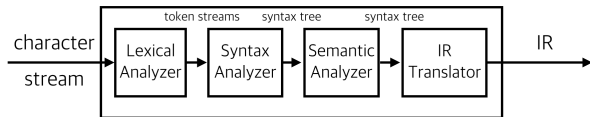


EC3204: Programming Languages and Compilers

Lecture 14 — IR Translation (1): *Automatic Translation*

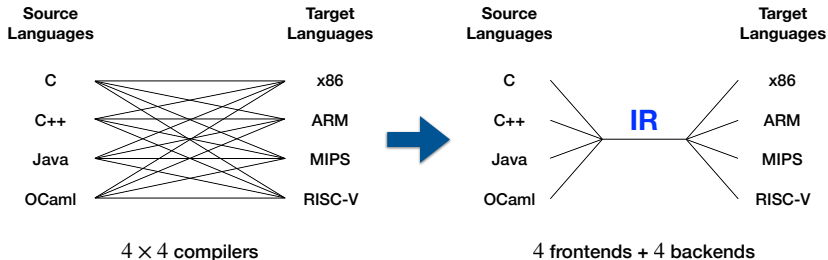
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Fall 2024

Translation from AST to IR



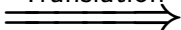
Why do we use IR?

- IR reduces the complexity of compiler design.



Translation Example (1)

```
{  
  int x;  
  x = 0;  
  print (x+1);  
}
```

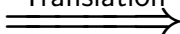
Translation


```
0 : x = 0  
0 : t1 = 0  
0 : x = t1  
0 : t3 = x  
0 : t4 = 1  
0 : t2 = t3 + t4  
0 : write t2  
0 : HALT
```

Translation Example (2)

```
{  
    int sum;  
    int i;  
  
    i = 0;  
    sum = 0;  
    while (i < 10) {  
        sum = sum + i;  
        i++;  
    }  
  
    print (sum);  
}
```

Translation



```
0 : sum = 0  
0 : i = 0  
0 : t1 = 0  
0 : i = t1  
0 : t2 = 0  
0 : sum = t2  
2 : SKIP  
0 : t4 = i  
0 : t5 = 10  
0 : t3 = t4 < t5  
0 : ifFalse t3 goto 3  
0 : t7 = sum  
0 : t8 = i  
0 : t6 = t7 + t8  
0 : sum = t6  
0 : t10 = i  
0 : t11 = 1  
0 : t9 = t10 + t11  
0 : i = t9  
0 : goto 2  
3 : SKIP  
0 : t12 = sum  
0 : write t12  
0 : HALT
```

Goal: define a translation procedure that converts a S program into a semantically equivalent T program.

- Define our source language (S) and target language (T).
- Define an automatic translation procedure from S to T .

Programming Language

A programming language is defined by:

- **Syntax:** a set of rules that define the structure of a program
- **Semantics:** a set of rules that define the meaning of a program execution

The syntax is divided into two kinds:

- **Concrete Syntax:** defines the full structure of a program.
 - ▶ Used when writing a program or reading a program.
- **Abstract Syntax:** defines the abstract, core structure of a program.
 - ▶ Used when automatically generating, analyzing, and optimizing a program.

`if (b) {c1} else {c2}`

`if b c1 c2`

Concrete Syntax of S

<i>program</i>	→	<i>block</i>	
<i>block</i>	→	{ <i>decls stmts</i> }	
<i>decls</i>	→	<i>decls decl</i> ϵ	
<i>decl</i>	→	<i>type x</i> ;	
<i>type</i>	→	int int[<i>n</i>]	
<i>stmts</i>	→	<i>stmts stmt</i> ϵ	
<i>stmt</i>	→	<i>lv = e</i> ;	
		<i>lv++</i> ;	
		if(<i>e</i>) <i>stmt</i> else <i>stmt</i>	
		if(<i>e</i>) <i>stmt</i>	
		while(<i>e</i>) <i>stmt</i>	
		do <i>stmt</i> while(<i>e</i>);	
		read(<i>x</i>);	
		print(<i>e</i>);	
		<i>block</i>	
<i>lv</i>	→	<i>x</i> <i>x</i> [<i>e</i>]	
<i>e</i>	→	<i>n</i>	integer
		<i>lv</i>	l-value
		<i>e</i> + <i>e</i> <i>e</i> - <i>e</i> <i>e</i> * <i>e</i> <i>e</i> / <i>e</i> - <i>e</i>	arithmetic operation
		<i>e</i> == <i>e</i> <i>e</i> < <i>e</i> <i>e</i> <= <i>e</i> <i>e</i> > <i>e</i> <i>e</i> >= <i>e</i>	conditional operation
		! <i>e</i> <i>e</i> <i>e</i> <i>e</i> && <i>e</i>	boolean operation
		(<i>e</i>)	

