$$I_i\omega_i = I_f\omega_f$$

8.15. Analogy between translatory motion and rotational motion

Table 8.1 summarizes analogy between some linear and angular quantities and their connecting relations.

Linear	Angular	Connecting relation
x	θ	$x = r\theta$
v	ω	$v = r\,\omega$
a	α	$a_t = r\,\alpha$
m	I	$I = \sum m\,r^2$
F	τ	$\tau = r\,F\sin\theta$
$K = \frac{1}{2}mv^2$	$K_R = \frac{1}{2}I\omega^2$	
$W = Fd$	$W = \tau\theta$	
$P = Fv$	$P = \tau\omega$	
$\sum F = m\,a$	$\sum \tau = I\alpha$	

Table 8.1 *Analogy between some linear and angular quantities and their connecting relations*

EXERCISES

1. In a *HCl* molecule, the separation between the nuclei of two atoms is about 1.27×10^{-10}m. Find the approximate location of the center of mass of the molecule, given that chlorine atom is about 35.5 times as massive as hydrogen atom and nearly all the mass of an atom is concentrated in the nucleus. **[$1.24\times10^{--10}$m]**
2. Two masses 6 kg and 2 kg are at $6\boldsymbol{i} - 7\boldsymbol{j}$ and $2\boldsymbol{i} + 10\boldsymbol{j}$ respectively. What is the center of mass of the system of these two masses? **[$5\boldsymbol{i} - 2.75\boldsymbol{j}$]**
3. A grindstone has moment of inertia of 6 kg.m^2 about its axis. A constant torque is applied and the grindstone is found to acquire a speed of 150 r.p.m in 10 seconds after starting from rest. Calculate the torque. **[9.42 N.m]**
4. An automobile moves on a road with a speed of 54 km/h. The radius of its wheel is 0.35 m. What is the average negative torque transmitted by its brakes to a wheel if the vehicle is brought to rest in 15 seconds? The moment of inertia of the wheel about its axis of rotation is 3 kg.m^2. **[-8.58 N.m]**
5. A flywheel of mass 500 kg and one meter diameter makes 500 r.p.m. Assuming the mass to be concentrated at the rim, calculate

   (a) the angular velocity of the wheel
   (b) moment of inertia of the wheel
   (c) energy of the flywheel. **[(a)52.36 rad/s (b)125 kgm^2 (c)1.75×10^5 J]**

6. A pulley $\left(I = \frac{1}{2}Mr^2\right)$ of mass $M = 6$ kg and radius $R = 20$ cm is mounted on a frictionless axis, as shown in figure 8.5. A massless cord is wrapped around the pulley while its other end supports a block of mass $m = 3$ kg. Take $g = 10$ m/s^2. Find
   (a) the linear acceleration of the block
   (b) the angular acceleration of the pulley
   (c) the tension in the cord. **[(a)5 m/s^2; (b)25 rad/s^2; (c)15 N]**

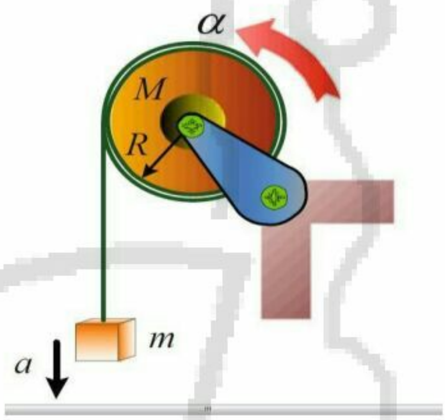


Figure 8.5 *See exercise 6*

7. A soldier stands with his arms stretched out at the center of a platform that rotates without friction with an angular speed $\omega_i = 1.8$ rev/s. The rotational inertia of the soldier and platform is $I_i = 6$ kg.m^2. When the soldier pulls his arms close to his body, he decreases the rotational inertia of the system to $I_f = 4$ kg.m^2.
   (a) What is the resulting final angular speed of the system?
   (b) Is there a gain or a loss in the rotational kinetic energy of the system; and which of the objects, soldier or platform, gained or lost this energy? **[2.7 rev/s]**

