

THE COPPERBELT UNIVERSITY
SCHOOL OF MATHEMATICS AND NATURAL SCIENCES
DEPARTMENT OF PHYSICS

PH 110: Introductory Physics

TEST 1

DATE: 11th MARCH 2021

TIME: 2 hours

INSTRUCTIONS: There are three (3) questions in this paper each carry 25 marks, answer **ALL**. Write down your **names**, **computer number** and **lecture group** on the front page of your answer booklet.

Where necessary use the following: $g = 9.8 \text{ m/s}^2$, $2.54 \text{ cm} = 1 \text{ inch}$, $1.609 \text{ km} = 1 \text{ mi}$, $1 \text{ ton} = 1000 \text{ kg}$.

Question One

- (a) The mass of the parasitic wasp can be as small as $5 \times 10^{-6} \text{ kg}$. What is this mass in (i) grams (g) (ii) milligrams (mg) (iii) micrograms (μg) [6]
- (b) State the number of significant figures in the following :
(i) 0.006 m^2 (ii) 0.2309 m^3 (iii) 0.006032 kg (iv) $2.75 \times 10^3 \text{ kg}$ [2]
- (c) If velocity (V), time (T) and force (F) were chosen as basic quantities, find the dimensions of mass. [3]
- (d) The time dependence of a physical quantity P is found to be of the form $P = P_0 e^{-\alpha t^2}$, where t is the time and α is some constant. What are the dimensions of α ? [2]
- (e) (i) state two applications of dimensional analysis [2]
(ii) The period of oscillation of a simple pendulum is assumed to depend on its length (l), mass of the bob (m) and acceleration due to gravity (g). Using dimensional analysis, derive its time period (T). [8]
(iii) Write down two limitations of dimensional analysis [2]

Question Two

- (a) A child obviously lost, walks 75 m at 25° north of east, then 100 m at 15° south of east, then 90 m south and finally 50 m 30° north of west. Choose the y-axis pointing north and the x-axis pointing east, and find:
(i) the total distance covered by the child. [2]
(ii) the magnitude and direction of the resultant displacement of the child. [10]
- (b) Given two vectors **A** and **B** where $\mathbf{A} = i + 2j + 3k$ and $\mathbf{B} = 3i + 2j + 3k$. Calculate the following:
 - (i) $\mathbf{A} \times \mathbf{B}$ [3]
 - (ii) The angle between **A** and **B**. [3]
 - (iii) Determine the unit vector perpendicular to $\mathbf{A} \times \mathbf{B}$. [2]

- (c) A force $\mathbf{F} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ pushes an object of mass 5 kg from the origin to a position vector $\mathbf{r} = 3\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$. Using the scalar product of vectors, determine the work done by the force on the object. [5]

Question Three

- a) Briefly discuss how average speed compares with average velocity [3].
- b) A ZAF Officer fires a bullet that moves along the x axis. Its velocity as a function of time is expressed as $v(t) = 4 + 8t$, where v is in m/s . The position of the bullet at $t = 1s$ is $25m$. Determine:
- (i) The acceleration at $t = 2s$ [2]
 - (ii) The position at $t = 1.5s$ [3]
- c) A teargas canister thrown vertically upward is held by a student after $2.0s$. Determine:
- (i) The speed with which the teargas canister was thrown [2]
 - (ii) The maximum height the teargas canister reaches. [2]
- (d) A golfer chooses a 7-iron club to 'chip' the ball a short distance onto the green. His shot gives the ball a velocity of $15 m \cdot s^{-1}$ at an angle of 35° to the horizontal. (Ignore air resistance for the following calculations.)
- (i) What are the horizontal and vertical components of the ball's velocity just after it is hit? [4]
 - (ii) How long will the ball take to reach its maximum height? [2]
 - (iii) Measured along the ground, how far will the ball have travelled when it hits the ground (i.e. what is its horizontal range?). [4]
 - (iv) How would this range change if this shot had been played on the Moon? Make sure to explain why it would change. [3]

PH110 TEST (I)

Solutions Prepared

By

PROF. G. G. NYAMBUYA¹

*The Copperbelt University,
School of Mathematics and Natural Sciences,
Department of Physics,
P. O. Box 21692,
Jambo Drive — Riverside — Kitwe,
Republic of Zambia.*

