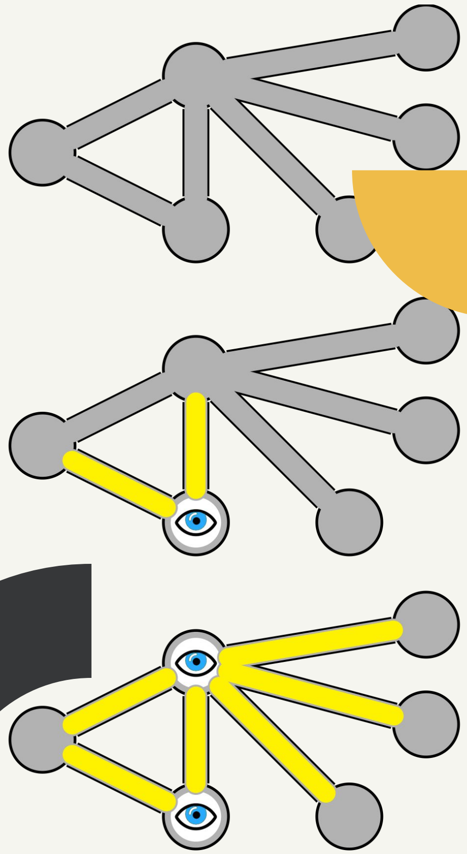
Vertex Cover

UNDER COVERING AND PARTITIONING

(GRAPH THEORY)



1

introduction

PRELIMINARIES and BACKGROUND OF THE PROBLEM

Important Terms

GRAPH - is a mathematical representation of a set of points, called nodes or vertices, which are interconnected by a set of lines called edges.

VERTICES/NODES - one of the points on which the graph is defined and which may be connected by graph edges.

EDGE - is one of the connections between the nodes (or vertices) of the network.

- **Undirected graphs** have edges that do not have a direction. The edges indicate a two-way relationship, in that each edge can be traversed in both directions.
- **Directed graphs** have edges with direction. The edges indicate a one-way relationship, in that each edge can only be traversed in a single direction.
- Graph edges sometimes have **Weights**, which indicate the strength (or some other attribute) of each connection between the nodes.

Background of the Problem

WHAT IS A VERTEX COVER?

- A vertex cover of a graph is a set of vertices that includes at least one endpoint of every edge of the graph.
- <u>FORMAL DEFINITION</u>: A vertex cover is a subset V' of the vertices of graph $G = (V,E)$ such that for every edge $(u,v) \in E$, either $u \in V'$ or $v \in V'$.
- A **minimum vertex cover** is a vertex cover of smallest possible size.

PROPERTIES OF A VERTEX COVER

- The set of all vertices is a vertex cover.
- A set of vertices is a vertex cover if and only if its complement is an independent set.
- Consequently, the number of vertices of a graph is equal to its minimum vertex cover number plus the size of a maximum independent set (Gallai 1959).

Idea of Proof

VERTEX COVER IS NP

- We could verify any solution in polytime with a simple n^2 examination of all the edges for endpoint inclusion in the given vertex cover.
- For a graph of n vertices it can be proved in $O(n^2)$. Thus, Vertex Cover is NP.

VERTEX COVER IS NP-HARD

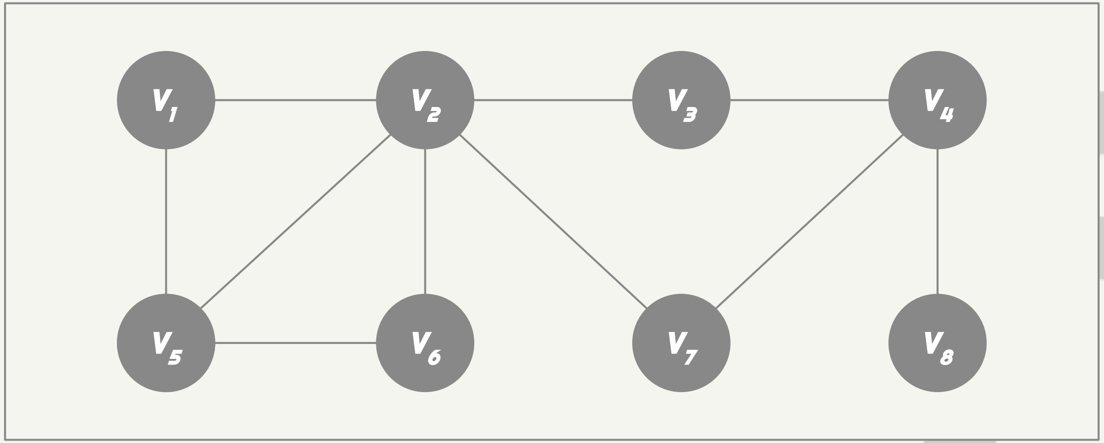
- Select a known NP-complete problem.
    - Independent Set (IS) and Clique problem are known NP-complete problems.
- Use IS or Clique to prove that Vertex Cover is NP-complete via Reduction.

