Solutions

QUESTION ONE

(a) (i) For any equation to be valid, it must be *homogeneous*. Explain what is meant by a *homogeneous* equation. [2 marks]

(ii) The period of a spring executing simple harmonic motion is given by the equation:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

where m is the mass and k is the spring constant given by $k = \frac{Force}{Displacement}$.

Show that the equation is homogeneous. [4 marks]

(b) What are the dimensions of a and b in the relation $F = at + bx$, where F is force, t is the time and x is the distance? [4 marks]

(c) The velocity v of water waves may depend on their wavelength λ, the density of water ρ and the acceleration due to gravity g. Establish dimensionally the relation between these quantities. [7 marks]

(d) If the velocity of light (3×10^8 m/s) is taken as the unit of velocity and a year is taken as the unit of time, what will be the unit of length? [3 marks]

(e) Density is defined as mass per unit volume. A neutron star has a density 2.8×10^{17} kg/m^3. Assume the star to be a perfect sphere. Find the radius of the neutron star whose mass is 4×10^{30} kg. [5 marks]

QUESTION TWO

(a) A particle undergoes three successive displacements in a plane as follows: 4.0 m southwest, 5.0 m east, 6.0 m in a direction 60° north of east. Choose the y-axis pointing north and the x-axis pointing east and find the

(i) magnitude and direction of the resultant displacement, and [7 marks]

(ii) displacement that would be required to bring the particle back to the starting point. [2 marks]

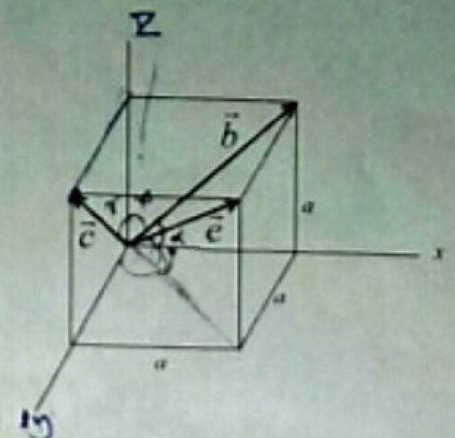
(b) When displacement $\vec{B}$ is added to displacement $\vec{A}$ the result is a displacement $\vec{C}$ that has components $C_x = -3.70\,cm$, $C_y = +2.25\,cm$ and $C_z = +4.60\,cm$. Displacements $\vec{A}$ and $\vec{B}$ are in the same direction, but the magnitude of $\vec{A}$ is only one-third that of $\vec{B}$. Find the components of $\vec{A}$. [4 marks]

(c) In the Figure below $\vec{b}$ and $\vec{c}$ are intersecting face diagonals of a cube of edge a. Find the

(i) components of the vector $\vec{d}$, where $\vec{d} = \vec{b} \times \vec{c}$. [4 marks]

(ii) the angle θ, between $\vec{b}$ and $\vec{c}$. [4 marks]

(iii) direction cosines of the body diagonal $\vec{e}$. [4 marks]



QUESTION THREE

(a) A ballast bag is dropped from a balloon that is 300 m above the ground and rising at 13 m/s. For the bag, find

(i) the maximum height reached, [2 marks]

(ii) its position and velocity 5 s after it is released, and [2 marks]

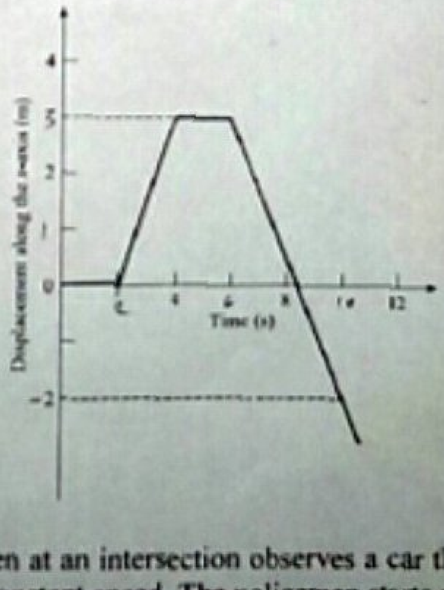
(iii) the time at which it hits the ground. [4 marks]

(b) A ball is thrown from the top of one building towards a very tall building 50 m away. The initial velocity of the ball is 20 m/s, 40° above the horizontal.

