THE COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

PHYSICS DEPARTMENT

08/06/2018 ACADEMIC YEAR

PH110 TEST 2

INTRODUCTORY PHYSICS

TIME: TWO (2) HOURS

INSTRUCTIONS:

THERE ARE FOUR (**4**) QUESTIONS IN THIS PAPER. EACH QUESTION CARRIES **25** MARKS. ATTEMPT **ALL** QUESTIONS.

MAXIMUM MARKS: 100

USE THE FOLLOWING DATA WHERE NECESSARY:

Acceleration due to gravity, $g = 9.8\ m/s^2$

QUESTION ONE

(a) State the law of conservation of linear momentum [3 marks]

(b) An estimated force-time graph for a hockey puck struck by a bat is shown in the Figure 1.1 below. From this graph, find the impulse delivered to the puck and the average force exerted on the puck. [5 marks]

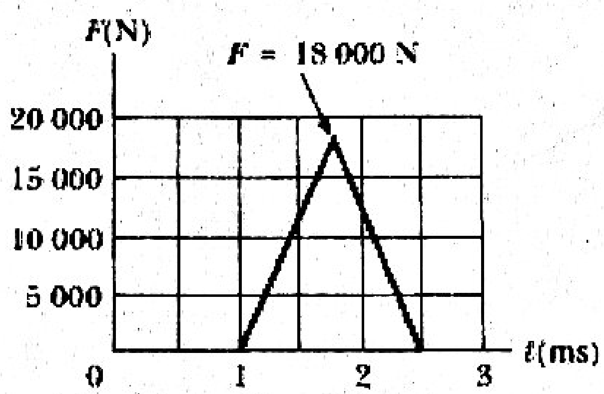


Figure 1.1

(c) A neutron having a mass of $1.67 \times 10^{-27} kg$ and moving at $10^8\ m/s$ collides with a deuteron of mass $3.34 \times 10^{-27} kg$, which is at rest and sticks to it.

(i) What type of collision is in between them? [1 marks]

(ii) What is the speed of the combination immediately after collision? [4 marks]

(iii) How much kinetic energy is lost during collision? [4 marks]

(d) A billiard ball of mass $m_1 = 0.4$ kg moving with a speed of 3 m/s to the right collides elastically with another ball of mass $m_2 = 0.6$ kg initially at rest. Find the speed of each ball after the collision. [8 marks]

QUESTION TWO

(a) A civil engineer tasked to construct an overhead bridge ensures that the bridge is in static equilibrium. State two conditions necessary for the bridge to be in static equilibrium. [2 marks]

(b) A box of mass 70 kg is suspended from a horizontally oriented ceiling by two tensions T_1 and T_2 developed in the cables of negligible mass. The tensions T_1 and T_2 make angles of 40° and 30°

receptively to the ceiling as shown in Figure 2.1. The system is in static equilibrium. Determine the tensions T_1 and T_2. [6 marks]

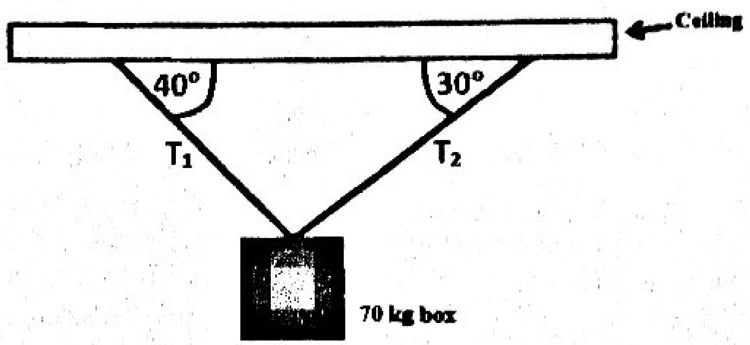


Figure 2.1

(c) The arrangement shown in Figure 2.2 below is in static equilibrium. A uniform beam of length 3 m and mass 50 kg is perpendicularly hinged to a vertically oriented wall at one end and suspended from the wall by a cable that is attached to the other end of the beam at an angle of 40°. A 150 kg box hangs from one end of the beam by a different cable. The hinge forms the rotational axis. Assume the cables have negligible masses and are inextensible. Calculate

