THE HONG KONG POLYTECHNIC UNIVERSITY 香港理工大學

PolyU 理大商學院 Business School
Innovation-driven Education and Scholarship

School of ACCOUNTING & FINANCE 會計及金融學院

Week 7: Introduction to Risk Measures and Measuring Algorithms Performance – Part 1

AF3214 Python Programming for Accounting and Finance

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R508, 8:30 am – 11:20 am, Wednesdays, Semester 2, AY 2024-25

Motivation

The purpose of these two weeks:

- to introduce risk and return; and
- how to quantify risk more analytically and put it all together in the very basics of modern portfolio theory;
- show you how to measure risk and return of portfolios using Python

Next three slides turn to some statistical background to talk about risk and return. To give you the measures that we're going to use for capturing risk and return.

Then apply it to stocks, and get a sense of what kinds of anomalies are out there that we should be aware of.

Eventually, take these measures to come up with the –
the cost of capital, the required rate of return, the risk-adjusted rate of return.

Statistical Background

Terminology

$$Realized\ Return\ R_{it} = \frac{D_{it}}{P_{it-1}} + \frac{P_{it} - P_{it-1}}{P_{it-1}} = \frac{D_{it} + P_{it}}{P_{it-1}} - 1$$

Dividend

dividend yield + cap gain

$Expected\ Return = E[R_{it}]$ ← Weighted average of its individual security's returns. Based on historical returns.

$Excess\ Return = R_{it} - r_f$ ← We're going to be looking at excess returns, which is in excess of the net risk-free rate

$Risk\ Premium = E[R_{it}] - r_f$ ← The average rate of return of a risky security minus the risk-free rate

So the excess return you can think of as a realization of that risk premium.

But on average over a long period of time, the number that we're going to be concerned with most is this risk premium number.

Statistical Background

Over the course of the last 100 years, US equity markets have provided an average rate of return minus the risk-free rate on the order of 7%.

Good? But that's a long-run average. So the technique for talking about the statistical aspects of using the expected rate of return.

- **Mean, variance, standard deviation:**

$$\mu_i = E[R_{it}] \qquad \sigma_i^2 = E[(R_{it} - \mu_i)^2] \qquad \sigma_i = \sqrt{\sigma_i^2}$$

expected rate of return

expected value of the squared excess return. fluctuation around the mean

the riskiness of returns use variance and standard deviation to proxy for

- **Sample estimators:**

$$\hat{\mu}_i = \frac{1}{T}\sum_{t=1}^{T} R_{it} \qquad \hat{\sigma}_i^2 = \frac{1}{T-1}\sum_{t=1}^{T}(R_{it} - \hat{\mu}_i)^2 \qquad \hat{\sigma}_i = \sqrt{\hat{\sigma}_i^2}$$

Statistical Background - Other Statistics

- **Median:** 50th percentile (probability of 1/2 that R_t< median)
- **Skewness** - which way the distribution leans
  - Is the distribution symmetric?
  - Negative: big losses are more likely than big gains
  - Positive: big gains are more likely than big losses
- **Correlation:** How closely do two variables move together?

Covariance(-∞ to +∞): $Cov[R_{it}, R_{jt}] = E[(R_{it} - \mu_i)(R_{jt} - \mu_j)]$

a systematic relationship between two random variables

Correlation [-1,1]: $Corr[R_{it}, R_{jt}] = \frac{E[(R_{it} - \mu_i)(R_{jt} - \mu_j)]}{\sigma_i \sigma_i}$

degree to which two random variables move in sequence

In our case, how closely do the returns of two investments move together.

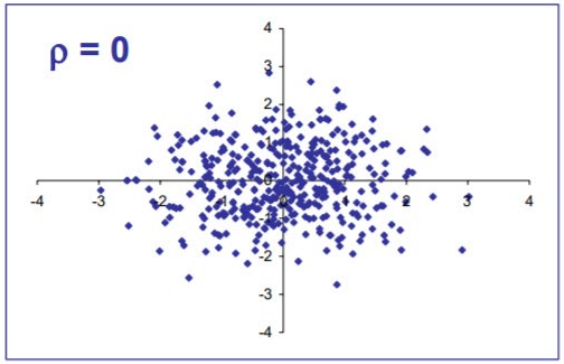
Correlation is a statistic that's a number between -1 and 1, that measures the degree of association between these two securities. If a new investment is either zero correlated or negatively correlated with your current portfolio, that's going to help in terms of dampening your fluctuations. But if two investments move at the same time, that's not only going to not help, that's going to actually add to your risks.

