Fair Constrained Spectral Clustering

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by

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<u>CERTIFICATE</u>

*This is to certify that the work described in this thesis, titled "**Fair Constrained Spectral Clustering**" was done by **Neha Afreen** (**Roll No. 214101034**), in the Department of Computer Science and Engineering, Indian Institute of Technology Guwahati, under my supervision, and has not been submitted for a degree elsewhere.*

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Declaration

This is to certify that the thesis entitled "**Fair Constrained Spectral Clustering**", submitted by me to the *Indian Institute of Technology Guwahati*, for the award of the degree of Master of Technology, is a bonafide work carried out by me under the supervision of Dr. V. Vijaya Saradhi. The content of this thesis, in full or in parts, have not been submitted to any other University or Institute for the award of any degree or diploma. I also wish to state that the work done is to the best of my knowledge and understanding nothing in this report amounts to plagiarism.

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Abstract

Spectral clustering outperforms any traditional clustering algorithms as it has the ability of clustering the data even with complex shapes. Spectral clustering projects the complex structure of data into an embedding space which is linearly separable by classical k means clustering algorithm. Spectral clustering with several constraints in the form of must-link and cannot-link often improves accuracy and gives better clustering of data points. Constraints that can be user defined or can be generated by the labeled data or by any other techniques, add prior knowledge to the data set hence improve the performance but these can decrease the fairness of the algorithm. Chierichetti et al(2017)[7] proposed that a clustering is fair if every demographic group is approximately proportionally represented in each cluster. We propose an algorithm to incorporate this fairness notion into constrained spectral clustering. We have implemented our algorithm in different experimental settings and have observed fairer clusters than normal constrained spectral clustering.

Contents

List of Figures

Chapter 1

Introduction

There is a very large volume of data that is being generated. These data often contain sensitive attributes like gender, race, genetic information etc. When sensitive information is fed in any ML model they tend to produce biased results. Similarly classical spectral clustering also produces unfair clusters. As spectral clustering is one of the most popular and widely used clustering techniques, researches have been done to make it perform better in terms of accuracy, fairness, cost etc. We have fair spectral clustering [15] which increases balance or fairness of the clusters. In constrained spectral clustering [9] with the addition of constraints the accuracy of the clusters can be increased but it may decrease the fairness of the clusters.

Therefore an spectral clustering algorithm that gives fairer clusters with improved accuracy was required. We proposed an algorithm to incorporate fairness into constrained spectral clustering.

1.1 Background

A data set $X = \{x_1, x_2, ..., x_n\}$ can be represented by a similarity graph $G = (V, E, W)$ where $V = \{x_1, x_2, ...x_n\}$ or it can also be represented as $V = \{v_1, v_2, ...v_n\}$, E is the set of edges between the data points in X and W is the edge weights representing the pairwise similarity between data points. K way spectral clustering divide the graph into K non-empty clusters denoted by $\{C_1, C_2, ..., C_K\}$. Let $H \in \mathbb{R}^{nxk}$ be the encoding of clustering $V = C_1 \cup C_2 \cup ... \cup C_k$ such that

$$H_{il} = \begin{cases} \frac{1}{\sqrt{|C_l|}}, & i \in C_l \\ 0, & \text{otherwise} \end{cases} \tag{1.1}$$

The objective function of spectral clustering [18] can be written as

$$\min_H Tr(H^T LH) \text{ subject to } H^T H = I_k \tag{1.2}$$

Where L is Laplacian matrix and can be defined as

$$L = D - W \tag{1.3}$$

D is degree matrix such that

$$d_{ii} = \sum_{j=1}^{n} W_{ij} \qquad \text{is the degree of vertex } v_i \tag{1.4}$$

Spectral clustering finds the solution of 1.2 as an embedding space in which the data points can be easily clustered. First K eigen vectors of L forms the embedding space in which the data points are linearly separable. Then K means algorithm can be employed to obtain clusters.

As the data contains sensitive attributes the clusters from classical spectral clustering may be biased towards certain groups and may not produce fair clusters.

Several notions have been formulated in order to quantify fairness of any ML algorithm. Some of the notions are agnostic fairness notion that is defined only for an specific algorithm like social fairness is for K means algorithm, Max fairness cost is for hierarchical agglomerative clustering algorithm etc. For spectral clustering one of the widely used notion is Balance. This notion was proposed by Chierichetti et al [7]. Later this notion was adopted as one of the methods to incorporate fairness in spectral clustering [15]. Different methods of imposing fairness in spectral clustering are mentions in chapter 2.

After fairness, one of the most important parameters to be considered in spectral clustering is accuracy or performance. The performance of spectral clustering can be improved by the addition of constraints. Many researches have been done on constrained spectral clustering (see chapter 3). In constrained spectral clustering we impose some information as constraints to the data in the form of **Must-link** and **Cannot-link** to enhance accuracy. If M is the set of must-link constraints then

$$M = (x_i, x_j) \quad | \quad x_i \text{ and } x_j \text{ are similar} \tag{1.5}$$

If C is the set of cannot-link constraints then

$$C = (x_i, x_j) \quad | \quad x_i \text{ and } x_j \text{ are dissimilar} \tag{1.6}$$

1.2 Challenges and Motivation

Constrained spectral clustering outperforms naive spectral clustering in terms of accuracy but fairness become dependent on the constrained supplied. In simple words fairness is ensured if the proportion of demographic group in each cluster becomes nearly equal to the proportion of demographic group in in the entire data [7] and presence of must-link and cannot-link does not guarantees the equality of these two proportions. Some constraints may increase the fairness whereas some constraints can lead to a purely unfair algorithm. Hence imposing fairness in constrained spectral clustering is the need of the moment.

1.3 Problem Statement

To propose an algorithm which incorporates fairness in constrained spectral clustering.