(i) the tension T in the cable supporting the beam and [3 marks]

(ii) the component of the force due to the hinge in the x direction (F_{HX}) and the component of the force due to the hinge in the y direction (F_{HY}) that the hinge exerts on the beam. [4 marks]

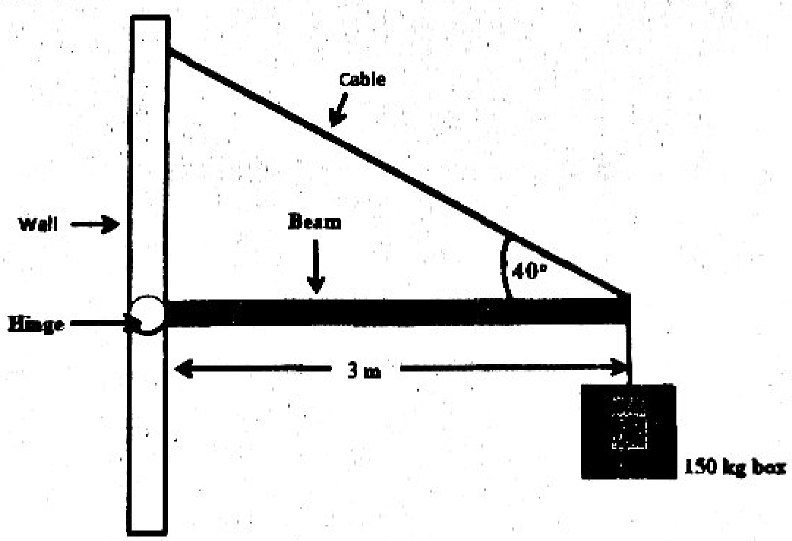


Figure 2.2

(d) A uniform ladder, of length 10 m and mass 22 kg, leans against a smooth wall while its bottom rests on rough ground 2.8 m from the wall. The ladder begins to slip when a 70 kg person climbs 90 percent of the way to the top of the ladder.

(i) Draw the free body diagram. [2 marks]

(ii) What is the coefficient of static friction between the ground and the ladder? [10 marks]

QUESTION THREE

(a) Express (i) 40 deg/s into rev/min [2 marks]

(ii) 1500 rpm into rad/s [2 marks]

(b) Consider a conical pendulum with an 80 kg bob on a 10 m wire making an angle of 5° with the vertical as shown in the Figure 3.1 below. Determine

(i) the horizontal and vertical components of the force exerted by the wire on the bob. [3 marks]

(ii) the radial acceleration of the bob. [2 marks]

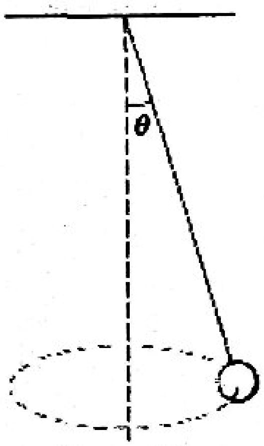


Figure 3.1

(c) A crate of eggs is located in the middle of the flat bed of a pickup truck as the truck negotiates an unbanked curve in the road. The curve may be regarded as an arc of a circle of radius 35 m. If the coefficient of static friction between crate and truck is 0.6, how fast can the truck be moving without the crate sliding?