2

illustration

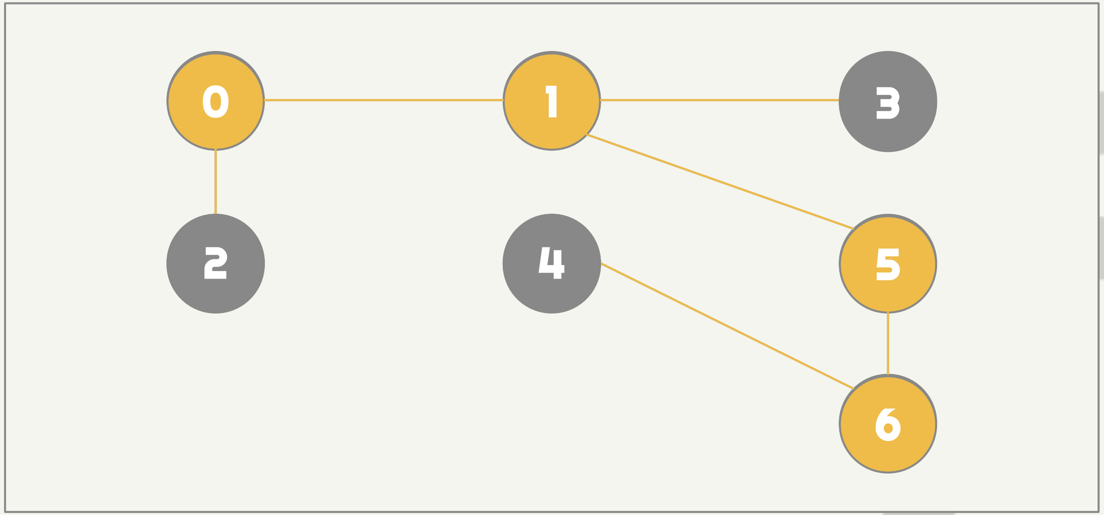
SAMPLE PROBLEM, EXAMPLES and APPLICATIONS

Illustration



- $\{v_1, v_3, v_5, v_6, v_7, v_8\}$ is a vertex cover.
- $\{v_2, v_4, v_5\}$ is a vertex cover.
- $\{v_2, v_3, v_8\}$ is **NOT** a vertex cover.

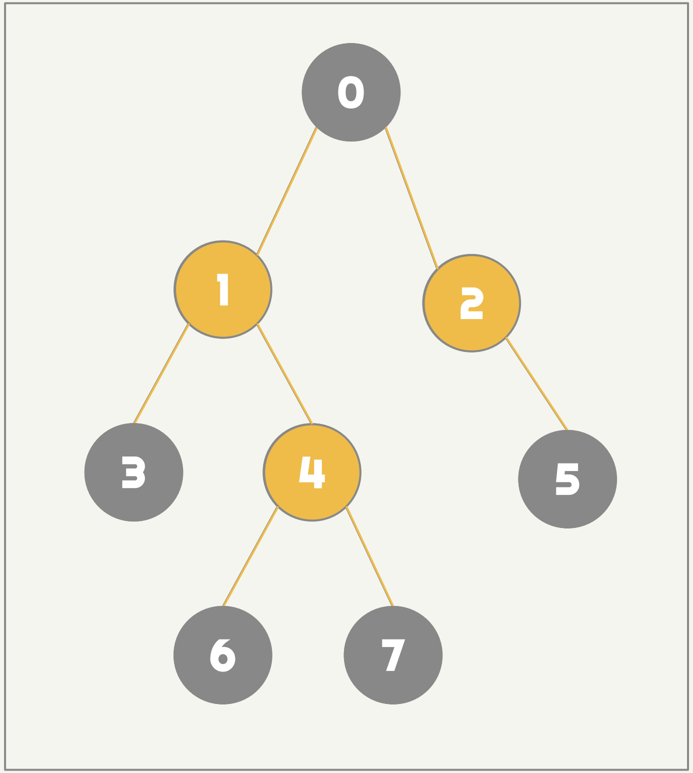
Example #1



The goal is to find the minimum vertex cover of the given graph.

MINIMUM VERTEX: *{0, 1, 5, 6}*

Example #2



For this example, the goal is to find the size of the minimal vertex cover.

MINIMUM VERTEX: *{1, 2, 4}*
Thus, the size of the smallest vertex cover is:
Three (3)

Applications of Vertex Cover

1

Biology

Has seen potential in some fields of computational biology, synthetic biology, and metabolic engineering (elimination of repetitive DNA sequences model)

2

Security

a commercial establishment interested in installing the fewest possible closed circuit cameras covering all hallways (edges) connecting all rooms (nodes) on a floor

2

demonstration

CODE DEMONSTRATION (answer to examples)

References

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- https://cstheory.stackexchange.com/questions/6670/vertex-cover-applications-in-the-real-world

Thank you!

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CMSC 142 MP - Vertex Cover