Chapter 2

Review of Prior Works

Sensitive attributes present in the data may cause clustering algorithm to produce unfair results. Incorporating fairness into spectral clustering was really the need of time. Researches have been done to improve spectral clustering in terms of fairness. The process of incorporating fairness into clustering can be divided into three categories

1. Pre-processing approaches
2. In-processing approaches
3. Post-processing approaches

2.1 Pre-processing approaches

In this approach the data X is transformed into another data X' and then it is given to any clustering approaches which then produces fair clusters. This method employed fairlet decomposition and converts data into micro-clusters or fairlets that meets the fairness requirements. Many works[7],[4],[16],[17] have been done to impose fairness using fairlet decomposition.

2.2 In-processing approaches

In this approach the clustering algorithm is modified to ensure fairness [11],[6],[15].

As we have used one of the in-processing approaches [15] in our proposed algorithm, let us discuss about it briefly. Let us assume that a data set contains $V = \{v_1, v_2, ..., v_n\}$ as n instances with h demographic groups V_s such that

$$V = \cup_{s \in [h]} \quad V_s \tag{2.1}$$

According to Chierichetti et al. (2017) [8], fairness notion is ensured if every cluster contains nearly the same number of data from each group V_s. Balance of any cluster C_l is defined as

$$Balance(C_l) = \min_{s \neq s' \in [h]} \frac{|V_s \cap C_l|}{|V_s' \cap C_l|} \in [0, 1] \tag{2.2}$$

In simple words fairness notion is actually asking for a clustering in which the proportion of every group in each cluster is same as their proportion in the entire data set.

Let $f^{(s)}$ defines the group membership of V_s such that

$$f_s^{(i)} = \begin{cases} 1, & \text{if } v_i \in V_s \\ 0, & \text{otherwise} \end{cases} \tag{2.3}$$

Let $H \in \mathbb{R}^{nxk}$ be the encoding of clustering $V = C_1 \cup C_2 \cup ... \cup C_k$ such that

$$H_{il} = \begin{cases} \frac{1}{\sqrt{|C_l|}}, & i \in C_l \\ 0, & \text{otherwise} \end{cases} \tag{2.4}$$

then the bellow equation guarantees fairness

$$\forall s \in [h-1] : \sum_{i=1}^{n} (f_i^s - \frac{V_s}{n}) H_{il} = 0 \implies \forall s \in [h] : \frac{|v_s \cap C_l|}{|C_l|} = \frac{|V_s|}{n} \tag{2.5}$$

In order to make the spectral clustering objective as fair as possible, we have to solve the following equation as given by [15]

$$\min_{H} Tr(H^T LH) \;\; \text{subject to} \;\; H^T H = I_k \;\; \text{and} \;\; F^T H = 0_{(h-1)xk} \tag{2.6}$$

Where $F \in R^{nx(h-1)}$ having column vectors $f^{(s)} - \frac{|V_s|}{n}.1_n$ $s \in [h-1]$

We can substitute H $= ZY$, where Z $\in$ R$^{n*(n-h+1)}$ and forms the orthonormal subspace of F^T and Y $\in$R$^{(n-h+1)*k}$

Then 2.6 will become

$$\min_{Y} Tr(Y^T Z^T LZY) \;\; \text{subject to} \;\; Y^T Y = I_k \tag{2.7}$$

Solution to the above equation is k smallest eigenvectors of $Z^T LZ$ written in the columns of Y. Then we can apply k means on H = ZY.

Algorithm(FSC)

- **Input:**
  1. Data set $V = x_1, x_2, ..., x_n$
  2. Group membership vectors f_s as given by 2.3
- Make similarity graph
- Find laplacian matrix
- Find $F \in R^{nx(h-1)}$ having column vectors $f^{(s)} - \frac{|V_s|}{n}.1_n$ $s \in [h-1]$
- Compute Z whose columns form the orthonormal basis of F^T
- Compute the k smallest eigenvectors of $Z^T LZ$ and put them as column vectors of another matrix Y.
- Apply k-means clustering to the rows of H = ZY
- **Output:**
  k Clusters

The above algorithm works and gives fairer clusters without significant increase in the cost of clustering.

2.3 Post-processing approaches

In post processing approaches vanilla clustering algorithm is applied on data and the obtained clusters are processed to make them fair [12],[13],[14],[5].

2.4 Constrained spectral clustering

Imposing domain knowledge of data into spectral clustering results in better performance or improves accuracy. One way to incorporate domain knowledge is to give must-link and cannot-link constraints. As finding pairwise relationship is much convenient than labelling all the data as it only requires if the two data are similar or not. If they are similar must-link constraints should be generated and if they are dissimilar cannot-link must be generated corresponding to that pair.

Constraints can be given in two forms

- adding constraints into affinity matrix
- adding constraints into optimisation function

Adding constraints into affinity matrix

In this approach the similarity matrix itself is changed. It incorporates the pairwise constraints in the affinity matrix.

$$W_{ij} = \begin{cases} 0, & \text{if } (x_i, x_j) \in C \\ +1, & \text{if } (x_i, x_j) \in M \\ W_{ij} & \text{otherwise} \end{cases} \tag{2.8}$$

[10] employed this approach. After modifying the graph laplacian the classical spectral clustering is followed. In this method the original affinity matrix is lost and a more natural way is to explicitly encode the constraints without modifying the original laplacian matrix.

Adding constraints into optimisation function

Constraints can be explicitly encoded without changing the graph laplacian. In this case the constrained matrix can be given as follows [9]

$$q_{ij} = \begin{cases} -1, & \text{if } (x_i, x_j) \in C \\ +1, & \text{if } (x_i, x_j) \in M \\ 0 & \text{otherwise} \end{cases} \tag{2.9}$$

[19] combines constraints and spectral clustering in a flexible manner. They used a parameter α for the trade-off between spectral clustering and constraints. It explicitly encodes

the constraints into optimization function without changing the affinity matrix.

[9] also modified the optimization function to incorporate fairness. As we have applied this method in our proposed model, let us discuss about it briefly.

According to [9] constrained spectral clustering, J_{cSC} can be written as the combination of naive spectral clustering,JSC and a penalisation term J_{CM} as follows

$$J_{cSC} = \gamma J_{SC} + (1 - \gamma) J_{CM} \tag{2.10}$$

γ is acting as a regularization term.