(d) A stone weighing 2 kg is whirled in a vertical circle at the end of a rope of length 1000 mm. Find the tension in the string and velocity of the stone at

(i) the lowest position,

(ii) midway when the string is horizontal and

(iii) the topmost position to just complete the circle. [4, 4, 4 marks]

QUESTION FOUR

(a) Define work, kinetic energy, and potential energy. [2, 2, 2 marks]

(b) An object of mass m slides down a hill of height h and length l. Show that when the object reaches the bottom of the hill its speed is

$$v = \sqrt{2gh - \frac{2fl}{m}}$$

where f is the average force retarding the motion. [5 marks]

(c) How much work is done by the force $\vec{F} = (-4.0\hat{i} - 6.0\hat{j})\,\text{N}$ on a particle that moves through displacement $\Delta\vec{r} = (-3.0\hat{i} + 2.0\hat{j})\,\text{m}$? [3 marks]

(d) A rectangular block of mass 15 g rests on a rough plane which is inclined to the horizontal at an angle of $\sin^{-1}(0.0525)$. A force of 0.05 N, acting in a direction parallel to the line of greatest slope, is applied to the block so that the block moves up the plane.

When the block has moved a distance of 1.5 m from its initial position, the applied force is removed. The block moves on and comes to rest after travelling a further 0.25 m.

Calculate

(i) the work done by the applied force [3 marks]

(ii) the potential energy gain of the block [3 marks]

(iii) the value of the coefficient of sliding friction between block and the surface of the inclined plane [5 marks]

@@@@@@@@@@@@@@@@@ GOOD LUCK @@@@@@@@@@@@@@@@@@@@

Question (1)

(a) [3 marks]

The law of Conservation of linear momentum States that if no external force on a system of several particles, the total linear momentum of an Isolated system remains Costant.

(b) [5 marks]

For a force-time graph, the ~~under~~ area under the Curve/graph represents the impulse or change in momentum of a body

$$\text{Impulse} = \frac{1}{2}bh = \frac{1}{2} \times (1.5 \times 10^{-3}s) \times 18\,000N = \underline{13.5\ Ns}$$

The average force $\vec{F}$ is given by

$$\vec{F} = \frac{\text{Impulse}}{\text{time}} = \frac{13.5\ Ns}{1.5 \times 10^{-3}} = \underline{9000N}$$

(c) (i) [1 mark]

perfectly Inelastic collision or Inelastic collision

(ii) [4 marks]

By the law of conservation of linear momentum, total momentum before collision must be equal to total momentum after Collision i.e

$$\Sigma P_i = \Sigma P_f$$

$$m_1u_1 + m_2u_2 = (m_1+m_2)v$$

$$V = \frac{m_1u_1 + m_2u_2}{m_1+m_2}$$

Data

$m_1 = 1.67 \times 10^{-27}\ Kg$

$u_1 = 10^8 m/s$

$m_2 = 3.34 \times 10^{-27} Kg$

$u_2 = 0?$

$$V = \frac{(1.67\times10^{-27})\times(10^{8}) + 0}{1.67\times10^{-27} + 3.34\times10^{-27}}$$

$$\therefore\ V = 3.3\times10^{7}\,m/s$$

(iii) [4 marks]

Kinetic energy lost is given by

$$K.E\ lost = K_i - K_f$$

$$= \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \left(\frac{1}{2}(m_1+m_2)v^2\right)$$

$$= \left[\frac{1}{2}\times(1.67\times10^{-27})\times(10^{8}) + 0\right] - \frac{1}{2}\left(1.67\times10^{-27} + 3.34\times10^{-27}\right)\times(3.3^{2}$$

$$= 8.35\times10^{-12}J - 2.73\times10^{-12}J$$

$$\therefore\ KE\ lost = 5.62\times10^{-12}J$$

(d) [8 marks]