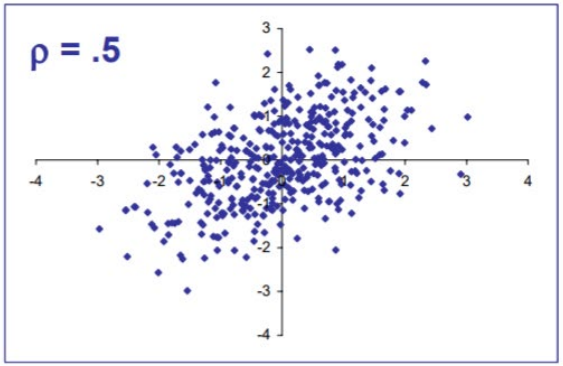
Correlation

An example here about what correlation looks like.

Examples of Correlation Between Two Random Variables

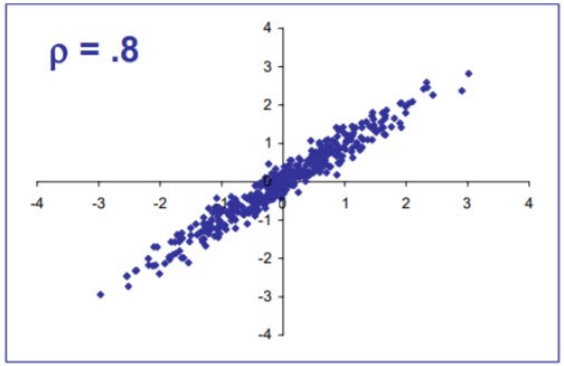
No correlation

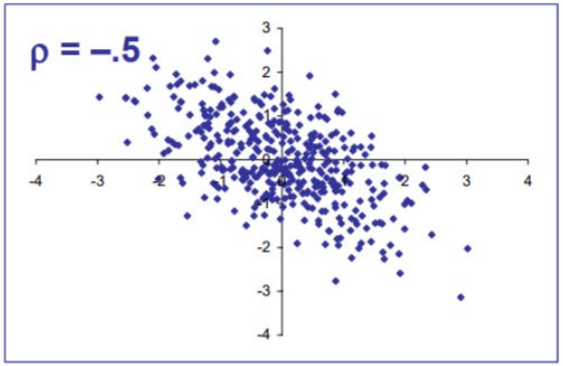




positive correlation 50%

high positive correlation 80%





negative correlation -50%

- Plotted four different scatter graphs of the return of one asset on the x-axis and the return of another asset on the y-axis.
- Dots represent those pairs of returns for different assumptions about correlation.

Empirical Properties of Stock Returns

We're going to rely on markets for information, because it's the wisdom of crowds that really gets us the information we need in order to make good financial decisions.

We're going to talk about how to quantify risk more analytically and put it all together in the very basics of modern portfolio theory.

