THE COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

DEPARTMENT OF PHYSICS

PH 110 TEST 1, 30 MARCH 2023

SOLUTIONS

QUESTION ONE.

(a) When taking measurements in an experiment, the results are expected to be both accurate and precise. Distinguish between **accuracy** and **precision**. **[3]**

Ans. **Accuracy:** How **close** the measurements are to the **true value.**
Precision: How **close** the measurements are to **each other** (**repeatability** of the measurement or **agreement** among the measurement)

(b) If the length and width of a rectangular plate are measured to be (17.30 ± 0.05) cm and (13.30 ± 0.05) cm, respectively, find the area of the plate and the approximate uncertainty in the calculated area. **[4]**

Ans.

$$\begin{aligned}
Area &= l \times b \\
&= (17.30 \pm 0.05)\, cm \times (13.30 \pm 0.05)\, cm \\
&= (17.30 \times 13.30) \pm 17.30 \times 0.05 \pm 13.30 \times 0.05 \pm 0.05 \times 0.05 \\
&= 230.09 \pm (0.863 \pm 0.663 \pm 0.0025) \\
&= (230.1 \pm 1.53)\; cm^2
\end{aligned}$$

(c) A solid cube is 350 g and each edge has length of 5.35 cm. Determine the density ρ of the cube in SI units. **[4]**

We first convert the mass m and the volume V in SI units:

$$m = 350\, g \times \frac{10^{-3}\, kg}{1 g} = 0.350\, kg$$ **[1mark]**

$$V = L^3 = \left(5.35 cm \times \frac{10^{-2}\, m}{1 cm}\right)^3 = (5.35)^3 \times 10^{-6}\, m^3$$ **[1mark]**

$$V = 1.53 \times 10^{-4}\, m^3.$$

$\therefore \quad \rho = \dfrac{0.350kg}{1.53\times10^{-4}m^3} = 2287.6kg/m^3 = 2.29\times10^3 kg/m^3$ **[2marks]**

(d) The speed v of an object is given by the equation $v = At^3 - Bt$, where t refers to time. What are the dimensions of A and B? **[4]**

Ans.

By the principle of homogeneity, physical quantities can only be add or subtract if they have same dimensions and units. Therefore, in the equation.

$v = At^3 - Bt$

$[v] = LT^{-1}$ and so $[A][t^3] = [v] = LT^{-1}$

$$[A] = \frac{LT^{-1}}{[t^3]} = \frac{LT^{-1}}{T^3} = LT^{-4}$$ **[2 MARKS]**

$$[A] = LT^{-4}$$

Similarly; $[B][t] = LT^{-1}$

$$[B] = \frac{LT^{-1}}{[t]} = \frac{LT^{-1}}{T} = LT^{-2}$$ **[2 MARKS]**

$$[B] = LT^{-2}$$

(e) (i) Write down the dimensions of **velocity, force and density** **[4]**

Ans.

$[v] = LT^{-1}$ **[1]** ; $F = m\times a$, $[F] = MLT^{-2}$ **[1.5]**; $\rho = m/V$, $[\rho] = ML^{-3}$ **[1.5]**

(ii) A Toyota Harrier moving with velocity v experiences a force F due to air resistance given by the express;

$$F = \tfrac{1}{5}C\rho^x v^y A^z$$

where C a dimensionless constant is called the drag coefficient, ρ is the density of the air and A is the cross section area of the vehicle. Use dimensional analysis to find the values of the powers of x, y and z. **[6]**

Ans.

$F = \tfrac{1}{5}C\rho^x v^y A^z$, Note that $\tfrac{1}{5}$ and C are dimensionless constants therefore we leave them amount during the dimensional analysis.

