THE COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS

TUTORIAL SHEET 1: **MA110 - Mathematical Methods** 2023/2024

1. A) List the elements of each of the following sets
   - i) $\{x: x \text{ is natural number less than } 5\}$
   - ii) $\{x: x \text{ is a negative integer greater than } -3\}$
   - iii) $\{x: x \text{ is a positive number less than } -5\} \cup \{1,2,3\}$
   - iv) $\{x: x \text{ is a positive even number less than } 10\} \cap \{x: x \text{ is an integer.}\}$
   - v) $\{x: x = 4k - 1, \text{where } k = 0,1,2,3,4,5\}$
   - vi) $\{x: x \text{ is an integer}\} \cap \{1, \sqrt{2},\ 3.14, 7\}$

   B) Given that $A = \{-2,-1,0,1,2\}$ .List the elements of the following sets

   i) $\{x^3 : x \in A\}$ ii) $\{x^2 + x : x \in A\}$ iii) $\left\{{}^{2}/_{x+1} : x \in A\right\}$ iv) $\{3x^2 + 1 : x \in A\}$

2. Describe each of the following in set builder notation
   a) $A = \{1,4,9,16,25\}$
   b) $B = \{-7,-5,-3,-1\}$
   c) C ={2,4,6,8,10,12,16}
   d) $D = \{1,2,4,8,16,32\}$

3. Let $A = \{1,2,3,4,5\}$, $B = \{2,4,6\}$,$C = \{3,4,5\}$ and let $E = \{0,1,2,3,4,5,6,7,8\}$
   Find i) B' ii) $A \cup B$ iii)$A \cap B$ iv)$(A \cup B)'$ v) $(A \cap B)'$ vi) $C - B$

   vii)$(U - A) \cap (B - C)'$ Viii) $A \cup (C - B)$

4. Verify or show the following properties:
   a) Associative Laws: $(A \cup B) \cup C = A \cup (B \cup C)$ and $(A \cap B) \cap C = A \cap (B \cap C)$
   b) Distributive Laws : $A \cap (B \cup C) = (A \cap B) U (A \cap C)$
   and $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

5. i) Prove the De Morgan's Laws a) $(B \cap C)' = B' \cup C'$ b) $(B \cup C)' = B' \cap C'$
   ii) Prove that $(A')' = A$
   iii) Verify or show the De Morgan's Laws a) $(B \cap C)' = B' \cup C'$ b) $(B \cup C)' = B' \cap C'$

6. If $C \subset D$, then simplify if possible
   i) $C \cap D$ ii) $C' \cup D'$ iii)$C \cup D'$ iv) $C' \cap (C \cup D)$

7. If C *and* D are disjoint, simplify if possible
   i)$C' \cap D'$ ii) $C' \cup D'$ iii) $(C \cap D)'$ iv) $(C \uplus D)'$

8. Represent each of the following on a Venn diagram

i) $A \cap B'$ ii) $(A \cap B)'$ iii) $(A \cap B') \cup (A' \cap B)$ iv) $(A \cup B) \cap (A \cup B')$
v) $[A' \cup (A \cap B)']'$ Vi) $A' \cap B' \cap C$

9. Using the associative and distributive properties of union and intersection of sets .Show that
i) $A = (A \cap B) \cup (A \cap B')$ ii) $A \cup B = (A \cap B) \cup (A \cap B') \cup (A' \cap B)$
iii) $A \cup (A' \cap B) = A \cup B$

10. a)Given that X, Y and Z are sets, simplify the following if possible
i) $[X' \cup (Y \cap Z)]'$ ii) $Y' \cap (X \cup Y)$ iii) $(X \cap Y) \cup (X \cap Y')$ iv) $(X \cup Y) \cap (X \cup Y')$

v) $[X' \cup (X - Z)]$

b) Given that X and Y are subsets of some universal set U, simplify the following:
(i) $X \cap (X' \cup Y)$.
(ii) $[(X \cap Y)' \cap (X' \cup Y)]'$.