(i) At what height, relative to the original level, will the ball strike the opposite wall? [4 marks]

(ii) Is the height in part (i) above or below the original level? Explain. [2 marks]

(c) The graph in the Figure below shows an object's one–dimensional motion along the x-axis. Describe its motion. [3 marks]



(d) A motorcycle policeman hidden at an intersection observes a car that ignores a stop sign, crosses the intersection, and continues on at constant speed. The policeman starts off in pursuit 2.0 s after the car has passed the stop sign, accelerates at 6.2 m/s^2 until his speed is 110 km/h, and then continues at this speed until he catches the car. At that instant, the car is 1.4 km from the intersection. How fast was the car traveling?

[8 marks]

QUESTION FOUR

(a) Action and reaction forces are equal and opposite, yet they do not cancel each otherresulting in zero net force. Why?

[2 marks]

(b) A mini bus is moving at the speed of 72 km/h on a horizontal straight rough road. Just before reaching the CBU station, the engine suddenly switches off and the bus skids to a stop through a distance of 50 m. Calculate the coefficient of kinetic frictionbetween the road and the bus's tyres. [5 marks]

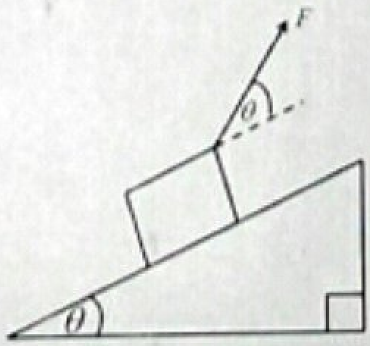
(c) An object of mass 10 g is moving along the x-axis. Its position as a function of time is given by

$$x = (4t^3 + 2t)m$$

Find the magnitude of the force acting on the object at $t = 2$ s. [4 marks]

(d) A man gardening pushes a lawnmower at a constant speed. To do this requires a force of 80 N directed along the handle, which is at an angle of 30^0 below the horizontal. The coefficient of kinetic friction between the lawnmower and the surface is 0.25. Find the mass of the lawnmower. [5marks]

(e) A block of mass m on a rough inclined surface is acted upon by a force F at an angle θ with the incline as shown below. The surface is inclined at an angle θ and the coefficient of kinetic friction between the block and the surface is μ_k



(i) Draw the free body diagram of the block. [2 marks]

(ii) Show that the acceleration of the block is

$$a = \frac{F(\cos\theta + \mu_k \sin\theta) - mg(\sin\theta + \mu_k \cos\theta)}{m}$$ [7 marks]

******************************* Good Luck *****************************

Question One

(a) (i) An Homogeneous equation is an equation in which each term on both sides of the equation has the same base units or dimensions.

(ii) To show that this equation is homogeneous, we need to substitute the quantities and replace them with their dimensions or base unit in the equation, as follows

Data

$[T] \rightarrow T$

$[m] \rightarrow M$

$[K] \rightarrow \frac{[F]}{[x]} \rightarrow \frac{MLT^{-2}}{L} \rightarrow MT^{-2}$

$[2\pi] \rightarrow 1$

$$T = 2\pi\sqrt{\frac{M}{K}}$$

$$= 1\sqrt{\frac{\cancel{M}}{\cancel{M}T^{-2}}}$$

$$= \sqrt{T^2}$$

$$T = T$$

Since the dimensions on both sides of the equation are the same, then the equation is homogeneous.