Question (1)

(a) The *mass* of the *parasitic wasp* can be as small as 5×10^{-6} kg. What is this mass in:

- i. grams (g). [2]
- ii. milligrams (mg). [2]
- iii. micrograms (μ g). [2]

ANSWER:

- i.
 - We know that: $1 \text{ kg} = 10^3 \text{ g}$. [1 mark]
 - Hence: $5 \times 10^{-6} \text{ kg} = \underline{5 \times 10^{-3} \text{ g}}$. [1 mark]
- ii.
 - We know that: $1 \text{ kg} = 10^6 \text{ mg}$. [1 mark]
 - Hence: $5 \times 10^{-6} \text{ kg} = \underline{5 \text{ mg}}$. [1 mark]
- iii.
 - We know that: $1 \text{ kg} = 10^9 \mu\text{g}$. [1 mark]
 - Hence: $5 \times 10^{-6} \text{ kg} = \underline{5 \times 10^3 \mu\text{g}}$. [1 mark]

¹E-mail: poet.ggn@gmail.com

(b) State the number of significant figures in the following quantities:

- | | |
|-------------------------------------|-------|
| i. 0.006 m^2 . | [1/2] |
| ii. 0.2309 m^3 . | [1/2] |
| iii. 0.006032 kg . | [1/2] |
| iv. $2.75 \times 10^3 \text{ kg}$. | [1/2] |

ANSWER:

- | | |
|---|------------|
| i. $0.006 \text{ m}^2 = 6 \times 10^{-3} \text{ m}^2$, hence <u>1 s.f.</u> | [1/2 mark] |
| ii. $0.2309 \text{ m}^3 = 2.309 \times 10^{-1} \text{ m}^3$, hence <u>4 s.f.</u> | [1/2 mark] |
| iii. $0.006032 \text{ kg} = 6.032 \times 10^{-3} \text{ kg}$, hence <u>4 s.f.</u> | [1/2 mark] |
| iv. $2.75 \times 10^3 \text{ kg} = 2.75 \times 10^3 \text{ kg}$, hence <u>3 s.f.</u> | [1/2 mark] |

(c) If velocity (V), time (T) and force (F) were chosen as basic quantities, find the dimensions of mass. [3]

ANSWER:

- We know that: $F = ma$, [1 mark]
- and that: $a = v/t$. [1 mark]
- From the above, it follows that: $m = Ft/v$, hence:

$$[m] = \frac{[F][t]}{[V]} = \text{FTV}^{-1}.$$

[1 mark]

(d) The time dependence of a physical quantity, P , is found to be of the form: $P = P_0 e^{\alpha t^2}$, where t is the time and α is some constant. What are the dimensions of α ?

ANSWER:

- The argument (αt^2) of the function $e^{\alpha t^2}$ is a dimensionless quantity, i.e.: $[\alpha t^2] = 1$. [1 mark]
- From this, it follows that: $[\alpha] = [t^{-2}] = T^{-2}$. [1 mark]

- (e) i. State two applications of dimensional analysis. [2]

ANSWER:

A mark for each of the following:

- To check the dimensional correctness of an equation. [1 mark]
- To derive equations from a sufficient base of knowledge of known variables under a set given of assumptions. [1 mark]
- To determine the units of constants in equations. [1 mark]

- ii. The period (T) of oscillation of a simple pendulum is assumed to depend on its length (l), mass of the bob (m) and the acceleration due to gravity (g). Use the *dimensional analysis approach* to derive an expression for the period. [8]

ANSWER:

Dimensional analysis requires that:

$$T = km^x l^y g^z,$$

where: k is a dimensionless quantity and: x, y, z , are unknown powers that make this equation dimensionally consistent. We know from dimensional analysis that:

$$[T] = [m]^x [l]^y [g]^z = M^x L^y (LT^{-2})^z = M^x L^{y+z} T^{-2z}.$$

Equating the powers, we will have:

$$\begin{array}{rclcl} x & = & 0 & \dots & (a) \quad [1 \text{ mark}] \\ y + z & = & 0 & \dots & (b) \quad [1 \text{ mark}] \\ -2z & = & 1 & \dots & (c) \quad [1 \text{ mark}] \end{array}.$$