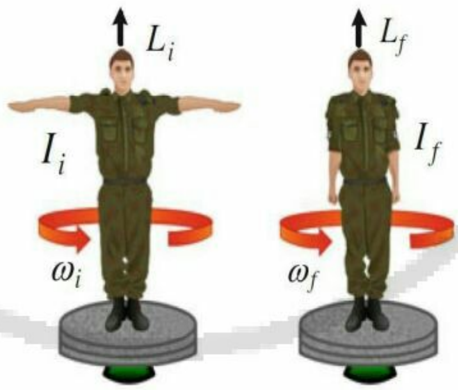


Figure 8.6 *See exercise 7*

8. Consider a disk of radius r and mass m initially at rest on an inclined surface. Show that if the disk rolls without slipping from a height h, it reaches the bottom of the incline with speed

$$v = \sqrt{\frac{4}{3}gh}$$

THE COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

DEPARTMENT OF PHYSICS

TUTORIAL SHEET 9_2023: **ROTATIONAL MOTION**

1. Consider an oxygen molecule (O_2) rotating in the *xy* plane about the *z* axis. The axis passes through the center of the molecule, perpendicular to its length. The mass of each oxygen atom is 2.66 x 10^{-26} kg, and at room temperature the average separation between the two atoms is d =1.21 x 10^{-10} m (the atoms are treated as point masses). (a) Calculate the moment of inertia of the molecule about the *z* axis. (b) If the angular speed of the molecule about the *z* axis is 4.60 x 10^{12} rad/s, what is its rotational kinetic energy?

2. Find the rotational kinetic energy of the earth due to its daily rotation on its axis. Assume it to be a uniform sphere. (M_E = 5.98 x 10^{24} kg, R_E =6.37 x 10^6).

3. A pulley of mass M = 15 kg and R = 20 m is mounted on a frictionless axis, as shown in Fig.3.1. A massless cord is wrapped around the pulley while its other end supports a block of mass m = 5 kg. If the cord does not slip, find: (i) the linear acceleration of the block (ii) the angular acceleration of the pulley and (iii) the tension in the cord.

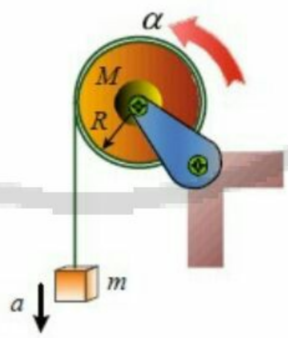


Fig.3.1

4. A 500-g uniform sphere of 7.0 cm radius spins at 30 rev/s on an axis through its center. Find its (a) rotational kinetic energy, (b) angular momentum, and (c) radius of gyration.

5. A disk of mass $m = 0.2$ kg and radius $R = 5$ cm is attached coaxially to the massless shaft of an electric motor, as shown in Fig.5.1. The motor runs steadily at 900 rpm and delivers 5 hp. (a) What is the angular speed of the disk in SI units? (b) What is the rotational kinetic energy of the disk? (c) How much torque does the motor deliver?

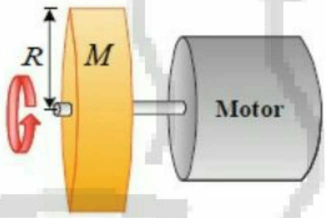


Fig.5.1

6. Fig 6.1 a disk and a shaft is coasting with angular speed ω_1. The combined moment of inertia of it and its shaft is I_1. A second disk of moment of inertia I_2 is dropped onto the first disk and ends up rotating with it. Find the angular velocity of the combination if the original angular velocity of the upper disk was (a) zero (b) ω_2 in the same direction as ω_1 (c) ω_2 in a direction opposite ω_1.

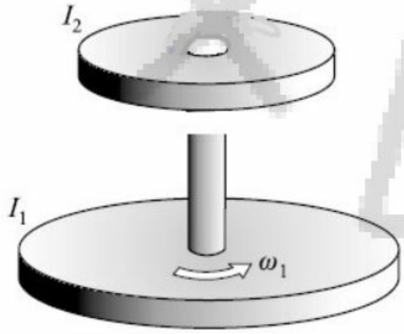
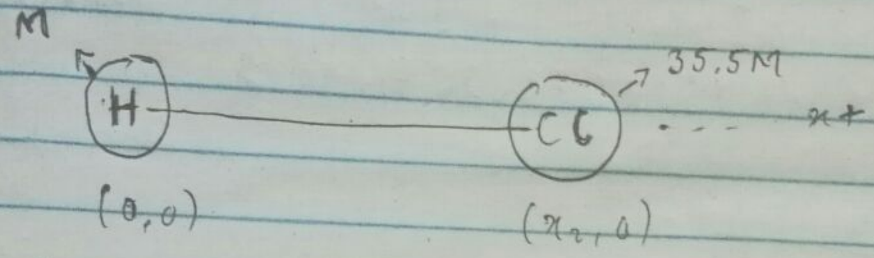


