THE COPPERBELT UNIVERSITY
SCHOOL OF MATHEMATICS AND NATURAL SCIENCES
DEPARTMENT OF PHYSICS

PH 110 INTRODUCTORY PHYSICS

TUTORIAL SHEET 1 2022: Units and Measurements

1. A carpet is to be installed in a room whose length is measured to be 13.71 m and whose width is measured to be 4.46 m. Find the area of the room.

2. A worker is to paint the walls of a square room 8.00 ft high and 12 ft along each side. What surface area in square meters must she cover?

3. An object in the shape of rectangular parallelepiped measures 2.0 in x 3.5 in x 6.5 in. Determine the volume of the object in m^3.

4. One gallon of paint (volume = 3.78×10^{-3} m^3) covers an area of 25.0 m^3. What is the thickness of the paint on the wall?

5. A rectangular plate has a length of (21.3 ± 0.2) cm and a width of (9.8 ± 0.1) cm. Calculate the area of the plate including its uncertainty.

6. How many significant figures are in the following numbers? (a) 78.9 ± 0.2 (b) 3.788×10^9 (c) 2.26×10^{-6} (d) 0.0053

7. The radius of a solid sphere is measured to be (6.50±0.20) cm, and its mass is measured to be (1.85 ± 0.02) kg. Determine the density of the sphere in kg/m^3 and the uncertainty in the density.

8. Compute the value of $\sum_{i=1}^{4} x_i$ if $x_i = (2i+1)$.

9. The square of the speed of an object undergoing a uniform acceleration a is some function of a and the displacement s, according to the expression given by:

   $$v^2 = ka^m s^n$$

   where k is a dimensionless constant. Show by dimensional analysis that this expression is satisfied only if $m = n = 1$.

10. The period T of a simple pendulum is measured in time units and is given by:

    $$T = 2\pi\sqrt{\frac{l}{g}}$$

where l is the length of the pendulum and g is the acceleration due to gravity in units of length divided by the square of time. Show that this equation is dimensionally consistent.

11. Convert these measurements into the new given units. Give answers in standard scientific notation.
   (a) 204.0 kg = _______________ mg
   (b) 503.8 cm^3 = _______________ litres
   (c) 0.0025 m^3 = _______________ dm^3
   (d) 25.9 litres = _______________ kilolitres
   (e) 60 km/h = _______________ m/s

12. Express the following in terms of prefixes
   (a) 0.00085 l (b) 5.44 x 10^{-11} g (c) 73,000,000 m (d) 9.450 s

13. Identify the number of significant figures in the following:
   (a) 9 500 s (b) 0.00702 cm (c) 1.040 x 10^{-5} kg (d) 7.600 ml

14. Express the following
   a) 100L/hour to cubic meters per second
   b) 1 atmosphere to pascal
   c) 100 Dyne to Newtons

15. Estimate how many people if they stood on top of each other would reach the moon.

16. If the unit of force is 100 N, unit of length is 10 m and unit of time is 100 s. What is the unit of mass in this system of units?

17. A furlong is 220 yards, a mile is 1760 yards or 1609 meters, and a fortnight is 14 days. In 1991, the Zambian athlete, Samuel Matete won an Olympic gold medal, in Zurich, Switzerland, when he represented Zambia in the 400 m hurdles. His average speed was 8.5 meters per second. Express his speed in

   (a) kilometer per minute
   (b) mile per hour
   (c) furlong per fortnight

18. A unit of area, often used in measuring land areas, is the *hectare*, defined as 10^4 m^2. A new open-pit copper mine in North-Western Province plans to excavate 75 hectares of land, down to a depth of 26 m, each year. What volume of earth, in cubic kilometers, would be removed during this time?

19. A given formula is derived for the velocity of an object v that varies with time expressed as $v = at^2 + bt + c$ where velocity v and time t are expressed in terms of SI units; determine the units of constants a, b and c in the given equation.

20. The centripetal force F acting on a particle depends on the mass m of the particle, its velocity v and radius r of the circle. Derive dimensionally the formula for force F.

21. The speed of sound υ might plausibly depend on the pressure p, the density ρ, and the volume V of the gas. Use dimensional analysis to determine the exponents x, y and z in the formula:

$$v = C\, p^x\, \rho^y\, V^z,$$

where C is a dimensionless constant. Hence write down the relationship between the said quantities based on the derived exponents.