$[F] = [\rho]^x[v]^y[A]^z$ **[1 Mark]**

$[MLT^{-2}] = [ML^{-3}]^x[LT^{-1}]^y[L^2]^z$

$MLT^{-2} = M^x L^{-3x} L^y T^{-y} L^{2z}$

$MLT^{-2} = M^x L^{-3x+y+2z} T^{-y}$ **[1 Mark]**

Matching like terms right and left

$M: \quad 1 = x$ **[1 Mark]**

$T: \quad -2 = -y$

$\therefore \quad y = 2$ **[1 Mark]**

$L: \quad 1 = -3x + y + 2z$

$1 = -3(1) + 2 + 2z$

$1 = -3 + 2 + 2z$ **[2 Marks]**

$1 = -1 + 2z$

$\therefore z = 1$

$F = \frac{1}{5} C\rho v^2 A$ **[1 Mark]**

QUESTION TWO

a) Triangle method, parallelogram method and polygon method. **[3 Marks]**

b) *Guide: Give one mark for defining and the other for two correct examples in each case.*

i) Scalar quantities are quantities which are completely specified by a certain number associated with a suitable unit *without* any mention of direction in space. *Examples*: time, mass, length, volume, density, temperature, energy, distance, speed, etc. **[2 Marks]**

ii) Vector quantities are quantities which are completely described only when *both* their magnitude and direction are specified. *Examples*: force, velocity, acceleration, displacement, torque, momentum, etc.

[2 Marks]

c)

vector	**x-component**	**y-component**
A =3.5	0	-3.5

B=8.2	$8.2\cos 45 = 5.798$ $= 5.8$	$8.2\sin 45 = 5.798$ $= 5.8$
C=?	C_x	C_y
R	-15	0

$C_x \Rightarrow \; -15 = 0 + 5.8 + C_x \quad , \quad C_y \Rightarrow \; 0 = -3.5 + 5.8 + C_y$

$C_x = -20.8\,\text{m}$ **[1 Mark]** , $C_y = -2.5\,\text{m}$ **[1 Mark]**

$\therefore C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-20.8)^2 + 2.3^2} = \sqrt{437.93} = 20.93\,\text{m}$

$\theta = \tan^{-1}\left(\dfrac{C_y}{C_x}\right) = \tan^{-1}\left(\dfrac{-2.3}{-20.8}\right) = 20.93°$ below $-x-axis$ or west of South.

(a) A unit vector is a quantity with direction and a magnitude of one. [2 **mark**]

(b)

(i) if $F = 5\hat{\imath} + A_y\hat{\jmath} + 4\hat{k}$) N, then $|F| = \sqrt{5^2 + A_y^2 + 4^2} = \sqrt{90}$ [**1 mark**]

$=> 25 + A_y^2 + 16 = 90 \qquad => A_y^2 = \sqrt{49}$

$\therefore A_y = 7$ [**1 mark**]

(ii) $\hat{f} = \frac{F}{|F|} = \frac{(5\hat{\imath}+7\hat{\jmath}+4\hat{k})N}{\sqrt{90}}$ [2 **mark**]

(iii) $Work = F.R = 5\hat{\imath} + 7\hat{\jmath} + 4\hat{k}\;).3\hat{\imath} - \hat{\jmath} + 2\hat{k}\;)$

$Work = 15 - 7 + 8 = 16J$ [**2 marks**]

(iv) $\boldsymbol{FxR} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 5 & 7 & 4 \\ 3 & -1 & 2 \end{vmatrix} = 18\hat{\imath} + 2\hat{\jmath} - 26\hat{k}$ or

$\boldsymbol{RxF} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -1 & 2 \\ 5 & 7 & 4 \end{vmatrix} = -(18\hat{\imath} + 2\hat{\jmath} - 26\hat{k})$ [3 **mark**]

(c) $|\boldsymbol{FxR}| = |F||R|sin(30) = \sqrt{2^2 + 1 + 2^2}\sqrt{3} * (0.5)$

$\sqrt{27} * 0.5 = 2.6Nm$ [**2 marks**]

QUESTION THREE

(a) Uniform motion is the type of motion in which the object travels in a straight line with constant speed or is the type of motion in which the object's velocity remains constant. [2 marks]

(ii) Acceleration is the rate of change of velocity of an object. [2 marks]

(b) The stone has the same velocity (v_i) as the balloon at an instant it was released.

