EC3204: Programming Languages and Compilers

Lecture 6 — Syntax Analysis (2): *Top-down Parsing*

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Top-Down Parsing

- Parsing is a process of constructing a parse tree of a given input string.
- Top-down parsing begins with the root of the parse tree, and extends the tree downward in preorder (root-left-right), until leaves match the input string.
- Equivalently, top-down parsing can be viewed as finding a leftmost derivation for an input string.

Rewriting Grammars for Top-Down Parsing

Unfortunately, not all grammars can be parsed by top-down parsing algorithms. To enable top-down parsing, we should first rewrite the grammar. Two representative transformations are:

- Eliminating ambiguity
- Eliminating left-recursion

Eliminating Ambiguity

Consider the grammar[1]:

$$\boldsymbol{E \to E + E \mid E * E \mid (E) \mid \mathbf{id}}$$

The grammar is ambiguous because it permits multiple parse trees, e.g.,

- $\mathbf{1+2*3}$: $\mathbf{(1+2)*3}$ vs. $\mathbf{1+(2*3)}$
- $\mathbf{1+2+3}$: $\mathbf{(1+2)+3}$ vs. $\mathbf{1+(2+3)}$

To eliminate the ambiguity, we should impose:

- (precedence) bind $*$ tighter than $+$
  - $\mathbf{1+2*3}$ is always parsed by $\mathbf{1+(2*3)}$
- (associativity) make $*$ and $+$ left-associative
  - $\mathbf{1+2+3}$ is always parsed by $\mathbf{(1+2)+3}$

Q. How to remove the ambiguity?

[1] In this course, we rely on $\boldsymbol{P}$ to define $\boldsymbol{G = (V, T, S, P)}$, if the first three are clear.

General facts about Ambiguity

We hope to transform ambiguous grammars into unambiguous ones. However,

- There is no algorithm to remove ambiguity from a CFG.
- There is no algorithm that can even tell us whether a CFG is ambiguous or not.
- There are context-free languages that are inherently ambiguous, i.e., there are context-free languages for which removing the ambiguity is impossible.

In practice, we can manually design an unambiguous grammar.

- Our original ambiguous grammar:

$$E \to E + E \mid E * E \mid (E) \mid \mathbf{id}$$

- An equivalent unambiguous grammar:

$$\begin{aligned} &E \to E + T \mid T \\ &T \to T * F \mid F \quad \text{/* introduce terms } (T) \text{ to prefer } * \text{ */} \\ &F \to \mathbf{id} \mid (E) \end{aligned}$$

Exercises

- Draw a parse tree for $\mathbf{id} + \mathbf{id} + \mathbf{id}$.

- Draw a parse tree for $\mathbf{id} + \mathbf{id} * \mathbf{id}$.

- Transform the grammar

$$\begin{aligned} \boldsymbol{E} &\rightarrow \boldsymbol{E} + \boldsymbol{T} \mid \boldsymbol{T} \\ \boldsymbol{T} &\rightarrow \boldsymbol{T} * \boldsymbol{F} \mid \boldsymbol{F} \\ \boldsymbol{F} &\rightarrow \mathbf{id} \mid (\boldsymbol{E}) \end{aligned}$$

so that $*$ associate to the right.

Exercises

- Draw a parse tree for $\mathbf{id} + \mathbf{id} + \mathbf{id}$.

$$E \Rightarrow_l E{+}T \Rightarrow_l E{+}T{+}T \Rightarrow_l T{+}T{+}T \Rightarrow_l \cdots \Rightarrow_l \mathbf{id}{+}\mathbf{id}{+}\mathbf{id}$$

- Draw a parse tree for $\mathbf{id} + \mathbf{id} * \mathbf{id}$.

$$\begin{aligned} & E \Rightarrow_l E + T \Rightarrow_l T + T \Rightarrow_l F + T \Rightarrow_l \mathbf{id} + T \\ & \Rightarrow_l \mathbf{id} + (T * F) \Rightarrow_l \mathbf{id} + (F * F) \Rightarrow_l \cdots \Rightarrow_l \mathbf{id} + \mathbf{id} * \mathbf{id} \end{aligned}$$

- Transform the grammar

$$\begin{aligned} E &\to E + T \mid T \\ T &\to T * F \mid F \\ F &\to \mathbf{id} \mid (E) \end{aligned}$$

so that $*$ associate to the right.
Replace $T * F$ with $F * T$.