22. A Copperbelt University railways engineering student, uses dimensional analysis to find the distance d over which a signal can be seen clearly in foggy conditions. The student assumes that the distance depends on the frequency f of the signal, the density ρ of the fog, and intensity of light (power/area) I from the signal. Show that

$$d = k\frac{1}{f}\left(\sqrt[3]{\frac{I}{\rho}}\right)$$

where k, is a dimensionless constant of proportionality

23. In the gas equation $\left(p + \frac{a}{v^2}\right)(v - b) = RT$, where p is the pressure, v is the volume R is the universal gas constant and T the temperature, what are the dimensions of a and b?

24. The viscosity η of a gas depends on the mass m, the effective diameter d and the mean speed of the molecules v. Use dimensional analysis to find an expression for η.

TUTORIAL SHEET 1 2022: UNITS & MEASUREMENTS

SOLUTION MANUAL.

(1) Finding the area in m^2

Data given

$l = 13.71 m$

$w = 4.46 m$

$A = m^2 = ?$

Solution

From the given data, Area in m^2 is found as follows.

Area (A) = length (l) x width (w)

$$A = lw$$
$$= 13.71 m \times 4.46 m$$
$$= (13.71 \times 4.46) m^2$$
$$= 61.1466 m^2$$
$$= 61.1 m^2$$

[our ANS remains correct to 3 s.f according to multiplication rule]

(2) Finding the surface area in square meters of the walls of the room.

Data given

$h = 8.00 ft$

$l = 12 ft$

$A = ft^2 = ?$

$A = m^2 = ?$

Solution

We must convert the area from $feets^2$ to $meters^2$. Then in this case we have 4 sides of equal measurements so we multiply our answer by 4 sides.

Area (A) = height (h) x length (l)

$$A = hl$$
$$A = 8.00 ft \times 12 ft$$
$$A = 96.00 ft^2 = \text{*Area for one side}$$
$$A\ (4\ \text{sides}) = 4 \times 96.00 ft^2$$
$$= 384.00 ft^2$$

$$A\ (m^2) = 384.00 ft^2 \times \left(\frac{1 m}{3.28084 ft}\right)^2$$
$$= 35.67467508 m^2$$
$$= 35.675 m^2$$

③ Finding the volume of the given object in m^3.

Data given:

$l = 2.0 in$
$b = 3.5 in$
$h = 6.5 in$
$V = in^3 = ?$
$V = m^3 = ?$

solution.

here we first find the volume in inches and convert it to meters as required.

So Volume (V) = length (l) x breadth (b) x ~~width~~ height (h)

$$V = lbh$$
$$= 2.0in \times 3.5in \times 6.5in$$
$$= (2.0 \times 3.5 \times 6.5)in^3$$
$$= 45.5in^3$$

$$V(m^3) = 45.5in^3 \times \left(\frac{0.0254m}{1in}\right)^3$$

$$V = 0.000745611412m^3$$
$$= \underline{\underline{0.0007456m^3}}$$

Note: When converting units if the first digit of the converted figure or value is less than the first digit of the original value (metric value), we increase the number of siginificant figures (s.f) in the converted unit (value) by one (more than the original value). So here we have

$\underline{4}5.5$ (original value) and $\underline{0}.000745....$ (converted value)

So $4 > 0$

hence the answer has 4 s.f instead of 3 s.f

(4) Finding the thickness of the paint on a wall.

<u>Data given</u>

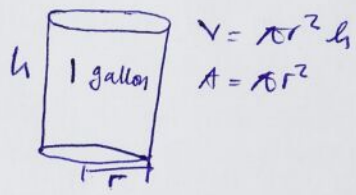
Volume $(V) = 1$ gallon

$1 \text{ gallon} \rightarrow 3.78 \times 10^{-3}\, m^3$

Area $= 25.0\, m^2$

height $= m = ?$

Diagramatically



<u>Solution</u>

let the volume of the paint

$(V) =$ length $\times$ breadth $\times$ height

or specifically, let

Volume $= \pi \times (\text{radius})^2 \times$ height

$V = \pi r^2 \times h$ — (1)

where Area $(A) = \pi r^2$ — (2)

(2) into (1)

$V = A h$ — (3)

making h the subject gives

$$h = \frac{V}{A}$$

Substituting the given values now gives

$$h = \frac{3.78 \times 10^{-3}\, m^3}{25.0\, m^2}$$

$$= \frac{3.78 \times 10^{-3}\, m}{25.0}$$

$$= 0.0001512\, m$$

$$= \underline{0.000151\, m}$$

[final should have 3 s.f according the division rules]

⑤ Calculating the Area of the plate with its uncertainty

data given

length = (21.3 ± 0.2) cm

width = (9.8 ± 0.1) cm

$A = m^2 = ?$

Area + uncertainty $= m^2 = ?$

Solution

Here we first find the Area of the plate without the uncertainty, we the find the uncertainty in measurements and add them. finally we multiply the added (sum) of the uncertaimitjes with the earlier found Area and write them down as follows.

