THE COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS & NATURAL SCIENCES

DEPARTMENT OF PHYSICS

PH 110: INTRODUCTORY PHYSICS 2021/2022 TEST 1
DURATION: $2\frac{1}{2}$ HOURS **TOTAL MARKS**: 100 MARKS

Mr Mukwa Chafe.

INSTRUCTIONS:

1. Write your Names, **Student Identification Number and Lecture Group** on the front page of your answer booklet and possibly your ID on all your scripts.
2. There are four (4) questions in this test; **ANSWER ALL.**
3. The marks for each question are shown in the square brackets **[ ]**, show your working to avoid loss of marks.

CONSTANTS:

1. Acceleration due to gravity $g = 9.81\ m/s^2$
2. Gravitational constant $G = 6.673 \times 10^{-11}\ N.m^2.kg^{-2}$
3. Mass of the Earth $M_E = 5.98 \times 10^{24}\ kg$

Where necessary, use

1.0 inch =2.54 cm, 1.609 km= 1.0 miles, 7.48 gallons =0.0283 m^3, 1.0 cm^3=1.0 ml,

746 W = 1.0 horsepower, 1000 kg = 1.0 tones.

QUESTION ONE.

a) One gallon of paint, that is $3.78\times10^{-3}m^3$, covers an area of $25m^2$. What is the thickness of paint on the wall? Give your answer in 5 significant figures. **[3]**

b) A plate rectangular in form has a length $(21.4\pm0.3)cm$ and a width of $(8.8\pm0.2)cm$. Determine the values of the *best area* of the plate and its *uncertainty*. **[4]**

c) A mile is 1760 yards or 1609 meters or 5280 feet, and a fortnight is 14 days. In 1991, the Zambian athlete, Samuel Matete won an Olympic gold medal, in Zurich, Switzerland, when he represented Zambia in the **400***m* hurdles. His average speed was 8.5 meters per second.
Give his speed in: (i) kilometers per second; (ii) yards per hour; and (iii) feet per fortnight. **[6]**

$\left(\frac{m}{s}\right)^6\left(\frac{m}{L^3}\right)\left(\frac{m}{s^2}\right)^{-3}$

$\frac{m^6}{s^6} \times \frac{mass}{m^2} \times \frac{s^6}{m^3}$

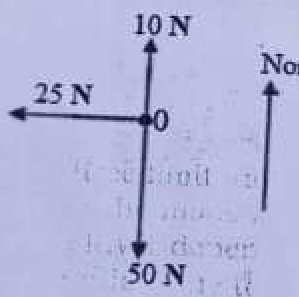
(d) (i) What are the three limitations of dimensional analysis? [3]

(ii) Assuming that the mass M of the big stone that can be moved by the flowing Kafue River depends on 'v' the velocity, 'ρ' the density of water and 'g', the acceleration due to gravity. Use dimensional analysis to find an expression for M. [9]

QUESTION TWO

(a) Give *two* examples in each case of a
  (i) scalar quantity [2]
  (ii) vector quantity [2]

(b) Three boys are pulling an object with forces acting in different directions as shown in the diagram to the right:
  (i) Draw a vector diagram and show the resultant force. [3]
  (ii) Calculate the magnitude and direction of the resultant [5]



(c) For what value of λ are the vectors $2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $3\mathbf{i} + 2\lambda\mathbf{j}$ perpendicular? [4]

(d) If vector $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$, and vector $\mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$, Find
  (i) $\mathbf{a} \times \mathbf{b}$ [3]
  (ii) the Sine of the angle between these vectors. [3]
  (iii) the Unit vector perpendicular to each vector. [3]

QUESTION THREE

(a) A runner travels 1.5 laps around a circular track in a time of 50 s. The diameter of the track is 45 m and its circumference is 142 m. Find :
  (i) The average speed of the runner [3]
  (ii) The magnitude of the runner's average velocity. [4]

(b) A particle moving along the x-axis has a displacement as a function of time given by:

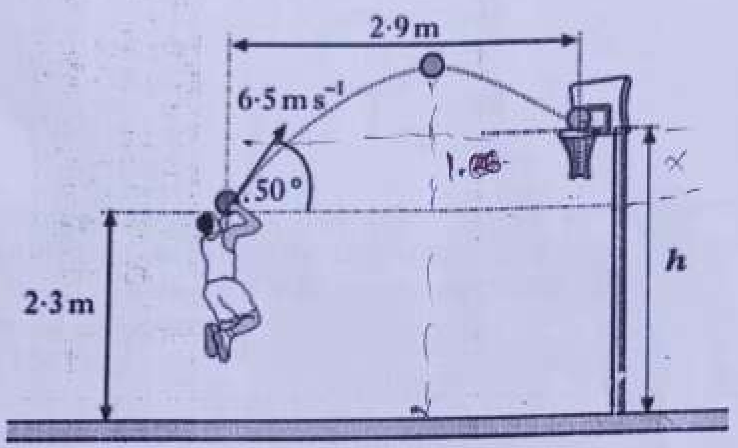
$x(t) = 30 + 20t - 15t^2$, where x is in m and t is in s. Find:
  (i) The velocity at $t = 0.5$ s. [3]
  (ii) The acceleration at $t = 3$ s. [4]

2 -3λ4

3

Mr. MUKWAGULE

(c) A basketball player throws a ball with an initial velocity of 6.5 m/s at an angle of 50° to the horizontal. The ball is 2.3 m above the ground when released, and travels a horizontal distance of 2.9 m to reach the top of the basket. See the figure below (Not drawn to scale). The effects of air resistance can be ignored.



x+2.3
1.26
2.3
3.56
h=

Find the

(i) horizontal and vertical components of the initial velocity of the ball. **[2]**
(ii) time taken by the ball to reach the basket. **[3]**
(iii) magnitude of the velocity of the ball as it reaches the top of the basket. **[4]**
(iv) height h as depicted from the figure. **[4]**

QUESTION FOUR

(a) Let 35° be the critical angle of an inclined plane on which a 10 kg block of wood slides down. Taking g = 9.81 m/s², determine the:

(i) Coefficient of static friction **[2]**
(ii) Static frictional force **[2]**

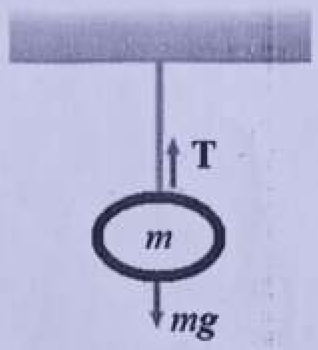
(b) Consider the diagram below with a 12 kg mass (m) vertically hanging on a rope that is attached to an upper platform. If the whole system accelerates upwards at 5.5 m/s², calculate the tension (**T**) in the rope. Take g = 9.81 m/s². **[3]**

x+2.3

1.26-x

x^2 =

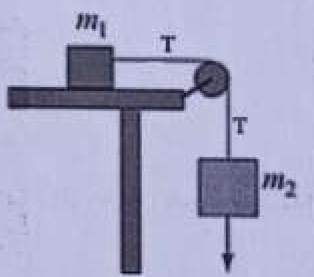
x^2



(c) Two blocks of mass m_1 = 4 kg and m_2 = 10 kg are connected via a cord through the pulley as shown. The coefficient of kinetic friction between m_1 and the table is 0.2. Taking $g = 9.81\ m/s^2$, determine the,

(i) Acceleration of the system [4]

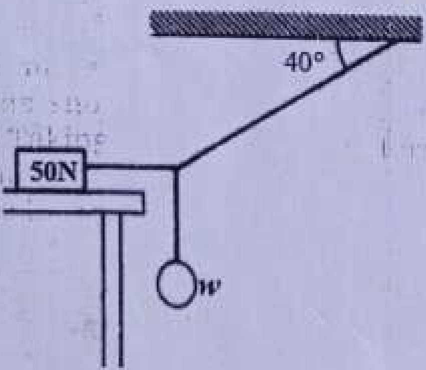
(ii) Tension in the cord. [2]



(d) The system shown in the Figure below is in equilibrium.