Data

$m_1 = 0.4\,Kg$

$u_1 = 3\,m/s$

$M_2 = 0.6\,Kg$

$u_2 = 0$

$$\Sigma P_i = \Sigma P_f$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$(0.4\times3) + (0.6\times0) = 0.4v_1 + 0.6v_2$$

$$1.2 = 0.4v_1 + 0.6v_2 \quad\text{———} (1)$$

Since the collision is elastic, the coefficient of restitution e is 1, given by

$$e = \frac{V_2 - V_1}{u_1 - u_2}$$

$$1 = \frac{V_2 - V_1}{3 - 0}$$

$$V_2 - V_1 = 3 \quad \text{——— (2)}$$

Solving eqn (1) and (2) simultaneously gives

$$V_2 = 3 + V_1 \quad \text{——— (3)}$$

(3) into (1)

$$1.2 = 0.4V_1 + 0.6(3 + V_1)$$

$$1.2 = 0.4V_1 + 1.8 + 0.6V_1$$

$$\therefore V_1 = -0.6\,m/s$$

And

$$V_2 = 3 - 0.6$$

$$\therefore V_2 = 2.4\,m/s$$

Question ②

ⓐ [2marks]

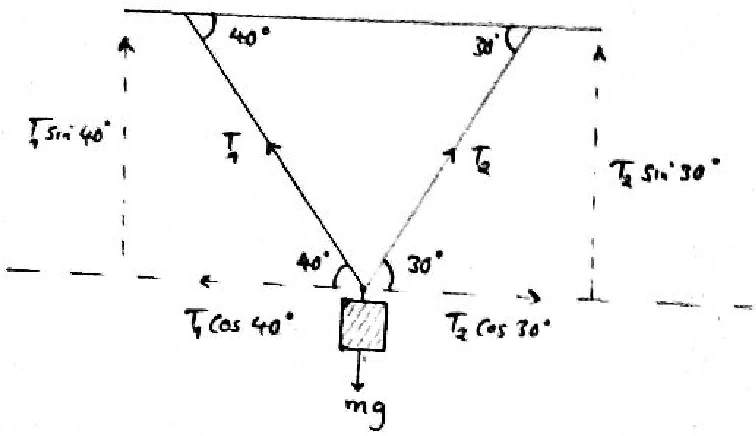
(i) the resultant of all external forces acting on the object must be zero

$$\Sigma F_{net} = 0$$

(ii) the sum of torques (τ) about any point must be equal to zero

$$\Sigma \tau = 0$$

ⓑ FBD [1mark]



- Horizontal direction

$$\Sigma F_x = 0$$

$$T_2 \cos 30° - T_1 \cos 40° = 0$$

$$T_2 \cos 30° = T_1 \cos 40°$$

$$T_2 = \frac{T_1 \cos 40°}{\cos 30°} \quad [1mark]$$

$$T_2 = \frac{0.77}{0.87} T_1$$

$$T_2 = 0.89 T_1 \quad \text{——— } ①$$

• Vertical motion

$$\sum F_y = 0$$

$$T_1 \sin 40^\circ + T_2 \sin 30^\circ - mg = 0$$

$$T_1 \sin 40^\circ + T_2 \sin 30^\circ = mg$$

$$0.64 T_1 + 0.5 T_2 = 686$$

$$0.64 T_1 + 0.534 T_1 = 686$$

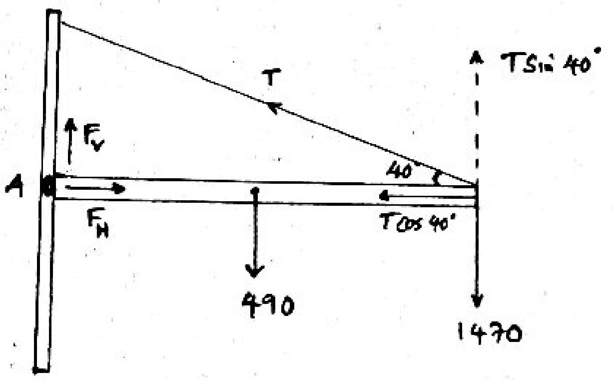
$$1.17 T_1 = 686$$

$$\therefore \underline{T_1 = 635 N} \quad [2 \text{ marks}]$$

And

$$\therefore \underline{T_2 = 561 N} \quad [2 \text{marks}]$$

⑥ FBD [1 mark]



