- Maintain Academic Honesty. You can discuss with others but the solution should be yours.
- For the written assignment, write it in hand and submit a scanned version (pdf) on moodle. Make sure that your scan is readable so that it can be corrected easily.
- **Simulation:** Submit solutions for simulation problems on Colab using python.

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1. [5 points] The system function of a LTI system has the pole-zero plot shown in Fig. 1. Specify whether each of the following statements is true, false, or cannot be determined from the information given.
   (a) The system is causal.
   (b) The system is stable.
   (c) If the system is causal, then it must be stable.
   (d) If the system is stable, then it must have a two-sided impulse response.

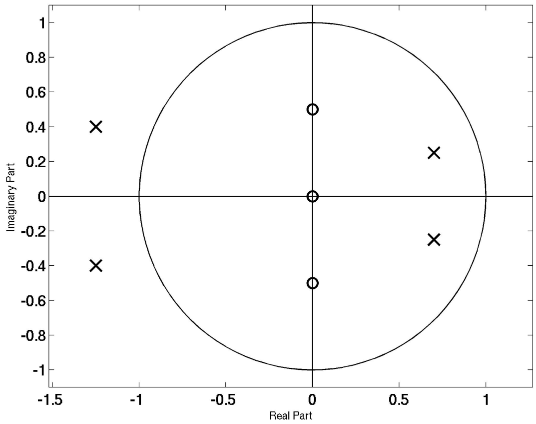

Figure 1: Pole-zero plot for question 1.

2. [5 points] A causal LTI system has system function

$$H(z) = \frac{(1 - 0.5z^{-1})(1 + 4z^{-2})}{1 - 0.64z^{-2}}.$$

   (a) Find expressions for a minimum-phase system $H_1(z)$ and an all-pass system $H_{ap}(z)$ such that

$$H(z) = H_1(z)H_{ap}(z).$$

   (b) Find expressions for a different minimum-phase system $H_2(z)$ and a generalized linear-phase FIR system $H_{lin}(z)$ such that

$$H(z) = H_2(z)H_{lin}(z).$$

3. [5 points] The following three things are known about a signal $x[n]$ with z-transform $X(z)$:
   - $x[n]$ is real valued and minimum-phase
   - $x[n]$ is zero outside the interval $0 \leq n \leq 4$.
   - $X(z)$ has a zero at $z = 0.5e^{j\pi/4}$ and a zero at $z = 0.5e^{j3\pi/4}$.

   Based on this information, answer the following questions:
   (a) Is $X(z)$ rational ? Justify.
   (b) Sketch the complete pole-zero plot for $X(z)$ and specify its ROC.

(c) If $y[n] * x[n] = \delta[n]$ and $y[n]$ is right-sided, sketch the pole-zero plot for $Y(z)$ and specify its ROC.

4. [5 points] Let $h[n]$ and $H(z)$ denote the impulse response and system function of stable all-pass LTI system. Let $h_i[n]$ denote the impulse response of the (stable) LTI inverse system. Assume that $h[n]$ is real. Show that $h_i[n] = h[-n]$.

5. [5 points] An LTI system has generalized linear phase and system function

$$H(z) = a + bz^{-1} + cz^{-2}.$$

The impulse response has unit energy, $a \geq 0$, and $H(e^{j\pi} = H(e^{jo} = 0$.

(a) Determine the impulse response $h[n]$.

(b) Plot $|H(e^{j\omega})|$.

6. Simulation: Consider a causal LTI system described by the following transfer function,

$$H(z) = \frac{\frac{1}{6} + \frac{1}{2}z^{-1} + \frac{1}{2}z^{-2} + \frac{1}{6}z^{-3}}{1 + \frac{1}{3}z^{-2}}$$

(a) Sketch the magnitude response $H(e^{j\omega})$ (linear scale, not dB) from the z-transform. What type of filter is $H(z)$?

(b) Sketch the pole-zero plot. Is the system stable ?

Now consider the following length-128 input signal,

$$x[n] = \begin{cases} 0 & n = 1, \ldots, 50 \\ 1 & n = 51, \ldots, 128. \end{cases} \tag{1}$$

(c) Plot the magnitude of $X(e^{j\omega})$.

(d) We want to filter $x[n]$ with $H(z)$ to obtain $y[n]$. Compute and plot $y[n]$ using the Matlab function $filter$. Plot the magnitude of $Y(e^{j\omega})$. Can you think of other ways of realizing this filtering operation ?

(e) Explain qualitatively the form of $y[n]$.

7. Simulation: Look at the example plot that shows the group delay when a Gaussian pulse is filtered using a rectangular window here `https://www.ee.iitb.ac.in/~akumar/courses/ee603-2020/sampling.html#group-delay-and-linear-phase`. Now, modify the example in the following way.

(a) Construct an ideal low-pass filter with cut-off $\frac{\pi}{2}$ and truncate it to keep the middle 201 points (i.e., take $\sin(\frac{\pi}{2}n)/\pi n$ for $n \in -100, -99, \ldots, 99, 100$. Use this to filter a Gaussian pulse. Where is the new peak ? How much is the delay ?

(b) Repeat for cut-off frequencies $\frac{pi}{4}$ and $\frac{3\pi}{4}$. Where are the peaks ?

(c) Transform the filter considered in part (a) to a high-pass filter by multiplying the coefficients by $(-1)^n$. Use this to filter the Gaussian pulse. What do you observe ?