(i) Determine the maximum value of w if the friction force on the 50 N block cannot exceed 15 N. [9]

(ii) What is the coefficient of static friction between the block and the table top? [3]



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QUESTION ONE.

a) Assume layer of paint to have equal thickness everywhere the surface. Thus,

$Volume\ = thickness \times area$ [1.5 marks]

$$thickness = \frac{volume}{aread}$$

$$thichness = \frac{3.78 \times 10^{-3} m^3}{25m^2}$$

$thicknes = 0.0001520m$ (5 significant figures) [1.5 marks]

b) Treat the best value with its uncertainty as a binomial, thus we expand area as:

$A = (21.4 \pm 0.3)cm \times (8.8 \pm 0.2)cm$ [1 mark]

$A = [21.4(8.8) \pm 21.4(0.2) \pm 0.3(8.8) \pm 0.3(0.2)]$ [1 mark]

$A = [188.3 \pm 4.28 \pm 2.64 \pm 0.06]$

Last term is negligible, therefore:

$best\ area = 188cm^2$ [1 mark]

$uncertainty = \pm 7cm^2$ [1 mark]

c)

i)

$$\frac{8.5m}{1s} \times \frac{1km}{1000m} = 0.0085km/s \quad \text{[1 mark]}$$

$$\frac{8.5m}{1s} \times \frac{1\ mile}{1609m} \times \frac{1760yard}{1\ mile} \times \frac{3600s}{1hr} = 33{,}471.7\ yards/hour \quad \text{[2 Marks]}$$

ii)

$$\frac{8.5m}{1s}\times\frac{5280feet}{1mile}\times\frac{1mile}{1609m}\times\frac{14\ day}{1\ fortnight}\times\frac{24hr}{1day}\times\frac{3600s}{1hr}$$
$$= 3.374\times10^{7} feet/fortnight$$

[3 Marks]

1(d) (i)

Some limitations of dimensional analysis:

1. It does not give any information about the dimensional constants in the formula.

2. It cannot be used to derive relations other than product of power functions. For example $v = u + t$.

3. It cannot be used to derive the relationship involving more than three physical quantities.

4. It cannot be used to derive formulae containing trigonometric, exponential and logarithmic functions. **[3 marks]**

1 (d) ii

Let $M = Kv^a\rho^b g^c$ (eqn1) **[1 mark]**

Where K= a dimensionless constant.

Dimensions of the quantities : $velocity(v)$ $density(\rho)$ and $acceleration\ due\ to\ gravity\ (g)$ are

$M = [M], v = [LT^{-1}]$,

$\rho = [ML^{-3}]\ and\ \ g = [LT^{-2}]$ **[2 marks]**

Substituting these dimensions in the equation (1), we get

$M = [LT^{-1}]^a\,[ML^{-3}]^b[LT^{-2}]^c$

$M^1L^0T^0 = L^aT^{-a}M^bL^{-3b}L^cT^{-2c}$

$M^1L^0T^0 = M^bL^{a-3b+c}T^{-a-2c}$ **[2 marks]**

Equating the powers of M, L and T, we get

$b = 1,\ a - 3b + c = 0,\ \ -a - 2c = 0,$

$a + c = 3,\ a = -2c$

$a = 3 - c = 6,\ c = -3$ **[3 marks]**

Therefore, substituting the values of a, b and c into equation (1) we get

$M = Kv^6\rho^1 g^{-3}$ **[1 mark]**

QUESTION TWO

Solutions:

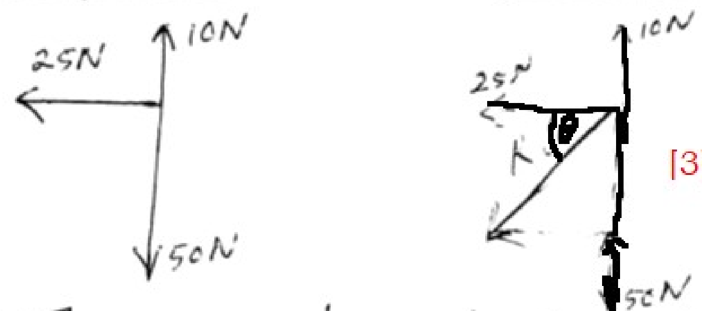
2 (a) (i) Examples of scalar quantities are: temperature, pressure, speed, distance, mass, energy, volume, time, etc (any two) [2]

(ii) Examples of vector quantities are; velocity, displacement, acceleration, weight, force, momentum, Impulse, etc (any two) [2]

2. (b)