1^{st} condition

$$\sum F_{net} = 0$$

• Horizontal ~~dir~~ direction

$$\sum F_x = 0$$

$$F_H - T\cos 40^\circ = 0$$

$$F_H = T\cos 40^\circ \quad \text{———} \quad (1)$$

- Vertical direction

$$\Sigma F_y = 0$$

$$F_v - 490 - 1470 + T\sin 40^\circ = 0$$

$$F_v + T\sin 40^\circ = 1960$$

$$F_v = 1960 - T\sin 40^\circ \quad \text{———} \quad (2)$$

2^{nd} – Condition;

$$\Sigma \tau = 0$$

$$(T\sin 40^\circ \times 3) - (490 \times 1.5) - (1470 \times 3) = 0$$

$$1.93T = 5145$$

$$\therefore \underline{T = 2665.80\text{N}} \quad [3 \text{ marks}]$$

Therefore

$$F_v = 1960 - (2665.80)\sin 40^\circ$$

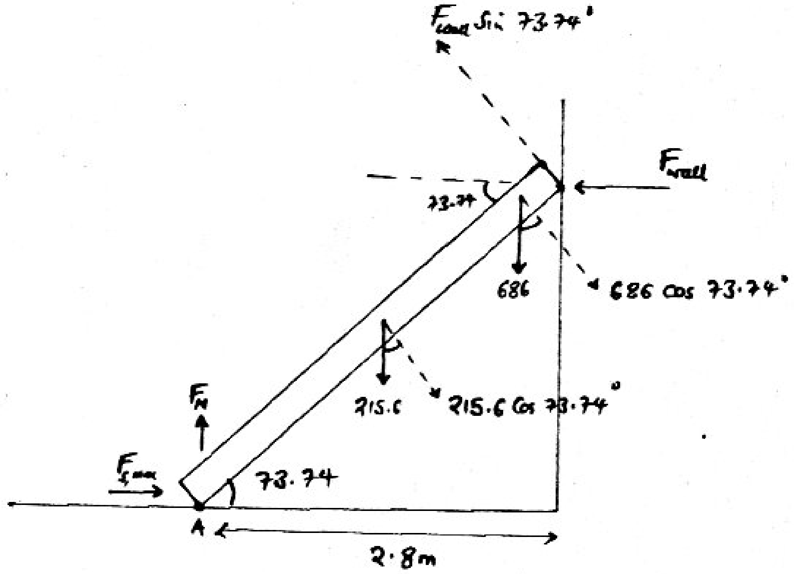
$$\therefore \underline{F_v = 246.46\text{ N}} \quad [2\text{marks}]$$

And

$$F_H = (2665.80)\cos 40^\circ$$

$$\therefore \underline{F_H = 2042.12\text{N}} \quad [2\text{marks}]$$

ⓓ FBD [2marks]



2nd - Condition ; $\Sigma\tau = 0$

$$\left(F_\omega \sin 73.74° \times 10\right) - \left(215.6 \cos 73.74° \times 5\right) - \left(686 \cos 73.74° \times 9\right) = 0$$

