$k(k+2)$ $(k+1)(k+3)$ 36 2×18 3×12

$2(k+1)+7$ b^2-4ac

$2k+2+7$

$2k+9$ $(k+2)(2k+9)$

$k(2k+9)+2(2k+9)$

$P=36$ $S=13$ $F=4,9$

MA110 - MATHEMATICAL METHODS TEST 2

Time allowed: Two hours thirty minutes (2:30)

Instructions:

1. You must write your **Name**, your **Computer Number** and **programme of study** on your answer sheet.
2. Calculators are **not** allowed in this paper.
3. There are four (4) questions in this paper, Attempt **All** questions and show **detailed** working for full credit

---

QUESTION ONE

a) Express $\frac{2x+1}{x^3-1}$ in partial fractions (5marks)

b) Find the centre and length of a radius of the given circle and graph it $x^2+y^2-10x=0$. (5marks)

c) Prove the result by induction: $1\times 3+2\times 4+\cdots+n(n+2)=\frac{1}{6}n(n+1)(2n+7)$ (5marks)

d) Find the 4^{th} term in the binomial expansion $\left(2-\frac{x}{2}\right)^9$ (5marks)

e) If $xy=64$ and $log_x y+log_y x=\frac{5}{2}$, Find x and y. (5marks)

QUESTION TWO

a) A is the point $(-1, 2)$, B is the point $(2, 3)$ and C is the point $(3, 5)$. P is a point which divides BC in the ratio 3 : 4 and Q lies on AB such that $AQ=\frac{2}{5}AB$.

(i) Find the coordinates of P (2.5 marks)

(ii) Find the coordinates of Q. (2.5 marks)

b) Find λ for which the matrix $\lambda I-A$ is a singular matrix if where I is an identity Matrix given that $A=\begin{pmatrix}1 & 0 & 2\\ 0 & -1 & -2\\ 2 & -2 & 0\end{pmatrix}$ (5 marks)

c) Show that the sum of the integers from 1 to n is $\frac{1}{2}n(n+1)$ (5marks)

$S_n=\frac{1}{2}n(n-1)d$

d) Solve the logarithmic equation : $\log(x-4)+\log(x-1)=1$ (5marks)

e) In the expansion of $(1+ax)^n$, the first three terms in ascending power of x are $1-\frac{5}{2}x+\frac{75}{8}x^2$, Find the values of n and a, and state the range of values of x for which the expansion is valid. (5marks)

QUESTION THREE

a) Find the radius of the circle with center at $C(-2,5)$ if the line $x+3y=9$ is a tangent line. (5marks)

b) Using geometrical progression, change $0.2\overline{14}$ to $\frac{a}{b}$ form , where a and b are integers and $b \neq 0$. (5marks)

c) Use mathematical induction to prove that the statement is true for all positive integers n given that 4^n-1 is divisible by 3 (5marks)

d) Graph $f(x)=log_{\frac{1}{2}}x$ by reflecting the graph of $g(x)=\left(\frac{1}{2}\right)^x$ a cross the line $y=x$ (5marks)

e) (i) Find the inverse of the matrix $A=\begin{pmatrix}3 & -1 & 2\\ 1 & 1 & 1\\ 2 & 2 & -1\end{pmatrix}$ (5marks)

(ii) Use your inverse to solve the system of linear equations

$$3x-y+2z=4$$
$$x+y+z=2$$
$$2x+2y-z=3$$

(3marks)

QUESTION FOUR

a) Write the following in sigma notation

(i) $1-\frac{2}{3}+\frac{4}{9}-\frac{8}{27}+\cdots$ (3marks)

(ii) $1^4+2^4+3^4+\cdots+n^4+(n+1)^4$ (2marks)

b) The number of grams of a certain radioactive substance present after t hours is given by the equation $Q=Q_0e^{-0.45t}$, where Q_0 represents the initial number of grams. How long will it take 2500 grams to be reduced to 1250 given $ln\left(\frac{1}{2}\right)=-0.693$ (5marks)

c) (i) Expand $(1+2x)^4$ and $(1-2x)^4$ in ascending powers of x. (5marks)

(ii) Hence reduce $(1+2x)^4-(1-2x)^4$ to its simplest form. (3marks)

(iii) Using the results in (ii) evaluate $(1.002)^4-(0.998)^4$ (5marks)