(b) According to the principle of homogeneity of dimensions, both the terms on the right hand side of the equation must have the same dimensions as F on the left hand side. Then

$$F = at + bx,$$

the dimensions for each term are

$[F] \rightarrow MLT^{-2}$

$[t] \rightarrow T$

$[x] \rightarrow L$

Equating each term on the right hand to the left hand side :

for a :

$$F = at$$

$$a = \frac{F}{t} = \frac{[F]}{[t]} = \frac{MLT^{-2}}{T}$$

$$\therefore \underline{a = MLT^{-3}}$$

for b :

$$F = bx$$

$$b = \frac{F}{x} = \frac{[F]}{[x]} = \frac{MLT^{-2}}{L}$$

$$\underline{b = MT^{-2}}$$

(c) The relation between the given quantities can be found by using dimension Analysis. The relation is of the form ;

$$v \propto \lambda^x \rho^y g^z$$

$$v = K \lambda^x \rho^y g^z \text{ ——— } (*)$$

where K is a dimensionless Costant

$[v] \rightarrow LT^{-1}$

$[\lambda] \rightarrow L$

$[\rho] \rightarrow ML^{-3}$

$[g] \rightarrow LT^{-2}$

$[K] \rightarrow 1$

Replacing the physical quantities in the above relation by their dimension,

$$LT^{-1} = L^x (ML^{-3})^y (LT^{-2})^z$$

$$LT^{-1} = L^x M^y L^{-3y} L^z T^{-2z}$$

$$LT^{-1} = M^y L^{x-3y+z} T^{-2z}$$: using the laws of Indices

By ~~defini~~ identification

$$M^0 = M^y$$
$$L^1 = L^{x-3y+z}$$
$$T^{-1} = T^{-2z}$$

Then, one obtains the following simultaneous equations;

$$y = 0$$
$$x - 3y + z = 1$$
$$-2z = 1$$

Solving the equations, one gets the following values $x = \frac{1}{2}$, $y = 0$, $z = \frac{1}{2}$

therefore;

$$v = K \lambda^{1/2} \rho^0 g^{\frac{1}{2}} = K \lambda^{1/2} g^{1/2}$$

$$\underline{v = K\sqrt{\lambda g}}$$

(d) By definition

$$\text{Speed} = \frac{\text{Distance}}{\text{time}}$$

$$\text{Distance} = \text{Speed} \times \text{time}$$

unit of Speed $= 3 \times 10^8$ m/s

unit of time $= 1$ year

$$1 \text{ year} = 1 \text{ yr} \times \frac{365.25 \text{ dy}}{1 \text{ yr}} \times \frac{24 \text{ hr}}{1 \text{ dy}} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{60 \text{ s}}{1 \text{ min}} = 31557600 \text{s}$$

$$\text{Distance} = 3 \times 10^8 \text{ m/s} \times 31557600 \text{ s}$$

$$\text{Distance} = 9.46 \times 10^{15} \text{ m (Light-year)}$$

$\therefore$ $\underline{\text{unit of Distance} = 9.46 \times 10^{15} \text{ m (Light-year)}}$

(e) By definition

$$\text{Density } (\rho) = \frac{\text{Mass}}{\text{Volume}} = \frac{m}{V}$$

Since the neutron star is assumed to be a perfect sphere, the volume of the sphere is given by;

$$V = \frac{4}{3}\pi r^3$$

where r is the radius of the sphere then;