11 a) Using the symbol " $\subset$ " put the set of numbers in ascending order given

$\mathbb{C}, \mathbb{N}, \mathbb{R}, \mathbb{Z}$ *and* $\mathbb{Q}$

b) Give the definition of the following sets $\mathbb{Q}^*, \mathbb{R}^*, \mathbb{C}^*$and $\mathbb{Z}^*$

12.a) Let $A = \{x \in \mathbb{R}: -4 \leq x < 2\}$ and $B = \{x \in \mathbb{R}: x \geq -1\}$. Find a) $A \cap B$ b) A'
b) Let $U = (-6,9]$ be the universal set, $= [-2,4], B = (-1,5)$ *and* $C = (-6,9]$.
Find the following sets:
i) $A \cap B$ ii) $U - C$ iii) $B'UA$ iv) $(A \cup C)'$
c) Given that $\mathbb{R}$, the set of real numbers is the universal set, $A = (-4,7]$ *and* $B = [4, \infty)$,
Find
i)A' ii)B' iii)$A - B$ iv) $B - A$
d) Let $A = (-9,9)$ be the universal set and $X = (-1,5]$, $Y = [-5,3]$ *and* $Z = [-1,7)$.
Find each of the following sets and display it on the number line:
i)X' ii) $A - X$ iii) $(X \cap Z)'$ iv) $(Y - X) \cap Z$
e) Let A = {1,2, 3, 4,5,6,7,8,9} ; B = [1, 5) and C= (3,8). The universal set

is a set of real numbers . If necessary use the real number line and find:

(i). (A ∩ B) ∪(A ∩ C) (ii). B ∩ C (iii). (B ∪ C)'

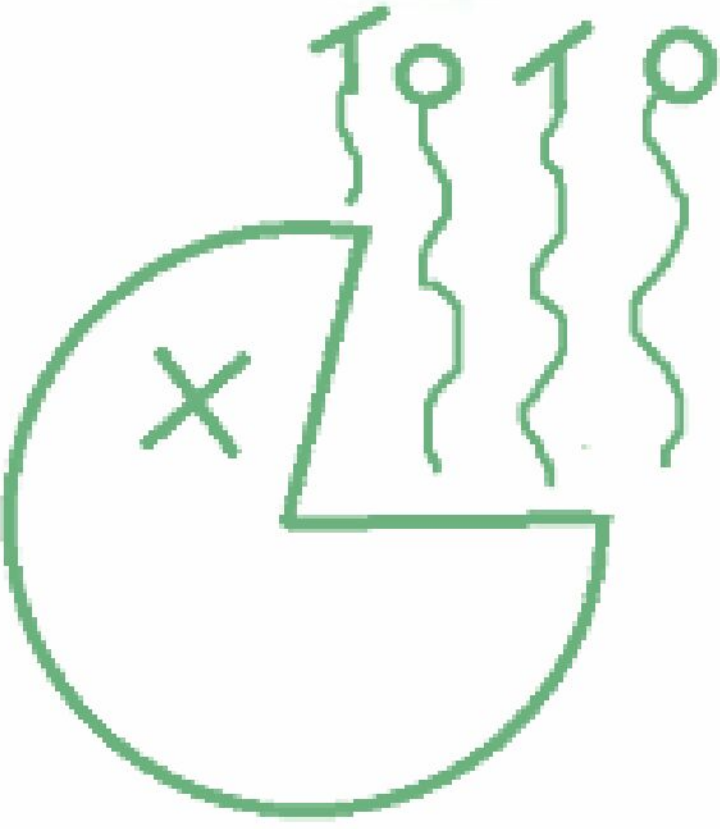
(iv). (B' ∪ C') ∩ A

13. a) Express the following in the form of $\frac{a}{b}$ where a and b are intergers, $b \neq 0$.
i) $0.\overline{33}$ ii) $0.\overline{16}$ iii) $2.\overline{143}$ iv) $3.\overline{7}$ v)1.171717....... vi) $2.5\overline{90}$

b) Prove that i) $\sqrt{3}$ is an irrational number.
ii) $\sqrt{2}$ is an irrational number

c) Given that $\sqrt{3}$ and $\sqrt{5}$ are irrational, show that the following are not rational numbers
i) $\sqrt{3}$ +5 is an irrational number.
ii) $\sqrt{5} - 1$ is an irrational number iii) $1 - \sqrt{3}$ is an irrational number

MA110 Tutorial sheets 📝 1



MA 110 Tutorial Sheet 1

Question 1

(A)