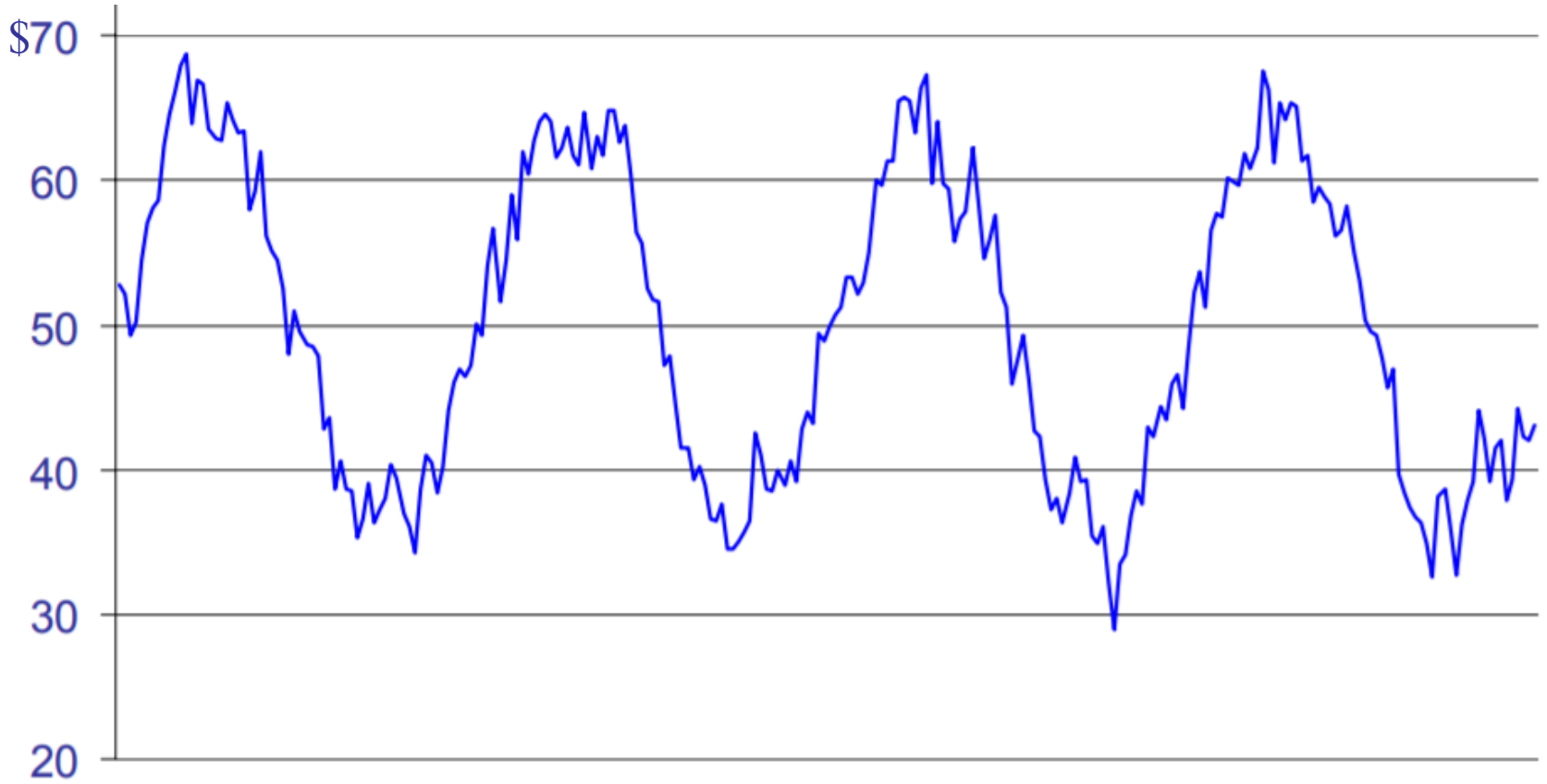
What Properties Should Stock Prices Have in "*Efficient*" Markets?

- Random, unpredictable
- Prices should react quickly and correctly to new information
- Investors cannot earn abnormal, risk-adjusted returns (in other words, once risk adjustment is taken into account, there shouldn't be any additional return left over.)

That's what we think of as a well-functioning & highly competitive market.