$$\rho = \frac{m}{V}$$

$$\rho = \frac{m}{\frac{4}{3}\pi r^3}$$

$$r = \sqrt{\frac{3m}{4\pi\rho}}$$

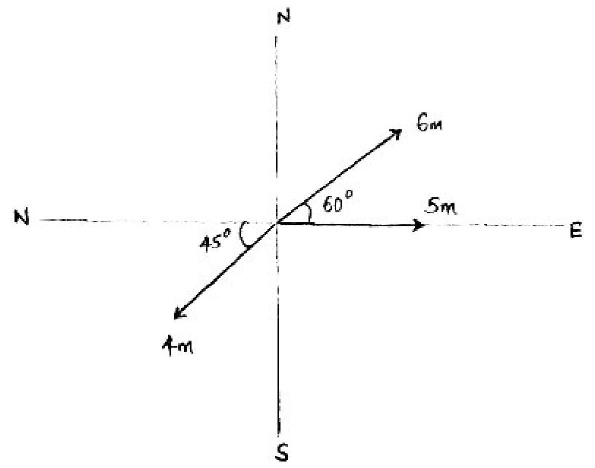
Given ; $m = 4 \times 10^{30}\,Kg$ and $\rho = 2.8 \times 10^{17}\,Kg/m^3$

$$r = \sqrt[3]{\frac{3 \times 4 \times 10^{30}}{4\pi \times 2.8 \times 10^{17}}}$$

$$\underline{r = 15000m = 15\,Km}$$

Question Two

(a) (i)



Obtain the x-component and y-component for the resultant.

x-component: $R_x = (5\cos 0^\circ + 6\cos 60^\circ - 4\cos 45^\circ)\hat{i}$

$R_x = (5+3-2.83)\hat{i}$

$R_x = 5.2\hat{i}$

y-component:

$R_y = (5\sin 0^\circ + 6\sin 60^\circ - 4\sin 45^\circ)\hat{j}$

$R_y = (5.2 - 2.8)\hat{j}$

$R_y = 2.4\hat{j}$

The magnitude for the Resultant is then;

$|R| = \sqrt{(R_x)^2 + (R_y)^2}$

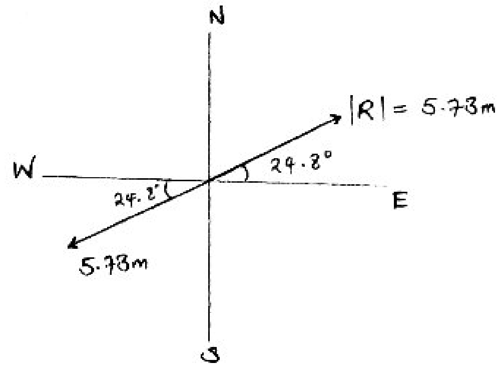
$= \sqrt{(5.2)^2 + (2.4)^2}$

$\therefore |R| = 5.73\,m$

The direction of the resultant is

$\tan\theta = \frac{R_y}{R_x}$ $\theta = \tan^{-1}\left(\frac{2.4}{5.2}\right)$ $\therefore \theta = 24.8^\circ$

(ii) The equilibrant or the Displacement that would be required to bring the particle back to the starting point is;



The equilibrant is $5.73m$, $\theta = 24.8°$ West of South

(b) the Resultant $\vec{C}$ is the sum of vector $\vec{A}$ and $\vec{B}$, meaning;

$$\vec{C} = \vec{A} + \vec{B} = (C_x\hat{i} + C_y\hat{j} + C_z\hat{k})$$

$$\vec{A} + \vec{B} = (-3.70\hat{i} + 2.25\hat{j} + 4.60\hat{k})\ cm$$

Since Displacements $\vec{A}$ and $\vec{B}$ are in the same direction, and

$$A = \frac{1}{3}B$$

This means that $\vec{A}$ and $\vec{B}$ are parallel, ~~the~~ and the Individual Components of $\vec{A}$ are one-third of $\vec{B}$, meaning

$$B = 3A$$

$$\vec{A} + 3\vec{A} = (-3.70\hat{i} + 2.25\hat{j} + 4.60\hat{k})\ cm$$

$$4\vec{A} = (-3.70\hat{i} + 2.25\hat{j} + 4.60\hat{k})\ cm$$

$$\vec{A} = \frac{1}{4}(-3.70\hat{i} + 2.25\hat{j} + 4.60\hat{k})\ cm$$

$$\vec{A} = (-0.925\hat{i} + 0.563\hat{j} + 1.15\hat{k})\ cm$$

$\therefore$ $A_x = -0.925\hat{i}$, $A_y = 0.563$ and $A_z = 1.15$

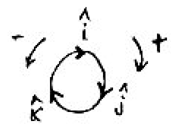
(c) (i) Components of the vector $\vec{d}$, where $\vec{d} = \vec{b} \times \vec{c}$