Area (A) = length (l) x width (w)

$= (21.3 \times 9.8)\,cm^2$

$= 208.74\,cm^2$

$= 210\,cm^2$ [rounded off to the nearest 10 so that we should have 2 s.f according to multiplication rules]

uncertainty $\frac{0.2}{21.3} \times 100\% + \frac{0.1}{9.8} \times 100\%$

$= 0.938967136\% + 1.020408163\%$

$= 1.959375299\%$

$= 2\%$ [Correct to 1 s.f]

finally $210\,cm^2 \pm \frac{2\%}{100\%} \times 210\,cm^2$

$= 210\,cm^2 \pm 4.2\,cm^2$

$= 210\,cm^2 \pm 4\,cm^2$

(6) Determining the number of significant figures (s.f)

(b) 3.788×10^4 here 10^4 does not count it just indicates the value of the number.

ANS: 4 s.f

(c) 2.26×10^{-6} same as (b)

ANS: 3 s.f

(a) Sorry (a) was skipt

78.9 ± 0.2 here ± 0.2 does not play a role in counting # of S.f it simply shows the uncertainty. So we only considere 78.9

ANS: 3 s.f

(d) 0.0053

ANS: 2 s.f

(1) Zero before the decimal point (d.p) is not a s.f provided that there are no other numbers before the d.p other than zero.

(1) Provided that we only a zero before the (d.p) then zero next to or just after the (d.p) is not a (s.f).

(7) Determining the density of the sphere with its uncertainty

Data given

Sphere's radius $(r) = (6.50 \pm 0.20)$ cm

Sphere's mass $(m) = (1.85 \pm 0.02)$ kg

Volume $(V) = m^3 = ?$

density $= kg/m^3 = ?$

density and uncertainty $= ?$

Solution

We proceed just as we did in question (5) except that here we multiply the uncertainty of the radius measurement by three to get the uncertainty of volume according to the rules. So we proceed as follows.

Volume of the sphere $(V) = \frac{4}{3}\pi r^3$ —①

density $(d) = \frac{mass}{Volume} = \frac{m}{V}$ —②

① into ②

$d = \frac{m}{\frac{4}{3}\pi r^3}$, $r = \frac{6.50\, cm \times 10^{-2}\, m}{1\, cm}$

$V = \frac{4}{3}\pi \times \left(\frac{6.50\, cm}{1\, cm} \times 10^{-2}\, m\right)^3$

$V = 1.1503465 1 \times 10^{-3}\, m^3$

$d = \frac{1.85\, kg}{1.150346 51 \times 10^{-3}\, m^3}$

$= 1608.211077\, kg/m^3$

$= 1610\, kg/m^3$ [rounded to the nearest 10 in order to have 3 s.f according to division rules] we now find the uncertainty in measurements

$\left(\frac{0.2}{6.5} \times 100\%\right) \times 3 + \frac{0.02}{1.85} \times 100\%$

(for volume) (for mass)

$= 10.31185031\%$

finally ~~uncertainty~~ density and its uncertainty will be

density (d) $= 1610\ kg/m^3 \pm \frac{10.31185031\%}{100\%} \times 1610\ kg/m^3$

$= 1610\ kg/m^3 \pm 166.02079\ kg/m^3$

$= 1610\ kg/m^3 \pm 166$

$= \underline{(1.61 \pm 0.17) \times 10^3\ kg/m^3}$

(8) Computing the value of $\sum_{i=1}^{4} x_i$ if $x_i = (2i+1)$

Data given

$i = 1 \rightarrow 4$

$x_i = (2i+1)$

$\sum_{i=1}^{4} x_i = ?$

Solution

Since in any calculation the simbol $\sum$ stands for summation, the we add the values found as follows

for $i = 1;\ x_i = 2i+1 = (2\times1)+1 = 3$

$i = 2;\ x_i = 2i+1 = (2\times2)+1 = 5$

$i = 3;\ x_i = 2i+1 = (2\times3)+1 = 7$

$i = 4;\ x_i = 2i+1 = (2\times4)+1 = 9$

finally $\sum_{i=1}^{4} x_i = (2i+1) = 3+5+7+9 = 24$ //

(9) dimensional analysis; showing that $v^2 = ka^m s^n$ is dimensionally correct.

data given

v = speed

a = acceleration

s = displacement.

k = constant (dimensionless)

Solution

We first start by identifying the units & dimensions of each ~~and~~ given information.

