EC3204: Programming Languages and Compilers

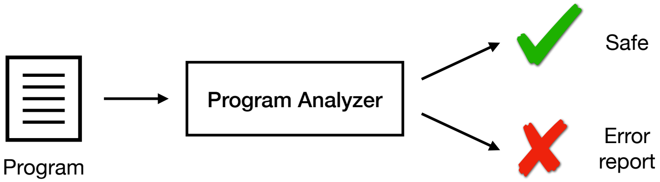
Lecture 10 — Semantic Analysis (1)

Introduction

Sunbeom So

Fall 2024

Semantic Program Analysis



- Program analysis is a technology for discovering bugs or proving safety.
- Widely used to find SW vulnerabilities and bugs in the industry.
  - VCC (Microsoft), Danfy (AWS), SAGE (Microsoft), SMTChecker (Ethereum Foundation), JavaPathFinder (NASA), and many others.

An Ideal Analyzer: Sound and Complete

An automatic analyzer A is ideal iff it is **sound** and **complete**.

- **Soundness**:

$$\text{for every program } p,\ A(p, \phi) = \texttt{safe} \implies p \text{ satisfies } \phi$$

  If p is proven to be safe by A, p is indeed safe.
  ($=$) If p is unsafe, A must return `unsafe` (contrapositive).
  ($=$) A misses no errors, i.e., A produces **no false negatives**.

An Ideal Analyzer: Sound and Complete

An automatic analyzer A is ideal iff it is **sound** and **complete**.

- **Soundness**:

$$\text{for every program } p,\ A(p, \phi) = \texttt{safe} \implies p \text{ satisfies } \phi$$

  If p is proven to be safe by A, p is indeed safe.
  ($=$) If p is unsafe, A must return `unsafe` (contrapositive).
  ($=$) A misses no errors, i.e., A produces **no false negatives**.

- **Completeness**:

$$\text{for every program } p,\ A(p, \phi) = \texttt{safe} \impliedby p \text{ satisfies } \phi$$

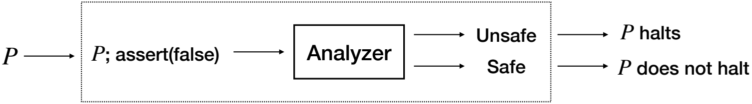
  If p is safe, A must prove its safety.
  ($=$) If A returns `unsafe`, p is unsafe (contrapositive).
  ($=$) A produces **no false positives**.

An ideal analyzer: No false negatives & No false positives

Hard Limit: Undecidability

We cannot have an ideal analyzer that can always produce a correct answer (`safe` or `unsafe`) for any program.

(Proof by Contradiction) Suppose exact analysis is possible. Then, we can solve the Halting problem!



Theorem (Rice Theorem)

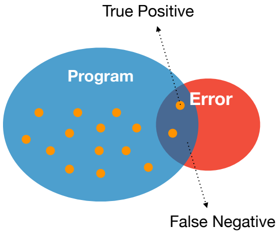
Let $\mathbb{L}$ be a Turing-complete language. Let ϕ be a nontrivial semantic property, i.e., there are $\mathbb{L}$ programs that satisfy ϕ and $\mathbb{L}$ programs that do not satisfy ϕ. There exists no algorithm $\boldsymbol{A}$ such that

$$\text{for every program } \boldsymbol{p} \in \mathbb{L},\ \boldsymbol{A}(\boldsymbol{p}, \phi) = \texttt{true} \iff \boldsymbol{p} \text{ satisfies } \phi.$$

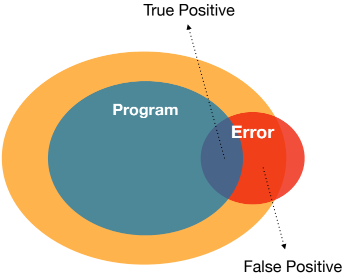
Side-Stepping Undecidability via Approximation

Any automatic analyzer compromises soundness or completeness.

	Under-approximation	**Over-approximiation**
Analysis Goal	complete but unsound	sound but incomplete
Examples	fuzzing, symbolic execution	formal verification, abstract interpretation



Under-approximation



Over-approximation

Fuzzing (Random Testing)

```
1 void testme (int x, int y) {
2   z = 2 * y;
3   if (z == x) {
4     if (x > y + 10) {
5       /* some error */
6     }
7   }
```

Suppose our goal is to find an input that triggers the error at line 5.