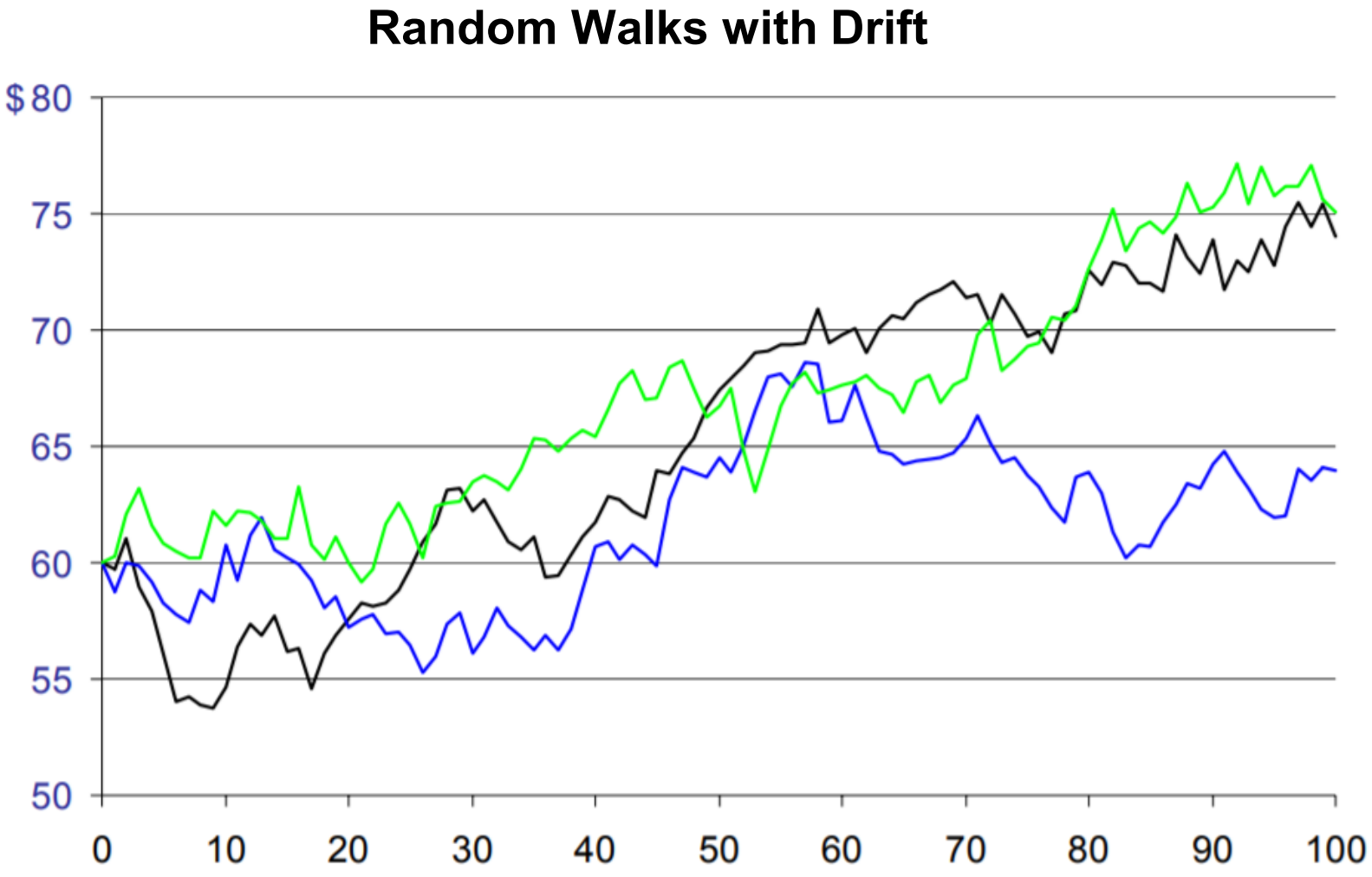
Empirical Properties of Stock Returns – Cont'd

Predictable Price Changes



Can you come up with a prediction for this?
How do you predict the behavior of this kind of a stock market?
What kind of curve would you fit to this?
Is this a good model for a market?

Empirical Properties of Stock Returns – Cont'd

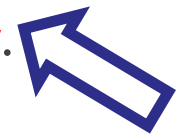


Drift meaning positive trends or negative trends

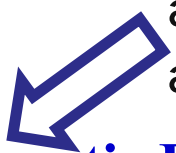
Categories of Risk

Broadly speaking, there are two main categories of risk: systematic and unsystematic.

- **Systematic Risk** – *Market Risks / Undiversifiable Risk / Volatility Risk* : these are risks that affect the overall market of the security. It is unpredictable and undiversifiable. The risk can be mitigated through *hedging*.



1. portfolio construction, 2. options, and 3. volatility indicators



asset-specific or company-specific uncertainty that can affect the performance of an investment.

- **Unsystematic Risk** – The risks that are associated with a company or sector. These are known as diversifiable risks and can be mitigated through *asset diversification*.