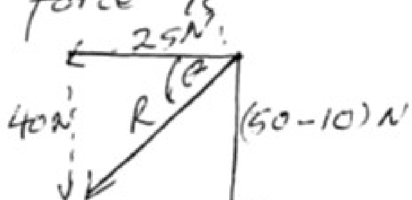
(i) A vector diagram showing the resultant of forces is as follows:

Originally — Diagram with Resultant



[3]

(ii) The magnitude & direction of the resultant force is



$\therefore R^2 = 25^2 + 40^2$

$\Rightarrow R = \sqrt{25^2 + 40^2} = \sqrt{625 + 1600}$

Magnitude $= \underline{47.2N}$ [3]

$$\text{Direction} = \theta = \tan^{-1}\left(\frac{40}{25}\right)$$
$$= \tan^{-1}(1.6)$$
$$\therefore \underline{\theta = 58.0^{\circ}}$$ [2]

Or Bearing of $\underline{\boldsymbol{\theta} = \mathbf{212}^{\mathbf{0}}}$

2 (c)

Let $\overline{a} = 2i - j + 2k$ and $\overline{b} = 3i + 2\lambda j$

Since $\overrightarrow{a}$ and $\overrightarrow{b}$ are perpendicular,

So $\overrightarrow{a} \cdot \overrightarrow{b} = 0$

$(2i - j + 2k) \cdot (3i + 2\lambda j) = 0$

$6 - 2\lambda = 0$

Or $\lambda = 3$

2 (d)

(i) $$\overrightarrow{a} \text{ x } \overrightarrow{b} = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 1 & -1 & 1 \end{vmatrix}$$

$$\overrightarrow{a} \text{ x } \overrightarrow{b} = i(3+4) - j(2-4) + k(-2-3)$$
$$= 7i + 2j - 5k$$

(ii) $$\text{Sin } \theta = \frac{\left|\overrightarrow{a} \text{ x } \overrightarrow{b}\right|}{\left|\overrightarrow{a}\right|\left|\overrightarrow{b}\right|} = \frac{\sqrt{7^2 + 2^2 + (-5)^2}}{\sqrt{2^2 + 3^2 + 4^2}.\sqrt{1^2 + (-1)^2 + 1^2}}$$
$$= \frac{\sqrt{78}}{\sqrt{29}\sqrt{3}}$$
$$\text{Sin } \theta = \sqrt{\frac{26}{29}}$$

(iii) If $\hat{n}$ is the unit vector perpendicular to $\overrightarrow{a}$ and $\overrightarrow{b}$ then

$$\hat{n} = \frac{\overrightarrow{a} \text{ x } \overrightarrow{b}}{\left|\overrightarrow{a} \text{ x } \overrightarrow{b}\right|} = \frac{7i + 2j - 5k}{\sqrt{78}}$$

QUESTION THREE

3 (a) i) average speed $= \frac{d}{t} = \left(\frac{1.5laps}{50s}\right)\left(\frac{142m}{lap}\right) = 4.26m/s$ **[3 marks]**

ii) Average velocity is a vector. In 1.5 laps, the displacement points from the starting point on the track to a point on the track 0.5 lap away which is directly across a diameter of the track.

$$\therefore \quad \bar{v} = \frac{45m}{50s} = 0.9m/s$$ **[3 marks]**

3 b) i)

$$x(t) = 30 + 20t - 15t^2$$

$$v(t) = \frac{dx(t)}{dt} = \frac{d}{dt}\left(30 + 20t - 15t^2\right) = 20 - 30t$$ **[3 marks]**

$$v(0.5) = (20 - 30(0.5))m/s = 20 - 15 = 5m/s$$

3 b) ii)

$$v(t) = 20 - 30t$$

$$a(t) = \frac{dv(t)}{dt} = \frac{d}{dt}\left(20 - 30t\right) = -30\,m/s^2$$ **[3 marks]**

$$a(3) = -30m/s^2. \textit{This means acceleration is consant.}$$

3 c)

(i) The horizontal component of the initial velocity of the ball is

$$v_{ox} = v_o \cos\theta = 6.5\cos 50° = \mathbf{4.18\ m/s} \quad [1\ mark]$$

The vertical component of the initial velocity of the ball is

$$v_{oy} = v_o \sin\theta = 6.5\,sin\,50° = \mathbf{4.98\ m/s} \quad [1\ mark]$$

(ii) The time taken by the ball to reach the basket is given by

$$x = v_{ox}t$$

$$t = \frac{x}{v_{ox}} = \frac{2.9}{4.18} = \mathbf{0.694\ s} \qquad [3\ marks]$$