Eliminating Left-Recursion

- A grammar is **left-recursive** if it has a non-terminal A such that there A appears as the first right-hand-side symbol in the production of A. For example, the grammar below is left-recursive.

$$E \rightarrow E + T \mid T$$

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- The left-recursive grammars are not suitable for top-down parsing; we may go into an infinite loop. For example, to parse T, we may apply $E \to E + T$ forever.

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- The left-recursive grammars are not suitable for top-down parsing; we may go into an infinite loop. For example, to parse T, we may apply $E \rightarrow E + T$ forever.
- We can remove left-recursion using transformation rules that produce non-left-recursive productions. For example, we can transform the left-recursive production $A \rightarrow A\alpha \mid \beta$ into

$$A \rightarrow \beta A', \quad A' \rightarrow \alpha A' \mid \epsilon$$

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$$A \rightarrow \beta A', \quad A' \rightarrow \alpha A' \mid \epsilon$$

- Using the transformation above, we can rewrite the grammar that has the right recursion:

$$E \rightarrow T\ E', \quad E' \rightarrow +\ T\ E' \mid \epsilon$$

Exercise

Describe an equivalent grammar without the left recursion.

$$\begin{aligned} E &\rightarrow E + T \mid T \\ T &\rightarrow T * F \mid F \\ F &\rightarrow \mathbf{id} \mid (E) \end{aligned}$$

Your answer:

Exercise

Describe an equivalent grammar without the left recursion.

$$\begin{aligned} E &\rightarrow E + T \mid T \\ T &\rightarrow T * F \mid F \\ F &\rightarrow \mathbf{id} \mid (E) \end{aligned}$$

Your answer:

$$\begin{array}{lcl} E & \rightarrow & T\ E' \\ E' & \rightarrow & +\ T\ E' \mid \epsilon \\ T & \rightarrow & F\ T' \\ T' & \rightarrow & *\ F\ T' \mid \epsilon \\ F & \rightarrow & (E) \mid \mathbf{id} \end{array}$$

Recap

So far, we learned transformations to obtain grammars suitable for top-down parsing.
Expression grammar:

$$E \to E + E \mid E * E \mid (E) \mid \mathbf{id}$$

Unambiguous version:

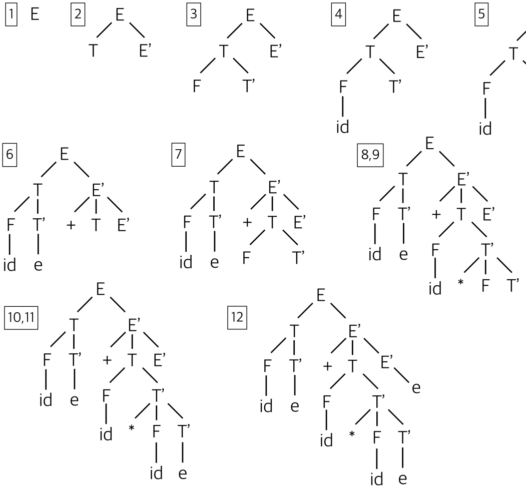
$$\begin{aligned} E &\to E + T \mid T \\ T &\to T * F \mid F \\ F &\to \mathbf{id} \mid (E) \end{aligned}$$

Non-left-recursive version:

$$\begin{array}{lcl} E & \to & T\ E' \\ E' & \to & +\ T\ E' \mid \epsilon \\ T & \to & F\ T' \\ T' & \to & *\ F\ T' \mid \epsilon \\ F & \to & (E) \mid \mathbf{id} \end{array}$$

Example

Top-down parsing sequence for the input string $\mathbf{id} + \mathbf{id} * \mathbf{id}$:



The Key Problem in Top-Down Parsing

Top-down parsing replaces the leftmost nonterminals with the body of some production. How to determine which production to use?