Common Risk Measures

Standard Deviation - it measures the **dispersion** of data from its mean. In our case, it is used in making an investment decision to measure the amount of historical **volatility** associated with an investment relative to its annual rate of return.

Beta (*β*) – beta measures the amount of **systematic risk** an individual security has relative to the whole market (e.g., S&P 500).

- The market has a beta of 1
- A security with a beta>1(<1) indicates that it is more(less) volatile than the stock market.

$$\textit{Beta coefficient } (\beta) = \frac{cov(R_s, R_m)}{var(R_m)}$$

R_s = the return on an individual stock
R_m = the return on the overall market

Alpha (*α*) – the active return on an investment, or an excess return (abnormal rate of return) above the benchmark.
or the portion of the excess return that is NOT explained by systematic risk

Beta (*β*)

Common Risk Measures

Beta as a Measure of Risk

Selected Betas and Associated Interpretations

Beta	Comment	Interpretation
2.0	Move in same direction as the market	Twice as responsive as the market
1.0		Same response as the market
0.5		One half as responsive as the market
0.0		Unaffected by market movement
−0.5	Move in opposite direction of the market	One-half as responsive as the market
−1.0		Same response as the market
−2.0		Twice as responsive as the market

- Higher stock betas should result in higher expected returns due to greater risk
- If the market is expected to increase 10%, a stock with a beta of 1.50 is expected to increase 15%
- If the market went down 8%, then a stock with a beta of 0.50 should only decrease by about 4%

Beta values for specific stocks can be obtained from Value Line reports or websites such as www.yahoo.com.

Relationship of Risk and Performance

- The risk-return tradeoff states that the potential return rises with an increase in risk.
- High Risk gives you the possibility of High Returns
- Investors often use the risk-return tradeoff as one of the essential components of each investment decision, and it is also used to assess the portfolio.
- So why is it best to use a portfolio of assets?
- The goal of any investment strategy is to maximize returns while at the same time minimizing risk.

Don't put all your eggs in one basket

Portfolios

What is a Portfolio?

- A portfolio is simply a specific combination of securities, usually defined by portfolio weights that sum to 1.
- Such as *gold, stocks, funds, derivatives, property, bonds*, etc.

To develop a profitable portfolio, it is essential to become familiar with its fundamentals and the factors that influence it.

- Portfolio Trading is not just about multi-tasking.
- Significant risk reductions can be achieved by tracking and managing portfolio risk

Portfolio weights can sum to 0 (dollar-neutral portfolios)
Weights can be positive (long positions) or negative (short positions).

Portfolios – Cont'd

- n assets, each with share price P_i $(i = 1, 2, \ldots n)$
- A portfolio is a collection of N_i shares of each asset i
- Total value of portfolio:

real number, e.g., 0, 100, 500, -200

$$V = N_1P_1 + N_2P_2 + \cdots + N_nP_n = \sum_{i=1}^{n} N_iP_i$$

$N_i = -200$?

shorts would be able to buffer some of the losses on the long side.

- A typical portfolio has $V > 0$. Define portfolio weights:

$$w_i = \frac{N_iP_i}{N_1P_1 + N_2P_2 + \cdots + N_nP_n} = \frac{N_iP_i}{V}$$

- A portfolio can then also be defined by its asset weights

$$\{w_1, w_2, \ldots, w_n\}, \quad w_1 + w_2 + \cdots + w_n = 1$$

portfolio weights that sum to 1

Portfolios – Cont'd

Example 1. Your investment account of $100,000 consists of three stocks: 200 shares of stock A, 1,000 shares of stock B, and 750 shares of stock C. Your portfolio is summarized by the following weights:

Asset	Shares	Price/Share	Dollar Investment	Portfolio Weight
A	200	$50	$10,000	10%
B	1,000	$60	$60,000	60%
C	750	$40	$30,000	30%
Total			$100,000	100%

Pretty straightforward how you get those weights

Portfolios – Cont'd

Example 2. You decide to purchase a home that costs $500,000 by paying 20% of the purchase price and getting a mortgage for the remaining 80%. What are your portfolio weights for this investment?

Asset	Shares	Price/Share	Dollar Investment	Portfolio Weight
Home	1	$500,000	$500,000	500%
Mortgage	1	-$400,000	-$400,000	-400%
Total			$100,000	100%

down payment

The weight of a particular asset in a portfolio can be negative

A liability such as a loan or a short position

Q: what happens if your home price declines by 10%?