Fig.6.1

7. A solid sphere, a hollow sphere, a solid disc and a hoop with the same mass and radius are spinning freely about a diameter with the same angular speed on a table. Which of these objects will one do maximum work to stop it?
8. A flywheel of mass 4 kg and radius 0.5 m rotates freely on a horizontal axis. A block of mass 2.0 kg hangs by a string that is tightly wrapped around the flywheel. Find the angular velocity of the flywheel and the speed of the block when the block has fallen through 1.6 m.

Chapter 8 - Exercise

1.



Here we are taking (H) to be the starting point/center/(origin) and thats why its labeled (0,0)

$$Cm = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$\begin{bmatrix} m_1 = M \\ m_2 = 35.5M \\ x_1 = 0 \\ x_2 = 1.27 \times 10^{-10} \end{bmatrix}$$

$$Cm = \frac{(M \times 0) + (35.5M \times 1.27 \times 10^{-10})}{M + 35.5M}$$

$$= \frac{35.5M \times 1.27 \times 10^{-10}}{36.5M}$$

$$Cm = 1.235 \times 10^{-10}\ m$$

2. 6kg and 2kg of $6i - 7j$ na $2i + 10j$ respectively

$$Cm_x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \qquad Cm_y = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

$$= \frac{(6 \times 6) + (2 \times 2)}{6+2} \qquad = \frac{(6 \times -7) + (2 \times 10)}{6+2}$$

$$Cm_x = 5 \qquad Cm_y = -2.75$$

$$\therefore\ cm = (5i, -2.75j)$$

3 $150 \text{ r.p.m} = \frac{150 \text{ rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{ min}}{60 \text{ s}}$

$150 \text{ r.p.m} = 15.70796327 \text{ rad/s}$

$\alpha = \frac{\omega_f - \omega_i}{t}$

$= \frac{15.70796327 - 0}{10}$

$\alpha = 1.570796327$

$\therefore \tau = I\alpha$

$= 6 \times 1.570796327$

$\underline{\underline{\tau = 9.42 \text{ N.m}}}$

4 $54 \text{ km/h} = \frac{54 \text{ km}}{\text{h}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ hour}}{3600 \text{ s}}$

$54 \text{ km/h} = 15 \text{ m/s}$

$v = r\omega$

$\omega = \frac{v}{r}$

$\omega = \frac{15 \text{ m/s}}{0.35 \text{ m}}$

$\omega = 42.85714286 \text{ rad/s}$

$\alpha = \frac{\Delta\omega}{t}$

$\alpha = \frac{0 - 42.85714286}{15}$

$\alpha = -2.857142857$

$\therefore \tau = I\alpha$

$= 3 \times -2.857142857$

$\underline{\tau = -8.57 \text{ N.m}}$

5 (a) $500 \text{ r.p.m} = \frac{500 \cancel{\text{revs}}}{\cancel{\text{min}}} \times \frac{2\pi \text{ rad}}{1 \cancel{\text{rev}}} \times \frac{1 \cancel{\text{min}}}{60 \text{ s}}$

$$\underline{\underline{\omega = 52.36 \text{ rad/s}}}$$

b.) $I = mr^2$ # since diameter = 1m

$= 500 \times (0.5)^2$

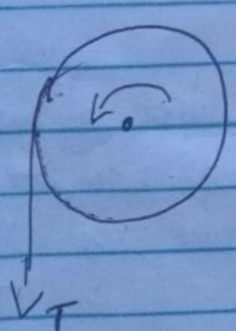
$$\underline{\underline{I = 125 \text{ kg.m}^2}}$$

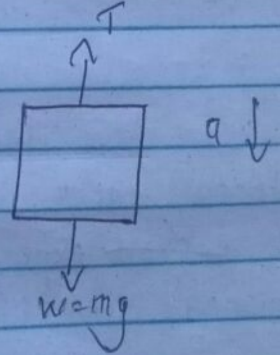
c $K.E = \frac{1}{2} I \omega^2$

$= \frac{1}{2}(125) \times (52.36)^2$

$$\underline{\underline{K.E = 1.7 \times 10^5 \text{ J}}}$$

6 ssss These one are actually a bit easy, lol, just resolve the forces properly then you are safe.