(i) $x = \{1, 2, 3, 4\}$

(ii) $x = \{-2, -1\}$

(iii) $x = \emptyset$ or $\{\ \}$ u {1,2,3 }

x = { 1, 2, 3}

A positive number less than -5 doesn't exist

(iv) $x = \{2, 4, 6, 8\}$

(v.) # when $k = 0$

$x = 4(0) - 1 = -1$

when $k = 1$

$x = 4(1) - 1 = 3$

when $k = 2$

$x = 4(2) - 1 = 7$

And so on till you reach when $k = 5$

$\therefore x = \{-1, 3, 7, 11, 15, 19\}$

(vi) $x = \{1, 7\}$

(B) $A = \{-2, -1, 0, 1, 2\}$

(i) $\{x^3 : x \in A\}$

$A = \{(-2)^3, (-1)^3, (0)^3, (1)^3, (2)^3\}$

$= \{-8, -1, 0, 1, 8\}$

(ii) $x^2 + x$

when $x = -2, -1, 0, 1, 2$

$(-2)^2 + (-2) = 4 - 2 = 2$

$(-1)^2 + (-1) = 1 - 1 = 0$

$(0)^2 + (0)^2 = 0 + 0 = 0$

$(1)^2 + (1)^2 = 1 + 1 = 2$

$(2)^2 + (2) = 4 + 2 = 6$

$A = \{0, 2, 6\}$

(iii) $\frac{2}{x+1}$

when $x = -2, -1, 0, 1, 2$

$\frac{2}{-2+1} = \frac{2}{-1} = -2$

$\frac{2}{-1+1} = \frac{2}{0}$ = undefined

$\frac{2}{0+1} = \frac{2}{1} = 2$

$\frac{2}{1+1} = \frac{2}{2} = 1$

$$\frac{2}{2+1} = \frac{2}{3} = \frac{2}{3}$$

$\therefore A = \{-2, 1, 2, \frac{2}{3}\}$

(iii) $A = \{-2, -1, 0, 1, 2\}$

$3x^2 + 1$

$x=-2$: $3(-2)^2 + 1 = 13$

$x=-1$: $3(-1)^2 + 1 = 4$

$x=0$: $3(0)^2 + 1 = 1$

$x=1$: $3(1)^2 + 1 = 4$

$x=2$: $3(2)^2 + 1 = 13$

$A = \{1, 4, 13\}$

Q.2

(a) $A = \{1, 4, 9, 16, 25\}$

$A = \{x^2 : x \in N, 1 \leq x \leq 5\}$

(b) $B = \{-7, -5, -3, -1\}$

Now note that there are no standard letters for odd, even and prime numbers, therefore

$B = \{2x-1 : x \in Z, -3 \leq x \leq 0\}$

(c). $C = \{2, 4, 6, 8, 10, 12, 16\}$

$C = \{2x : x \in N, 1 \leq x \leq 8, x \neq 7\}$

(d) $D = \{1, 2, 4, 8, 16, 32\}$

$D = \{2^x : x \in Z, 0 \leq x \leq 5\}$

Q.3

$A = \{1, 2, 3, 4, 5\}$

$B = \{2, 4, 6\}$ $C = \{3, 4, 5\}$

$E = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$

Find

(i) $B' = \{0, 1, 3, 5, 7, 8\}$

(ii) $A \cup B = \{1, 2, 3, 4, 5, 6\}$

(iii) $A \cap B = \{2, 4\}$

(iv) $(A \cup B)' = \{0, 7, 8\}$

(v) $(A \cap B)' = \{0, 1, 3, 5, 6, 7, 8\}$

(vi) $C - B = C \cap B'$

$\{3, 5\}$

(vii) $(U - A) \cap (B - C)'$

First simplify it !

$(U - A) \cap (B - C)'$

$(U \cap A') \cap (B \cap C')'$

$A' \cap (B \cap C')'$

$A' = \{0, 6, 7, 8\}$

$B \cap C' = \{2, 6\}$

$(B \cap C')' = \{0, 1, 3, 4, 5, 7, 8\}$

$\underline{A' \cap (B \cap C')' = \{0, 7, 8\}}$

(viii) $A \cup (C - B)$

$A \cup (C \cap B')$

$C \cap B' = \{3, 5\}$

$A = \{1, 2, 3, 4, 5\}$.

