80) + \varepsilon_{new}$$

Goal:

$$SE(\hat{y}(x=80) - (y_{new} \mid x=80)) = \sqrt{Var(\hat{y}(x=80) - (y_{new} \mid x=80))}$$

$$Var(\hat{y}(x=80) - (y_{new} \mid x=80)) = \underset{①}{Var(\hat{y}(x=80))} + \underset{②}{Var(\underline{y_{new}} \mid x=80)}$$

$$\cancel{- 2Cov(\hat{y}(80), (y_{new} \mid x=80))} \to 0$$

b/c $y_{new} \perp\!\!\!\perp y_1, \dots, y_n$

$$= \sigma^2\left(\frac{1}{n} + \frac{(80-\bar{x})^2}{SSX}\right) + \sigma^2$$

$$= \sigma^2\left(1 + \frac{1}{n} + \frac{(80-\bar{x})^2}{SSX}\right)$$

$$SE(\hat{y}(80) - y_{new}(80)) = \sqrt{Var(\dots)}$$

$$= \sqrt{\sigma^2\left(1 + \frac{1}{n} + \frac{(80-\bar{x})^2}{SSX}\right)} = \sigma\sqrt{1 + \frac{1}{n} + \frac{(80-\bar{x})^2}{SSX}}$$

For a new obs $\Longleftrightarrow$ PI