Portfolios – Cont'd

Question: A portfolio contains a cash account holding US$2,000 at the beginning of the period. The same portfolio also contains a US$1,000 loan at the start of the period. The net value of the portfolio at the beginning of the period is US$2,000 - US$1,000 = US$1,000.

At the end of the period, 1 percent interest has accrued on the cash account, and 5 percent has accrued on the loan. There have been no transactions over the period.

So what is the weight for both accounts and what is the portfolio return?

Portfolios – Cont'd

Types of Portfolio

1. Income portfolio - securing a steady flow of income. Such as stocks that generate regular dividends, rather than a track of price appreciation.

2. Growth portfolio - parks money into growth stocks of a company who are in their active growth stage. May subject to higher risks.

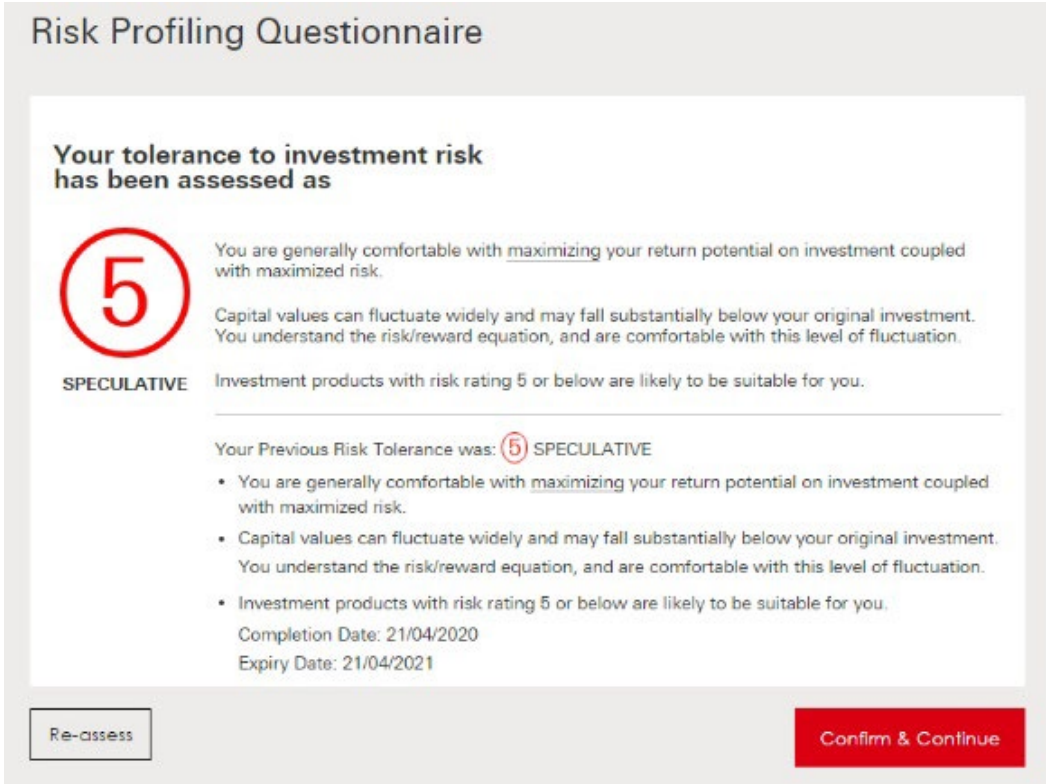
3. Value portfolio - puts money into cheap assets in valuation and focuses on securing bargains in the investment market.

During economic recession, investors look for profitable firms whose shares are priced lower than their fair value. When the market revives, value portfolio holders generate substantial earnings.

Portfolios – Cont'd

Factors that Affect Portfolio Allocation?

1. Risk Tolerance ⇨



2. Time horizon – when to put money on a particular investment option is crucial for building a profitable portfolio

Variance

$$S^2 = \frac{\sum (x_i - \bar{x})^2}{n - 1}$$

S^2 = sample variance

x_i = the value of the one observation

$\bar{x}$ = the mean value of all observations

n = the number of observations

To assess the degree of dispersion of the returns of a portfolio.