$\underline{A \cup (C \cap B') = \{1, 2, 3, 4, 5\}}$

Q.4

[a] $(A \cup B) \cup C = A \cup (B \cup C)$

Here you can either chose to verify or to ~~pro~~ show

With verifying you come with your own universal set, your own set A, B and C then find $(A \cup B) \cup C$ and also $A \cup (B \cup C)$ and just make sure the answer comes out the same.

Then if you want to show you do thiss

$(A \cup B) \cup C = A \cup (B \cup C)$

L.H.S

let $x \in (A \cup B) \cup C$

$x \in (A \cup B)$ or $x \in C$

$x \in A$ or $x \in B$ or $x \in C$

$x \in A$ or $x \in (B \cup C)$

$x \in A \cup (B \cup C)$

$\therefore$ since $(A \cup B) \cup C \subseteq A \cup (B \cup C)$

then $(A \cup B) \cup C = A \cup (B \cup C)$

$(A \cap B) \cap C = A \cap (B \cap C)$

Again verifying here you'll need to do exactly as you did in the previous ~~ex~~ question, hope you did it right.

Showing nomba !!

$(A \cap B) \cap C = A \cap (B \cap C)$

2.4.3

let $x \in (A \cap B) \cap C$

$x \in (A \cap B)$ and $x \in C$
$x \in A$ and $x \in B$ and $x \in C$
$x \in A$ and $x \in (B \cap C)$
$x \in A \cap (B \cap C)$

$\therefore$ since $(A \cap B) \cap C \subseteq A \cap (B \cap C)$
then $(A \cap B) \cap C = A \cap (B \cap C)$

(b.) Distributive laws

??? On the next two pages i have verified and shown $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

try doing up ?

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

proving the distributive law

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$x.$ 1.1.5

let $x \in A$ and $x \in B \cup C$

$x \in A$ and $x \in B$ or $x \in C$

$x \in A$ and $x \in B$ or $x \in A$ and $x \in C$

$x \in A \cap B$ or $x \in A \cap C$

$(A \cap B) \cup (A \cap C)$

verifyng the distributive law

Distributive laws

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

let $E = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 4, 6\}$, $B = \{2, 4, 6, 8, 10\}$ $C = \{4, 5, 9\}$

First lets find $A \cap (B \cup C)$

$B \cup C = \{2, 4, 5, 6, 8, 9, 10\}$

$\underline{A \cap (B \cup C) = \{4, 6\}}$

$\cancel{A \cap B}$

Now lets find $(A \cap B) \cup (A \cap C)$

$A \cap B = \{4, 6\}$ $\qquad$ $A \cap C = \{4\}$

$\underline{(A \cap B) \cup (A \cap C) = \{4, 6\}}$

$\therefore \quad A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$(A\cap B)\cap C = A\cap(B\cap C)$

Again verifying here you'll need to do exactly as you did in the previous ~~a~~ question, hope you did it right.

Showing nomba !!

$(A\cap B)\cap C = A\cap(B\cap C)$

L.H.S

let $x \in (A\cap B)\cap C$

$x \in (A\cap B)$ and $x \in C$
$x \in A$ and $x \in B$ and $x \in C$
$x \in A$ and $x \in (B\cap C)$
$x \in A\cap(B\cap C)$

$\therefore$ since $(A\cap B)\cap C \subseteq A\cap(B\cap C)$
then $(A\cap B)\cap C = A\cap(B\cap C)$

(b.) Distributive laws

On the next two pages i have verified and shown $A\cap(B\cup C) = (A\cap B)\cup(A\cap C)$

try doing up
$A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$

(5, iii)

a) $(B\cap C)' = B'\cup C'$

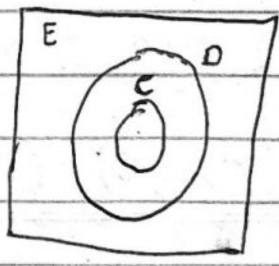
Verifying again is coming up with your sets and find $(B\cap C)'$ and $B'\cup C'$ then jxt making sure thy giveth you the same result.