Solving the above set of simultaneous equations, one obtains:

$$\begin{array}{rclcl} x & = & 0 & \dots & (a) \quad [1 \text{ mark}] \\ y & = & \frac{1}{2} & \dots & (b) \quad [1 \text{ mark}] \\ z & = & -\frac{1}{2} & \dots & (c) \quad [1 \text{ mark}] \end{array}.$$

hence:

$$T = k \left(\frac{l}{g} \right)^{1/2} = k \sqrt{\frac{l}{g}}.$$

[3 marks]

iii. Write down two limitations of dimensional analysis.

[2]

ANSWER:

A mark for each of the following:

- It does not tell us the value of the dimensionless constant involved in the derived equation or expression. [1 mark]
- It does not always tell us the exact form of the relation. [1 mark]
- It does not tell whether a given physical quantity is a scalar or vector. [1 mark]

Question (2)

(a) A child obviously lost, walks 75 m at 25° North of East, then 100 m at 15° South of East, then 90 m South and finally 50 m 30° North of West. Choose the y -axis pointing North and the x -axis pointing East, and find:

(i) The total distance covered by the child.

[2]

ANSWER:

- $d = 75 \text{ m} + 100 \text{ m} + 90 \text{ m} + 25 \text{ m}$ [1 mark]
- Hence: $d = 315 \text{ m}$ [1 mark]

(ii) The magnitude and direction of the resultant displacement of the child.

[10]

ANSWER:

As shown in Figure (1), let the displacement vectors be A , B , C , and D . Further, let the resultant displacement of the four vectors be R .

- X-Component of R :

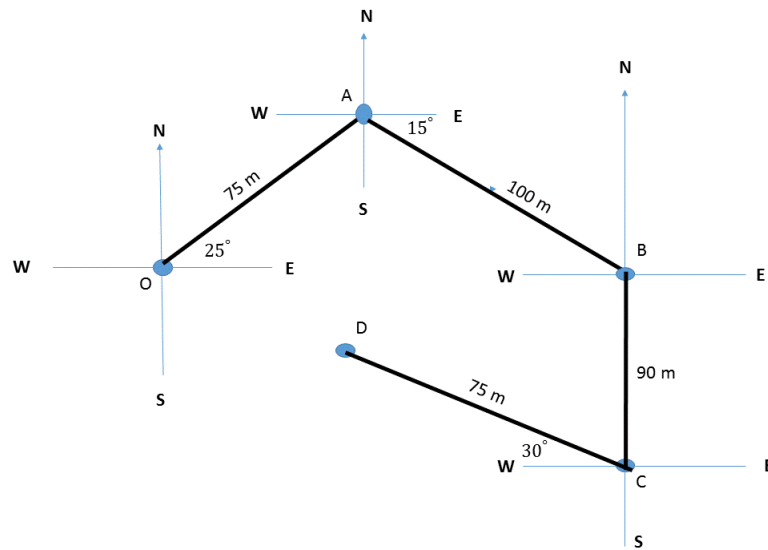


Figure (1): Vector Diagram of the Wandering Lost Child.

- The *x*-component of the resultant displacement vector, \mathbf{R} , is such that: $R_x = A_x + B_x + C_x + D_x$. [1 mark]
- That is to say: $R_x = (75 \text{ m}) \cos(75^\circ) + (100 \text{ m}) \cos(345^\circ) + (90 \text{ m}) \cos(270^\circ) + (50 \text{ m}) \cos(150^\circ)$ [1 mark]
- Therefore: $\underline{R_x = 121.3 \text{ m}}$. [1 mark]

- Y-Component of \mathbf{R} :
 - The *y*-component of the resultant displacement vector, \mathbf{R} , is such that: $R_y = A_y + B_y + C_y + D_y$. [1 mark]
 - That is to say: $R_x = (75 \text{ m}) \sin(75^\circ) + (100 \text{ m}) \sin(345^\circ) + (90 \text{ m}) \sin(270^\circ) + (50 \text{ m}) \sin(150^\circ)$ [1 mark]
 - Therefore: $\underline{R_x = 59.2 \text{ m}}$. [1 mark]