$$9.60\,F_\omega = 301.84 + 1728.70$$

$$F_{wall} = 2030.54\,N / 9.60 = 211.51\,N$$

1st - Condition ; $\Sigma F_x = 0$ and $\Sigma F_y = 0$

$\Sigma F_x = 0$

$F_{s,max} - F_\omega = 0$

$F_s = 2030.54/9.60$

$$\mu_s = \frac{2030.54/9.60}{F_N}$$

$$\mu_s = \frac{211.51}{F_N} = \frac{211.51}{901.6}$$

$\therefore \mu_s = 0.23$

$\Sigma F_y = 0$

$F_N = 215.6 + 686$

$\therefore F_N = 901.6$

Question ③

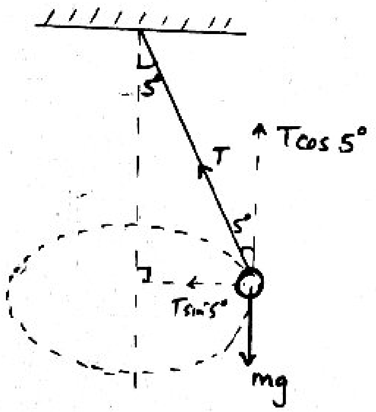
ⓐ ① [2marks]

$$40\ \text{deg/s} = \frac{40\ \cancel{\text{deg}}}{\cancel{s}} \times \frac{1\ \text{rev}}{360\ \cancel{\text{deg}}} \times \frac{60\ \cancel{s}}{1\ \text{min}} = 6.67\ \text{rev/min}$$

ⓘⓘ [2marks]

$$1500\ \text{rad/min} = \frac{1500^{\times 2\pi}\ \text{rad}}{\cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60\text{s}} = 157.1\ \text{rad/s}$$

ⓑ ① [3 marks]



• Horizonatal component

$$T\sin 5° = \frac{mg}{\cos 5°}\ \sin 5°$$

$$HC = mg \tan 5°$$

$$= (80)\times(9.8)\tan 5°$$

$\therefore$ HC = 68.59N [1½ mark]

• Vertical component

$$T\cos 5° - mg = 0$$

$$T = \frac{mg}{\cos 5°} = 787.15$$

$$VC = mg = T\cos 5°$$

$\therefore$ VC = 784 [1½ mark]

ⓘⓘ [2marks]

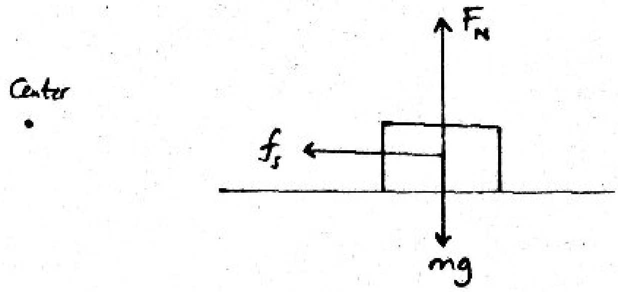
$$T\sin 5° = \frac{mv^2}{r} \quad \left(a_r = \frac{v^2}{r}\right)$$

$$T\sin 5° = m a_r$$

$$a_r = \frac{T\sin 5°}{m}$$

$$\therefore a_r = 0.857\ \text{m/s}^2$$

(c) FBD [1 mark]



- Vertical direction

$$\sum F_y = 0$$

$$F_N = mg \quad \text{———} \quad (1)$$

- toward the Center

$$f_s = \frac{mv^2}{r}$$

$$\mu_s F_N = \frac{mv^2}{r} \qquad (F_N = mg)$$

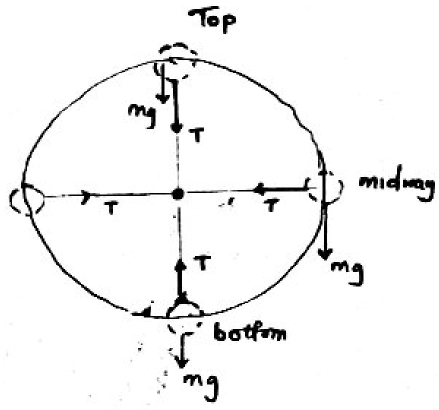
$$\mu_s \cancel{m} g = \frac{\cancel{m} v^2}{r}$$

$$V = \sqrt{\mu_s r g} \qquad [1 \text{ mark}]$$

$$V = \sqrt{(0.6) \times (35) \times (9.8)}$$

$$\therefore V = 14.35 \text{ m/s}$$

(d) FBD



(i) [4marks]