$\cancel{\Sigma F \; \tau = I\alpha \quad TR = \frac{1}{2} MR^2}$

ΣF ; $mg - T = ma$

$\therefore [T = mg - ma]$

ΣF ; $\tau = I\alpha$

where $\begin{bmatrix} \tau = TR \\ I = \frac{1}{2} MR^2 \\ \alpha = \frac{a}{R} \end{bmatrix}$

this one from $a = r\alpha$

~~$\tau = I\alpha$~~

$\tau = I\alpha$

$T \cdot \cancel{R} = \left(\frac{1}{2}M\cancel{R^2}\right)\left(\frac{a}{\cancel{R}}\right)$

$T = \frac{1}{2}Ma$

but $T = mg - ma$

$mg - ma = \frac{1}{2}Ma$

$mg = \frac{1}{2}Ma + ma$

$mg = \left(\frac{1}{2}M + m\right)a$

$a = \frac{mg}{\frac{1}{2}M + m}$

$a = \frac{3 \times 10}{\frac{1}{2}(6) + 3}$

$\underline{a = 5 m/s^2}$ -

(b.) $a = r\omega$

$\omega = \frac{a}{r}$

$\omega = \frac{5}{0.2}$

$\underline{\omega = 25 rad/s^2}$

(c.) $T = mg - ma$

$= (g - a)m$

~~$= (9.8 - 5)3$~~

$= (10 - 5)3$

$\underline{T = 15 N}$

7. (a) In this illustration the soldier is using his own internal energy (muscles) therepore no external torque is ~~express~~ experienced an thus we see Angular moment being conserved

$L_i = L_f$

$I_i\omega_i = I_f\omega_f$

$\frac{6 \times 1.8}{4} = \frac{4 \times \omega_f}{4}$

$\underline{\omega_f = 2.7 rev/s}$

(b) $K.E_i = \frac{1}{2}I\omega_i^2$

$K.E = \frac{1}{2}(6) \times (1.8)^2$

$\underline{K.E_i = 9.72\ J}$

$K.E_f = \frac{1}{2}(4)(2.7)^2$

$\underline{K.E_f = 14.58\ J}$

(x)(iii) Inspite of angular momentum being conserved K.E was not conserved as we can see that $K.E_f > K.E_i$.

Understanding that the solder was using his muscles (Internal energy) to pull his arms close to him, we can conclude that some of the internal energy was used to increase the K.E. of the platform

Try researching more on this one!!

8 $E_i = E_f$

$\frac{1}{2}mv_i^2 + \frac{1}{2}I\omega_i^2 + mgh = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 + mgh$

$mgh = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$

Where ; $I = \frac{1}{2}mr^2$

$\omega_f = \frac{v}{r}$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2$$

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 + \frac{1}{4}\cancel{m}v^2$$

$$gh = \frac{3}{4}v^2$$

$$\sqrt{v^2} = \sqrt{\frac{4}{3}gh}$$

$$v = \sqrt{\frac{4}{3}gh}$$

hence shown.

Thought of adding tutorial sheet solutions as well, enjoy!!

1 $$I = \sum_{i=1}^{n} m_i r_i^2$$

$$I = m_1 r^2 + m_2 r^2$$

$$\left[\begin{array}{l} \text{where ; } m = 2.66 \times 10^{-26} \\ r = \frac{1.21 \times 10^{-10}}{2} = 6.05 \times 10^{-11}\,m \end{array}\right]$$

$$\therefore I = (2.66 \times 10^{-26}) \times (6.05 \times 10^{-11})^2 + (2.66 \times 10^{-26}) \times (6.05 \times 10^{-11})^2$$

$$I = 1.95 \times 10^{-46}\ kg.m^2$$

(b) $$K.E = \frac{1}{2} I \omega^2$$

$$= \frac{1}{2}(1.95 \times 10^{-46}) \times (4.6 \times 10^{12})^2$$

$$K.E = 2.06 \times 10^{-21}\ J$$

2 $K.E = \frac{1}{2} I \omega^2$

but;