$1+2x=1.002$
$2x=0.002$
$\boxed{x=0.001}$
$1-2x=0.998$
$-2x=-0.002$
$x=0.001$

$y=3\left(-\frac{12}{5}\right)+\frac{11}{4}$

$\boxed{\frac{19}{5}}$

THE COPPERBELT UNIVERSITY

School of Mathematics and Natural Sciences

99%

Course: MA 110

Program: Natural Science (N.Q)

Sin: 22111573

Date: 22nd May 2023

Q	MARKS
1	25
2	25
3	26
4	23
Total	99

(a) $\dfrac{2x+1}{x^3-1}$

taking the denominator

$$x^3-1 = x^3+0x^2+0x-1$$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 0 & -1 \\ & \downarrow & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 0 \end{array}$$

$$(x^2+x+1)(x-1)$$

$$\frac{2x+1}{x^3-1} = \frac{2x+1}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{(x^2+x+1)} \Bigg] (x-1)(x^2+x+1)$$

$$2x+1 = A(x^2+x+1) + (Bx+C)(x-1)$$

$$= Ax^2+Ax+A + Bx^2-Bx+Cx-C$$

$$2x+1 = (A+B)x^2 + (A-B+C)x + A-C$$

$(A+B)x^2 = 0x^2$

$A+B=0$

$A=-B$ ...(i)

$(A-B+C)x = 2x$

$A-B+C = 2$

$-B-B+C = 2$

$-2B+C = 2$ ...(ii)

$A-C = 1$

$\cancel{A=1-C}$

$A = 1+C$ ...(iii)

eqn (iii) in to eq (i)

$1+C = -B$

$1 = -C-B$

$B+C = -1$ ...(iv)

$$-\left|\begin{array}{l} B + C = -1 \\ -2B + C = 2 \end{array}\right.$$

$$3B = -3$$

$$\underline{\underline{B = -1}}$$

$$A = -B$$

$$A = -(-1)$$

$$\underline{\underline{A = 1}}$$

$$A = 1 + C$$

$$1 = 1 + C$$

$$\underline{\underline{C = 0}}$$

$$\therefore \frac{2x+1}{x^3-1} = \frac{1}{x-1} + \frac{-x}{x^2+x+1}$$

5

25/25

Q. 1

$$x^2+y^2-10x=0$$

$$x^2+y^2-10x+0y+0=0$$

$$x^2+y^2+2gx+2fy+c=0$$

$2gx=-10x$ $\qquad$ $2fy=0y$

$\frac{2g}{2}=\frac{-10}{2}$ $\qquad$ $2f=0$

$g=-5$ $\qquad$ $f=0$

$h=-g$ $\qquad$ $k=-f$

$h=-(-5)$ $\qquad$ $k=-(0)$

$h=5$ $\qquad$ $k=0$

$\therefore$ the centre is $(5,0)$

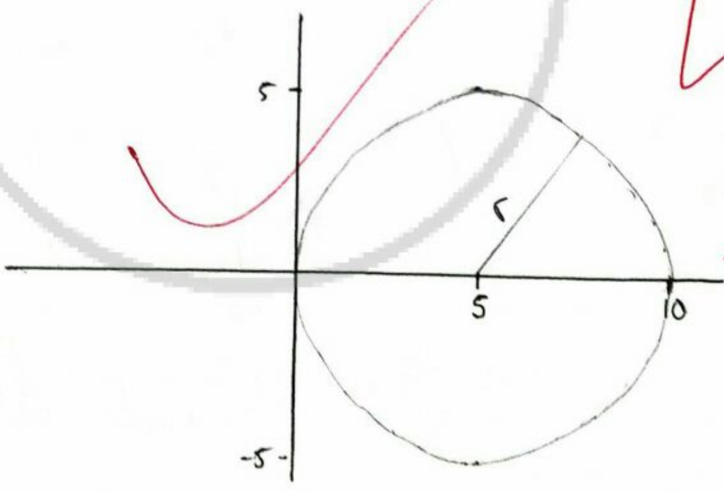
radius $=\sqrt{f^2+g^2-c}$

$=\sqrt{(0)^2+(-5)^2-0}$

$=\sqrt{25}$

$r=5$

$\therefore$ radius $=5$



(c) $1\times3+2\times4+\dots+n(n+2)=\frac{1}{6}n(n+1)(2n+7)$

Step 1: It is true for $n=1$ since

$1\times3+2\times4+\dots+1(1+2)=\frac{1}{6}(1)(1+1)(2(1)+7)$

$1\times3+2\times4+\dots+3=3$

ep 2. ; We assume it true for $n=k$

i.e $1\times3 + 2\times4 + \dots + k(k+2) = \frac{1}{6}k(k+1)(2k+7) \dots (i)$

We need to prove it true for $n=k+1$ by adding the $(k+1)^{th}$ term to both sides of equation (i)