Abstract Syntax of S

program → *block*
block → *decls stmts*
decls → *decls decl* | ϵ
decl → *type x*
type → *int* | *int*[*n*]
stmts → *stmts stmt* | ϵ

stmt → *lv = e*
| *if e stmt stmt*
| *while e stmt*
| *do stmt while e*
| *read x*
| *print e*
| *block*

lv → *x* | *x*[*e*]

<i>e</i>	→	<i>n</i>	integer
		<i>lv</i>	l-value
		<i>e+e</i> <i>e-e</i> <i>e*e</i> <i>e/e</i> <i>-e</i>	arithmetic operation
		<i>e==e</i> <i>e<e</i> <i>e<=e</i> <i>e>e</i> <i>e>=e</i>	conditional operation
		<i>!e</i> <i>e e</i> <i>e&&e</i>	boolean operation

Semantic Domain of S

A memory state m is a mapping from locations (Loc) to values ($Value$).

$$m \in Mem = Loc \rightarrow Value$$

$$l \in Loc = Var + Addr \times Offset$$

$$v \in Value = \mathbb{N} + Addr \times Size$$

$$a \in Addr = \text{MemoryAddress}$$

$$Offset = \mathbb{N}$$

$$Size = \mathbb{N}$$

cf) $+$ denotes a disjoint union operator.

Inference Rule

We define program executions using **big-step operational semantics**, where the meanings are specified based on overall execution results. In particular, we define the semantics using **inference rules**. An inference rule is of the form:

$$\frac{A}{B}$$

- Interpreted as: “if A is true then B is also true”.
- A : hypothesis (antecedent)
- B : conclusion (consequent)
- Inference rules without hypotheses are called axioms (e.g., B):

$$\overline{B}$$

- The hypothesis may contain multiple statements, e.g.,

$$\frac{A \quad B}{C}$$

Interpreted as: “If both A and B are true then so is C ”.

Example: Semantics of S

The execution rules of if-statements:

$$\frac{M \vdash e \Rightarrow n \quad n \neq 0 \quad M \vdash stmt_1 \Rightarrow M_1}{M \vdash \text{if } e \text{ } stmt_1 \text{ } stmt_2 \Rightarrow M_1}$$

$$\frac{M \vdash e \Rightarrow 0 \quad M \vdash stmt_2 \Rightarrow M_1}{M \vdash \text{if } e \text{ } stmt_1 \text{ } stmt_2 \Rightarrow M_1}$$

- The semantics consists of judgments of the form:

$$M \vdash stmt \Rightarrow M'$$

which can be read as “Executing *stmt* under *M* results in a new memory *M'*”.

- Similarly, we have the following judgments for *decl*, *lv*, and *e*:

$$M \vdash decl \Rightarrow M', \quad M \vdash e \Rightarrow v, \quad M \vdash lv \Rightarrow l$$

Semantics of S

$$\boxed{M \vdash \text{decl} \Rightarrow M'}$$

$$\overline{M \vdash \text{int } x \Rightarrow M[x \mapsto 0]}$$

$$\frac{M \vdash e \Rightarrow n \quad (a, 0), \dots, (a, n-1) \notin \text{Dom}(M)}{M \vdash \text{int}[e] \ x \Rightarrow M[x \mapsto (a, n), (a, 0) \mapsto 0, \dots, (a, n-1) \mapsto 0]} \quad n > 0$$

$$\boxed{M \vdash \text{stmt} \Rightarrow M'}$$

$$\frac{M \vdash lv \Rightarrow l \quad M \vdash e \Rightarrow v}{M \vdash lv = e \Rightarrow M[l \mapsto v]}$$

