- Maintain Academic Honesty. You can discuss with others but the solution should be yours.
- For the written assignment, write it in hand and submit a scanned version (pdf) on moodle. Make sure that your scan is readable so that it can be corrected easily.
- **Simulation:** Submit solutions for simulation problems on Colab using python.

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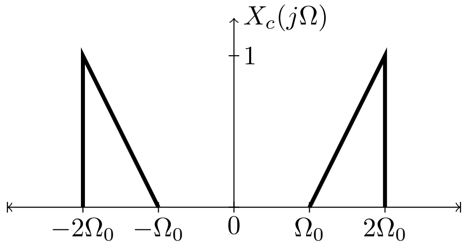
1. [10 points] Consider the continuous-time signal $x(t) = \cos(200\pi t)$, where t is measured in seconds. Answer the following questions:
   (a) [2 points] Sketch the continuous-time Fourier transform of this signal, marking all the salient axis values.
   (b) [2 points] What is the minimum sampling rate needed for this signal to be recovered from its samples without any distortion, if uniform sampling is performed?
   (c) [3 points] This signal is sampled at 150 Hz and then reconstructed using sinc interpolation using the reconstruction pulse $\text{sinc}(150t)$. What are the frequencies present in the reconstructed output?
   (d) [3 points] Continuing with the 150 Hz sampling and reconstruction approach, specify all frequencies f_0 such that, if we sample $\cos 2\pi f_0 t$ at 150 Hz and reconstruct using $\text{sinc}(150t)$, the same output frequencies as in (c) is obtained.

2. [5 points] Consider a real, continuous-time signal $x_c(t)$ with the following spectrum $X_c(j\Omega)$:
   (a) [1 point] What is the bandwidth of the signal ? What is the minimum sampling period in order to satisfy the sampling theorem ?



   (b) [2 points] Take a sampling period $T_s = \pi/\Omega_0$; clearly, with this sampling period, there will be aliasing. Plot the DTFT of the discrete-time signal $x_a[n] = x_c(nTs)$.
   (c) [2 points] Suggest a block diagram to reconstruct $x_c(t)$ from $x_a[n]$.

3. [5 points] Let $x(t) = \text{rect}(t)$, where t is measured in seconds. That is, $x(t) = 1$ for $t \in [-0.5, 0.5]$, and 0 otherwise. This signal is sampled at the following rates and then reconstructed as $\hat{x}(t)$ using a corresponding sinc reconstruction filter:
   (a) $f_s = 1$ Hz
   (b) $f_s = 10$ Hz
   (c) $f_s = 100$ Hz

   Plot the reconstructed output $\hat{x}(t)$ for each case above, and compute the sum squared error $\int_{-\infty}^{\infty} |x(t) - \hat{x}(t)|^2 dt$ in each case (you can approximate this using the `numpy.trapz` function). What is your observation? Explain.