- Maintain Academic Honesty. You can discuss with others but the solution should be yours.
- For the written assignment, write it in hand and submit a scanned version (pdf) on moodle. Make sure that your scan is readable so that it can be corrected easily.
- **Simulation:** Submit solutions for simulation problems on Colab using python.

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1. [5 points] Two sequences $x_1[n]$ and $x_2[n]$ are defined as follows.

$$x_1[n] = \begin{cases} 1, & 0 \leqslant n \leqslant 99, \\ 0, & \text{otherwise.} \end{cases} \quad \text{and}$$

$$x_2[n] = \begin{cases} 1, & 0 \leqslant n \leqslant 9, \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Determine and sketch the linear convolution $x_1[n] * x_2[n]$.
   (b) Determine and sketch the 100-pt circular convolution $x_1[n] \circledast_{100} x_2[n]$.
   (c) Determine and sketch the 110-pt circular convolution of $x_1[n] \circledast_{110} x_2[n]$.

2. [5 points] Find the N-point DFT of the sequence

$$x[n] = \cos(\omega_0 n), \quad 0 \leqslant n \leqslant N-1.$$

   Compare the DFT coefficients obtained when $\omega_0 = \frac{2\pi k_0}{N}$ with those obtained when $\omega_0 \neq \frac{2\pi k_0}{N}$.

3. [5 points] A sequence $x[n]$ of length 8 has as its 8-pt DFT $X[k]$.
   (a) A new sequence $y[n]$ of length 16 is defined as

$$y[n] = \begin{cases} x[n/2] & n \text{ is even} \\ 0 & n \text{ is odd .} \end{cases}$$

   Find $Y[k]$ in terms of $X[k]$.
   (b) A new sequence $z[n]$ of length 16 is defined as

$$z[n] = \begin{cases} x[n] & n = 0, \ldots, 7 \\ x[n-8] & n = 8, \ldots, 15. \end{cases}$$

   Find $Z[k]$ in terms of $X[k]$.

4. [5 points] The unit sample response of a single-pole filter is

$$h[n] = \left(\frac{1}{3}\right)^n u[n].$$

   The frequency response of this filter is sampled at $\omega_k = \frac{2\pi k}{16}, k = 0, \ldots, 15$ to obtain $G[k]$. Find $g[n]$, the 16-point inverse DFT of $G[k]$.

5. Simulation: Consider the following infinite non-periodic DT signal

$$x[n] = \begin{cases} 0 & n < 0 \\ 1 & 0 \leqslant n < a \\ 0 & n \geqslant a. \end{cases}$$

   (a) Compute its DTFT $X(e^{j\omega})$.

(b) Next we want to visualize the magnitude of $X(e^{j\omega})$. Plot 10,000 points of one period of $|X(e^{j\omega})|$ (from 0 to 2π) for $a = 20$.

(c) Next we will compute DFT. Generate a finite sequence $x_1[n]$ of length $N = 30$ such that $x_1[n] = x[n]$ for $n = 1, \ldots, N$. Note $a = 20$. Compute its DFT and plot its magnitude (as `stem` plot). Compare it with the plot obtained in (b), by overlaying the two plots.

(d) Repeat (c) for different values of $N = 50, 100, 1000$. What can you conclude ?

6. Simulation: A continuous-time signal $x(t) = \cos(2\pi 3010t)$ is sampled at f_s and the N-point DFT is computed using a rectangular window. Sketch the magnitude DFT in each of the following cases. Label the X-axis in terms of the DFT sample numbers $k = 0, \ldots, N-1$.

   (a) $f_s = 20$ kHz, $N = 2000$

   (b) $f_s = 40$ kHz, $N = 4000$

   (c) $f_s = 20$ kHz, $N = 1000$

   (d) $f_s = 20$ kHz, $N = 2000$ with zero-padding,i.e.,

$$x[n] = \begin{cases} x(nT), & n = 0, \ldots, 999 \\ 0 & n = 1000, \ldots, 1999. \end{cases}$$