If you're still lost text me on whatsApp
0965091047

Q.6

$C \subset D$

C is a subset of D as in



(i) $C\cap D = C$

(ii) $C'\cup D' = C'$

(iii) $C\cup D'$ =
This one can't be simplified !!
bakamba ati if possible

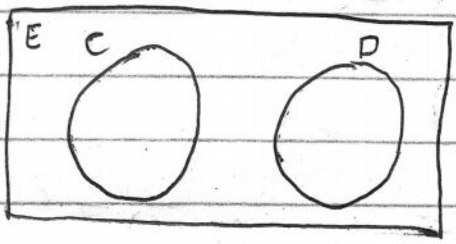
(iv) $C' \cap (C \cup D)$

$C \cup D = D$

$\therefore \underline{\underline{C' \cap D}}$

Q. 7

C na D bali disjoint



(i) $C' \cap D'$ → Can't be simplified

(ii) $C' \cup D' = \underline{\underline{U \text{ or } E}}$

(iii) $(C \cap D)' =$

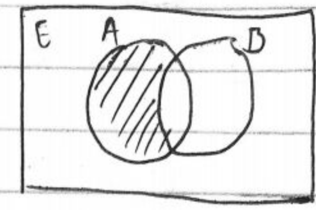
$C \cap D = \phi$

$(\phi)' = \underline{\underline{U \text{ or } E}}$

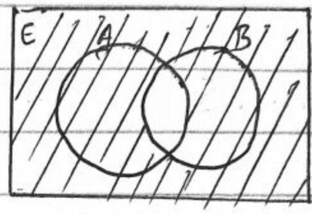
(iv) $(C \cup D)'$ → can't be simplified

Q. 8

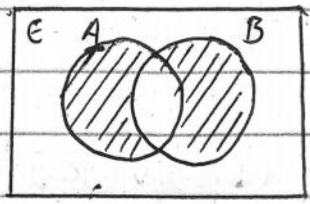
(i) $A \cap B'$



(ii) $(A \cap B)'$



(iii) $(A \cap B') \cup (A' \cap B)$

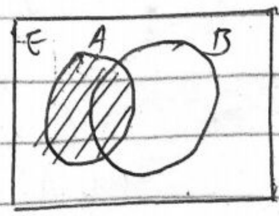


(iv) $(A \cup B) \cap (A \cup B')$

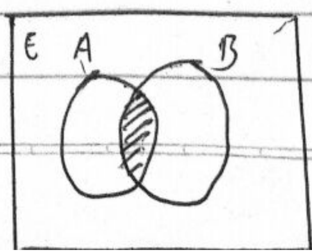
$A \cup (B \cap B')$

$A \cup \phi$

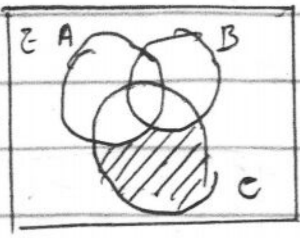
$\underline{A}$



(v) $[A' \cup (A \cap B)']'$



(vi) $A' \cap B' \cap C$



Q.9

(i) $A = (A \cap B) \cup (A \cap B')$

R.H.S

$A \cap (B \cup B')$

$A \cap U$

$\underline{\underline{A}}$ Hence shown

(ii) $A \cup B = (A \cap B) \cup (A \cap B') \cup (A' \cap B)$

R.H.S

$(A \cap B) \cup (A \cap B') \cup (A' \cap B)$

$A \cap (B \cup B') \cup (A' \cap B)$

$A \cap U \cup (A' \cap B)$

$A \cup (A' \cap B)$

$(A \cup A') \cap (A \cup B)$

$U \cap (A \cup B)$

$\underline{\underline{A \cup B}}$

(iii) $A \cup (A' \cap B) = A \cup B$

L.H.S

$A \cup (A' \cap B)$

$(A \cup A') \cap (A \cup B)$

$U \cap (A \cup B)$

$\underline{\underline{A \cup B}}$

Q.10

(a)

i) $[X' \cup (Y \cap Z)]'$

$\underline{\underline{X \cap (Y \cap Z)'}}$

(ii) $Y' \cap (X \cup Y)$

$(Y' \cap X) \cup (Y' \cap Y)$

$(Y' \cap X) \cup \phi$

$\underline{\underline{Y' \cap X}}$

(iii) $(X \cap Y) \cup (X \cap Y')$

$X \cap (Y \cup Y')$

$X \cap U$

$\underline{\underline{X}}$

(iv) $(X \cup Y) \cap (X \cup Y')$

$X \cup (Y \cap Y')$

$X \cup \varnothing$

$\underline{\underline{X}}$

(v.) $[X' \cup (X - Z)]$

$[X' \cup (X \cap Z')]$

$(X' \cup X) \cap (X' \cup Z')$

$U \cap (X' \cup Z')$

$\underline{\underline{X' \cup Z'}}$

(b.)