from the diagram

$$\vec{b} = a\hat{i} + a\hat{j}$$
$$\vec{c} = a\hat{j} + a\hat{k}$$

$$\vec{d} = \vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & a & 0 \\ 0 & a & a \end{vmatrix}$$

$$\therefore \underline{\vec{b} \times \vec{c} = a^2\hat{i} - a^2\hat{j} + a^2\hat{k}}$$

By using 

(ii) between $\vec{b}$ and $\vec{c}$, meaning the θ between $\vec{b}$ and $\vec{c}$

$$\cos\theta = \frac{\vec{b} \cdot \vec{c}}{|\vec{b}| \times |\vec{c}|}$$

$$\vec{b} \cdot \vec{c} = (a\hat{i} + a\hat{j} + 0\hat{k}) \cdot (0\hat{i} + a\hat{j} + a\hat{k})$$
$$\vec{b} \cdot \vec{c} = a^2$$

Note that:

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$
$$\hat{i} \cdot \hat{j} = \hat{i} \cdot \hat{k} = \hat{j} \cdot \hat{k} = 0$$

the magnitude fore vector $\vec{b}$ and $\vec{c}$

$$|\vec{b}| = \sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2}$$
$$|\vec{c}| = \sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2}$$

$$\cos\theta = \frac{a^2}{2a^2} = \frac{1}{2}$$

$$\therefore \underline{\theta = \cos^{-1}\left(\tfrac{1}{2}\right) = 60^\circ}$$

(iii) from the diagram, the vector diagonal of the Cube $\vec{e}$ is

$$\vec{e} = a\hat{i} + a\hat{j} + a\hat{k}$$

Since $\vec{e}$ a body diagonal of a Cube, $\cos\alpha = \cos\beta = \cos\gamma$

then the direction Cosines of the body diagonal $\vec{e}$ is.

$$\cos\alpha = \cos\beta = \cos\gamma = \frac{a}{|\vec{e}|}$$

the magnitude of $\vec{e}$ is

$$|\vec{e}| = \sqrt{a^2+a^2+a^2} = \sqrt{3a^2} = a\sqrt{3}$$

$$\cos\alpha = \cos\beta = \cos\gamma = \frac{\cancel{a}}{\cancel{a}\sqrt{3}}$$

$$\cos\alpha = \frac{1}{\sqrt{3}}$$

$$\underline{\alpha = 54.74^\circ}$$

Question Three part ⓐ

ⓐ The Initial Velocity of the bag when released Is the Same as that of the balloon, 13 m/s upward. Let us Choose upward as positive and take $y=0$ at the point of release.

(i) At the height point, $V_f = 0$. From $V_{fy}^2 = V_{iy}^2 + 2ay$; for vertical motion

$$0 = (13\,m/s)^2 + 2(-9.81\,m/s^2)y \quad \text{or} \quad y = 8.6\,m$$

the maximum height is $300 + 8.6 = 308.6\,m$ or $0.31\,Km$

(ii) Take the end point to be its position at $t = 5.0s$. Then, using one of the vertical kinematic equation, from

$$y = V_{iy}t + \frac{1}{2}a_y t^2$$

$$y = (13\,m/s)(5.0s) + \frac{1}{2}(-9.81\,m/s^2)(5.0s)^2$$

$$y = -57.5\,m \quad \text{or} \quad -58\,m$$

so its height is $300 - 57.5 = 242.5\,m$

Also, ~~we can use~~

$$V_{fy} = V_{iy} + at$$

$$V_{fy} = 13\,m/s + (-9.81\,m/s^2)(5.0s) = -36\,m/s$$

it is on its way down with a velocity of 36 m/s – Downward.