$$1\times3 + 2\times4 + \dots + k(k+2) + (k+1)(k+3) = \frac{1}{6}(k+1)(k+2)(2k+9)$$

$$\frac{1}{6}k(k+1)(2k+7) + (k+1)(k+3)$$

$$\frac{1}{6}(k+1)\left[k(2k+7) + 6(k+3)\right]$$

$$\frac{1}{6}(k+1)\left[2k^2 + 7k + 6k + 18\right]$$

$$2k^2 + 4k + 9k + 18$$

$$\frac{1}{6}(k+1)\left[2k(k+2) + 9(k+2)\right]$$

$$\left[(2k+9)(k+2)\right]$$

$$\frac{1}{6}(k+1)(k+2)(2k+9)$$

which is equal to $n=k+1$ when $k+1$ is replaced by n.

5

Conclusion:

Since it is true for $n=1$. It is also true for $n=k$ making it true also for $n=k+1$. Therefore it is true for all natural numbers n.

(d) $$\left(2 - \frac{x}{2}\right)^9 = \binom{n}{r}(a)^{n-r}(b)^r$$

$$\frac{9!}{6!\,3!} = \frac{\overset{3}{\cancel{9}}\times\overset{4}{\cancel{8}}\times7\times\cancel{6!}}{\cancel{6!}\times\cancel{3}\times\cancel{2}\times1} = 84 \quad = \binom{9}{3}(2)^{9-3}\left(-\frac{x}{2}\right)^3$$

$$= 84\,(2)^6\left(-\frac{x^3}{2^3}\right)$$

$84 \times 2^{6} \times -\frac{x^3}{2^3}$

$84 \times 2^3 \times -x^3$

$\underline{\underline{-672x^3}}$ ✓ 5

(e) $xy = 64$, $\log_x y + \log_y x = \frac{5}{2}$

$\log_x y + \frac{1}{\log_x y} = \frac{5}{2} \Big] \times 2$

$2\log_x y + \frac{2}{\log_x y} = 5$

let $a = \log_x y$

$2a + \frac{2}{a} = 5 \Big] \times a$

$2a^2 + 2 = 5a$

$2a^2 - 5a + 2 = 0$

$p = 4$ $s = -5$ $f = -1, -4$

$2a^2 - 4a - a + 2 = 0$

$2a(a-2) - 1(a-2) = 0$

$(2a-1)(a-2) = 0$

$2a - 1 = 0$, $a - 2 = 0$

$a = \frac{1}{2}$ $a = 2$

$\log_x y = \frac{1}{2}$ $\log_x y = 2$

$y = x^{\frac{1}{2}}$ $y = x^2$

when $y = x^{1/2}$

$xy = 64$

$x \cdot x^{1/2} = 64$

$x^{3/2 \times 2/3} = (64)^{2/3}$

$x = \left(\sqrt[3]{64}\right)^2$

$x = (4)^2$

$\underline{x = 16}$

$y = 16^{1/2}$

$\underline{y = 4}$

when $y = x^2$

$xy = 64$

$x \cdot x^2 = 64$

$\sqrt[3]{x^3} = \sqrt[3]{64}$

$\underline{x = 4}$

5

$y = x^2$

$y = 4^2$

$\underline{y = 16}$

$\boxed{Q. 3}$

(a.) $x + 3y = 9$ ---- i

$\frac{3y}{3} = \frac{9}{3} - \frac{x}{3}$

$y = 3 - \frac{x}{3}$

$\therefore m_1 = -\frac{1}{3}$

$m_1 m_2 = -1$

$-\frac{1}{3} m_2 = -1$

$m_2 = 3$

$y - y_1 = m(x - x_1)$

$y - 5 = 3(x + 2)$

$y - 5 = 3x + 6$

$y = 3x + 11$ ... (ii)