(i) $X \cap (X' \cup Y)$

$(X \cap X') \cup (X \cap Y)$

$\varnothing \cup (X \cap Y)$

$\underline{\underline{X \cap Y}}$

(ii) $[(X \cap Y)' \cap (X' \cup Y)]'$

$[(X' \cup Y') \cap (X' \cup Y)]'$

$[X' \cup (Y' \cap Y)]'$

$[X' \cup \varnothing]'$

$[X']'$

$\underline{\underline{X}}$

Q. 11

(a.) $N \subset Z \subset Q \subset R \subset C$

(b.) Q^* -> Rational numbers

R^* -> Real numbers

Z^* -> Intergers

C^* -> Complex Numbers

Q. 12

(a.) $A = \{x \in R : -4 \leq x < 2\}$

~~A~~ # $A = [-4, 2)$

$B = \{x \in R : x \geq -1\}$

$B = [-1, \infty)$

(a) $A \cap B = \underline{[-1, 2)}$

$A = [-4, 2)$

$A' = (-\infty, -4) \cup [2, \infty)$

(b.) $U = (-6, 9]$ $\quad A = [-2, 4]$

$B = (-1, 5)$ $\quad C = (-6, 9]$

(i) $A \cap B = (-1, 4]$

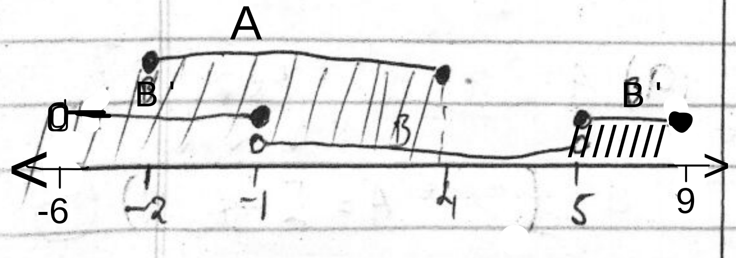
(ii) $U - C = U \cap C'$

$C' = \emptyset$

since $U = C$

(iii) ~~A B' ∪ A~~

(iii) $B' \cup A$



$B' \cup A = [-6, 4] \cup [5, 9]$

(iv) $(A \cup C)'$

simplifying this will give you

$(C)'$

since $U = C$

$C' = \emptyset$

(c.) $A = (-4, 7]$ $\quad B = [4, \infty)$

(i) $A' = (-\infty, -4] \cup (7, \infty)$

(ii) $B' = (-\infty, 4)$

(iii) $A - B$

$A \cap B' = (-4, 4)$

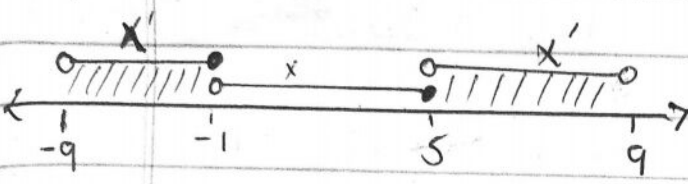
(iv) $B - A$

$B \cap A' = (7, \infty)$

(d) $A = (-9, 9)$ $x = (-1, 5]$

$\therefore$ $y = [-5, 3]$ and $z = [-1, 7)$

(i) $x' = (-9, -1] \cup (5, 9)$



(ii) $A - x = A \cap x'$

$= x'$ ... since A ni universal set

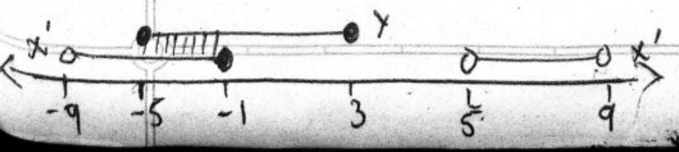
so solve then same a (i) above

(iii) $(x \cap z)' = (-9, -1] \cup (5, 9)$

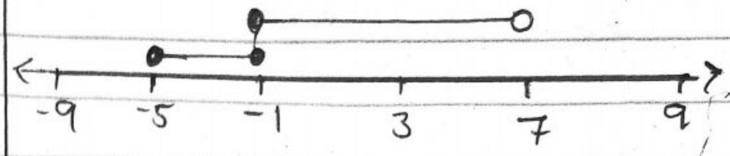
(iv) $(y - x) \cap z$

$(y \cap x') \cap z$

$y \cap x' = [-5, -1]$



$(y \cap x') \cap z = -1$



(e.)

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$B = [1, 5)$ and $C = (3, 8)$