(iii) Just as it hits the ground, the bag's displacement is $-300\,m$. then

$$y = V_{iy}t + \frac{1}{2}at^2 \quad \text{becomes}$$

$$-300\,m = (13\,m/s)t + \frac{1}{2}(-9.81\,m/s^2)t^2$$

or $4.90t^2 - 13t - 300 = 0$. Solving the quadratic gives $t = 9.3s$ and $-6.6s$. Only the positive time has the physical meaning, so the required answer is 9.3s.

we could have avoided the quadratic formula by first computing v_f:

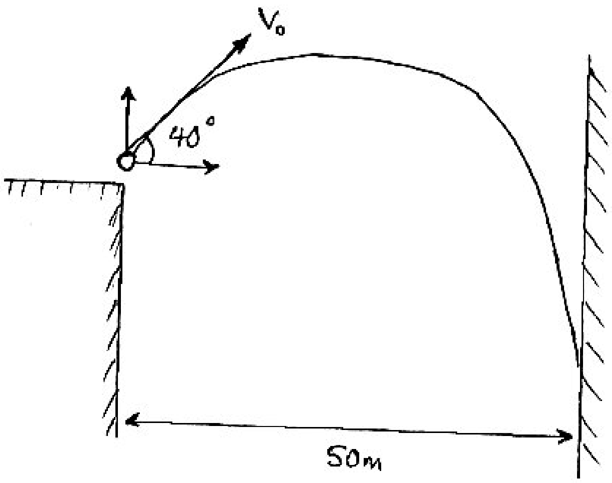
$$v_{fy}^2 = v_{iy}^2 + 2as \quad \text{becomes}$$

$$v_{fy}^2 = (13\,m/s)^2 + 2(-9.81\,m/s^2)(-300m)$$

So that $v_{fy} = \pm 77.8\,m/s$. Then, using the negative value for v_f (why?) in

$v_{fy} = v_{iy} + at$ gives $t = 9.8s$, as before.

ⓑ ⓘ Diagram



In the horizontal motion

$$V_{ox} = V_{fx} = \bar{V}_x = 20\cos 40^\circ = 15.3\,m/s$$

In the vertical motion

$$V_{oy} = 20\sin 40^\circ = 12.9\,m/s$$

Since in the horizontal motion

$$V_{ox} = V_{fx} = \bar{V}_x = 15.3\,m/s$$

then

$$x = \bar{V}_x t \Rightarrow 50m = 15.3\,m/s\,t$$

$$t = \frac{50\cancel{m}}{15.3\cancel{m}/s} = 3.27s$$

In the vertical motion, taking upward as positive

$$y = V_{oy}t - \frac{1}{2}a_y t^2$$

$$y = (12.9) \times (3.27) - \frac{1}{2}(9.8)(3.27)^2$$

$$y = 42.18 - 52.38$$

$\therefore$ $\underline{y = -10.2m}$: the height is below the horizontal level since we upward as positive and our height is -ve

(ii) The height is below the horizontal level since we took upward +ve and calculated height is negative.

(c) The object is stationary (at rest) during the first 2 seconds. Between 4 seconds and 2 seconds the object is moving to the right with speed $\left(\frac{3-0}{4-2}\right)$ m/s = 1.5 m/s. Between 6 seconds and 4 seconds the object is at rest; the slope of the graph is zero and x does not change for that interval. From 6 seconds to 10 seconds and beyond, the object is moving in the negative x-direction; the slope and the velocity are negative. We have $V_{av} = \text{slope} = \frac{x_f - x_i}{t_f - t_i}$

$$V_{av} = \frac{-2-3}{10-6} = \frac{-5\,m}{4.0\,s} = -1.3\,m/s$$

the average velocity is then $V_{av} = 1.3\,m/s$ (negative x-direction)