$$\frac{M \vdash e \Rightarrow n \quad n \neq 0 \quad M \vdash \text{stmt}_1 \Rightarrow M_1}{M \vdash \text{if } e \ \text{stmt}_1 \ \text{stmt}_2 \Rightarrow M_1} \quad \frac{M \vdash e \Rightarrow 0 \quad M \vdash \text{stmt}_2 \Rightarrow M_1}{M \vdash \text{if } e \ \text{stmt}_1 \ \text{stmt}_2 \Rightarrow M_1}$$

$$\frac{M \vdash e \Rightarrow 0}{M \vdash \text{while } e \ \text{stmt} \Rightarrow M} \quad \frac{M \vdash e \Rightarrow n \quad n \neq 0 \quad M \vdash \text{stmt} \Rightarrow M_1 \quad M_1 \vdash \text{while } e \ \text{stmt} \Rightarrow M_2}{M \vdash \text{while } e \ \text{stmt} \Rightarrow M_2}$$

$$\frac{M \vdash \text{stmt} \Rightarrow M_1 \quad M_1 \vdash e \Rightarrow 0}{M \vdash \text{do } \text{stmt} \ \text{while } e \Rightarrow M_1} \quad \frac{M \vdash \text{stmt} \Rightarrow M_1 \quad M_1 \vdash e \Rightarrow n \quad n \neq 0 \quad M_1 \vdash \text{do } \text{stmt} \ \text{while } e \Rightarrow M_2}{M \vdash \text{do } \text{stmt} \ \text{while } e \Rightarrow M_2}$$

$$\overline{M \vdash \text{read } x \Rightarrow M[x \mapsto n]} \quad \overline{M \vdash \text{print } e \Rightarrow M}$$

Semantics of S

$$\boxed{M \vdash lv \Rightarrow l}$$

$$\frac{}{M \vdash x \Rightarrow x} \quad \frac{M \vdash e \Rightarrow n_1 \quad M(x) = (a, n_2) \quad n_1 \geq 0 \wedge n_1 < n_2}{M \vdash x[e] \Rightarrow (a, n_1)}$$

$$\boxed{M \vdash e \Rightarrow v}$$

$$\begin{array}{c} \frac{}{M \vdash n \Rightarrow n} \quad \frac{M \vdash lv \Rightarrow l}{M \vdash lv \Rightarrow M(l)} \\ \frac{M \vdash e_1 \Rightarrow n_1 \quad M \vdash e_2 \Rightarrow n_2}{M \vdash e_1 + e_2 \Rightarrow n_1 + n_2} \quad \frac{M \vdash e \Rightarrow n}{M \vdash -e \Rightarrow -n} \\ \frac{M \vdash e_1 \Rightarrow n_1 \quad M \vdash e_2 \Rightarrow n_2 \quad n_1 = n_2}{M \vdash e_1 == e_2 \Rightarrow 1} \quad \frac{M \vdash e_1 \Rightarrow n_1 \quad M \vdash e_2 \Rightarrow n_2 \quad n_1 \neq n_2}{M \vdash e_1 == e_2 \Rightarrow 0} \\ \frac{M \vdash e_1 \Rightarrow n_1 \quad M \vdash e_2 \Rightarrow n_2 \quad n_1 > n_2}{M \vdash e_1 > e_2 \Rightarrow 1} \quad \frac{M \vdash e_1 \Rightarrow n_1 \quad M \vdash e_2 \Rightarrow n_2 \quad n_1 \leq n_2}{M \vdash e_1 > e_2 \Rightarrow 0} \\ \frac{M \vdash e_1 \Rightarrow n_1 \quad M \vdash e_2 \Rightarrow n_2 \quad n_1 \neq 0 \vee n_2 \neq 0}{M \vdash e_1 || e_2 \Rightarrow 1} \\ \frac{M \vdash e_1 \Rightarrow n_1 \quad M \vdash e_2 \Rightarrow n_2 \quad n_1 \neq 0 \wedge n_2 \neq 0}{M \vdash e_1 \&\& e_2 \Rightarrow 1} \\ \frac{M \vdash e \Rightarrow 0}{M \vdash !e \Rightarrow 1} \quad \frac{M \vdash e \Rightarrow n \quad n \neq 0}{M \vdash !e \Rightarrow 0} \end{array}$$

Syntax of T

program \rightarrow *LabeledInstruction**

LabeledInstruction \rightarrow *Label* \times *Instruction*

Instruction \rightarrow skip
| $x = \text{alloc}(n)$
| $x = y \text{ bop } z$
| $x = y \text{ bop } n$
| $x = \text{uop } y$
| $x = y$
| $x = n$
| goto *L*
| if *x* goto *L*
| ifFalse *x* goto *L*
| $x = y[i]$
| $x[i] = y$
| read *x*
| write *x*

bop \rightarrow + | - | * | / | > | >= | < | <= | == | && | ||

uop \rightarrow - | !