$I = \frac{2}{5} MR^2$

$= \frac{2}{5} (5.98 \times 10^{24})(6.37 \times 10^6)^2$

$I = 9.70599448 \times 10^{37}$

$T = \frac{2\pi}{\omega}$

$\omega = \frac{2\pi}{T}$, ∴

where $T = 24 \text{ hours} \times \frac{3600s}{1 \text{ hours}}$

$T = 86400$

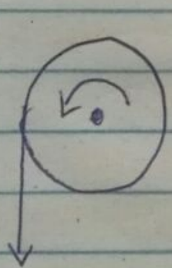
$\omega = \frac{2\pi}{86400}$

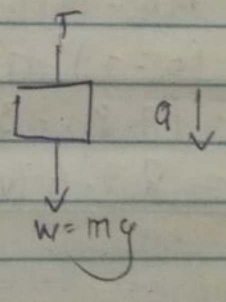
$\omega = 7.2722 \times 10^{-5}$ rad/s

$\therefore K.E = \frac{1}{2} (9.70599448 \times 10^{37}) \times (7.2722 \times 10^{-5})^2$

$K.E = 2.5665 \times 10^{29}$ J

3.





Σ_P; $\tau = I\alpha$

where; $\tau = TR$

$I = \frac{1}{2} MR^2$

$\alpha = \frac{a}{R}$

Σ_F; $F = ma$

$mg - T = ma$

$mg - ma = T$ ...(ii)

$T\cancel{R} = \frac{1}{2} MR^{\cancel{2}} \left(\frac{a}{R}\right)$

$T = \frac{1}{2} Ma$ ...(i)

$$mg - ma = \frac{1}{2}Ma$$
$$mg = \frac{1}{2}Ma + ma$$
$$mg = \left(\frac{1}{2}M + m\right)a$$
$$a = \frac{mg}{\left(\frac{1}{2}M + m\right)}$$
$$a = \frac{5 \times 9.8}{\frac{1}{2}(15) + 5}$$
$$\underline{\underline{a = 3.92\,m/s^2}}$$

(ii). $a = r\omega$

$$\omega = \frac{a}{r}$$
$$\omega = \frac{3.92}{20}$$
$$\underline{\underline{\omega = 0.196\,rad/s^2}}$$

(iii) $T = mg - ma$

$$T = (g - a)\,m$$
$$T = (9.8 - 3.92)\,5$$
$$\underline{\underline{T = 29.4\,N}}$$

4. $K.E = \frac{1}{2}I\omega^2$, where $I = \frac{2}{5}Mr^2$

$$\omega = \frac{30\,rev}{5} \times \frac{2\pi}{1\,rev} = 188.5\,rad/s$$

$$= \frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2$$
$$= \frac{1}{5}mr^2\omega^2$$

$\frac{1}{5}(0.5)(0.07)^2(188.5)^2$

$K.E = 17.41\ J$

(b.) $\cancel{K = \frac{1}{2} I \omega}$ $L = I\omega$

where $I = \frac{2}{5} Mr^2$

$= \frac{2}{5}(0.5)(0.07)^2$

$I = 9.8 \times 10^{-4}$

$L = (9.8 \times 10^{-4}) \times (188.5)$

$L = 0.18473\ J.s$

(c.) $k = \sqrt{\frac{I}{m}}$

$= \sqrt{\frac{(9.8 \times 10^{-4})}{0.5}}$

$k = 0.0443\,m$

5 (a) $\omega = 900\ r.p.m = \frac{900\ rev}{min} \times \frac{1\ min}{60\ sec} \times \frac{2\pi\ rad}{1\ rev}$

$= 94.25\ rad/s$

(b.) $K.E = \frac{1}{2} I \omega^2$, where $I = \frac{1}{2} Mr^2$

$= \frac{1}{2}(0.2)(0.05)^2$

$I = 2.5 \times 10^{-4}$

$K.E = \frac{1}{2}(2.5 \times 10^{-4}) \times (94.25)^2$

$K.E = 1.11\ J$

c. $P = \tau\omega$

where $P = 5hp \times \frac{746W}{1hp} = 3730$

$$\frac{P}{\omega} = \frac{\tau\omega}{\omega}$$

$$\tau = \frac{P}{\omega}$$

$$\tau = \frac{3730}{94.25}$$

$$\tau = 39.58\,m.N$$

6. $L_i = L_f$

$$I_{1i}\omega_i + I_{2i}\omega_i = I_{1f}\omega_f + I_{2f}\omega_f$$

$$I_{1i}\omega_i + 0 = (I_{1f} + I_{2f})\omega_f$$

$$\omega_f = \frac{I_{1i}\omega_{1i}}{I_{1f} + I_{2f}}$$

(b) $I_1\omega_1 + I_2\omega_2 = I_1'\omega_f' + I_2'\omega_f'$

$$I_1\omega_1 + I_2\omega_2 = (I_1' + I_2')\omega_f$$

$$\omega_f = \frac{I_1\omega_1 + I_2\omega_2}{I_1' + I_2'}$$

(c) $I_1\omega_1 - I_2\omega_2 = I_1'\omega_f + I_2'\omega_f$

$$\omega_f = \frac{I_1\omega_1 - I_2\omega_2}{I_1' + I_2'}$$

7 $\overset{\$\$\$}{x}$ As these objects spin freely on thy table the work done to stop them will depend on the kinetic energy of their rotation;