(d) Let t_1 be the time it takes the policeman to catch the car,

$$\frac{110\,Km}{h} = \frac{110\,000\,m}{3600\,s} = 30.56\,m/s$$

Let's find the time it took the policeman to attain a speed of 110 Km/h = 30.56 m/s

$$V_f = V_0 + at_1 \quad (V_0 = 0)$$

$$V_f = at_1$$

$$t_1 = \frac{V_f}{a}$$

$$t_1 = \frac{30.56}{6.2}$$

$$t_1 = 4.93\,s$$

And the distance travelled in 4.93s is

$$x_{P_1} = V_0 t_1 + \frac{1}{2} a t_1^2$$

$$= 0 + \frac{6.2}{2} \times (4.93)^2$$

$$\therefore x_{P_1} = 75.3452\,m$$

~~Remaing~~ the Remaining distance before Catching the Car

$$x_2 = 1400m - 75.3452 = 1324.6548m$$

and the time Remaining to Catch the Car is

$$t_2 = \frac{1324.6548m}{30.56\,m/s} = 43.35s$$

$$\therefore \quad t_{total} = t_1 + t_2 \quad (\text{policeman})$$

$$= 43.35 + 4.93$$

$$= 48.28s$$

Therefore the Car had been travelling for

$$t = t_p - 2 = 48.28s - 2s$$

$$t_{car} = 46.28s$$

And the speed of the Car is

$$V_{car} = \frac{\text{Distance}}{\text{time}}$$

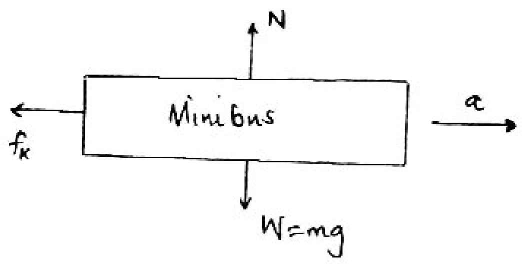
$$= \frac{1400m}{46.28s}$$

$$\therefore V_{car} = 30.25\,m/s \quad \text{or} \quad 108.9\ Km/h$$

Question Four

ⓐ This is because action and reaction forces act on different objects in the interaction and they do not cancel each other. Note that two equal and opposite forces acting on the same object do not make an action-reaction pair.

ⓑ The free-body diagram for the bus is as follows:



Given; final Velocity $V = 0$

Initial velocity $V_o = 72\ Km/h = 20m/s$

Displacement $S = 50m$

1st we find the acceleration a of the minibus by using displacement-velocity relation for a body moving in a straight line, given by:

$$V_f^2 = V_o^2 + 2as$$

$$a = \frac{V_f^2 - V_o^2}{2s}$$

$$a = \frac{0 - 20^2}{2 \times 50} = -4\ m/s^2$$

We now apply Newton's Second law of motion to determine the Coefficient of Kinetic friction μ_K, as follows

$$\Sigma F_x = ma$$

$$-f_K = ma$$

where $f_K = \mu_K N$ and $N = W = mg$, then

$$-\mu_K mg = ma$$

$$\mu_K = -\frac{a}{g}$$

$$\mu_K = -\frac{(-4)}{9.8}$$

$$\mu_K = 0.4$$

(c) Applying Newton's second law of motion, the resultant force F is given by

$$F = ma$$

where m is the mass of an object and a is its acceleration.