Semantic Domain of T

A T program is executed under the following semantic domain (the same with that of S).

$$\begin{aligned}m \in Mem &= Loc \rightarrow Value \\l \in Loc &= Var + Addr \times Offset \\v \in Value &= \mathbb{N} + Addr \times Size \\a \in Addr &= \text{MemoryAddress} \\Offset &= \mathbb{N} \\Size &= \mathbb{N}\end{aligned}$$

Semantics of T

$$\overline{M \vdash \text{skip} \Rightarrow M}$$

$$\frac{(l, 0), \dots, (l, s-1) \notin \text{Dom}(M)}{M \vdash x = \text{alloc}(n) \Rightarrow M[x \mapsto (l, s), (l, 0) \mapsto 0, (l, 1) \mapsto 1, \dots, (l, s-1) \mapsto 0]}$$

$$\overline{M \vdash x = y \text{ bop } z \Rightarrow M[x \mapsto M(y) \text{ bop } M(z)]}$$

$$\overline{M \vdash x = y \text{ bop } n \Rightarrow M[x \mapsto M(y) \text{ bop } n]}$$

$$\overline{M \vdash x = \text{uop } y \Rightarrow M[x \mapsto \text{uop } M(y)]}$$

$$\overline{M \vdash x = y \Rightarrow M[x \mapsto M(y)]} \quad \overline{M \vdash x = n \Rightarrow M[x \mapsto n]}$$

$$\overline{M \vdash \text{goto } L \Rightarrow M} \quad \overline{M \vdash \text{if } x \text{ goto } L \Rightarrow M} \quad \overline{M \vdash \text{ifFalse } x \text{ goto } L \Rightarrow M}$$

$$\frac{M(y) = (l, s) \quad M(i) = n \quad 0 \leq n \wedge n < s}{M \vdash x = y[i] \Rightarrow M[x \mapsto M((l, n))]}$$

$$\frac{M(x) = (l, s) \quad M(i) = n \quad 0 \leq n \wedge n < s}{M \vdash x[i] = y \Rightarrow M[(l, n) \mapsto M(y)]}$$

$$\overline{M \vdash \text{read } x \Rightarrow M[x \mapsto n]} \quad \overline{M \vdash \text{write } x \Rightarrow M} \quad M(x) = n$$

Execution of a T Program

- ① Set *instr* to the first instruction of the program.
- ② Set the initial memory state M to the empty mapping, i.e., $M = []$.
- ③ Repeat:
 - ① If *instr* is HALT, terminate the execution.
 - ② Update M by M' such that $M \vdash instr \Rightarrow M'$
 - ③ Update *instr* by the next instruction.
 - ★ When the current instruction is goto L, if x goto L, or ifFalse x goto L, the next instruction is L.
 - ★ Otherwise, the next instruction is what immediately follows.

Translation of Expressions

$$\mathbf{trans}_e : e \rightarrow \mathit{Var} \times \mathit{LabeledInstruction}^*$$

$$\begin{aligned}\mathbf{trans}_e(n) &= (t, [t = n]) && \dots \text{new } t \\ \mathbf{trans}_e(x) &= (t, [t = x]) && \dots \text{new } t \\ \mathbf{trans}_e(x[e]) &= \text{let } (t_1, \mathit{code}) = \mathbf{trans}_e(e) && \\ &\quad \text{in } (t_2, \mathit{code}@[t_2 = x[t_1]]) && \dots \text{new } t_2 \\ \mathbf{trans}_e(e_1 + e_2) &= \text{let } (t_1, \mathit{code}_1) = \mathbf{trans}_e(e_1) && \\ &\quad \text{let } (t_2, \mathit{code}_2) = \mathbf{trans}_e(e_2) && \\ &\quad \text{in } (t_3, \mathit{code}_1@\mathit{code}_2@[t_3 = t_1 + t_2]) && \dots \text{new } t_3 \\ \mathbf{trans}_e(-e) &= \text{let } (t_1, \mathit{code}_1) = \mathbf{trans}_e(e) && \\ &\quad \text{in } (t_2, \mathit{code}_1@[t_2 = -t_1]) && \dots \text{new } t_2\end{aligned}$$

The first component (e.g., t) in the output is a temporary variable that stores the value of an original expression.

cf) We omit instruction labels if they are not relevant to the discussion.