$$K.E = \frac{1}{2} I \omega^2$$

- And since ω is the same for all, then their k.E must depend on their "I" -> [inertia]
- The more the inertia the more the K.E, the more work is required to stop it.

I of solid sphere;

$$I_{SS} = \frac{2}{5} MR^2$$

I of hollow sphere

$$I_{HS} = \frac{2}{3} MR^2$$

I of solid disc

$$I_{SD} = \frac{1}{2} mR^2$$

I of hoop

$$I_H = mR^2$$

$\overset{\$\$\$}{x}$ Again keeping in mind that there mass (M) and radius (R) are all the same, thereforce, we can conclude to say the Factor coefficient is going to determine the difference in inertia's of these object

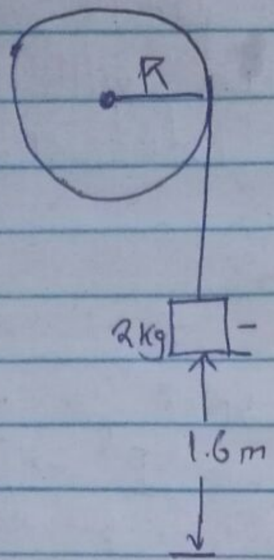
- Coefficient factore of; $I_{SS} = \frac{2}{5} = 0.4$

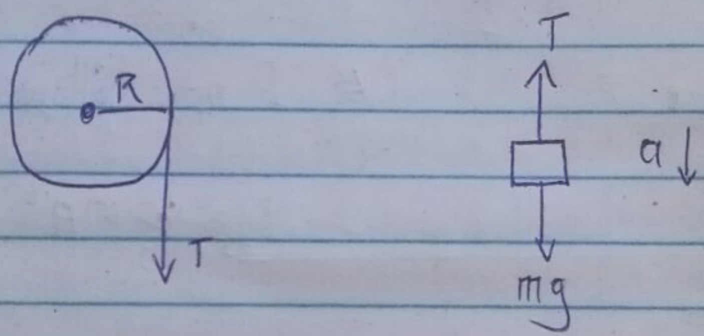
$$I_{HS} = \frac{2}{3} = 0.67$$

$$I_{SD} = \frac{1}{2} = 0.5$$

$$I_H = 1$$

$\therefore$ The hoop will have the greatest inertia giving us the biggest K.E which will require the max work to stop it.





ΣF ; $\tau = I\alpha$

$$T\cancel{R} = \frac{1}{2}M\cancel{R}^{2}\left(\frac{a}{\cancel{R}}\right)$$

$$T = \frac{1}{2}Ma \quad \text{---(i)}$$

ΣF ; $F = ma$

$$mg - T = ma$$

$$T = mg - ma \quad \text{--(ii)}$$

$$mg - ma = \frac{1}{2}Ma$$

$$mg = \left(\frac{1}{2}M + m\right)a$$

$$a = \frac{mg}{\frac{1}{2}M + m}$$

$$a = \frac{2 \times 9.8}{\frac{1}{2}(4) + 2}$$

$$a = 4.9 \ m/s^2$$

∴ To find its speed

$$v^2 = u^2 + 2as$$
$$v^2 = 0^2 + 2as$$
$$v = \sqrt{2as}$$
$$= \sqrt{2 \times 4.9 \times 1.6}$$
$$\underline{\underline{v = 3.96\, m/s}}$$

∴ To Find the Angular velocity !!

$$s = ut + \tfrac{1}{2}at^2$$
$$1.6 = (0)t + \tfrac{1}{2}(4.9)t^2$$
$$t^2 = 0.653061224$$
$$t = 0.808122035\, s$$

My apologies this part wasn't actually that neccesary !!

$$v = r\omega$$
$$3.96 = 0.5\,\omega$$
$$\underline{\underline{\omega = 7.9\, rad/s}}$$

* ~~What did the switch~~

What did the bulb say to the switch, "You turn me on."