$v = m/s = LT^{-1}$

$a = m/s^2 = LT^{-2}$

$s = m = L$

$k = 0 = 0$ = dimensionless so it can be neglected.

now

$v^2 = ka^m s^n$

$v^2 = a^m s^n$

Putting dimensions gives

~~$(LT^{-1})^2 = (LT^{-2})^m$~~

$[LT^{-1}]^2 = [LT^{-2}]^m [L]^n$

$[L]^2[T^{-1}]^2 = [L]^{m+n}[T^{-2}]^m$

L.H.S	R.H.S
$L = L^2$	$L = L^{m+n}$
$T = T^{-2}$	$T = T^{-2}$

lets now find the values of m and n

$L^2 = L^{m+n}$

$2 = m + n$ — ①

$T^{-2} = T^{-2m}$

$-2 = -2m$

$m = 1$ — ②

② m to ①

$2 = m + n$

$2 = 1 + n$

$n = 2 - 1$

$n = 1$

now $v^2 \propto a^m s^n$

$v^2 = k a^m s^n$ let $k = 1$

$L^2T^{-2} = [LT^{-2}]^1 [L]^1$

$L^2T^{-2} = L^{1+1} T^{-2}$

$L^2T^{-2} = L^2T^{-2}$

dimensionally correct.

LHS = RHS

So since $m = n = 1$ in this case we have shown that the given equation is dimensionally correct.

(10) Showing that $T = 2\pi\sqrt{l/g}$ is dimensionally correct.

<u>data given</u>

T = period of simple pendulum

l = length of the pendulum

g = acceleration due to gravity

<u>Solution</u>

We proceed similarly as in the previous question except that here the power for the RHS units is

$T = s = T^1$

$l = m = L$

$g = m/s^2 = LT^{-2}$

$[T] = [L]^{\frac{1}{2}} [L^{+1} T^{-2}]^{-\frac{1}{2}}$

$[T] = [L]^{-\frac{1}{2}+\frac{1}{2}} [T]^{-2 + (-\frac{1}{2})}$

Note: $\sqrt{\frac{l}{g}} = \sqrt{l g^{-1}} = (l g^{-1})^{\frac{1}{2}}$
$= [L]^{\frac{1}{2}} [L T^{-2}]^{-\frac{1}{2}}$

$(g^{-1})^{\frac{1}{2}}$
$g^{-\frac{1}{2}}$

$T = L^0 T^1$

$T = T$

. LHS = RHS hense its dimensionally correct

(11) Conversion of units giving answers in standard notation

(a) 204.0 kg to mg

ANS: $\frac{1 \, mg}{0.001 kg} \times 204.0 \, kg = 204000 \, mg$

$= 2.04 \times 10^5 \, mg$

Note Conversion rules must be applied here for determining the number of significant figures.

let A be the first digit of the number to be converted. And B to be the first digit of the converted number. (i.e) [$\underset{A}{2}00$ converted to $\underset{B}{1}00$]

Rules now.

when (1) A = B maintain the number of significant figures in converted value as of the initial value

(2) A < B also use rule number (1) above

(3) A > B drop the number of significant figures by one in the final value.

(11) b $503.8 cm^3 \rightarrow$ litres

ANS: $503.8 cm^3 \times \frac{1 \, Litre}{1000 \, cm^3} = 0.5038$ litres
$= 0.50380$ litres

So $s > 0$ we increase s.f by one

5.0380×10^{-1} litres

Note ANS = Answer

(c) $0.0025\,m^3$ to dm^3 (decimeters)

ANS: $0.0025\,m^3 \times \left(\frac{1\,dm}{0.1\,m}\right)^3 = 2.5\,dm^3$

$= \underline{25 \times 10^{-1}\,dm^3}$

(d) 25.9 litres to kilo litres

ANS: 25.9 litres $\times \frac{0.001 \text{ kilolitres}}{1 \text{ litre}} = 0.0259$ kilolitres

$= 0.02590$ kilolitres

$= \underline{2.590 \times 10^{-2} \text{ kilolitres}}$

(e) 60 km/h to m/s.

ANS: $\frac{60\,km}{h} \times \frac{1\,h}{60\,s} \times \frac{1000\,m}{1\,km} = 1000\,m/s$

$= \cancel{1} = \underline{\underline{1 \times 10^3\,m/s}}$

(12) Expressing the values given in terms of prefixes

GIVEN VALUE	ANSWER
(a) 0.00085 litres	= 0.85 ml = 0.85 milli litres
(b) 5.44×10^{-11} g	= 5.44×10^{-2} nano grams
(c) 73 000 000 m	= 73 Mega meters
(d) 9.450 s	= 9450 milli seconds

(13) Identifying the number of significant figures.