- Q. Test cases for reaching line 5?
  -
- Q. Success probability of random testing? (assume $0 \leq x, y \leq 100$)
  -

Fuzzing (Random Testing)

```
1 void testme (int x, int y) {
2   z = 2 * y;
3   if (z == x) { /* x = 2 * y */
4     if (x > y + 10) { /* y > 10 */
5       /* some error */
6     }
7   }
```

Suppose our goal is to find an input that triggers the error at line 5.

- Q. Test cases for reaching line 5?
  - **(x,y) :** $\mathbf{(22, 11), (24, 12), \cdots, (100, 50)}$
- Q. Success probability of random testing? (assume $\mathbf{1 \leq x, y \leq 100}$)
  - $\mathbf{0.4\%}$

Symbolic Execution

Testing methods that analyze program behavior based on logical formulas.

```
void testme (int x, int y) {
  z = 2 * y;
  if (z == x) {
    if (x > y + 10) {
      /* some error */
    }
  }
```

1. Generate the constraint (α, β: symbolic inputs for x and y).

$$(x = \alpha) \wedge (y = \beta) \wedge (z = 2 * y) \wedge (z = x) \wedge (x > y + 10)$$

2. Solve the constraint using an SMT solver.[1]

$$[x \mapsto 22, y \mapsto 11, z \mapsto 22, \underline{\alpha \mapsto 22, \beta \mapsto 11}]$$

[1] E.g., Z3 – https://github.com/Z3Prover/z3

Formal Verification

Symbolic execution with **invariant inference**.

```
@pre: n ≥ 0 /* precondition: assumed to be true at the entry */
@post: j = n /* postcondition: expected to be true at the exit */
void testme (int n) {
  int i := 0;
  int j := 0;
  while (i ≠ n) {
    i := i + 1;
    j := j + 1;
  }
  return;
}
```

- Fuzzing and symbolic execution cannot prove the postcondition.
- To prove it, we must compute the **loop invariant** that summarizes the behavior of infinite iterations.

$$\underline{i = j} \land i = n \to j = n$$

Verification Challenge: Invariant Inference

```
@pre: ⊤
@post: sorted(rv, 0, |rv| − 1)
bool BubbleSort (int a[]) {
  int[] a := a0;
  @L1 : { −1 ≤ i < |a|
          ∧ partitioned(a, 0, i, i + 1, |a| − 1)
          ∧ sorted(a, i, |a| − 1) }
  for (int i := |a| − 1; i > 0; i := i − 1) {
    @L2 : { 1 ≤ i < |a| ∧ 0 ≤ j ≤ i
            ∧ partitioned(a, 0, i, i + 1, |a| − 1)
            ∧ partitioned(a, 0, j − 1, j, j)
            ∧ sorted(a, i, |a| − 1) }
    for (int j := 0; j < i; j := j + 1) {
      if (a[j] > a[j + 1]) {
        int t := a[j];
        a[j] := a[j + 1];
        a[j + 1] := t;
      }
    }
  }
  return a;
}
```

$$\textsf{sorted}(a, l, u) \iff \forall i, j. l \leq i \leq j \leq u \rightarrow a[i] \leq a[j]$$

$$\textsf{partitioned}(a, l_1, u_1, l_2, u_2) \iff \forall i, j. l_1 \leq i \leq u_1 < l_2 \leq j \leq u_2 \rightarrow a[i] \leq a[j]$$

Abstract Interpretation

Execute the program with abstract inputs.

```
void testme (int x) {
  y = x * 12 + 9 * 11;
  assert (y % 2 == 1); /* Goal: prove the safety */
}
```

- Underapproximation-based approaches cannot prove the safety.
- Verification approaches can prove the safety, but exact symbolic encoding on realistic programs is too expensive.
- We can prove the assertion using more lightweight reasoning.

$$\text{even number} + \text{odd number} = \text{odd number}$$

Abstract Interpretation

Of course, abstract interpretation is not a panacea.

```
void testme (int x) {
  y = x;
  y = y + x + 1;
  assert (y % 2 == 1); /* false alarm */
}
```

$$\text{any number} + \text{any number} + \text{odd number} = \text{any number}$$

Summary

- Achieving an *ideal* analyzer is undecidable, i.e., impossible.
- Principles of program analysis: approximate behavior.

	Under-approximation	**Over-approximiation**
Analysis Goal	complete but unsound	sound but incomplete
Examples	fuzzing, symbolic execution	formal verification, abstract interpretation

- In this course, we will focus on **abstract interpretation**, as it is arguably the most common and cost-effective approach in compilers.
- If you are interested in other analysis approaches, consider joining our lab or taking my Software Engineering course.