$\therefore$ we now put eq (ii) into eq (i) to find the point they meet

$x + 3y = 9$

$x + 3(3x + 11) = 9$

$x + 9x + 33 = 9$

$10x = 9 - 33$

$\frac{10x}{10} = \frac{-24}{10}$

$x = -\frac{12}{5} = -2.4$

$y = 3x + 11$

$y = 3\left(-\frac{12}{5}\right) + 11$

$y = -\frac{36}{5} + \frac{11}{1}$

$\frac{-36 + 55}{5}$

$y = \frac{19}{5}$

$\therefore \left(-\frac{12}{5}, \frac{19}{5}\right)$ is the point where the tangent meets the circle

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad (\underset{x_1}{-2}, \underset{y_1}{5}) \left(\underset{x_2}{-\tfrac{12}{5}}, \underset{y_2}{\tfrac{19}{5}}\right)$$

$$r = \sqrt{\left(-\tfrac{12}{5} + 2\right)^2 + \left(\tfrac{19}{5} - 5\right)^2}$$

$$-\frac{12}{5} + \frac{2}{1} = \frac{-12+10}{5} = -\frac{2}{5} \qquad \frac{19}{5} - \frac{5}{1} = \frac{19-25}{5} = -\frac{6}{5}$$

$$r = \sqrt{\left(-\tfrac{2}{5}\right)^2 + \left(-\tfrac{6}{5}\right)^2}$$

$$= \sqrt{\frac{4}{25} + \frac{36}{25}}$$

$$= \sqrt{\frac{40}{25}}$$

$$r = \frac{\sqrt{40}}{5} = \frac{\sqrt{4 \times 10}}{5}$$

$$r = \frac{2\sqrt{10}}{5}$$

5

(b.) $0.2\overline{14} = 0.2 + 0.0\overline{14}$

$0.0\overline{14} = 0.014 + 0.00014 + 0.0000014 + \cdots$

$r = \dfrac{0.00014}{0.014} = \dfrac{14}{1400} = \dfrac{1}{100} = 0.01$

$\therefore S_\infty = \dfrac{a_1}{1-r}$

$= \dfrac{0.014}{1-0.01}$

$= \dfrac{0.014}{0.99} = \dfrac{14}{1000} \div \dfrac{99}{100} = \dfrac{7}{500} \div \dfrac{99}{100}$

$= \dfrac{7}{\cancel{500}_5} \times \dfrac{\cancel{100}^1}{99}$

$S_\infty = \dfrac{7}{495}$

$\therefore 0.2 + \dfrac{7}{495}$

$= \dfrac{1}{5} + \dfrac{7}{495}$

$$\frac{1}{5} + \frac{7}{495}$$

$$\frac{99+7}{495}$$

$$= \frac{106}{495}$$

(e.) $4^n - 1$ divisible by 3

Step 1 : It is true for $n=1$ since $4^1 - 1 = 3$ which is divisible by 3

Step 2 : Assume it true $n=k$ i.e there exists an integer x such that

$$4^k - 1 = 3x \quad \ldots (i)$$

we need to prove it true for $n=k+1$ ($4^{k+1} - 1 = 3x$) starting with eq (i).

$$4^k - 1 = 3x$$

$$4^k = 3x - 1 \quad \ldots (ii)$$

multiply 4 throughout equation (ii)