$$T - mg = \frac{mv^2}{r}$$

$$T = \frac{mv^2}{r} + mg$$

Answer

$$T_L = \frac{m}{r}(u^2 + gr)$$

$$= \frac{m}{r}(5gr + gr)$$

$$T_L = 6mg = (6 \times 2 \times 9.8)$$

$$\therefore \underline{T_L = 117.6N} \quad [2marks]$$

And $V_L = \sqrt{5gr}$

$$V_L = \sqrt{5 \times 9.8 \times 1}$$

$$\therefore \underline{V_L = 7m/s} \quad [2marks]$$

(ii) midway when the string is horizontal

$$T_{horizontal} = \frac{M}{r}(u^2 - 3gh + gr) = \frac{M}{r}(u^2 - 3gr + gr)$$

$$= 3mg, \quad u^2 = 5gr$$

$$\therefore \underline{T_H = 58.5N} \quad [2marks]$$

And

$$V_H = \sqrt{u^2 - 2gh}$$

$$= \sqrt{3gr}$$

$$\therefore \underline{V_H = 5.42m/s}$$

(iii) the topmost position to just complete the circle

$$T_t = \frac{m}{r}(u^2 - 5gr)$$

$$= \frac{m}{r}(5gr - 5gr)$$

$$\therefore T_t = 0N \quad [2marks]$$

And

$$V_t = \sqrt{u^2 - 2gh}$$

$$= \sqrt{5gr - 4gr} \qquad h = 2r$$

$$= \sqrt{gr}$$

$$\therefore V_t = 3.13 m/s$$

Question (4)

(a) (i) [2marks]

- Work is the product of the displacement and the component of a force in the direction of the displacement. i.e

$$W = \vec{F} \cdot \vec{s} = Fs \cos \theta$$

(ii) [2marks]

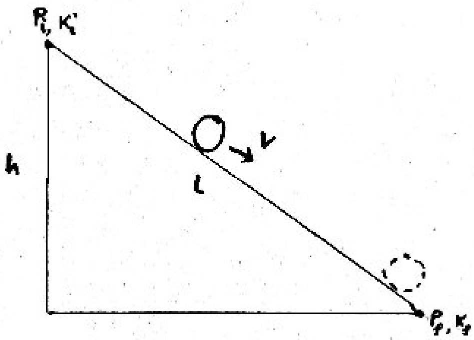
- Kinetic energy is the energy an object possesses by virtue of its motion.

(iii) [2marks]

- potential energy is the energy an object possesses by virtue of its position below or above some reference point.

(b) [5marks]

FBD [1 mark]



From the work energy theorem the change in mechanical energy of the object is equal to the work done against friction.

$$\Delta ME = -W_f$$

$$\Delta PE + \Delta KE = -fl$$

$$\left(\tfrac{1}{2}mv^2 - 0\right) + \left(0 - mgh\right) = -fl$$

$$\left[\tfrac{1}{2}mv^2 - mgh = -fl\right] \times 2$$

$$mv^2 - 2mgh = -2fl$$

$$\frac{\cancel{m}v^2}{\cancel{m}} = \frac{2\cancel{m}gh}{\cancel{m}} - \frac{2fl}{m}$$

$$v^2 = 2gh - \frac{2fl}{m}$$

$$\therefore V = \sqrt{2gh - \frac{2fl}{m}}$$ Hence shown

(c) $\vec{F} = (-4\hat{i} - 6\hat{j})N$

$\Delta\vec{r} = (-3\hat{i} + 2\hat{j})m$

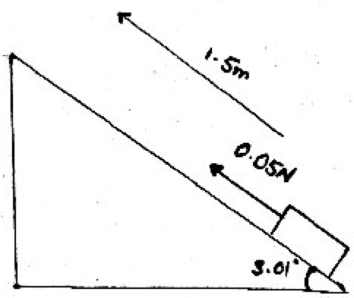
$W = \vec{F} \cdot \Delta\vec{r}$ [1 mark]