- Solutions

We shall be using rules as follows

(a)(i) Zeroes at the end of a number with no decimal point are not s.f

so 9500 s has 2 s.f

(b)(ii) If we only have a zero before the decimal point then zero before the decimal point and zero (s) after the decimal point are not s.f.

So 0.00702 has 3 s.f

(iii) Zero (s) in between numbers (non zeroes) is counted as a s.f.

(c) 1.040×10^{-5} kg has 4 s.f

(iv) If the number before the decimal point is a non-zero digit, the all zeroes in that number are counted (as in c above)

(d) 7.600 ml has 4 s.f (rule iv applies here)

(14) Conversion of units.

(a) 100 L/hour to cubic meters / second

ANS: $\frac{100L}{h} \times \frac{h}{L} \times 2.778 \times 10^{-7} m^3/s$

$= 2.778 \times 10^{-5} m^3/s$

(b) 1 atm to Pascal

ANS: This is just a conversion factor we have here

1 atm = 101325 Pa

(C) 100 Dyne to Newtons

ANS: $100\,\text{dyne} \times \dfrac{1N}{100000\,\text{dyne}} = 0.001N$

(15) Estmating the number of people to reach the moon from the earth.

Solution

Given that the distance from the earth to the moon is about 384.4 million meters. And given also that the average height of a human being is about 1.72m, then we estimate as follows

$$\text{People required} = \frac{384.4 \text{ million } \cancel{\text{meters}}}{1.72\ \cancel{\text{meters}}}$$

$$= \frac{384.4 \text{ million}}{1.72}$$

$$= \frac{384.4 \times 1\,000000}{1.72}$$

$$= 223488372.1$$

$$\simeq 223.5 \text{ million people}$$

(16) Finding the unit of mass in the given system.

Data given

Force (F) = 100N

distance = 10m

time = 100 seconds

acceleration = ?

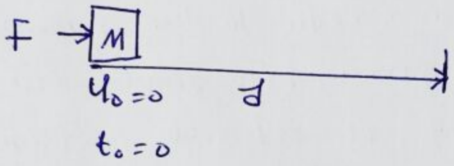
Mass = ?

9. **Solution**

given that force (F) = mass × acceleration

$$f = ma \quad -①$$

lets have a clear picture of this given system. We assume we need to push a box of mass m with force f horizontally with a distance d.



let the initial velocity of the box u_0 at time to be equal to zero. given the distance d the box is to cover, let it undergo a certain acceleration a during the given period of time t.

then $d = u_0 t_0 + \frac{1}{2}at^2 \quad -②$

$$d = \frac{1}{2}at^2 \quad -③ \qquad \text{for } u_0 = t_0 = 0$$

making a the subject gives

$$a = \frac{2d}{t^2} \quad -④$$

④ into ①

$$f = ma$$

$$f = m\frac{2d}{t^2} \quad -⑤$$

making m the subject gives

$$m = Ft^2/2d \quad -⑥$$

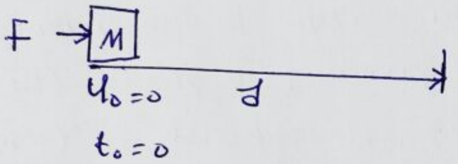
lets now use ⑥ to find m.

9. Solution

given that force (F) = mass × acceleration

$$F = ma \quad -① \quad ,$$

lets have a clear picture of this given system. We assume we need to push a box of mass m with force F horizontally with a distance d.



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$$d = \tfrac{1}{2}at^2 \quad -③ \qquad \text{for } u_0 = t_0 = 0$$

making a the subject gives

$$a = \frac{2d}{t^2} \quad -④$$

④ into ①

$$f = ma$$

$$f = m\frac{2d}{t^2} \quad -⑤$$

making m the subject gives

$$m = Ft^2/2d \quad -⑥$$

lets now use ⑥ to find m.