- **Recursive-decent parsing** uses backtracking.
- **Predictive parsing** uses a parsing table without backtracking (more efficient).

In particular, we will cover $\boldsymbol{LL(1)}$[2] parsing, a type of predictive parsing looking at $\mathbf{1}$ symbol ahead in the input.[3]

[2] **L**eft-to-right scanning, **L**eftmost derivation, **1**-symbol lookahead

[3] Predictive parsers looking at $\boldsymbol{k}$ symbols ahead are called $\boldsymbol{LL(k)}$ parsers.

Parsing Table

To look ahead at the input string, a predictive parser uses a **parsing table** for the expression grammar:

	id	$+$	$*$	(	)	\$
E	$E \to T\ E'$			$E \to T\ E'$		
E'		$E' \to +\ T\ E'$			$E' \to \epsilon$	$E' \to \epsilon$
T	$T \to F\ T'$			$T \to F\ T'$		
T'		$T' \to \epsilon$	$T' \to *\ F\ T'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \to \textbf{id}$			$F \to (E)$		

- The leftmost column indicates non-terminal symbols.
- The topmost row indicates the next input symbols (look-ahead tokens).
- \$ is a special "endmarker" to indicate the end of an input string.
- If a nonterminal symbol X is given and the next input symbol is y, apply the production in the entry (X, y) of the table.

Predictive Parsing Example

The sequence of predictive parsing for $\mathbf{id} + \mathbf{id} * \mathbf{id}$:

	id	+	*	(	)	$
E	$E \to T\ E'$			$E \to T\ E'$		
E'		$E' \to +\ T\ E'$			$E' \to \epsilon$	$E' \to \epsilon$
T	$T \to F\ T'$			$T \to F\ T'$		
T'		$T' \to \epsilon$	$T' \to *\ F\ T'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \to \mathbf{id}$			$F \to (E)$		

Stack	Input	Action
$E\$$	$\mathbf{id} + \mathbf{id} * \mathbf{id}\$$	
$TE'\$$	$\mathbf{id} + \mathbf{id} * \mathbf{id}\$$	
$FT'E'\$$	$\mathbf{id} + \mathbf{id} * \mathbf{id}\$$	
$\mathbf{id}T'E'\$$	$\mathbf{id} + \mathbf{id} * \mathbf{id}\$$	match
$T'E'\$$	$+\mathbf{id} * \mathbf{id}\$$	
$E'\$$	$+\mathbf{id} * \mathbf{id}\$$	
$+TE'\$$	$+\mathbf{id} * \mathbf{id}\$$	match
$TE'\$$	$\mathbf{id} * \mathbf{id}\$$	
$FT'E'\$$	$\mathbf{id} * \mathbf{id}\$$	
$\mathbf{id}T'E'\$$	$\mathbf{id} * \mathbf{id}\$$	match
$T'E'\$$	$*\mathbf{id}\$$	
$*FT'E'\$$	$*\mathbf{id}\$$	match
$FT'E'\$$	$\mathbf{id}\$$	
$\mathbf{id}T'E'\$$	$\mathbf{id}\$$	match
$T'E'\$$	$\$$	
$E'\$$	$\$$	
$\$$	$\$$	accept

Predictive Parsing Algorithm

Input: a string w and a parsing table M for grammar G
Output: a leftmost derivation of w or an error indication

let a be the first symbol of w
let X be the top stack symbol
while ($X \neq \$$) { /* repeat until stack becomes empty */
    if ($X = a$) pop the stack and let a be the next symbol of w
    else if (X is a terminal) error
    else if ($M[X, a]$ is empty) error
    else if ($M[X, a] = X \rightarrow Y_1 Y_2 \cdots Y_k$) {
        output the production $X \rightarrow Y_1 Y_2 \cdots Y_k$
        pop the stack (remove the top stack symbol)
        push $Y_k, Y_{k-1}, \ldots, Y_1$ onto the stack (with Y_1 on top)
    }
    let X be the top stack symbol /* update the top stack symbol */
}

Constructing Parsing Table

A predictive parser processes an input string using a parsing table. How to construct the parsing table?