Portfolio Variance:

$$w_1{}^2\sigma_1{}^2 + w_2{}^2\sigma_2{}^2 + 2w_1w_2Cov_{1,2}$$

- Variance is a statistical measure of how much observations *differ* from each other.
- In accounting and finance, variance is a measure of *volatility*, and it measures how much a stock tends to deviate from its mean. *Or the sense of fluctuation around the mean.*
- The higher the variance, the more wildly the stock fluctuates.

Standard Deviation

$$s = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N - 1}}$$

- Standard deviation is a measure of how much an investment's returns can vary from its average return.
- The smaller an investment's standard deviation, the less volatile it is.
- Standard deviation help traders quantify certain outcomes.
- implied volatility (IV) refers to the implied magnitude, or one standard deviation range, of potential movement ***away from the stock price*** in a year's time.
- a low implied volatility means market is not expecting the stock price to move much away from the current stock price

So What's the Difference

$$S^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} \qquad s = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N-1}}$$

Both concepts are of paramount importance as they are used to measure security and market ***volatility***.

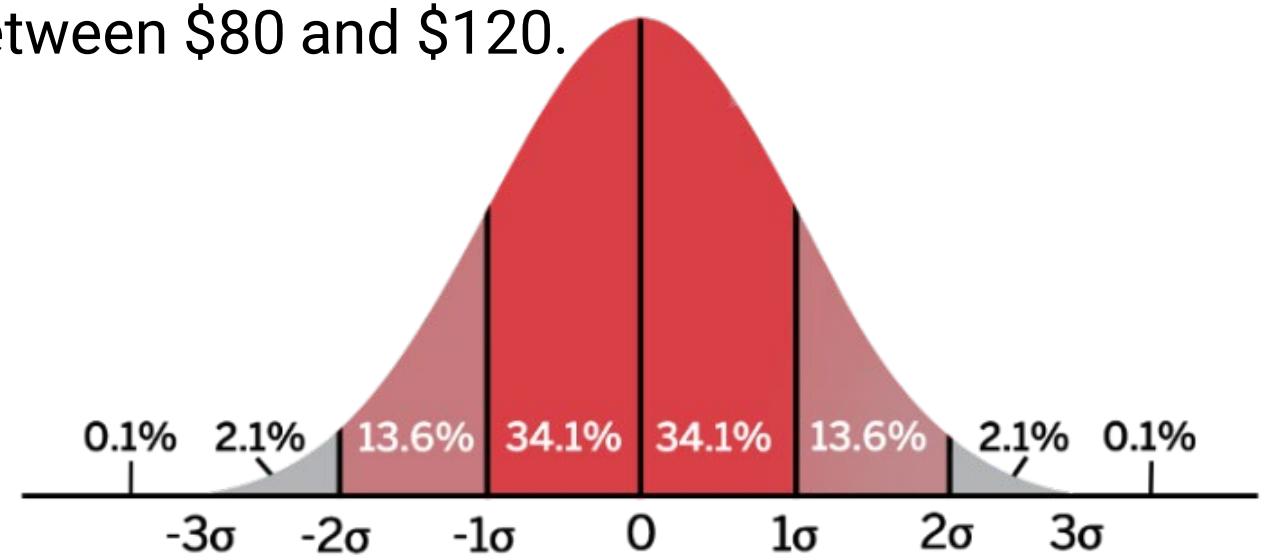
The calculation of variance uses squares because it *weighs outliers* more *heavily* than data closer to the mean.

Standard deviation looks at how far from the mean a group of numbers is. It is one of the key methods that analysts, portfolio managers, and advisors use *to determine risk*.

When the group of numbers is *closer* to the mean, the investment is *less risky*; when the group of numbers is *far away* from the mean, the investment is of *greater risk* to an investor.

Standard Deviation – Cont'd

For example, the 1SD expected move of a $100 stock with an IV% of 20% is between +- $20 of the current stock price, or a range between $80 and $120.