Let u_k be the indicator vector and $V = V_1 \cup V_2 \cup ... \cup V_k$ are the clusters such that

$$u_{ik} = \begin{cases} 1, & v_i \in V_k \\ 0, & \text{otherwise} \end{cases} \tag{2.11}$$

From [18] we can write J_{SC} as

$$J_{SC} = u^T L u \tag{2.12}$$

We can write J_{CM} as

$$J_{CM} = - \sum_{(x_i, x_j) \in C} (u_i - u_j)^2 + \sum_{(x_i, x_j) \in M} (u_i - u_j)^2 \tag{2.13}$$

$$J_{CM} = \sum_{i,j} (u_i - u_j)^2 q_{ij} \tag{2.14}$$

where

$$q_{ij} = \begin{cases} -1, & \text{if } (x_i, x_j) \in C \\ +1, & \text{if } (x_i, x_j) \in M \\ 0 & \text{otherwise} \end{cases} \tag{2.15}$$

$$J_{CM} = u^T L_Q u \tag{2.16}$$

Here $L_Q = D_Q - Q$ is the laplacian matrix of constraint graph with D_Q as the degree matrix of constraint graph

Substituting J_{SC} and J_{CM} from 2.12 and 1.16 in 2.10

$$J_{cSC} = u^T(\gamma L + (1-\gamma)L_Q)u = u^T L_{cSC} u \tag{2.17}$$

where

$$L_{cSC} = D_{cSC} - W_{cSC} \tag{2.18}$$

with

$$D_{cSC} = \gamma D + (1-\gamma)D_q \tag{2.19}$$

$$W_{cSC} = \gamma W + (1-\gamma)Q \tag{2.20}$$

The linearly separable embedding space is given by the eigenmap of L_{cSC}

L_{cSC} contains negative entries. In order to remove it we can use D_{cSc} as follows

$$d_{cSC}(i,j) = \sum_{j=1}^{n} |W_{ij}| \tag{2.21}$$

Algorithm(CSC)

- **Input:**
  1. Data set $V = x_1, x_2, ..., x_n$
  2. Constraints set in the form of ML and CL
- Make similarity graph, W
- Make Constraint martix Q from 2.15
- Compute modified constrained laplacian Matrix, L_{cSC} from 2.18
- Compute the k smallest eigenvectors of L_{cSC} and put them as column vectors of another matrix Y.
- Apply k-means clustering to the rows of Y.
- **Output:**
  k Clusters

[9] shows that incorporating fairness increases accuracy of spectral clustering algorithm. Increasing the percentage of constraints might sometimes decrease the performance due to the addition of some constraints. In [9], constraints are selected randomly purely on the basis of labels. Hence an intelligent selection of constraints is required to make constrained spectral clustering improve the performance.

Chapter 3

Incorporating Fairness in constrained spectral clustering

Till now we saw incorporation of fairness as well as constraints in spectral clustering. We see from [9] that the addition of constraints into spectral clustering enhances the performance but at the same time these constraints can drastically decrease the fairness or balance.

Balance of clustering algorithm can be defined as follows

$$Balance = \min_{i \in k}(\min_{s \in [h]} \frac{|C_i \cap V_s|}{|C_i|}) \tag{3.1}$$

The Balance of individual cluster, C_l can be given as

$$Balnce(C_l) = \min_{s \in [h]} \frac{|C_l \cap V_s|}{|C_l|} \tag{3.2}$$

Definition of fairness in constraints set:

In order to improve fairness, the fraction of (v_i, v_j) *where* $i \neq j$ *data pairs in must-link constraints set should be close to the fraction of* (v_l, v_l) *data pairs in cannot-link constraints set if* V *contains* h *groups* V_s *and* $1 =< i, j, l <= h$.

Explanation: There could be two possible conditions between any pair of data points:

1. They are in Must-link constraints
2. They are in cannot-link constraints

If they are in must-link constraints ie; they should be in the same cluster then to increase balance of individual clusters given in equation 3.2 they should belong to different demographic groups.

If they are in cannot-link constraints ie; they should be in different clusters then to increase balance of individual clusters given in equation 3.2 they should belong to same demographic groups.

To increase the balance of overall clustering as in eq 3.1 the difference of the fractions of (v_i, v_j) where i≠j data pairs in must-link constraints set and (v_l, v_l) data pairs in cannot-link constraints set should be minimum. □

If constraints are given such that these two fractions are almost equal then the constraint set can be referred as fair constraints. We want to minimize the difference of these fractions along with

1. the objective function from eq 1.2 of classical spectral clustering (Spectral clustering with fair constraints) and
2. the objective function from eq 2.6 of fair spectral clustering(Fair spectral clustering with fair constraints).

Let the data $V = v_1, v_2, ..., v_n$ has [h] groups. The constraint matrix Q is defined as

$$q_{ij} = \begin{cases} -1, & \text{if } (x_i, x_j) \in C \\ +1, & \text{if } (x_i, x_j) \in M \\ 0 & \text{otherwise} \end{cases} \tag{3.3}$$

$group(v_i)$ indicates the group to which the data point v_i belongs to. Let us defined two separate matrix Q_M and Q_C

$$Q_M = \begin{cases} +1, & \text{if } (x_i, x_j) \in M \\ 0 & \text{otherwise} \end{cases} \tag{3.4}$$

and

$$Q_C = \begin{cases} -1, & \text{if } (x_i, x_j) \in C \\ 0 & \text{otherwise} \end{cases} \tag{3.5}$$

Let us define a vector as follows

$$e = [1, 1, ..., 1]_{1*n} \tag{3.6}$$

Let Q_M^S can be given as

$$Q_M^S(i,j) = \begin{cases} 1, & \text{if } group(v_i) \neq group(v_j), (v_i, v_j) \in M \\ 0 & \text{otherwise} \end{cases} \tag{3.7}$$

Let Q_C^S can be given as

$$Q_C^S(i,j) = \begin{cases} -1, & \text{if } group(v_i) = group(v_j), (v_i, v_j) \in C \\ 0 & \text{otherwise} \end{cases} \tag{3.8}$$

$$\text{Total number of must-link constraints} = e^T Q_M e \tag{3.9}$$

$$\text{Total number of cannot-link constraints} == e^T Q_C e \tag{3.10}$$

Proportion of (v_i, v_j) where i≠j data pairs in must-link constraints set

$$= \frac{e^T (Q_M \odot Q_M^S) e}{e^T Q_M e} \tag{3.11}$$

Proportion of (v_l, v_l) data pairs in cannot-link constraints set

$$= \frac{e^T (Q_C \odot Q_C^S) e}{e^T Q_C e} \tag{3.12}$$

Difference between the fractions

$$= |\frac{e^T (Q_M \odot Q_M^S) e}{e^T Q_M e} - \frac{e^T (Q_C \odot Q_C^S) e}{e^T Q_C e}| \tag{3.13}$$

We need to minimise eq 3.13 in order to increase fairness.