1. Compute $FIRST$ and $FOLLOW$ sets of the grammar.
   - Both sets are used to look 1 symbol ahead.
2. Construct the parsing table using these sets.

$FIRST$ and $FOLLOW$

Definition

Given a string α of terminal and non-terminal symbols, $FIRST(\alpha)$ is the set of all terminal symbols that can begin any string derived from α.

- If $\alpha \Rightarrow^* c\beta$, then $c \in FIRST(\alpha)$.
- As an exception, if $\alpha \Rightarrow^* \epsilon$, $\epsilon \in FIRST(\alpha)$.

Definition

For a non-terminal X, $FOLLOW(X)$ is the set of terminals a that can appear immediately to the right of X in some sentential form.

- If $S \Rightarrow^* \alpha X a \beta$, then $a \in FOLLOW(X)$.
- As an exception, if $S \Rightarrow^* \alpha X$, $\$ \in FOLLOW(X)$.

Example

$$
\begin{array}{lcl}
E & \to & T\ E' \\
E' & \to & +\ T\ E' \mid \epsilon \\
T & \to & F\ T' \\
T' & \to & *\ F\ T' \mid \epsilon \\
F & \to & (E) \mid \mathbf{id}
\end{array}
$$

- $FIRST(E)$
- $FIRST(E')$
- $FIRST(T)$
- $FIRST(T')$
- $FIRST(F)$
- $FOLLOW(E)$
- $FOLLOW(E')$
- $FOLLOW(T)$
- $FOLLOW(T')$
- $FOLLOW(F)$

Example

$$
\begin{array}{lcl}
E & \rightarrow & T\ E' \\
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T & \rightarrow & F\ T' \\
T' & \rightarrow & *\ F\ T' \mid \epsilon \\
F & \rightarrow & (E) \mid \mathbf{id}
\end{array}
$$

- $FIRST(E) =$

Example

$$
\begin{array}{lcl}
E & \rightarrow & T\ E' \\
E' & \rightarrow & +\ T\ E' \mid \epsilon \\
T & \rightarrow & F\ T' \\
T' & \rightarrow & *\ F\ T' \mid \epsilon \\
F & \rightarrow & (E) \mid \mathbf{id}
\end{array}
$$

- $FIRST(E) = FIRST(TE') = FIRST(FT') = \{(, \mathbf{id}\}$
- $FIRST(E') =$

Example

$$
\begin{array}{lcl}
E & \rightarrow & T\ E' \\
E' & \rightarrow & +\ T\ E' \mid \epsilon \\
T & \rightarrow & F\ T' \\
T' & \rightarrow & *\ F\ T' \mid \epsilon \\
F & \rightarrow & (E) \mid \mathbf{id}
\end{array}
$$

- $FIRST(E) = FIRST(TE') = FIRST(FT') = \{(, \mathbf{id}\}$
- $FIRST(E') = FIRST(+TE') \cup \{\epsilon\} = \{+, \epsilon\}$
- $FIRST(T) =$

Example

$$
\begin{array}{lcl}
E & \rightarrow & T\ E' \\
E' & \rightarrow & +\ T\ E' \mid \epsilon \\
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- $FIRST(T) = FIRST(FT') = \{(, \mathbf{id}\}$
- $FIRST(T') = FIRST(*FT') \cup \{\epsilon\} = \{*, \epsilon\}$
- $FIRST(F) =$

Example

$$
\begin{aligned}
E &\rightarrow T\ E' \\
E' &\rightarrow +\ T\ E' \mid \epsilon \\
T &\rightarrow F\ T' \\
T' &\rightarrow *\ F\ T' \mid \epsilon \\
F &\rightarrow (E) \mid \mathrm{id}
\end{aligned}
$$