$m = Ft^2/2s$

$= \frac{100N \times (100)^2 s^2}{2 \times 10m}$, $N = kg\,m/s^2$

$= \frac{100\,kg\,m/s^2 \times (100)^2 s^2}{2 \times 10m}$

$= \frac{100\,kg\,\cancel{m}\,\cancel{s^{-2}} \times (100)^2 \cancel{s^2}}{2 \times 10\,\cancel{m}}$

$= \underline{50000\,kg}$

(17) Conversion of units. (8.5 meters per second) to

(a) kilometer per meter

ANS: $\frac{8.5m}{s} \times \frac{60s}{1\,minute} \times \frac{1\,Km}{1000m} = \underline{0.51Km/minute}$

(b) mile per hour

here we use the solution in (a) above

ANS: $\frac{0.51km}{minute} \times \frac{60\,minutes}{1\,hour} \times \frac{1\,mile}{1.609km}$

$= 19.01802362\,miles/hour$

$= \underline{19\,miles/hour}$

(c) ~~we~~ furlong per fortnight

ANS: We first convert miles per hour to yards/hour so

$$\frac{19.01802362 \text{ miles}}{\text{hour}} \times \frac{1760 \text{ yards}}{1 \text{ mile}}$$

$\doteq 33471.72157$ yards / hour

then we convert to furlong / hour as follows

$$\frac{33471.72157 \cancel{\text{yards}}}{\text{hour}} \times \frac{\text{furlong}}{220 \text{ yards}}$$

$= 152.1441889$ furlong / hour

finally we convert furlong / hour to furlong / fortnight

$$\frac{152.1441889 \text{ furlong}}{\text{hour}} \times \frac{14 \text{ days}}{1 \text{ fortnight}} \times \frac{\text{hour}}{0.041666666 \text{ day}}$$

$= 51120.44747$ furlong / fortnight

$= \underline{\underline{51000 \text{ furlong / fortnight}}}$

Note: 24 hours → 1 day

so 1 hour → x days

[Rounded off to the nearest hundred to make it become correct to 2. sf]

(18) Determining the volume of land to be removed for a year.

<u>Data given</u>

height = 26m

Area = ?

1 hectare $= 10^4 m^2$

Volume $= \cancel{k}m^3 = ?$

Volume $= km^3 = ?$

Solution

Since we don't know specifically the shape of the land to be ~~cattle~~ removed but just a few information, lets solve it in this form.

let Volume (V) = length x breadth x height , length x breadth = Area

$$V = lbh$$

$$V = Ah \quad -①$$

given that 1 hectare $= 10^4 m^2$

the we are told 75 hectares well be removed, so

$$A = 75 \text{ hectares} \times \frac{10^4 m^2}{\text{hectare}}$$

$$A = 75 \times 10^4 m^2 \quad -ⓑ$$

$$h = 26m \quad -ⓒ$$

ⓑ and ⓒ into ①

$$V = 75 \times 10^4 m^2 \times 26m$$

$$V = 260000 m^3$$

We convert now

$$V = 260000 m^3 \times \left(\frac{1km}{1000m}\right)^3$$

$$= 2.6 \times 10^{-4} km^3$$

(19) Dimensional analysis: finding the dimensions of a, b and c in the equation $V = at^2 + bt + c$

Data given

V = velocity

t = time

a = b = c = constant

Solution

Accordition to the principle of homogenity for dimensional analysis, units (similar) are addedup divided, subtracted and ~~divide~~ multiplied.

So a & c should have similar units as those of t.
And c hould have units as those of V
This is because LHS should equalize RHS. In terms of dimensions.

$V = at^2 + bt + c$

Lets solve individually as follows

① $V = at^2$, $V = LT^{-1}$

$t = T$

$LT^{-1} = aT^2$

$\frac{LT^{-1}}{T^2} = \frac{a\cancel{T^2}}{\cancel{T^2}}$

$a = LT^{-1-2}$

$a = LT^{-3}$ //

② $V = bt$

$LT^{-1} = bT$

$\frac{LT^{-1}}{T} = \frac{bT}{T}$

$b = LT^{-1-1}$

$b = LT^{-2}$ //

③ $V = c$

$LT^{-1} = c$

$\therefore c = LT^{-1}$

$\therefore a = LT^{-3}$

$b = LT^{-2}$

$c = LT^{-1}$

* Lets try to prove.

$V = at^2 + bt + c$

$[LT^{-1}] = [LT^{-3}][T]^2 + [LT^{-2}][T] + [LT^{-1}]$

$[L][T^{-1}] = [L][T^{-3+2}] + [L][T^{-2+1}] + [L][T^{-1}]$

$[L][T^{-1}] = [L][T^{-1}] + [L][T^{-1}] + [L][T^{-1}]$

$LT^{-1} = LT^{-1} + LT^{-1} + LT^{-1}$

$LT^{-1} = 3LT^{-1}$, 3 is a dimensionally constant and so it doesn't count here

$\therefore$ LHS = RHS

$L = L$

$T^{-1} = T^{-1}$

hense the dimensions for a, b and c are correct.

(20) ~~Dimensional~~ analysis: Deriving dimensions of force f.