Given; $\Delta y = -72.5\, m$, $g = -9.8\, m/s^2$, $t = 5\, s$

$$\Delta y = v_i t + \frac{1}{2} g t^2$$

$$-72.5 = v_i\,(5) + \frac{1}{2}(-9.8)(5)^2$$

$$\boldsymbol{v_i = 10\, m/s} \text{ [3 marks]}$$

(c) Let us model the patrol car and the bus as particles. It is convenient to choose the position of the intersection as the origin. The patrol car catches the bus at the instant its position matches that of the bus, i.e. $\Delta x_{car} = \Delta x_{bus} = 240\, m$

Let the time taken by the patrol car to catch up with bus be t. Then time taken by the bus is $t + 2$.

For the patrol car, $\Delta x_{car} = 240\, m$, $v_i = 0$, $v_f = 48\, m/s$

$$\Delta x_{car} = \left(\frac{v_i + v_f}{2}\right) t$$

$$t = \frac{2\Delta x_{car}}{v_i + v_f} = \frac{2(240)}{0 + 48} = 10\, s \text{ [3marks]}$$

For the bus, $\Delta x_{bus} = 240\, m$, $a = 0$, $v = cst$

$$\Delta x_{bus} = v(t + 2)$$

$$v = \frac{\Delta x_{bus}}{t + 2} = \frac{240}{10 + 2} = \boldsymbol{20\, m/s} \text{ [3 marks]}$$

Alternative method

For the patrol car, $\Delta x_{car} = 240\, m$, $v_i = 0$, $v_f = 48\, m/s$

$$v_f{}^2 = v_i{}^2 + 2a\Delta x_{car} \iff a = \frac{v_f{}^2 - v_i{}^2}{2\Delta x_{car}} = \frac{48^2 - 0^2}{2(240)} = 4.8\, m/s^2 \text{ [2 marks]}$$

$$v_f = v_i + at \iff t = \frac{v_f - v_i}{a} = \frac{48 - 0}{4.8} = 10\, s \text{ [2 marks]}$$

For the bus, $\Delta x_{bus} = 240\, m$, $a = 0$, $v = cst$

$$x_{bus} = v(t + 2)$$

$$v = \frac{x_{bus}}{t+2} = \frac{240}{10+2} = \mathbf{20\ m/s}$$ [2 marks]

(d) (i) The horizontal component of the initial velocity of the coin is

$$v_{ox} = v_o \cos\theta = 7\cos 60° = \mathbf{3.5\ m/s}$$ [1 mark]

The vertical component of the initial velocity of the coin is

$$v_{oy} = v_o \sin\theta = 7\sin 60° = \mathbf{6.06\ m/s}$$ [1 mark]

(ii) The time to reach the dish

$$x = v_x t = v_{ox} t$$

$$t = \frac{x}{v_{ox}} = \frac{2.8}{3.5} = \mathbf{0.8\ s, hence\ shown}$$ [2 marks]

(iii) Height h

$$h = v_{oy}t + \frac{1}{2}gt^2 = 6.06(0.8) + \frac{1}{2}(-9.8)(0.8)^2$$

$$\boldsymbol{h = 1.71\ m}$$ [3 marks]

(iv) The horizontal component of the velocity remains constant, as it is never affected by gravity

$$v_x = v_{ox} = 3.5\ m/s$$

The vertical component of velocity of the coin when it enters the dish is

$$v_y = v_{oy} + gt = 7\sin 60° + (-9.8)(0.8) = -1.78\ m/s$$ [2 marks]

The magnitude of the velocity of the coin when it enters the dish is

$$v = \sqrt{{v_x}^2 + {v_y}^2}$$

$$v = \sqrt{(3.5)^2 + (-1.78)^2}$$

$$\boldsymbol{v = 3.93\ m/s}$$ [2 marks]

Therefore, the magnitude of the velocity has reduced from 7 m/s to 3.93 m/s. [1 mark]