We can define a matrix Q_e such that

$$Q_e = |\frac{Q_M \odot Q_M^S}{e^T Q_M e} - \frac{Q_C \odot Q_C^S}{e^T Q_C e}| \tag{3.14}$$

3.1 Spectral clustering with fair constraints

The optimisation function for constrained spectral clustering with fair constraints J_{cSfC} can be written in terms of spectral clustering J_{SC}, Constrained spectral clustering J_{cSC} and an extra term J_{Qe} derived from eq 3.13 as follows

$$\begin{gathered}\min_{H} \gamma_1(J_{SC}) + \gamma_2(J_{cSC}) + \gamma_3(J_{Qe}) \\ \text{subject to} \quad \gamma_1 + \gamma_2 + \gamma_3 = 1 \quad \text{and} \quad H^T H = I_k\end{gathered} \tag{3.15}$$

From eq 1.2

$$J_{SC} = Tr(H^T L H) \tag{3.16}$$

From eq 2.17

$$J_{cSC} = Tr(H^T L_{cSC} H) \tag{3.17}$$

From eq 3.14

$$J_{Qe} = Tr(H^T L_{Qe} H) \tag{3.18}$$

We can write 3.15 as

$$\begin{gathered}\min_{H} \gamma_1 Tr(H^T L H) + \gamma_2 Tr(H^T L_{cSC} H) + \gamma_3 Tr(H^T L_{Qe} H) \\ \text{subject to} \quad \gamma_1 + \gamma_2 + \gamma_3 = 1, \quad \text{and} \quad H^T H = I_k\end{gathered} \tag{3.19}$$

Taking $\gamma_1 = \gamma_2 = \gamma_3 = \frac{1}{3}$

Then 3.19 will become

$$\frac{1}{3} \min_{H} Tr(H^T(L + L_{cSC} + L_{Qe})H) \quad \text{subject to} \quad H^T H = I_k \tag{3.20}$$

$$\frac{1}{3} \min_{Y} Tr(H^T L_r H) \quad \text{subject to} \quad H^T H = I_k \tag{3.21}$$

Where $L_r = L + L_{cSC} + L_{Qe}$

Solution to the above equation is k smallest eigenvectors of L_r that can be written as the column vectors of Y. Then we can apply k means on Y to obtain clusters.

Algorithm(SCFC)

- **Input:**
  1. Data set $V = x_1, x_2, ..., x_n$
  2. Group membership vectors f_s as given by 2.3
  3. Constraints in the form of ML and CL.
- Make similarity graph
- Find laplacian matrix, L_r
- Compute the k smallest eigenvectors of L_r.(columns of Y)
- Apply k-means clustering to the rows of Y
- **Output:**
  Final Clusters

3.2 Fair spectral clustering with fair constraints

The optimisation function for fair constrained spectral clustering with fair constraints J_{fcSfC} can be written in terms of fair spectral clustering J_{fSC}, Constrained spectral clustering J_{cSC} and an extra term J_{Qe} derived from eq 3.13 as follows

$$\begin{gathered}\min_H \gamma_1(J_{fSC}) + \gamma_2(J_{cSC}) + \gamma_3(J_{Qe}) \\ \text{subject to} \quad \gamma_1 + \gamma_2 + \gamma_3 = 1 \quad \text{and} \quad H^T H = I_k\end{gathered} \tag{3.22}$$

From eq 2.6

$$J_{fSC} = Tr(H^T L H) \tag{3.23}$$

From eq 2.17

$$J_{cSC} = Tr(H^T L_{cSC})H \tag{3.24}$$

From eq 3.14

$$J_{Qe} = Tr(H^T L_{Qe} H) \tag{3.25}$$

We can write eq 3.22 as

$$\begin{gathered}\min_H \gamma_1 Tr(H^T L H) + \gamma_2 Tr(H^T L_{cSC} H) + \gamma_3 Tr(H^T L_{Qe} H) \\ \text{subject to} \quad \gamma_1 + \gamma_2 + \gamma_3 = 1,\ H^T H = I_k \quad \text{and} \quad F^T H = 0_{(h-1)xk}\end{gathered} \tag{3.26}$$

We can substitute H = ZY, where Z $\in$ R$^{n*(n-h+1)}$ and forms the orthonormal subspace of F^T and Y $\in$R$^{(n-h+1)*k}$ and taking $\gamma_1 = \gamma_2 = \gamma_3 = \frac{1}{3}$

Then 3.26 will become

$$\frac{1}{3} \min_Y Tr(Y^T Z^T (L + L_{cSC} + L_{Qe}) Z Y) \quad \text{subject to} \quad Y^T Y = I_k \tag{3.27}$$

$$\frac{1}{3} \min_Y Tr(Y^T Z^T L_p Z Y) \quad \text{subject to} \quad Y^T Y = I_k \tag{3.28}$$

Where $L_P = L + L_{cSC} + L_{Qe}$

Solution to the above equation is k smallest eigenvectors of $Z^T L_P Z$ that can be written as the column vectors of Y. Then we can apply k means on H = ZY to obtain clusters.