Data given

f = force

m = mass

v = velocity

r = radius of the circle

Solution

here we are told that

$f \propto mvr$

$f = k\,mvr$, k = dimensionally constant

lets assign the powers of mvr by x, y and z accordingly

so

$f = k\,m^x v^y r^z$ let $k=1$

$f = m^x v^y r^z$ — (1)

we now right down the ~~units~~ units and dimensions

$f = kgm/s^2 = kg\,m\,s^{-2} = MLT^{-2}$

$m = kg = M$

$v = m/s = LT^{-1}$

$r = m = L$

now we go back to the equation

$$f = [m]^x [v]^y [r]^z$$

$$= [M]^x [LT^{-1}]^y [L]^z$$

$$= [M]^x [L]^{y+z} [T^{-1}]^y$$

$$MLT^{-2} = [M^x][L^{y+z}][T^{-y}]$$

LHS = RHS

$M = M^x$

$\therefore\ x = 1$ (1)

$L = L^{y+z}$

$y + z = 1$ —②

$T^{-2} = T^{-y}$

$-2 = -y$

$\therefore y = 2$ —③

③ into ②

$2 + z = 1$

$z = 1 - 2$

$z = -1$

So we now have

$x = 1$

$y = 2$

$z = -1$

lets now recall the equation ①

$f = m^x v^y r^z$

we now replace x, y, z with 1, 2, -1

$f = m^1 v^2 r^{-1}$

$f = \frac{mv^2}{r}$

$\therefore f = \frac{mv^2}{r}$ This is the formula for Centripetal force F derived dimensionally.

(21) Determining the exponents of x, y and z in the given equation using dimensional analysis

<u>data given</u>

v = speed

P = pressure

ρ = density

V = Volume

C = dimensionless constant.

we proceed as follows

we write the dimensions and units of the given data.

$L = L^{y+z}$

$y + z = 1 \quad - (2)$

$T^{-2} = T^{-y}$

$-2 = -y$

$\therefore \; y = 2 \quad - (3)$

(3) into (2)

$2 + z = 1$

$z = 1 - 2$

$z = -1$

So we now have

$x = 1$

$y = 2$

$z = -1$

lets now recall the equation (1)

$f = m^x v^y r^z$

we now replace x, y, z with $1, 2, -1$

$f = m^1 v^2 r^{-1}$

$f = \frac{mv^2}{r}$

$\therefore \; f = \frac{mv^2}{r}$ This is the formula for Centripetal force F derived dimensionally.

(21) Determining the exponents of x, y and z in the given equation using dimensional analysis

<u>data given</u>

v = speed

P = pressure

ρ = density

V = Volume

C = dimensionless constant.

We proceed as follows

We write the dimensions and units of the given data.

$$V = m/s = LT^{-1}$$

$$\left.\begin{aligned} P &= Kg\,m\,s^{-2}m^{-2} = [M][LT^{-2}][L^{-2}] \\ \rho &= Kg\,m^{-3} = [M][L^{-3}] \\ V &= m^3 = [L^3] \end{aligned}\right\} ⓐ$$

now we have

$V = C P^x \rho^y V^z$ ① as the given equation, C a dimensionally constant.

so we replace ⓐ into ①

$$[LT^{-1}] = \{[M][LT^{-2}][L^{-2}]\}^x [ML^{-3}]^y [L^3]^z$$

$$[L][T^{-1}] = [M^{x+y}][L^{-x+3z-3y}][T^{-2x}]$$

$$[L][T^{-1}] = [M^{x+y}] = [L^{-x+3z-3y}][T^{-2x}]$$

we now proceed as follows

LHS = RHS

$$T^{-1} = T^{-2x}$$

$$-1 = -2x$$

$$x = \frac{1}{2} \quad ①$$

$$L = L^{-x+3z-3y}$$

$$1 = -x + 3z - 3y \quad , x = \frac{1}{2}$$

$$1 = -\frac{1}{2} + 3z - 3y$$

$$3z - 3y = \frac{3}{2} \quad -②$$

$$M^0 = M^{x+y}$$

$$0 = x + y$$

$$y = -x$$

$$y = -\frac{1}{2}$$

now

$1 = -x + 3z - 3y$

$1 = 3z - \frac{1}{2} + \frac{3}{2}$

$1 = 3z + 1$

$z = 0$

$\therefore x = \frac{1}{2}$

$y = -\frac{1}{2}$

$z = 0$

hence $v = C P^x \rho^y V^z$ let $C = 1$

$V = P^{1/2} \rho^{-1/2} V^0$

$V = \left(\frac{P}{\rho}\right)^{1/2}$

$$\underline{v = C\sqrt{\frac{P}{\rho}}}$$

(22) dimensional analysis; showing that distance equalizes the given units.