(iii)The horizontal component of the velocity remains constant, and is

$$v_x = v_{ox} = 4.18\ m/s$$

The vertical component of the velocity is

$$v_y = v_{oy} + gt = 4.98 + (-9.8)(0.694) =- 1.82\ m/s \quad [2\ marks]$$

The magnitude of the velocity is

$$v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{(4.18)^2 + (-1.82)^2} = \mathbf{4.56\ m/s} \quad [2\ marks]$$

(iv) The vertical distance travelled by the ball is

$$y = v_{oy}t + \frac{1}{2}gt^2 = 4.98(0.694) + \frac{1}{2}(-9.81)(0.694)^2 = 1.10\ m \quad [3\ marks]$$

The height $\boldsymbol{h}$ is therefore;

$$\boldsymbol{h} = 2.3\ m + y = 2.3\ m + 1.10\ m = \mathbf{3.40\ m} \quad [1\ mark]$$

QUESTION FOUR

(a)

(i) The coefficient of static friction $\mu_s = tan(\theta_c) = tan(35^0) = \mathbf{0.70}$ [2]

(ii) $f_s = \mu s.\vec{\boldsymbol{n}} = 0.7 * mgcos(35) = 0.7 * 10 * 9.81 * 0.81915 = \mathbf{56.25\ N}$ [2]

(b) $ma_y = \sum F_y$ [1]

$ma_y = T - mg$ [1]

$T = ma_y + mg = m(a_y + g) = 12(5.5 + 9.81) = \mathbf{183.72\ N}$ [1]

(c) (i) $m_1 a_x = \sum F_x \Rightarrow m_1 a_x = T - f_k \quad \ldots (1)$ [1]

$-m_2 a_y = \sum F_y \Rightarrow -m_2 a_y = T - m_2 g \quad \ldots (2)$ [1]

But $a_x = a_y = a$ and $f_k = \mu s.\vec{\boldsymbol{n}} = 0.2 * 4 * 9.81 = 7.848\ N$ [1]

Solving equations (1) and (2) simultaneously:

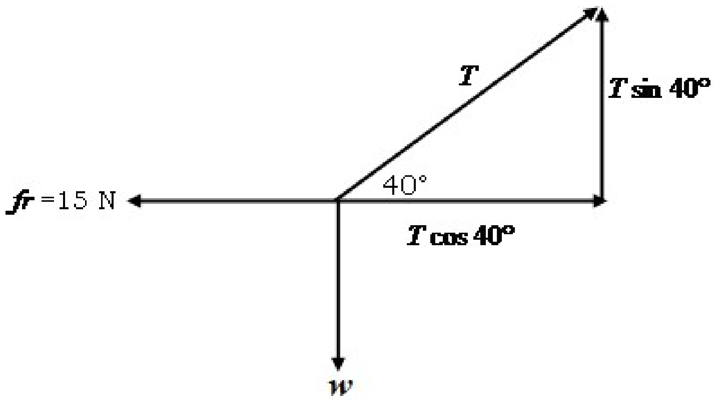
$m_1 a = T - 7.848$

$m_2 a = -T + 10 * 9.81 \quad \Rightarrow a = \frac{98.1 - 7.848}{m_1 + m_2} = \frac{98.1 - 7.848}{14} = \mathbf{6.45\ m/s^2}$ [1]

(ii) We use one of the two equations above;

$T = m_1 a + 7.848 = 4 * 6.45 + 7.848 = \mathbf{33.65\ N}$ [2]

4 (d)



[2 marks] free body diagram

For a system to be in equilibrium $\sum F = 0$ **[1 mark]**

$$\sum F_x = 0$$

$$T\cos 40° - 15N = 0$$

$$T\cos 40° = 15$$

$$T = \frac{15N}{\cos 40°} = 19.6N$$

[3 marks}

$$\sum F_y = 0$$

[1 mark]

$$w = T\sin 40°$$

$$w = 19.6\sin 40° = 12.6N$$

[2marks]

4 d ii)

$$fr = 15, \quad N = 50N$$

$$fr = \mu N$$

$$\mu = \frac{fr}{N} = \frac{15N}{50N} = 0.3$$

[3 marks]