Algorithm(FSCFC)

- **Input:**
  1. Data set $V = x_1, x_2, ..., x_n$
  2. Group membership vectors f_s as given by 2.3
  3. Constraints in the form of ML and CL.
- Make similarity graph
- Find laplacian matrix, L_p
- Find $F \in R^{nx(h-1)}$ having column vectors $f^{(s)} - \frac{|V_s|}{n}.1_n \; s \in [h-1]$
- Compute Z whose columns form the orthonormal basis of F^T.
- Compute the k smallest eigenvectors of $Z^T L_P Z$.(columns of Y)
- Apply k-means clustering to the rows of H = ZY
- **Output:**
  Final Clusters

Chapter 4

Experimental Setup

We evaluate our proposed fair constrained spectral clustering algorithm on four different data sets(Adult data set [1], bank marketing data set [1], Ricci data set [2], Hepatitis data set [3]) under different assumptions and constraint settings.
We choose the sensitive attribute of the data sets like gender in adult and hepatitis data set, marital status in bank marketing data set, race in Ricci data set. We take only one sensitive group under consideration.
The affinity matrix is weighted k nearest neighbor with weight equivalent to the Gausian similarity between data points taking σ^2 as the average of the variance of database features. In order to construct constraint matrix, we randomly take two data points and if these two have same label then we give a must-link constraint otherwise we give cannot constraint. We evaluate the experiment under different percentages of constraints. We took regularisation terms $\gamma_1, \gamma_2, \gamma_3$ values equal to $\frac{1}{3}$.

As constraint matrix is random we average the results of 50 iterations in every percentage of constraints.

We conduct our experiment with different ratios of the terms in eq 3.13 and also with different methods as discussed bellow

1. **Classical spectral clustering, SC**: Normal spectral clustering algorithm [18]

2. **Constrained spectral clustering, CSC**: Algorithm discussed in section 2.4

3. **Fair spectral clustering, FSC**: Algorithm discussed in section 2.2

4. **Spectral clustering with fair constraints in the ratio of 1:1, SCFCE**: Constrained spectral clustering with ratio of fair constraints or ratio of two terms in eq 3.13 to be equal to 1:1

5. **Fair spectral clustering with fair constraints in the ratio of 1:1, FSCFCE**: Fair constrained spectral clustering with ratio of fair constraints or ratio of two terms in eq 3.13 to be equal to 1:1

6. **Spectral clustering with fair constraints in the ratio of 1:9, SCFCU**: Constrained spectral clustering with ratio of fair constraints or ratio of two terms in eq 3.13 to be equal to 1:9

7. **Fair spectral clustering with fair constraints in the ratio of 1:9, FSCFCU**: Fair constrained spectral clustering with ratio of fair constraints or ratio of two terms in eq 3.13 to be equal to 1:9

Chapter 5

Results

We implemented all the seven settings mentioned in chapter 5 in different data sets. We considered one sensitive attribute at a time and conduct our experiment under different percentage of constraints. We graphically compare the balance and random index obtained from these algorithms .

For each data set we obtained four different observations(SCFCE vs FSCFCE, SCFCU vs FSCFCU, SCFCE vs SCFCU, FSCFCE vs FSCFCU) comparing balance.

We observed that our proposed algorithm gives fairer clusters with improved performance or random index.

Adult data set:

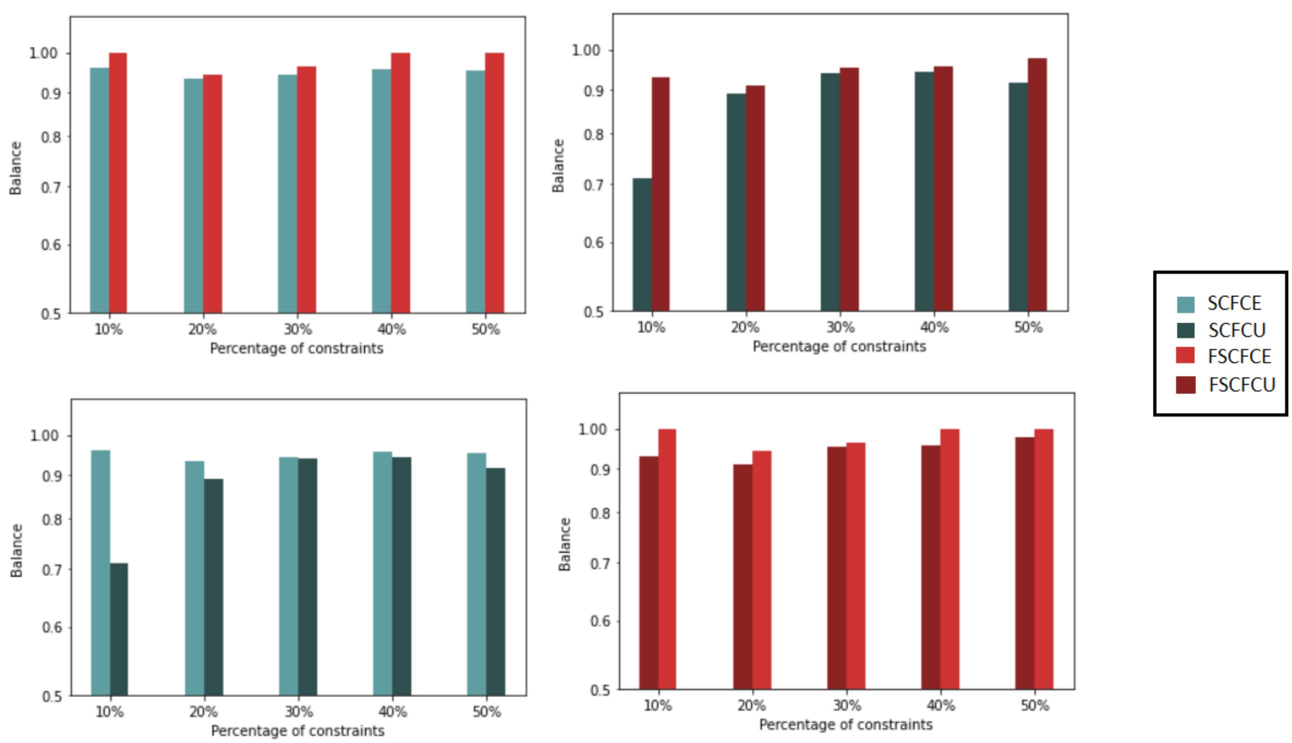

Fig. 5.1 Balance vs Percentage of constraints

Adult data set:

Algorithm	Balance (10%)	Balance (20%)	Balance (30%)	Balance (40%)	Balance (50%)
Classical spectral clustering	0.9267				
Constrained spectral clustering	0.9104	0.9027	0.86624	0.89256	0.89379
Fair spectral clustering	1.0				

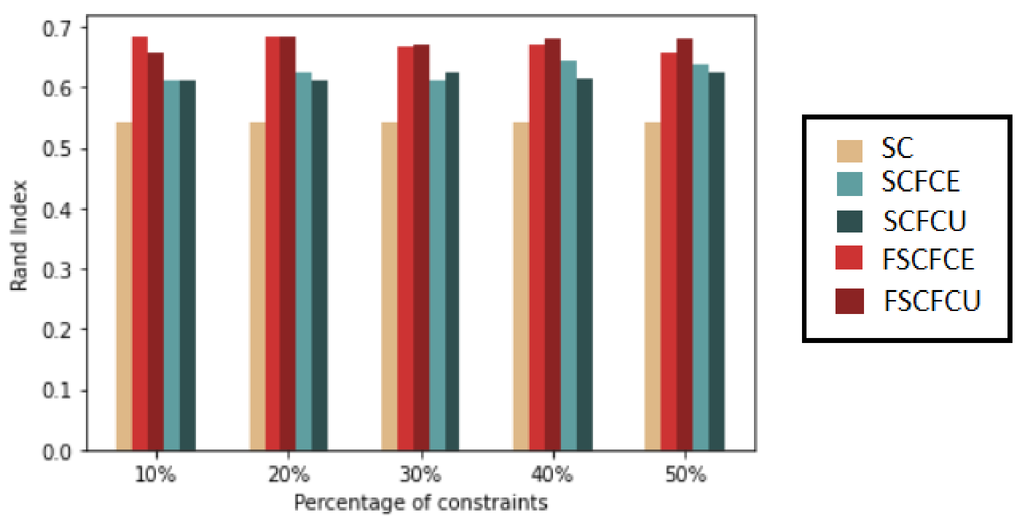

Fig. 5.2 Rand Index vs Percentage of constraints

Bank Marketing data set:

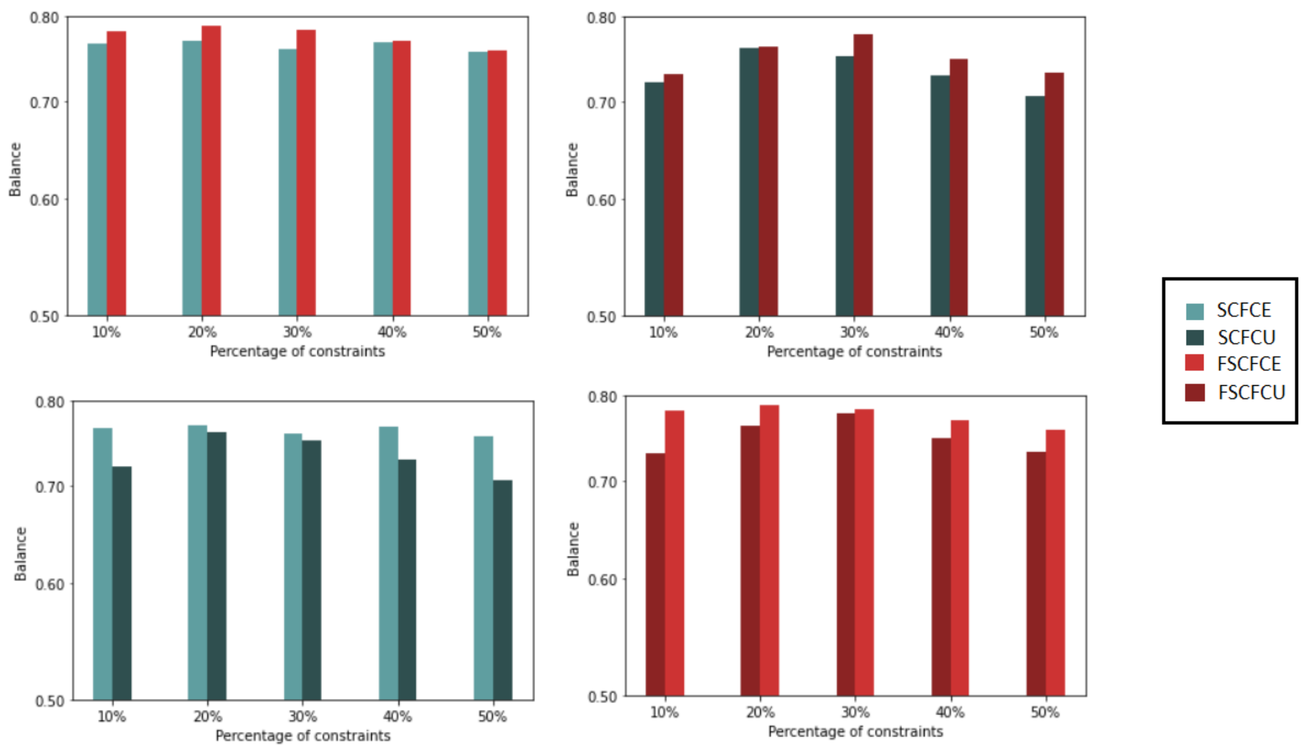

Fig. 5.3 Balance vs Percentage of constraints

Bank Marketing data set:

Algorithm	Balance (10%)	Balance (20%)	Balance (30%)	Balance (40%)	Balance (50%)
Classical spectral clustering	0.6065				
Constrained spectral clustering	0.7766	0.7372	0.7464	0.7391	0.8147
Fair spectral clustering	0.7209				