Given

$$x = (4t^3 + 2t)\,m$$

By definition

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt}$$

$$v = \frac{dx}{dt} = \frac{d}{dt}(4t^3 + 2t) = (12t^2 + 2)\,m/s$$

$$a = \frac{dv}{dt} = \frac{d}{dt}(12t^2 + 2) = 24t\ m/s^2$$

at $t = 2s$, the acceleration of the object is

$$a = 24 \times 2 = 48\,m/s^2$$

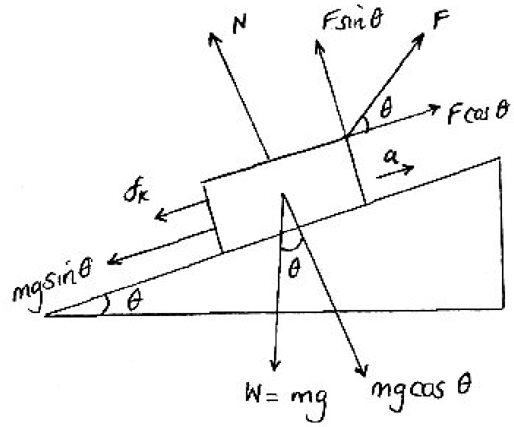
then:

$$\underline{F = ma = (0.01)(48) = 0.48\,N}$$

$$m = \frac{F(\cos 40° - u_K \sin 40°)}{u_K g}$$

$$\therefore \quad m = \frac{80(\cos 40° - 0.25 \sin 40°)}{0.25(9.8)} = 24.2\,\text{Kg}$$

(e) (i) The free-body diagram for block is as follows:



(ii) Applying Newton's second law of motion:

$$\sum F_x = ma$$

$$F\cos\theta - f_K - mg\sin\theta = ma$$

$$a = \frac{F\cos\theta - f_K - mg\sin\theta}{m}$$

where Kinetic friction $f_K = u_K N$

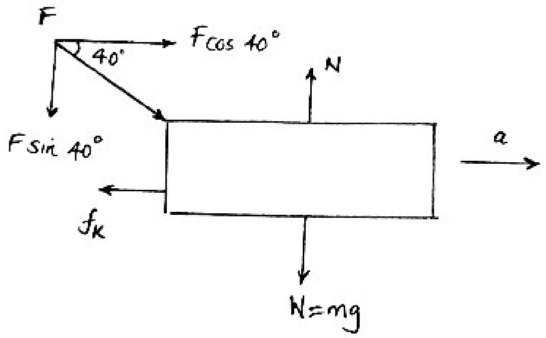
To find the normal force N, we proceed as follows

$$\sum F_y = 0$$

$$N + F\sin\theta - mg\cos\theta = 0$$

$$N = mg\cos\theta - F\sin\theta$$

(d) The free-body diagram for the lawnmower is



Applying the second law of ~~Newton~~ Newton, we get:

$$\Sigma F_x = ma$$

$$F\cos 40° - f_K = ma$$

Since the lawnmower is pushed at a costant speed, it means $a=0$, then;

$$F\cos 40° - f_K = 0$$

$$F\cos 40° = f_K$$

In the vertical direction

$$\Sigma F_y = 0$$

$$N - mg - F\sin 40° = 0$$

$$N = mg + F\sin 40°$$

therefor;

$$f_K = u_K N = u_K(mg + F\sin 40°)$$

$$F\cos 40° = u_K(mg + F\sin 40°)$$

$$F\cos 40° = u_K mg + u_K F\sin 40°$$

$$\cancel{u_K mg = F(}$$

$$u_K mg = F\cos 40° - u_K F\sin 40°$$

$$u_K mg = F(\cos 40° - u_K \sin 40°)$$

then

$$f_k = u_k N = u_k (mg \cos\theta - F \sin\theta)$$

$$a = \frac{F\cos\theta - u_k(mg\cos\theta - F\sin\theta) - mg\sin\theta}{m}$$

$$a = \frac{F\cos\theta - u_k mg\cos\theta + u_k F\sin\theta - mg\sin\theta}{m}$$

$$a = \frac{F\cos\theta + u_k F\sin\theta - mg\sin\theta - u_k mg\cos\theta}{m}$$

$$\therefore \quad a = \frac{F(\cos\theta + u_k\sin\theta) - mg(\sin\theta + u_k\cos\theta)}{m}$$ Hence shown.