# Derivative of tanh(x)

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$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$= \frac{f(x)}{g(x)} \quad \left| \begin{array}{l} f(x) = e^x - e^{-x} \\ g(x) = e^x + e^{-x} \end{array} \right.$$

$$\frac{d}{dx}\tanh(x) = \frac{d}{dx}\frac{f(x)}{g(x)} \quad | \text{ Quotient rule}$$

$$= \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{g(x)^2} \quad \left| \begin{array}{l} f(x) = e^x - e^{-x} \\ g(x) = e^x + e^{-x} \end{array} \right.$$

$$= \frac{(e^x+e^{-x})\cdot \frac{d}{dx}(e^x - e^{-x}) - (e^x - e^{-x})\cdot \frac{d}{dx}(e^x + e^{-x})}{(e^x+e^{-x})^2} \quad | \text{ Calculate derivative } \left(\tfrac{d}{dx}e^{-x} = -e^{-x}\right)$$

$$= \frac{(e^x+e^{-x})\cdot (e^x + e^{-x}) - (e^x - e^{-x})\cdot (e^x - e^{-x})}{(e^x+e^{-x})^2} \quad | \text{ simplify}$$

$$= \frac{(e^x+e^{-x})^2}{(e^x+e^{-x})^2} - \frac{(e^x-e^{-x})^2}{(e^x+e^{-x})^2} \quad | \text{ simplify}$$

$$= 1 - \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)^2 \quad | \text{ simplify}$$

$$= 1 - \tanh(x)^2$$

## Quotient rule

$$\frac{d}{dx}\frac{f(x)}{g(x)} \quad | \text{ product rule}$$

$$= f'(x)\cdot \frac{1}{g(x)} + f(x)\cdot \frac{d}{dx}\frac{1}{g(x)} \quad | \text{ chain rule}$$

$$= \frac{f'(x)}{g(x)} + f(x)\cdot g'(x)\cdot \left(-\frac{1}{g(x)^2}\right) \quad | \text{ simplify}$$

$$= \frac{f'(x)}{g(x)} - \frac{f(x)\cdot g'(x)}{g(x)^2} \quad | \text{ expand fracture}$$

$$= \frac{f'(x)}{g(x)}\cdot\frac{g(x)}{g(x)} - \frac{f(x)\cdot g'(x)}{g(x)^2} \quad | \text{ simplify}$$

$$= \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{g(x)^2}$$