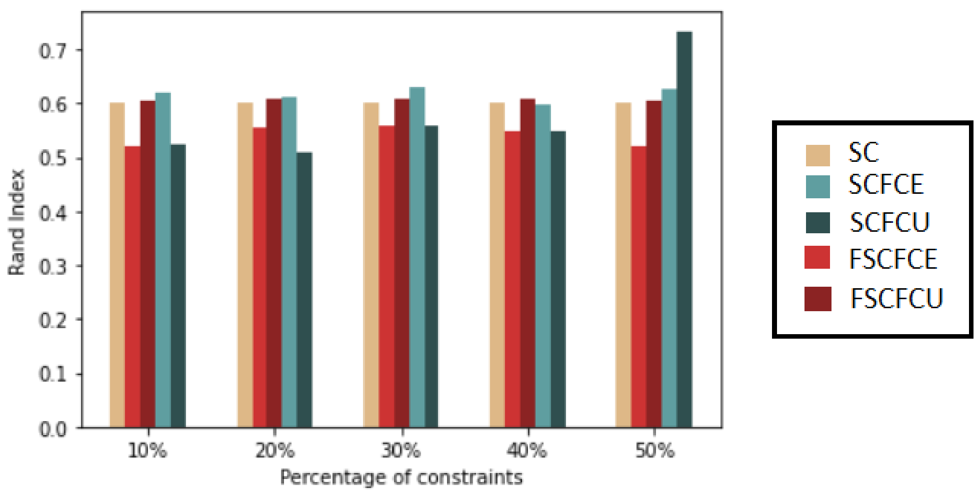

Fig. 5.4 Rand Index vs Percentage of constraints

Hepatitis data set:

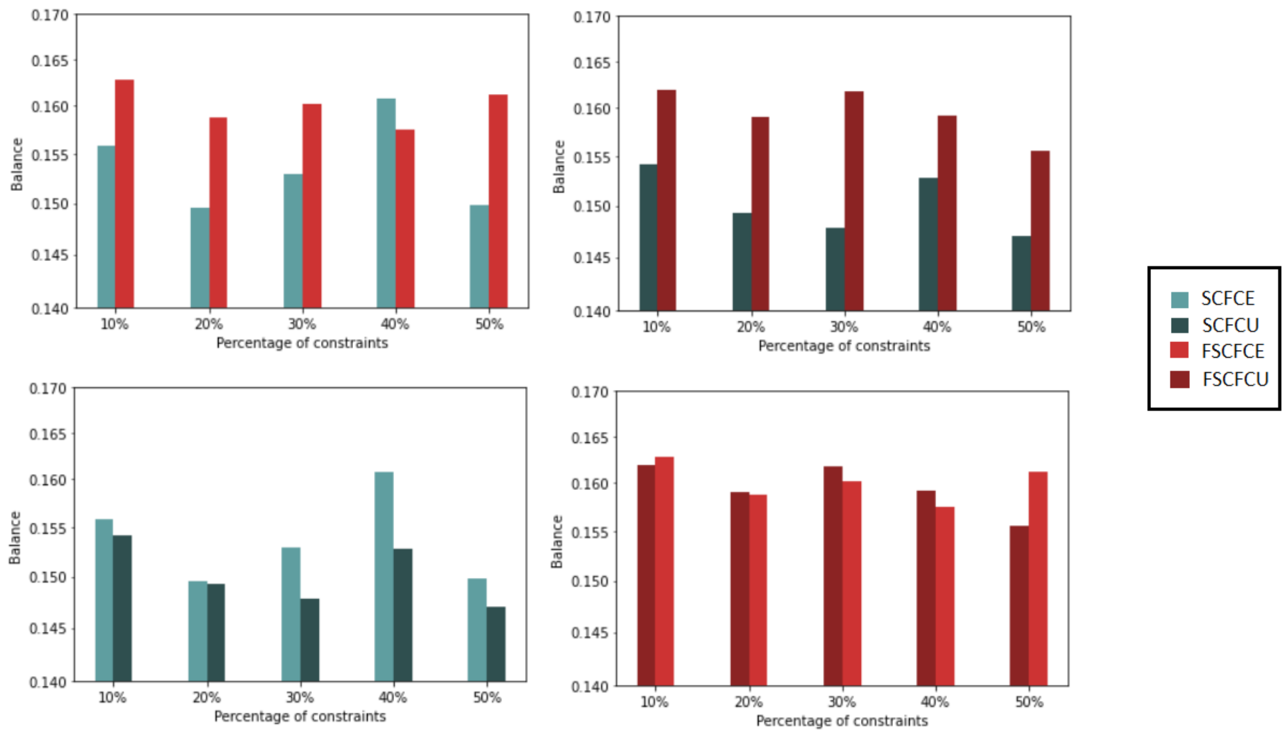

Fig. 5.5 Balance vs Percentage of constraints

Hepatitis data set:

Algorithm	Balance (10%)	Balance (20%)	Balance (30%)	Balance (40%)	Balance (50%)
Classical spectral clustering			0.1421		
Constrained spectral clustering	0.13658	0.14397	0.13300	0.13561	0.13856
Fair spectral clustering			0.2040		

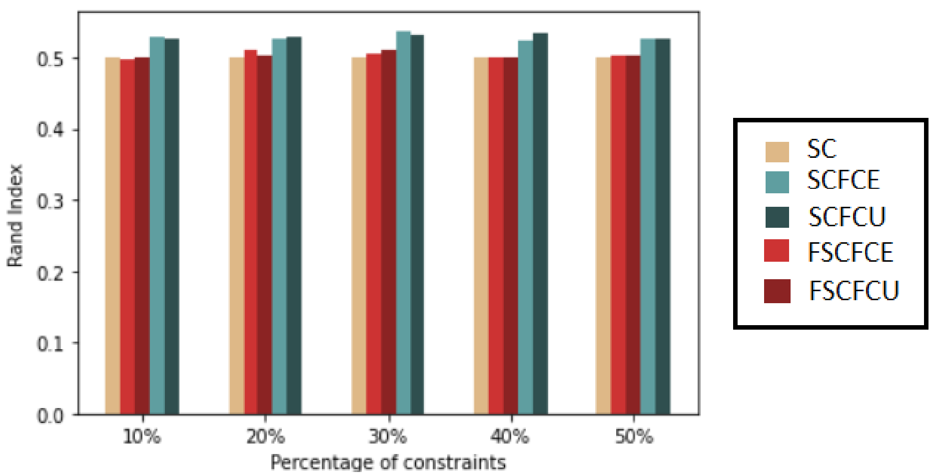

Fig. 5.6 Rand Index vs Percentage of constraints

Ricci data set:

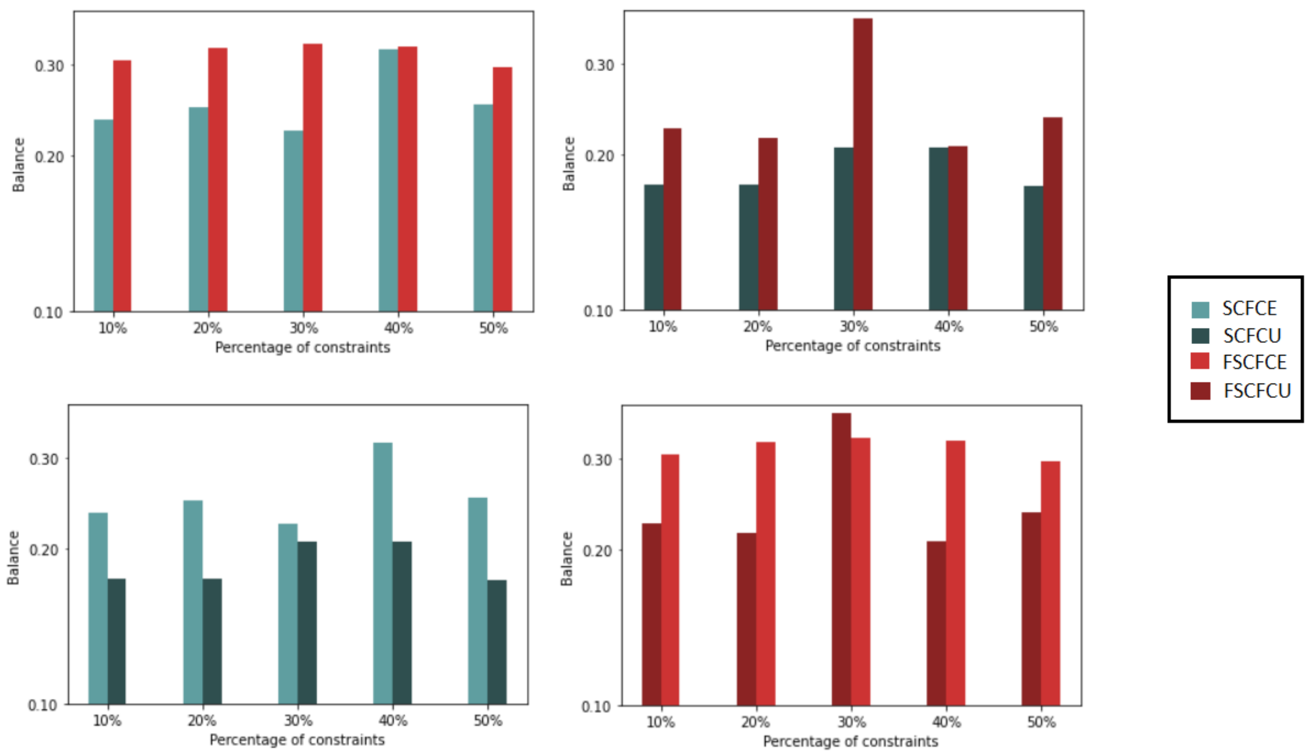

Fig. 5.7 Balance vs Percentage of constraints

<u>Ricci data set:</u>

Algorithm	Balance (10%)	Balance (20%)	Balance (30%)	Balance (40%)	Balance (50%)
Classical spectral clustering	0.2150				
Constrained spectral clustering	0.2290	0.2281	0.2058	0.2245	0.2500
Fair spectral clustering	0.3143				

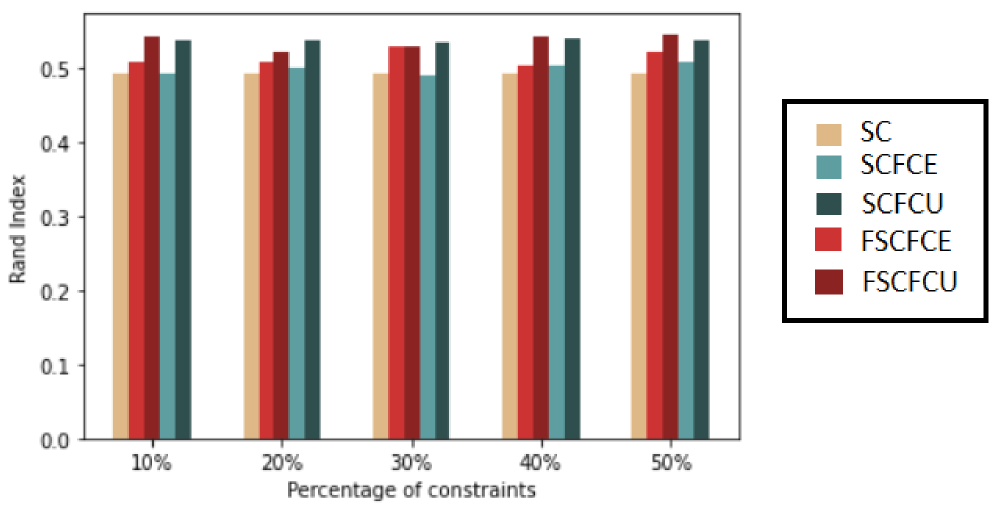

Fig. 5.8 Rand Index vs Percentage of constraints

Chapter 6

Discussions

We proposed a fairer version of constrained spectral clustering and applied in different data sets under various experimental settings. We also discussed the rigorous formulation of our algorithm. Result shows that we are able to generate fair clusters out of constrained spectral clustering. There might be the case that two data points having large similarity are having different labels. As the constraints generated is solely based on the output labels, it might sometimes decrease the rand Index. Hence selection of constraints should be done intelligently.

One of the directions of future work is that instead of using classical k means one can also use its fairer version but that totally depends on the cost one can bear to achieve fairness.

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