- Maintain Academic Honesty. You can discuss with others but the solution should be yours.
- For the written assignment, write it in hand and submit a scanned version (pdf) on moodle. Make sure that your scan is readable so that it can be corrected easily.
- **Simulation:** Submit solutions for simulation problems on Colab using python. Due date for this is 28 Aug. 2021

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1. [2 points] What is the condition on ω_0 for $\cos(\omega_0 n)$ to be periodic ?
2. [3 points] The unit sample response of a linear shift-invariant system is

$$h[n] = (\frac{1}{3})^n u[n].$$

Find the response of the system to

(a) the complex exponential $x[n] = e^{jn\pi/4}$

(b) $x[n] = (\frac{1}{6})^{n-6} u[n]$

3. [10 points] The DTFT pair

$$a^n u[n] \Leftrightarrow \frac{1}{1 - ae^{-j\omega}}, \quad |a| < 1$$

is given.

(a) Using the above equation, determine the DTFT, $X(e^{j\omega})$, of the sequence

$$x[n] = -b^n u[-n-1] = \begin{cases} -b^n, & n \leq -1 \\ 0, & \geq 0. \end{cases}$$

What restriction on b is necessary for the DTFT of $x[n]$ to exist?

(b) Determine the sequence $y[n]$ whose DTFT is

$$Y(e^{j\omega}) = \frac{2e^{-j\omega}}{1 + 2e^{-j\omega}}.$$

4. [5 points] Consider the LTI system with frequency response

$$H(e^{j\omega}) = e^{-j\left(\frac{\omega}{2} + \frac{\pi}{4}\right)}, \qquad -\pi \leq \omega < \pi$$

Determine $y[n]$, the output of the system, if the input to the system is

$$x[n] = \cos\left(\frac{15\pi n}{4} - \frac{\pi}{3}\right)$$

5. [15 points] Consider an ideal low-pass filter with impulse response $h_{\text{lp}}[n]$ and frequency response

$$H_{\text{lp}}(e^{j\omega}) = \begin{cases} 1, & |\omega| < 0.2\pi \\ 0, & 0.2\pi \leq |\omega| \leq \pi \end{cases}$$

(a) A new filter is designed by the equation $h_1[n] = (-1)^n h_{\text{lp}}[n] = e^{j\pi n} h_{\text{lp}}[n]$. Determine an equation for the frequency response of $H_1(e^{j\omega})$, and plot the equation for $|\omega| < \pi$. What kind of filter is this?

(b) A new filter is designed by the equation $h_2[n] = 2h_{\text{lp}}[n]\cos(0.5\pi n)$. Determine an equation for the frequency response of $H_2(e^{j\omega})$, and plot the equation for $|\omega| < \pi$. What kind of filter is this?

(c) A new filter is designed by the equation

$$h_3[n] = \frac{\sin(0.1\pi n)}{\pi n} h_{\text{lp}}[n]$$

Determine an equation for the frequency response of $H_3(e^{j\omega})$, and plot the equation for $|\omega| < \pi$. What kind of filter is this?

6. [5 points] A linear time-invariant system has frequency response

$$H(e^{j\omega}) = \begin{cases} e^{-j\omega 3}, & |\omega| < \frac{2\pi}{16}\left(\frac{3}{2}\right) \\ 0, & \frac{2\pi}{16}\left(\frac{3}{2}\right) \leq |\omega| \leq \pi. \end{cases}$$

The input to the system is a periodic unit-impulse train with period $N = 16$ i.e

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n + 16k]$$

Find the output of the system.

7. [10 points] The overall system in the dotted box in Fig. 1 can be shown to be linear and time invariant.

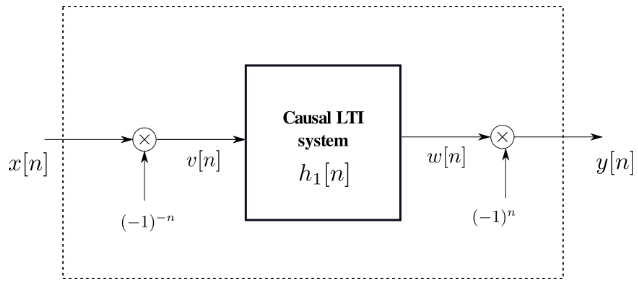

Figure 1: An example LTI system.

(a) Determine an expression for $H(e^{j\omega})$, the frequency response of the overall system from the input $x[n]$ to the output $y[n]$, in terms of $H_1(e^{j\omega})$, the frequency response of the internal LTI system. Remember that $(-1)^n = e^{j\pi n}$.

(b) Plot $H(e^{j\omega})$ for the case when the frequency response of the internal LTI system is

$$H_1(e^{j\omega}) = \begin{cases} 1, & |\omega| < \omega_c \\ 0, & \omega_c \leq |\omega| \leq \pi \end{cases}$$

8. Plot the following sequences for $n = -7, -6, \ldots, 6, 7$ using stem plots. Label the plots appropriately.
   (a) $\delta[n]$
   (b) $u[n]$
   (c) $\delta[n+1] - \delta[n-1]$
   (d) $(0.5)^n u[n]$
   (e) $(-0.5)^n u[n]$

9. Download the wave file from this URL: `https://upload.wikimedia.org/wikipedia/commons/b/b3/Mozart_Trio_from_Wind_Serenade_K388.wav` and open it using the `scipy.io.wavfile.read` function.
   (a) Plot the waveform using `Matplotlib`. Observe the variations of the waveform. What is the range of values that you observe ?

(b) Scale all values down by half; remember to keep the data types intact. Use `scipy.io.wavfile.write` to save the file and listen to it. What do you observe? Make more changes to the waveform and listen to see how it is affected.

10. Simulation: Write a program to generate a sinusoidal sequence

$$x[n] = A\sin(\omega_0 n + \phi),$$

and plot the sequence as a stem plot (lollipop like). The input data will be the desired length L, amplitude A, the angular frequency ω_0 (where $0 \le \omega_0 \le 2\pi$), and the phase ϕ (where $0 \le \phi \le 2\pi$) of the desired sequence.

(a) Generate sinusoidal sequences with $\omega_0 = 0.3\pi$ and $\omega_0 = 0.75\pi$. Determine the period of each sequence from the plot and verify the result theoretically.

(b) Generate and plot the sequence

$$x[n] = \sin(0.1\pi n + 0.75\pi) - 3\cos(0.8\pi n + 0.2\pi) + \cos(1.3\pi n).$$

What is its fundamental period ?

11. Simulation: Consider the following DT rectangular window,

$$w[n] = \begin{cases} 1 & -N \le n \le N \\ 0 & \text{otherwise.} \end{cases}$$

(a) Plot the DTFT (512 points in $-\pi \le \omega \le \pi$) of $w[n]$ for $N = 5, 20, 100$. Scale the DTFT so that $W(e^{j0}) = 1$. Label your plots. Describe the effect of increasing N on the DTFT. Plot the magnitude and phase of $W(e^{j\omega})$ in one plot and the real, imaginary parts in another.

(b) Repeat the previous part for the triangular window,

$$w_{\text{tri}}[n] = \left(1 - \frac{|n|}{N}\right) w[n].$$

(c) Mention the differences between the DTFTs in (a) and (b).