$$4^{k+1} = 4 \cdot 3x - 4$$

$$4^{k+1} = 4 \cdot 3x - 3 + 1$$

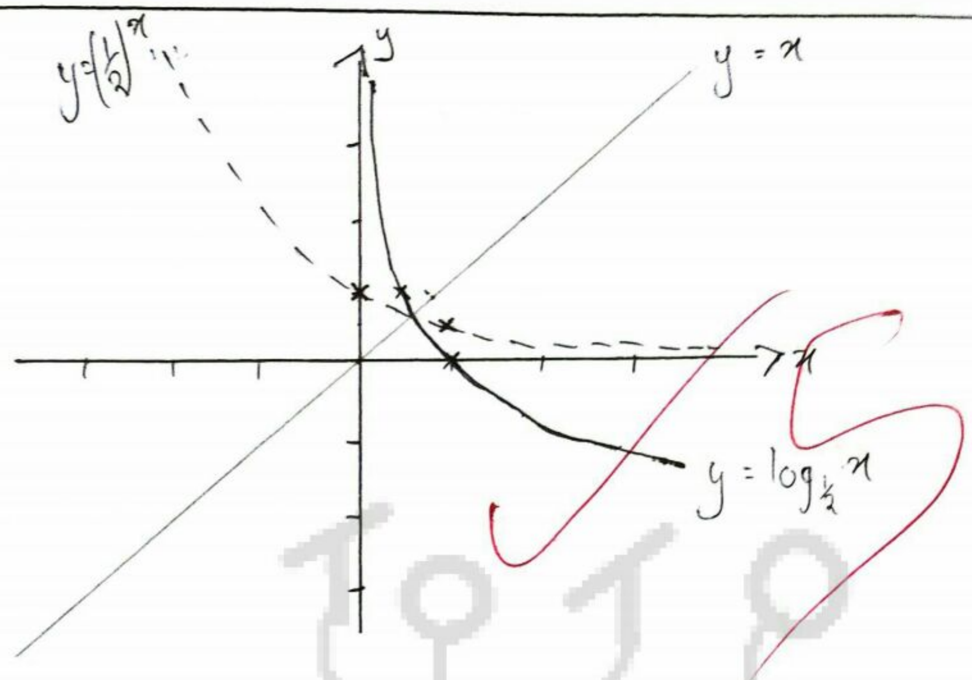
$$4^{k+1} = 3(4x - 1) + 1$$

$$4^{k+1} - 1 = 3(4x - 1)$$

From the last statement we see that it is divisible by 3

Conclusion :

Since it is true $n=1$, and true for $n=k$, it is also true for $n=k+1$. Therefore it is true for all positive integers n.



e. $A = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 1 & 1 \\ 2 & 2 & -1 \end{bmatrix}$

$$C = \begin{bmatrix} +\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} & +\begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} \\ -\begin{vmatrix} -1 & 2 \\ 2 & -1 \end{vmatrix} & +\begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} & -\begin{vmatrix} 3 & -1 \\ 2 & 2 \end{vmatrix} \\ +\begin{vmatrix} -1 & 2 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} & +\begin{vmatrix} 3 & -1 \\ 1 & 1 \end{vmatrix} \end{bmatrix}$$

$$C = \begin{bmatrix} (-1-2) & -(-1-2) & (2-2) \\ -(1-4) & (-3-4) & -(6+2) \\ (-1-2) & -(3-2) & (3+1) \end{bmatrix}$$

$$C = \begin{bmatrix} -3 & 3 & 0 \\ 3 & -7 & -8 \\ -3 & -1 & 4 \end{bmatrix} = C^T = \begin{bmatrix} -3 & 3 & -3 \\ 3 & -7 & -1 \\ 0 & -8 & 4 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -1 & 2 \\ 1 & 1 & 1 \\ 2 & 2 & -1 \end{vmatrix}$$

$$-1\begin{vmatrix} -1 & 2 \\ 2 & -1 \end{vmatrix} + 1\begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} - 1\begin{vmatrix} 3 & -1 \\ 2 & 2 \end{vmatrix}$$

$$-1(1-4) + 1(-3-4) - 1(6+2)$$

$$3 - 7 - 8$$

$$|A| \; -12$$

$$\therefore A^{-1} = \frac{1}{|A|} \times \operatorname{adj} A$$

$$A^{-1} = \frac{1}{-12}\begin{bmatrix} -3 & 3 & -3 \\ 3 & -7 & -1 \\ 0 & -8 & 4 \end{bmatrix}$$

5

(ii) $3x - y + 2z = 4$

$x + y + z = 2$

$2x + 2y - z = 3$

$$\begin{pmatrix} 3 & -1 & 2 \\ 1 & 1 & 1 \\ 2 & 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{-12}\begin{pmatrix} -3 & 3 & -3 \\ 3 & -7 & -1 \\ 0 & -8 & 4 \end{pmatrix}\begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{-12}\begin{pmatrix} -12 + 6 - 9 \\ 12 - 14 - 3 \\ 0 - 16 + 12 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{-12}\begin{pmatrix} -15 \\ -5 \\ -4 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{15}{12} \\ \frac{5}{12} \\ \frac{1}{3} \end{pmatrix}$$