Examples

- $2 \Rightarrow t = 2$, where t holds the value of the expression (label is omitted)
- $x \Rightarrow t = x$
- $x[1] \Rightarrow t1 = 1, t2 = x[t1]$
- $2+3 \Rightarrow t1 = 2, t2 = 3, t3 = t1 + t2$
- $-5 \Rightarrow t1 = 5, t2 = -t1$
- $(x+1)+y[2] \Rightarrow t1=x, t2=1, t3=t1+t2, t4=2, t5=y[t4], t6=t3+t5$

Translation of Statements

$\mathbf{trans}_s : stmt \rightarrow LabeledInstruction^*$

$\mathbf{trans}_s(x = e) = \text{let } (t_1, code_1) = \mathbf{trans}_e(e) \\ code_1@[x = t_1]$

$\mathbf{trans}_s(x[e_1] = e_2) = \text{let } (t_1, code_1) = \mathbf{trans}_e(e_1) \\ \text{let } (t_2, code_2) = \mathbf{trans}_e(e_2) \\ \text{in } code_1@code_2@[x[t_1] = t_2]$

$\mathbf{trans}_s(\text{read } x) = [\text{read } x]$

$\mathbf{trans}_s(\text{print } e) = \text{let } (t_1, code_1) = \mathbf{trans}_e(e) \\ \text{in } code_1@[write t_1]$

Translation of Statements

$\text{trans}_s(\text{if } e \text{ stmt}_1 \text{ stmt}_2) =$

let $(t_1, \text{code}_1) = \text{trans}_e(e)$
let $\text{code}_t = \text{trans}_s(\text{stmt}_1)$
let $\text{code}_f = \text{trans}_s(\text{stmt}_2)$
in $\text{code}_1 @$ $\dots \text{new } l_t, l_f, l_x$
 $[\text{if } t_1 \text{ goto } l_t] @$
 $[\text{goto } l_f] @$
 $[(l_t, \text{skip})] @$
 $\text{code}_t @$
 $[\text{goto } l_x] @$
 $[(l_f, \text{skip})] @$
 $\text{code}_f @$
 $[\text{goto } l_x] @$
 $[(l_x, \text{skip})]$

Translation of Statements

trans_s(while *e stmt*) =

let (*t*₁, *code*₁) = **trans_e**(*e*)

let *code*_{*b*} = **trans_s**(*stmt*)

in [(*l*_{*e*}, skip)]@

*code*₁@

[ifFalse *t*₁ *l*_{*x*}]@

*code*_{*b*}@

[goto *l*_{*e*}]@

[(*l*_{*x*}, skip)]

... new *l*_{*e*}, *l*_{*x*}

trans_s(do *stmt* while *e*) =

trans_s(*stmt*)@**trans_s**(while *e stmt*)

Declarations and Block (Program)

Declarations:

$$\begin{aligned}\mathbf{trans}_d(\text{int } x) &= [x = 0] \\ \mathbf{trans}_d(\text{int}[n] x) &= [x = \text{alloc}(n)]\end{aligned}$$

Blocks:

$$\begin{aligned}\mathbf{trans}_b(d_1, \dots, d_n \ s_1, \dots, s_m) = \\ \mathbf{trans}_d(d_1) @ \dots @ \mathbf{trans}_d(d_n) @ \mathbf{trans}_s(s_1) @ \dots @ \mathbf{trans}_s(s_m)\end{aligned}$$

Examples

- $x=1+2 \Rightarrow t_1 = 1; t_2 = 2; x = t_1 + t_2$
- $x[1]=2 \Rightarrow t_1 = 1; t_2 = 2; x[t_1] = t_2$
- `if (1) x=1; else x=2; \Rightarrow`
- `while (x<10) x++; \Rightarrow`

Summary

Define a translation procedure that converts a S program into a semantically equivalent T program.

- Defined a source language (S) and a target language (T).
 - ▶ syntax (concrete syntax, abstract syntax), semantics
- Defined an automatic translation procedure from S to T .
 - ▶ Key principle: every automatic translation from language S to T is done *recursively* on the structure of the source language S .