- $FIRST(E) = FIRST(TE') = FIRST(FT') = \{(, \mathrm{id}\}$
- $FIRST(E') = FIRST(+TE') \cup \{\epsilon\} = \{+, \epsilon\}$
- $FIRST(T) = FIRST(FT') = \{(, \mathrm{id}\}$
- $FIRST(T') = FIRST(*FT') \cup \{\epsilon\} = \{*, \epsilon\}$
- $FIRST(F) = FIRST(\text{“}(E)\text{”}) \cup FIRST(\mathrm{id}) = \{(, \mathrm{id}\}$
- $FOLLOW(E) =$

Example

$$
\begin{array}{lcl}
E & \rightarrow & T\ E' \\
E' & \rightarrow & +\ T\ E' \mid \epsilon \\
T & \rightarrow & F\ T' \\
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- $FIRST(F) = FIRST(\text{“}(E)\text{”}) \cup FIRST(\mathrm{id}) = \{(, \mathrm{id}\}$
- $FOLLOW(E) = \{\$\} \cup FIRST(\text{“})\text{”}) = \{\$,)\}$
- $FOLLOW(E') = FOLLOW(E) = \{\$,)\}$
- $FOLLOW(T) = FOLLOW(E)$

Example

$$\begin{array}{lcl} E & \rightarrow & T\ E' \\ E' & \rightarrow & +\ T\ E' \mid \epsilon \\ T & \rightarrow & F\ T' \\ T' & \rightarrow & *\ F\ T' \mid \epsilon \\ F & \rightarrow & (E) \mid \mathrm{id} \end{array}$$

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- $FIRST(E') = FIRST(+TE') \cup \{\epsilon\} = \{+, \epsilon\}$
- $FIRST(T) = FIRST(FT') = \{(, \mathrm{id}\}$
- $FIRST(T') = FIRST(*FT') \cup \{\epsilon\} = \{*, \epsilon\}$
- $FIRST(F) = FIRST(``(E)") \cup FIRST(\mathrm{id}) = \{(, \mathrm{id}\}$
- $FOLLOW(E) = \{\$\} \cup FIRST(``)") = \{\$,)\}$
- $FOLLOW(E') = FOLLOW(E) = \{\$,)\}$
- $FOLLOW(T) = FOLLOW(E) \cup (FIRST(E') \setminus \{\epsilon\}) = \{+, \$,)\}$
- $FOLLOW(T') = FOLLOW(T) = \{+, \$,)\}$
- $FOLLOW(F) =$
  $FOLLOW(T) \cup FOLLOW(T')$

Example

$$\begin{array}{lcl} E & \rightarrow & T\ E' \\ E' & \rightarrow & +\ T\ E' \mid \epsilon \\ T & \rightarrow & F\ T' \\ T' & \rightarrow & *\ F\ T' \mid \epsilon \\ F & \rightarrow & (E) \mid \mathrm{id} \end{array}$$

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- $FIRST(E') = FIRST(+TE') \cup \{\epsilon\} = \{+, \epsilon\}$
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- $FIRST(T') = FIRST(*FT') \cup \{\epsilon\} = \{*, \epsilon\}$
- $FIRST(F) = FIRST(\text{“}(E)\text{”}) \cup FIRST(\mathrm{id}) = \{(, \mathrm{id}\}$
- $FOLLOW(E) = \{\$\} \cup FIRST(\text{“})\text{”}) = \{\$,)\}$
- $FOLLOW(E') = FOLLOW(E) = \{\$,)\}$
- $FOLLOW(T) = FOLLOW(E) \cup (FIRST(E') \setminus \{\epsilon\}) = \{+, \$,)\}$
- $FOLLOW(T') = FOLLOW(T) = \{+, \$,)\}$
- $FOLLOW(F) =$
  $FOLLOW(T) \cup FOLLOW(T') \cup (FIRST(T') \setminus \{\epsilon\}) = \{*, +, \$,)\}$

Algorithm for computing $FIRST$

To compute $FIRST(X)$ for any string X from a grammar, apply the following rules until no more terminals or ϵ can be added to $FIRST$ set for every grammar symbol:

- If X is a terminal, then $FIRST(X) = \{X\}$.
- When X is a nonterminal and $X \rightarrow Y_1 Y_2 \cdots Y_k$ is a production, add the followings to $FIRST(X) = FIRST(Y_1 Y_2 \cdots Y_k)$.
  - $FIRST(Y_1) \setminus \{\epsilon\}$, i.e., all non-$\epsilon$ symbols of $FIRST(Y_1)$
  - $FIRST(Y_2) \setminus \{\epsilon\}$ if $\epsilon \in FIRST(Y_1)$
  - $FIRST(Y_3) \setminus \{\epsilon\}$ if $\epsilon \in FIRST(Y_1)$, $\epsilon \in FIRST(Y_2)$
  - $\cdots$
  - $FIRST(Y_k) \setminus \{\epsilon\}$ if $\epsilon \in FIRST(Y_j)$ for every $j \in [1, k-1]$
  - ϵ if $\epsilon \in FIRST(Y_j)$ for every $j \in [1, k]$
- If $X \rightarrow \epsilon$ is a production, then add ϵ to $FIRST(X)$.

Algorithm for computing $FOLLOW$

To compute $FOLLOW(X)$ for all nonterminals X, apply the following rules until nothing can be added to any $FOLLOW$ set.

Part 1. Compute an interim $FOLLOW(X)$.

- If X is the start symbol, add \$ to $FOLLOW(X)$.
- For every production $A \rightarrow \alpha X \beta$, add everything in $FIRST(\beta)$ except for ϵ to $FOLLOW(X)$.

$$FOLLOW(X) \leftarrow (FIRST(\beta) \setminus \{\epsilon\}) \cup FOLLOW(X)$$

Part 2. Propagate the interim result.

- For every production $X \rightarrow \alpha A$, add $FOLLOW(X)$ to $FOLLOW(A)$.
- For every production $X \rightarrow \alpha A \beta$, if $\epsilon \in FIRST(\beta)$, add $FOLLOW(X)$ to $FOLLOW(A)$.

Construction of Parsing Table

- **Goal:** Fill in each entry $M[A, a]$[4] of a predictive parsing table M by adding a production $A \rightarrow \alpha$ only when that production can be useful for processing the next input symbol a.
- **Input:** a context-free grammar G
- **Output:** a predictive parsing table M
- **Algorithm:** For each production $A \rightarrow \alpha$, do the following.
  1. For every terminal $a \in FIRST(\alpha)$, add $A \rightarrow \alpha$ to $M[A, a]$.
  2. If $\epsilon \in FIRST(\alpha)$, for each terminal $b \in FOLLOW(A)$, add $A \rightarrow \alpha$ to $M[A, b]$.
  3. Similarly, if $\epsilon \in FIRST(\alpha)$ and $\$ \in FOLLOW(A)$, add $A \rightarrow \alpha$ to $M[A, \$]$.

If a resulting parsing table for a grammar contains at most one production (i.e., does not contain conflicting productions), the grammar is in $LL(1)$ grammar.

[4] A is a nonterminal and a is a terminal or $\$$.

Example

	id	+	*	(	)	$
E	$E \to T\ E'$			$E \to T\ E'$		
E'		$E' \to +\ T\ E'$			$E' \to \epsilon$	$E' \to \epsilon$
T	$T \to F\ T'$			$T \to F\ T'$		
T'		$T' \to \epsilon$	$T' \to *\ F\ T'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \to \text{id}$			$F \to (E)$		

- $FIRST(F) = FIRST(T) = FIRST(E) = \{(, \text{id}\}$.
- $FIRST(E') = \{+, \epsilon\}$.
- $FIRST(T') = \{*, \epsilon\}$.
- $FOLLOW(E) = FOLLOW(E') = \{), \$\}$.
- $FOLLOW(T) = FOLLOW(T') = \{+,), \$\}$.
- $FOLLOW(F) = \{+, *,), \$\}$.

Summary

- Transformations for top-down parsing: eliminating ambiguity and left-recursion
- $LL(k)$ is a kind of top-down parsing, which predicts productions by looking at the next k input symbols.
- $LL(1)$ parsing algorithm: use $FIRST$ and $FOLLOW$ to predict a production by looking at the next input symbol.