$\therefore x = \frac{15}{12}, \ y = \frac{5}{12}, \ z = \frac{1}{3}$

26/28

Q.4

(a)(i) $1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \dots$

$r = \frac{-\frac{2}{3}}{1}$

$r = -\frac{2}{3}$

$\therefore a_n = ar^{n-1}$

$a_n = 1\left(-\frac{2}{3}\right)^{n-1}$

$a_n = \left(-\frac{2}{3}\right)^{n-1}$

$$\sum_{n=1}^{\infty}\left(-\frac{2}{3}\right)^{n-1}$$

23/23

(ii) $1^4 + 2^4 + 3^4 + \dots n^4 + (n+1)^4$

$$\sum_{k=0}^{k}(k+1)^4$$

$$\cancel{Q = Q_0 e^{-0.45t}}$$

$$\cancel{\frac{2500}{1250} = \frac{1250 e^{-0.45t}}{1250}}$$

$$Q = Q_0 e^{-0.45t}$$

$$\frac{1250}{2500} = \frac{\cancel{2500} e^{-0.45t}}{\cancel{2500}}$$

$$\ln \tfrac{1}{2} = \ln e^{-0.45t}$$

$$\frac{-0.693}{-0.45} = \frac{-\cancel{0.45}t}{-\cancel{0.45}}$$

$$t = \frac{0.693}{0.45} \text{ Hours}$$

$$t = \frac{77}{50} \text{ hors}$$

(c, i) $(1+2x)^4 = \binom{4}{0}(1)^4 + \binom{4}{1}(1)^3(2x)^1 + \binom{4}{2}(1)^2(2x)^2 + \binom{4}{3}(1)^1(2x)^3 + \binom{4}{4}(1)^0(2x)^4$

$$= 1 + 8x + 24x^2 + 32x^3 + 16x^4$$

$(1-2x)^4 = \binom{4}{0}(1)^4 + \binom{4}{1}(1)(-2x)^1 + \binom{4}{2}(1)^2(-2x)^2 + \binom{4}{3}(1)^1(-2x)^3 + \binom{4}{4}(1)^0(-2x)^4$

$$= 1 - 8x + 24x^2 - 32x^3 + 16x^4$$

(ii) $(1+2x)^4 - (1-2x)^4$

$1 + 8x + 24x^2 + 32x^3 + 16x^2 - (1 - 8x + 24x^2 - 32x^3 + 16x^4)$

$1 - 1 + 8x + 8x + 24x^2 - 24x^2 + 32x^3 + 32x^3 + 16x^4 - 16x^4$

$$16x + 64x^3$$

$(1.002)^4 - (0.998)^4$

$1 + 2x = 1.002$ $\quad\quad$ $1 - 2x = 0.998$

$2x = 1.002 - 1$ $\quad\quad$ $-2x = 0.998 - 1$

$2x = 0.002$ $\quad\quad$ $-2x = -0.002$

$x = 0.001$ $\quad\quad$ $x = 0.001$

$\therefore$ since :

$$(1+2x)^4 - (1-2x)^4 = 16x + 64x^3$$

$$(1+2(0.001))^4 - (1-2(0.001))^4 = 16(0.001) + 64(0.001)^3$$

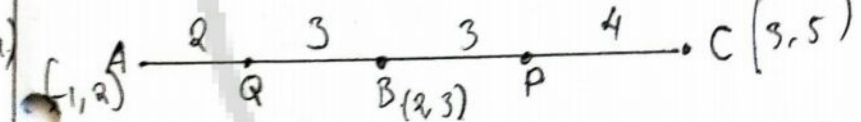
$$(1.002)^4 - (0.998)^4 = 0.016 + 64(0.000\,000\,001)$$

$$= 0.016 + 0.000\,000\,064$$

$$= \underline{\underline{0.016\,000\,064}}$$

5

Q.2

(a) 