- Magnitude:
 - The magnitude of the resultant displacement is: $R = \sqrt{R_x^2 + R_y^2}$. [1 mark]
 - That is to say: $R = \sqrt{(121.3 \text{ m})^2 + (59.2 \text{ m})^2}$. [1 mark]
 - Therefore: $\underline{R = 135 \text{ m}}$. [1 mark]

- Direction:

- The direction of the resultant displacement is such that:

$$\tan \theta = \frac{R_y}{R_x}.$$

[1 mark]

- Therefore:

$$\theta = \tan^{-1} \left(\frac{59.2 \text{ m}}{121.3 \text{ m}} \right) = 26^\circ.$$

Alternatively: -26° , or $334^\circ = 360^\circ - 26^\circ$.

[1 mark]

- (b) Given two vectors \mathbf{A} and \mathbf{B} , where: $\mathbf{A} = \hat{i} + 2\hat{j} + 3\hat{k}$, and: $\mathbf{B} = 3\hat{i} + 2\hat{j} + 3\hat{k}$. Calculate the following:

- (i) $\mathbf{A} \times \mathbf{B}$.

[3]

ANSWER:

- * We know that:

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & 2 & 3 \end{vmatrix}.$$

[1 mark]

- * Expanding:

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} 2 & 3 \\ 2 & 3 \end{vmatrix} \hat{i} - \begin{vmatrix} 1 & 3 \\ 3 & 3 \end{vmatrix} \hat{j} + \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} \hat{k}.$$

[1 mark]

- * Hence: $\mathbf{A} \times \mathbf{B} = 0\hat{i} + 6\hat{j} - 4\hat{k} = 6\hat{j} - 4\hat{k}$.

[1 mark]

- (ii) The angle between \mathbf{A} and \mathbf{B} .

[3]

ANSWER:

There are two ways to the answer:

- FIRST METHOD:

- We know that: $\mathbf{A} \times \mathbf{B} = |\mathbf{A}| |\mathbf{B}| \hat{\mathbf{n}} \cos \theta$. Taking the magnitude on both-sides and re-arranging, we will have that:

$$\theta = \sin^{-1} \left[\frac{|\mathbf{A} \times \mathbf{B}|}{|\mathbf{A}| |\mathbf{B}|} \right].$$

[1 mark]

- Therefore:

$$\theta = \sin^{-1} \left[\frac{|6\hat{\mathbf{j}} - 4\hat{\mathbf{k}}|}{|\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}| |3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}|} \right] = \sin^{-1} \left[\frac{\sqrt{6^2 + 4^2}}{\sqrt{(1^2 + 2^2 + 3^2)(3^2 + 2^2 + 3^2)}} \right].$$

[1 mark]

- Hence: $\theta = \sin^{-1}(\sqrt{13/77}) = 0.42 \text{ rad} = 24.3^\circ$.

[1 mark]

• SECOND METHOD:

- We know that: $\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}| |\mathbf{B}| \cos \theta$, hence:

$$\theta = \cos^{-1} \left[\frac{|\mathbf{A} \cdot \mathbf{B}|}{|\mathbf{A}| |\mathbf{B}|} \right].$$

[1 mark]

- Therefore:

$$\theta = \cos^{-1} \left[\frac{(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) \cdot (3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})}{|\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}| |3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}|} \right] = \cos^{-1} \left[\frac{(1)(3) + (2)(2) + (3)(3)}{\sqrt{(1^2 + 2^2 + 3^2)(3^2 + 2^2 + 3^2)}} \right].$$

[1 mark]

- Hence: $\theta = \cos^{-1}(8/\sqrt{77}) = 0.42 \text{ rad} = 24.3^\circ$.

[1 mark]

(iii) Determine the unit vector perpendicular to $\mathbf{A} \times \mathbf{B}$.