(i) $(A \cap B) \cup (A \cap C)$

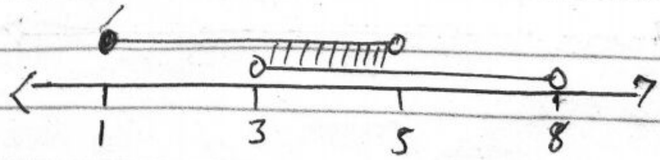
this can be simplified as:

$A \cap (B \cup C)$

where: $B \cup C = [1, 8)$

$\therefore$ $A \cap (B \cup C) = \{1, 2, 3, 4, 5, 6, 7\}$

(ii) $B \cap C = (3, 5)$:



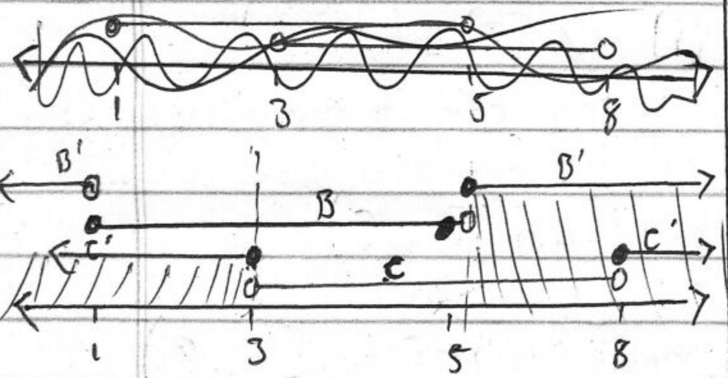
(iii) $(B \cup C)'$

$B \cup C = [1, 8)$

$\therefore (B \cup C)' = (-\infty, 1) \cup [8, \infty)$

(iv) $(B' \cup C') \cap A$

First lets find B' & C'



$\therefore B' \cup C' = (-\infty, 3] \cup [5, \infty)$

$(B' \cup C') \cap A$

$= \{1, 2, 3, 5, 6, 7, 8, 9\}$

Q.13

(a.)

(i) $0.\overline{33}$

$x = 0.\overline{33}$ ... (i)

$100x = 33.\overline{33}$ ... (ii)

equ ... ii − equ ... (i)

$100x - x = 33.\overline{33} - 0.\overline{33}$

$\frac{99x}{99} = \frac{33}{99}$

$x = \frac{1}{3}$

(ii) $0.\overline{16}$

$x = 0.\overline{16}$ ... (i)

$100x = 16.\overline{16}$ ... (ii)

$100x - x = 16.\overline{16} - 0.\overline{16}$

$\frac{99x}{99} = \frac{16}{99}$

$x = \frac{16}{99}$

(iii) $2.\overline{143}$

$x = 2.\overline{143}$

$1000x = 2143.\overline{143}$

$1000x - x = 2143.\overline{143} - 2.\overline{143}$

$\frac{\cancel{999}x}{\cancel{999}} = \frac{2141}{999}$

$x = \frac{2141}{999}$

(iv) $3.\overline{7}$

$x = 3.\overline{7}$

$10x = 37.\overline{7}$

$10x - x = 37.\overline{7} - 3.\overline{7}$

$\frac{9x}{9} = \frac{34}{9}$

$x = \frac{34}{9}$

(v.) $1.171717....$

$x = 1.\overline{17}$

$100x = 117.\overline{17}$

$100x - x = 117.\overline{17} - 1.\overline{17}$

$\frac{\cancel{99}x}{\cancel{99}} = \frac{116}{99}$

$x = \frac{116}{99}$

(vi) $2.5\overline{90}$

$x = 2.5\overline{90}$

$10x = 25.\overline{90}$

$1000x = 2590.\overline{90}$

$1000x - 10x = 2590.\overline{90} - 25.\overline{90}$

$\frac{990x}{990} = \frac{2565}{990}$

$x = \frac{2565}{990}$ or $57/22$

(b.)