QUESTION FOUR

(a) When a resultant force acts on an object, the object accelerates in the direction of the force. This acceleration is directly proportional to the force and inversely proportional to the mass of the object ✓✓✓ **[2]**

(b) (i) Kinetic frictional force $f_k = \mu_k N = \mu_k mg$ ✓

$= (0.13)(1)(9.8)$

$= 1.27\ N$ ✓ **[2]**

(ii) The acceleration of the system is;

Applying Newton's second law of motion on the 1 kg mass gives;

$F_{net} = ma$ ✓

$T - f_k = ma$ ✓

$T - 1.27 = (1)(a)$

$T - 1.27 = a$ ✓ (1)

Applying Newton's second law of motion on the 2 kg mass gives;

$F_{net} = ma$

$mg - T = ma$ ✓

$(2)(9.8) - T = (2)(a)$

$19.6 - T = 2a$ ✓ (2)

Solving equations (1) and (2) simultaneously gives;

$a = T - 1.27 = 19.6 - 2a - 1.27$

$3a = 18.33$

$a = 6.11\ m/s^2$ ✓ **[6]**

(iii) Tension in the string is;

Substituting the value of acceleration calculated into equation (1) gives;

$T = 1.27 + a = 1.27 + 6.11 = 7.38\ \text{N}$ ✓ ✓ **[2]**

c) i) Guide: Give a mark for either a statement or an equation

- First condition states that the net force acting upon the object must be zero. Mathematically expressed as:

$$\Sigma F = 0$$

[1 Mark]

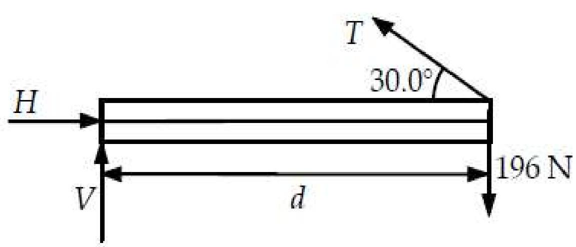
- Second condition states that the net torque acting upon the object must also be zero. Mathematically expressed as:

$$\Sigma \tau = 0$$

[1 Mark]

-

ii) Free-body diagram:



[1 Mark]

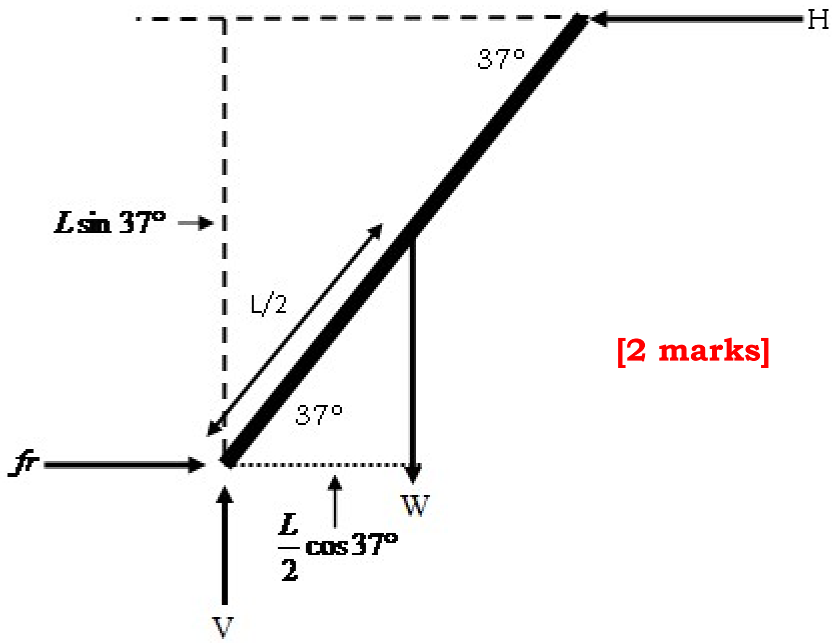
Consider the torques about an axis perpendicular to the page and through the left end of the horizontal beam.

$$\sum \tau = +(T \sin 30.0°)d - (196 \text{ N})d = 0,$$

giving $T = \boxed{392 \text{ N}}$.