[2]

ANSWER:

- We know that: $\hat{\mathbf{n}} |\mathbf{A} \times \mathbf{B}| = \mathbf{A} \times \mathbf{B}$, where $\hat{\mathbf{n}}$ is the required unit vector perpendicular to $\mathbf{A} \times \mathbf{B}$. [1 mark]
- Therefore:

$$\hat{\mathbf{n}} = \frac{\mathbf{A} \times \mathbf{B}}{|\mathbf{A} \times \mathbf{B}|} = \frac{6\hat{\mathbf{j}} - 4\hat{\mathbf{k}}}{\sqrt{(6)^2 + (-4)^2}} = \frac{6\hat{\mathbf{j}} - 4\hat{\mathbf{k}}}{\sqrt{52}}.$$

[1 mark]

- (c) A force: $\mathbf{F} = 2\text{N}\hat{i} + 3\text{N}\hat{j} + 4\text{N}\hat{k}$, pushes an object of mass 5 kg from the origin to a position vector: $\mathbf{r} = 3\text{m}\hat{i} - 3\text{m}\hat{j} + 5\text{m}\hat{k}$. Using the scalar product of vectors, determine the work done by the force on the object. [5]

ANSWER:

- We know that: $\text{Work} = \mathbf{F} \cdot \mathbf{r}$. [1 mark]
- Therefore: $\text{Work} = (2\text{N}\hat{i} + 3\text{N}\hat{j} + 4\text{N}\hat{k}) \cdot (3\text{m}\hat{i} - 3\text{m}\hat{j} + 5\text{m}\hat{k})$. [1 mark]
- Hence: $\text{Work} = (2) \times (3)\text{Nm} + (3) \times (-2)\text{Nm} + (4) \times (5)\text{Nm}$. [1 mark]
- Thus: $\text{Work} = 6\text{ J} - 6\text{ J} + 20\text{ J}$. [1 mark]
- Work done = 20 J. [1 mark]

Question (3)

- (a) Briefly discuss how average speed compares with average velocity. [3]

ANSWER:

i. A mark for any three of the following:

- Average speed is a scalar quantity while average velocity is a vector quantity both with same units ($\text{m} \cdot \text{s}^{-1}$) and dimensions (LT^{-1}). [1 mark]
- Average speed or velocity depends on time interval which it is defined. [1 mark]
- For a given time interval, average velocity is single value while average speed can have many values depending on the path followed. [1 mark]
- If after motion a particle comes back to its initial position, then the average velocity is zero (as displacement is zero) but the average speed is greater than zero. [1 mark]

- (b) A ZAF officer fires a bullet that moves along the x -axis. Its speed as a function of time is expressed as: $v(t) = 4 + 8t$, where, $v(t)$, is in $\text{m} \cdot \text{s}^{-1}$. The position of the bullet at: $t = 1\text{ s}$, is 25 m. Determine:

- (i) The acceleration at: $t = 2\text{ s}$. [2]

ANSWER:

- $a(t) = \frac{dv(t)}{dt} = \frac{d(4 + 8t)}{dt}$. [1 mark]
- Therefore: $a(t) = 8 \text{ m} \cdot \text{s}^{-2}$. [1 mark]

(ii) The position at: $t = 1.5 \text{ s}$. [3]

ANSWER:

- We know that: $\frac{dx(t)}{dt} = v(t)$. [1 mark]
- Therefore: $[x(t)]_{25 \text{ m}}^{x(t)} = x(t) - 25 \text{ m} = \int_1^t v(t) dt = \int_0^t (4 + 8t) dt = [4t + 4t^2]_1^t = 4t + 4t^2 - 8$, hence: $x(t) = 4t + 4t^2 + 17$. [1 mark]
- Therefore, at: $t = 1.5 \text{ s}$, we have that: $x = 32 \text{ m}$. [1 mark]

(c) A tear-gas canister thrown vertically upward is held by a student after 2.0 s of reaching its natural maximum height. Determine:

(i) The speed with which the tear-gas canister was thrown. [2]

ANSWER:

- We know that: $t_f = \frac{2V \sin \theta}{g}$. [1 mark]
- Hence: $V = \frac{gt_f}{2 \sin 90^\circ} = \frac{(9.8 \text{ m} \cdot \text{s}^{-2}) \times (2 \text{ s})}{2 \sin 90^\circ} = 9.8 \text{ m} \cdot \text{s}^{-1}$. [1 mark]

(ii) The maximum height the tear-gas canister reaches. [2]

ANSWER:

We have that: $v = 0 \text{ m} \cdot \text{s}^{-1}$ at: $y_{\max} = h_{\max}$ and that: $u = 9.8 \text{ m} \cdot \text{s}^{-1}$.