(i) $\sqrt{3}$

\# Proof by Contradiction

Assuming that $\sqrt{3}$ is rational such that it can be written in the form a/b where $a, b \in \mathbb{Z}$ and $b \neq 0$ and a and b no common Factor. thus

$\sqrt{3} = a/b$ ....(i)

$(\sqrt{3})^2 = \left(\frac{a}{b}\right)^2$

$$3 = \frac{a^2}{b^2}$$

$$a^2 = 3b^2 \quad \ldots (ii)$$

This statement (ii) implies that a^2 has a factor of **3**. Hence a also has a factor **3** which can be written as $a = 3k$ where k is an ~~it~~ interger

$$a^2 = 3b^2$$
$$(3k)^2 = 3b^2$$
$$\frac{9k^2}{3} = \frac{3b^2}{3}$$

$$b^2 = 3k^2 \ldots (iii)$$

Here in (iii) b^2 has a factor of 3 meaning b also has a factor of 3. This contradicts our earlier assumption that a and b has no common factor therefore by contradiction $\sqrt{3}$ is irrational.

(ii) $\sqrt{2}$

basically same thing as the $\sqrt{3}$. we have jst proved so am sure you got this

(c)

$\sqrt{3}$ na $\sqrt{5}$ $\rightarrow$ irrational

(i) $\sqrt{3} + 5$

Proof by Contradiction

Assuming that $\sqrt{3} + 5$ is rational such that it can be written in the form a/b where $a, b \in \mathbb{Z}$ and $b \neq 0$

$$\left(\sqrt{3} + 5\right)^2 = \left(\frac{a}{b}\right)^2$$

$$(\sqrt{3})^2 + 2(\sqrt{3})(5) + (5)^2 = \frac{a^2}{b^2}$$

$$3 + 10\sqrt{3} + 25 = \frac{a^2}{b^2}$$

$$10\sqrt{3} + 28 = \frac{a^2}{b^2}$$

$$10\sqrt{3} = \frac{a^2}{b^2} - 28$$

$$10\sqrt{3} = \frac{a^2 - 28b^2}{b^2} \quad \times \frac{1}{10}$$

$$\sqrt{3} = \frac{a^2 - 28b^2}{10b^2} \quad \ldots (i)$$

statement (i) implies that the intergers $a^2 - 28b^2$ and $10b^2$ is equal to an irrational number $\sqrt{3}$, which is a contradiction. Hence we have proved that $\sqrt{3} + 5$ is irrational

(ii) $\sqrt{5} - 1$

Proof by Contradiction

Suppose $\sqrt{5} - 1$ is rational such that is can be expressed as a/b where a and b are integers, $b \neq 0$

$$\left(\sqrt{5} - 1\right)^2 = \left(\frac{a}{b}\right)^2$$

$$(\sqrt{5})^2 + 2(\sqrt{5})(-1) + (-1)^2 = \frac{a^2}{b^2}$$

$$5 - 2\sqrt{5} + 1 = \frac{a^2}{b^2}$$

$$-2\sqrt{5} + 6 = \frac{a^2}{b^2}$$

$$-2\sqrt{5} = \frac{a^2}{b^2} - 6$$

$$-2\sqrt{5} = 6 - \frac{a^2}{b^2}$$

$$2\sqrt{5} = \left.\frac{6b^2 - a^2}{b^2}\right] \times \frac{1}{2}$$

$$\sqrt{5} = \frac{6b^2 - a^2}{2b^2} \quad \cdots (i)$$

the statement above means that the irrational number $\sqrt{5}$ is equal to the rational numbers $6b^2 - a^2$ & $2b^2$ which is a contradiction. Hence proved that $\sqrt{5} - 1$ is irrational

(iii) $1 - \sqrt{3}$

Am sure you have gotten the hang of these ma suppose nama assume ... lol

i'll leave this one for you!!