<u>data given</u>

$d =$ distance

$f =$ frequency

$I =$ intensity

$\rho =$ density

$k =$ dimensionally constant

<u>solution</u>

It's quiet the same as what we did in question 21

So let

$d \propto f \, I \, \rho$

$d = k f I \rho$

let assign the powers of the given units as follows

$d = k f^x I^y \rho^z$, k is dimensionless constant

$d = f^x I^y \rho^z$ — ①

So now we need to show that

$d = k \frac{1}{f} \left(\sqrt[3]{\frac{I}{\rho}} \right)$ as follows.

dimensions of the given units

$$\left.\begin{array}{l} f = T^{-1} \\ I = ML^2T^{-3} \\ \rho = ML^{-3} \end{array}\right\} ⓐ$$

ⓐ into ①

$d = f^x I^y \rho^z$

$L = [T^{-1}]^x [ML^2T^{-3}]^y [ML^{-3}]^z$

$L = [T^{-1}]^x [M^{y+z}][L^{2y-3z}][T^{-3y}]$

$L = [T]^{-x-3y} [M]^{y+z} [L]^{2y-3z}$ — ⓑ

we can also write ⓑ as

$[M^0][T^0][L] = [T]^{-x-3y}[M]^{y+z}[L]^{2y-3z}$

now lets find the values of x, y and z

LHS = RHS

$M^0 = M^{y+z}$

$0 = y+z$

$-y = z$ — ①

$T^0 = T^{-x-3y}$

$0 = -x-3y$

$x = -3y$ — ②

$L^1 = L^{0y-3z}$

$1 = -3z$

$z = -\frac{1}{3}$ — ③

③ into ①

$-y = z$

$-y = -\frac{1}{3}$

$y = \frac{1}{3}$ — ④

④ into ②

$x = -3y$

$x = -3 \times \frac{1}{3}$

$x = -1$

$\therefore x = -1$

$y = \frac{1}{3}$

$z = -\frac{1}{3}$

plug in the values of x, y and z into equation ⓒ gives

$$d = f^{x} I^{y} \rho^{z}$$

$$d = f^{-1} I^{1/3} \rho^{-1/3}$$

$$d = \frac{1}{f}\left(\frac{I}{\rho}\right)^{1/3}$$

$$d = \frac{1}{f}\sqrt[3]{\frac{I}{\rho}}$$

$$\therefore d = K \tfrac{1}{f}\left(\sqrt[3]{I/\rho}\right)$$

hence the above equation is dimensionally constant.

(23) dimensional analysis: finding the dimensions of a and b in the given equation

<u>data given</u>

P = pressure

V = volume

a = b = Constants

T = Temperature

R = Universal gas constant

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

<u>Solution</u>

We proceed as we did in question (19). So here the units b should be that of V and units of a should be that of P and V.

So $b = v$, $V = L^3$

$\therefore b = L^3$

$\frac{a}{V^2} + P$ here $a = PV^2$

So $a = \frac{F}{A} V^2$

$\boxed{= \frac{MLT^{-2}}{L^2}}$

$= [M][L][T^{-2}][L^2][L^3]^2$

$= [M][L^{1+2+6}][T^{-2}]$

$[M][L]^9 [T]^{-2}$

$\therefore a = ML^9 T^{-2}$

$b = L^3$

24 Dimensional analysis; find the expression for viscosity dimensionally.

<u>given data</u>

η = viscosity

m = mass

v = speed

d = distance (diameter)

Solution:

lets proceed as follows (Same as question 21)

$\eta \propto mdv$

$\eta = Kmdv$, K is dimensionless constant.

$\eta = m d v$

$\eta = m^x d^y v^z$

$\eta = [M]^x [d]^y [v]^z$

$[M][L^{-1}][T^{-1}] = [M]^x [L]^y [L T^{-1}]^z$

$[M]^1 [L]^{-1} [T]^{-1} = [M]^x [L]^{y+z} [T]^{-z}$

We now find the values of x, y and z as follows

LHS = RHS

$M^1 = M^x$

$1 = x$ —①

$L^{-1} = L^{y+z}$

$-1 = y+z$ —②

$T^{-1} = T^{-z}$

$-1 = -z$

$z = 1$ —③

then

$-1 = y + z$

$-1 = y + 1$

$y = -1 - 1$

$y = -2$

We now substitute the values of x, y and z in the equation

$\eta = m^x d^y v^z$

$\eta = m^1 d^{-2} v^1$

$\eta = \frac{mv}{d^2}$

As required