[2 Marks]

d) Or e)



[2 marks]

With reference to the free body diagram

Applying the first condition $\sum F = 0$

i) $\sum F_x = 0$, $H = fr$

$H = \mu N = \mu V$ .... (1) **[1 mark]**

$\sum F_y = 0$, $V = 300\,N$ .... (2) **[1 mark]**

ii) $\sum \tau = 0$

$\sum c\tau = \sum cc\tau$

$$300\,N \times \frac{L}{2}\cos 37° = HL\sin 37°$$ **[1 mark]**

$$300\,N \times \frac{6}{2}\cos 37° = 6H\sin 37°$$

$$900\cos 37° = 6H\sin 37°$$

$$\Rightarrow H = \frac{900\cos 37°}{6\sin 37°} = 199.1\,N$$

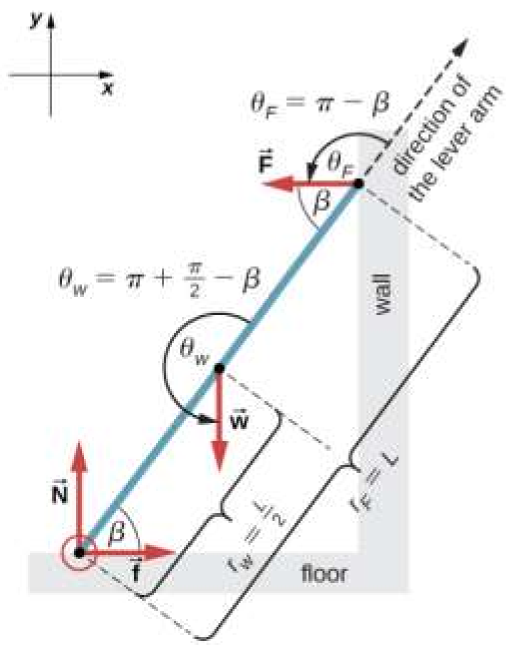
$H = 199.1N$ **[1 mark]**

From eqn. (2) we have $\mu = \frac{H}{300} = \frac{199.1}{300} = 0.66 = 0.66$ **[1 mark]**

METHOD TWO

d)

Given the free-body diagram



[1 Mark]

From the free-body diagram, the net force in the x direction is:

$$\Sigma F_x = 0$$
$$+f - F = 0 \ldots (1)$$

the net force in the y direction is:

$$+N - w = 0 \ldots (2)$$

[1 Mark]

and the net torque along the rotation axis at the pivot point is:

$$\Sigma \tau = 0$$
$$\tau_w + \tau_F = 0 \ldots (3)$$

[1 Mark]

And the torques for w and F are:

$$\tau_w = r_w w \sin\theta_w$$
$$\tau_w = r_w w \sin(180° + 90 - \beta)$$
$$\tau_w = -\frac{L}{2} w \sin(90° - \beta)$$
$$\tau_w = -\frac{L}{2} w \cos\beta \ldots (4)$$

[1 Mark]

then

$$\tau_F = r_F F \sin\theta_F$$
$$\tau_F = r_F F \sin(180° - \beta)$$
$$\tau_F = LF \sin\beta \ldots (5)$$

[1 Mark]

Eqn (4) and (5) into (3):

$$-\frac{L}{2} w \cos\beta + LF \sin\beta = 0$$
$$F = \frac{w}{2} \cot\beta$$
$$F = \frac{300N}{2} \cot 37° = \mathbf{199.1N}$$

(reaction force from the wall)

[1 Mark]

The normal reaction force with the floor is obtained by solving eqn(2):

$$N = w = \mathbf{300N}$$

Friction force is obtained from eqn(1):

$$f - F = 0$$
$$f = 113N$$

Thus,

$$f_s = \mu_s N$$
$$\mu_s = \frac{f_s}{N} = \frac{199.1N}{300N}$$
$$\boldsymbol{\mu_s} = \mathbf{0.66}$$

[1 Mark]