$= (-4\hat{i} - 6\hat{j}) \cdot (-3\hat{i} + 2\hat{j})$

$= (12 - 12)\ Nm$ [1 mark]

$\therefore W = 0J$ [1 mark]

(d) FBD [1 mark]



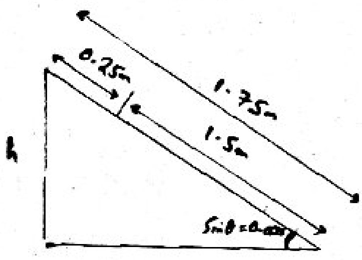
(i) $W = \vec{F} \cdot \vec{s}$

$= (0.05N) \times (1.5m)$ [1 mark]

$\therefore W = 0.075\ J$ [1 mark]

(ii) $P_i = 0$

$PE_f = ?$



$$\sin\theta = \frac{h}{1.75}$$

$$h = \sin\theta \times 1.75m$$

$$h = 0.0525 \times 1.75m$$

$$\therefore h = 0.09m$$

$$PE = mgh$$

$$PE = \left(\frac{15g}{1000}\right) \times (9.8) \times (0.09)$$

$\therefore PE = 0.013 J$ [3 marks]

(iii) Take the earth + block + inclined plane as the system. Just before the applied force is removed, the work energy theorem gives

$$W_{ext} = \Delta KE + \Delta PE + \Delta TE$$

$$\cancel{0.075} =$$

$$Fs = \frac{1}{2}m\left(V_2^2 - u_1^2\right) + mg\left(h_2 - h_1\right) + fs_1$$

$$0.075 = 0.5 \times 0.015\left(V_2^2 - 0\right) + 0.015 \times 9.8\left(0.09 - 0\right) + 1.5f$$

$$0.075 = 0.0075V_2^2 + 0.013 + 1.5f$$

$$0.0075V_2^2 = 0.075 - 1.5f - 0.013$$

$$V_2^2 = 8.27 - 200f \quad \text{------} \quad (1)$$

Just after the applied force is removed the work energy theorem gives

$$W_{ext} = \Delta KE + \Delta PE + \Delta TE$$

$$0 = \frac{1}{2}m\left(v_3^2 - v_2^2\right) + mg\left(h_3 - h_2\right) + fs_2$$

$$\cancel{0 = \frac{1}{2}\times(0.015)\times\left(0^2 - v_2^2\right) + 0.015\times 9.8\left(0.0525\times 1.75 - 0.0525\times 1.5\right)}$$

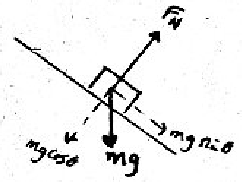
$$0 = 0.5\times(0.015)\times\left(0 - v_2^2\right) + 0.015\times 9.8\left(0.0525\times 1.75 - 0.0525\times 1.5\right) + 0.25f$$

$$0 = -0.0075v_2^2 + 0.014 - 0.012 + 0.25f$$

$$0 = -0.0075v_2^2 + 0.002 + 0.25f$$

$$v_2^2 = 0.27 + 33.33f \quad \text{———} \quad (2)$$

equating (1) and (2)



$$8.27 - 200f = 0.27 + 33.33f$$

$$-233.33f = -8$$

$$\therefore \; f = 0.034\text{ N} \quad (\text{friction force})$$

$F_N = mg\cos\theta$

$$\therefore \; \mu = \frac{f}{mg\cos\theta}$$

$$\mu = \frac{0.034}{(0.015)\times(9.8)\times\cos 3.01^\circ}$$

$$\therefore \; \mu = 0.22 \text{ or } 0.24$$

[5 marks]