- **One standard deviation** of a stock encompasses approximately **68.2%** of outcomes in a distribution of occurrences based on current implied volatility.
- **Two standard deviations** of a stock encompasses approximately **95.4%** of outcomes in a distribution of occurrences based on current implied volatility.
- **Three standard deviations** of a stock encompasses approximately **99.7%** of outcomes in a distribution of occurrences based on current implied volatility.

https://www.tastylive.com/concepts-strategies/standard-deviation

What is the difference?

- While both terms are related, volatility refers to the degree of variation in an asset's price over time, whereas standard deviation quantifies this variation statistically.
- The former is the broader concept, encompassing the overall fluctuations.
- While the latter provides a precise numerical measure of these fluctuations, offering traders a clearer understanding of market behavior and risk.

Covariance

$$cov_{x,y} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{N - 1}$$

$cov_{x,y}$ = covariance between variable a and y

x_i = data value of x

y_i = data value of y

$\bar{x}$ = mean of x

$\bar{y}$ = mean of y

N = number of data values

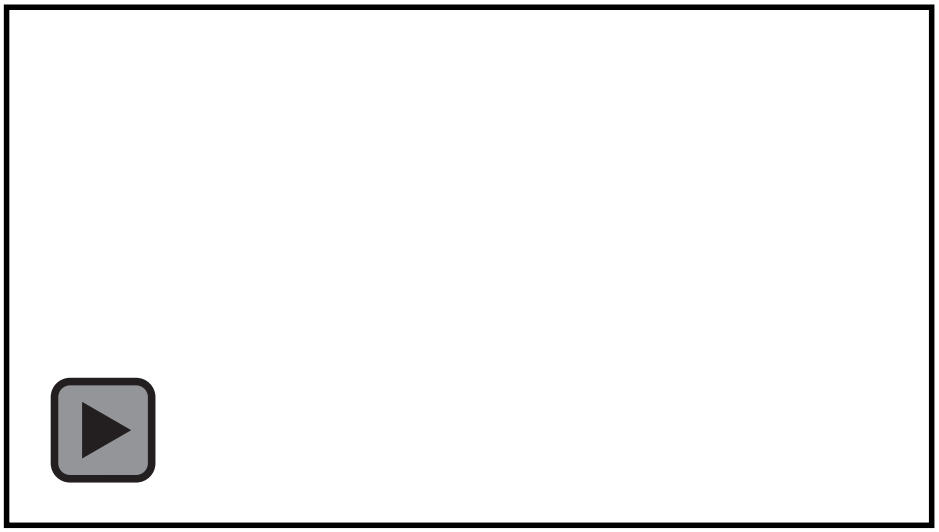
- Covariance measures the *directional relationship* between the returns on two assets.
- It is a statistical tool that is used to determine the *relationship* between the *movement of two assets prices*.
- When two stocks tend to move together, they are having a positive covariance; when they move inversely, the covariance is negative.
- Risk and volatility can be **reduced** in a portfolio by paring assets that have a **negative** covariance. ← Why? -> risk diversification

Correlation

Formula of Correlation:
(a.k.a Pearson correlation)

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}}$$

where
- r = correlation coefficient
- x_i = values of the x-variable in a sample
- $\bar{x}$ = mean of the values of the x-variable
- y_i = values of the y-variable in a sample
- $\bar{y}$ = mean of the values of the y-variable



- Correlation is a statistic that measures the degree to which two stocks move in relation to each other. correlation coefficient: [-1.0, +1.0].
- If they move together a lot, then we say that they're highly correlated. And in some cases, if they move in opposite directions, we say that they are negatively correlated.

Question: Is it good if the new investment is either zero correlated or negatively correlated with your current portfolio?

Anomalies

- **Size Effect**: Smaller stocks typically outperform larger stocks, especially in January.
- **January Effect**: Returns in January tend to be abnormally high.
- **Value Effect**: Low P/B (value) stocks typically outperform high P/B (growth) stocks.
- **Momentum**: Stocks with high returns over the past 12 months typically continue to outperform stocks with low past returns.
- **Accruals and Issuances**: Stocks with high past accruals and/or recent stock offerings typically underperform stocks with low past accruals and no stock offerings.

Short Summary

So we're going to use correlation, along with mean , standard deviation, and variance, to try to put together good collections of securities.

In other words, good portfolios of securities.

The End

Jupyter Notebook