- From the given information and from our knowledge that: $v^2 = u^2 - 2gy_{\max}$, it thus follows that: $0 = (9.8 \text{ m} \cdot \text{s}^{-1})^2 - 2(9.8 \text{ m} \cdot \text{s}^{-2})y_{\max}$. [1 mark]
- Therefore: $y_{\max} = 4.9 \text{ m}$. [1 mark]

(d) A golfer chooses a 7-iron club to ‘chip’ the ball a short distance onto the green. His shot gives the ball a velocity of $15 \text{ m} \cdot \text{s}^{-1}$ at an angle of 35° to the horizontal. Ignoring air resistance, answer the following questions:

(i) What are the horizontal and vertical components of the ball’s velocity just after it is hit? [4]

ANSWER:

- Horizontal component:
 - $v_x = v_0 \cos \theta$. [1 mark]
 - Hence: $v_x = (15 \text{ m} \cdot \text{s}^{-1}) \cos 35^\circ = 12.3 \text{ m} \cdot \text{s}^{-1}$. [1 mark]
- Vertical component:
 - $v_y = u_0 \sin \theta$. [1 mark]
 - Hence: $u_y = (15 \text{ m} \cdot \text{s}^{-1}) \sin 35^\circ = 8.60 \text{ m} \cdot \text{s}^{-1}$. [1 mark]

(ii) How long will the ball take to reach its maximum height? [2]

ANSWER:

- We know that: $v_y = v_0 \sin \theta - gt$, and that: $v_y = 0$, at maximum height. [1 mark]
 - Therefore: $t = \frac{v_0 \sin \theta}{g} = \frac{8.60 \text{ m} \cdot \text{s}^{-1}}{9.8 \text{ m} \cdot \text{s}^{-2}} = 0.88 \text{ s}$. [1 mark]
- (iii) Measured along the ground, how far will the ball have travelled when it hits the ground — *i.e.*, what is its horizontal range? [4]

ANSWER:

There are two ways to arrive at the answer and these ways are as follows:

1. First Method:

- We know that: $x(t) = v_0 t \cos \theta = (12.3 \text{ m} \cdot \text{s}^{-1})t$. [1 mark]
- To calculate t , we know that: $y = v_0 t \sin \theta - \frac{1}{2}gt^2$. At the point where the ball will hit the ground: $y = 0$, hence: $0 = (8.60 \text{ m} \cdot \text{s}^{-1})t - (9.8 \text{ m} \cdot \text{s}^{-2})t^2$, thus: $t = 1.76 \text{ s}$. Now — substituting this into the above, we will obtain that the ball will hit the ground at the point: $x = 21.6 \text{ m}$ [1 mark]

2. Second Method:

- From symmetry, taking the time obtain in 3 (d)(ii), and then multiplying this by 2 in-order to obtain the time for the full-length journey to the ground, we will have that: $t = 2 \times (0.88 \text{ s}) = 1.76 \text{ s}$. [1 mark]
- Hence: $x = (12.3 \text{ m} \cdot \text{s}^{-1}) \times (1.76 \text{ s}) = 21.6 \text{ m}$ [1 mark]

(iv) How would this range change if this shot had been played on the Moon? Explain why it would change and how it is going to change. [3]

ANSWER:

If this shot is played on the Moon, the gravitational field strength is going to be affected since the gravitational field strength on the surface of Moon, g_M , is from that on Earth, g_E : — *i.e.*, it is smaller than is on Earth: $g_M < g_E$. [1 mark]

One mark for each of the following:

- *Range* will be longer since: $R = \frac{V^2 \sin 2\theta}{g}$. [1 mark]
- *Maximum height* will be longer since: $h_{max} = \frac{V^2 \sin^2 \theta}{2g}$. [1 mark]
- *Time of flight* will be longer since: $t_f = \frac{2V \sin \theta}{g}$. [1 mark]