28/25

(i) $P = \left(\dfrac{mx_2 + nx_1}{m+n}, \dfrac{my_2 + ny_1}{m+n} \right)$

$= \left(\dfrac{3(3) + 4(2)}{3+4}, \dfrac{3(5) + 4(3)}{3+4} \right)$

$P = \left(\dfrac{17}{7}, \dfrac{27}{7} \right)$

$$Q = \left(\frac{mx_2 + nx_1}{m+n}, \frac{m(y_2) + ny_1}{m+n} \right)$$

$$= \left(\frac{2(2) + 3(-1)}{2+3}, \frac{2(5) + 3(2)}{2+3} \right)$$

$$Q = \left(\frac{1}{5}, \frac{12}{5} \right)$$

5

(b) $\lambda I - A$

$$\lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & -2 \\ 2 & -2 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} - \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & -2 \\ 2 & -2 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \lambda-1 & 0 & -2 \\ 0 & \lambda+1 & 2 \\ -2 & 2 & \lambda \end{pmatrix}$$

$$\begin{vmatrix} \lambda-1 & 0 & -2 \\ 0 & \lambda+1 & 2 \\ -2 & 2 & \lambda \end{vmatrix} = 0$$

$$\lambda-1 \begin{vmatrix} \lambda+1 & 2 \\ 2 & \lambda \end{vmatrix} + 0 \begin{vmatrix} 0 & -2 \\ 2 & \lambda \end{vmatrix} - 2 \begin{vmatrix} 0 & -2 \\ \lambda+1 & 2 \end{vmatrix}$$

$$\lambda-1(\lambda^2 + \lambda - 4) - 2(0 - (-2\lambda - 2))$$

$$\lambda-1(\lambda^2 + \lambda - 4) - 2(2\lambda + 2)$$

$$(\lambda-1)(\lambda^2 + \lambda - 4) - 4\lambda - 4$$

$x^3 + x^2 - 4x - x^2 - x + 4 - 4x - 4$

$x^3 - 9x = 0$

$x(x^2 - 9) = 0$

$x = 0 \quad , \quad x^2 - 9 = 0$

$\sqrt{x^2} = \sqrt{9}$

$\cancel{x = 9}$

$\underline{\underline{x = 3}}$

5

(d) $\log(x-4) + \log(x-1) = 1$

$\log(x-4)(x-1) = 1$

$(x-4)(x-1) = 10$

$x^2 - 5x + 4 = 10$

$x^2 - 5x - 6 = 10$

$p = -6 \quad s = -5 \quad f = 1, -6$

$x^2 - 6x + x - 6$

$x(x-6) + 1(x-6) = 0$

$(x+1)(x-6) = 0$

$x = -1 \quad , \quad x = 6$

$\underline{\underline{\therefore x = 6}}$

5

$(1 + ax)^n = 1 + nx + \frac{n(n-1)}{2!}x^2$

$= 1 + n(ax) + \frac{n(n-1)}{2}(ax)^2$

$= 1 + nax + \frac{n(n-1)}{2}a^2x^2$

$$1 + nax + \frac{n(n-1)a^2x^2}{2} = 1 - \frac{5}{2}x + \frac{75}{8}x^2$$

$1 = 1$, $\quad nax = -\frac{5}{2}x$, $\quad \frac{n(n-1)}{2}a^2x^2 = \frac{75}{8}x^2$

$na = -\frac{5}{2}$

$n = \frac{-5}{2a}$ ...(i)

$$\frac{n(n-1)a^2}{2} = \frac{75}{8} \quad \Big] \times 2$$

$$n(n-1)a^2 = \frac{75}{4}$$

where $n = -\frac{5}{2a}$

$$-\frac{5}{2a}\left(-\frac{5}{2a} - 1\right)a^2 = \frac{75}{4}$$

$$-\frac{5}{2a}\left(\frac{-5-2a}{2a}\right)a^2 = \frac{75}{4}$$

$$\left(\frac{20+10a}{4a^2}\right)a^2 = \frac{75}{4}$$

$$\frac{20+10a}{4} = \frac{75}{4} \quad \Big] \times 4$$

$$20 + 10a = 75$$

$$10a = 50$$

$$\underline{\underline{a = 5}}$$

$n = -\frac{5}{2a}$

$n = -\frac{5}{2(5)}$

$\underline{\underline{n = -\frac{1}{2}}}$

$\therefore (1 + 5x)^{-1/2}$

$|x| < 1$

$|5x| < 1$

$\frac{5|x|}{5} < \frac{1}{5}$

$\underline{\underline{|x| < \frac{1}{5}, \text{ is the validity of the expansion}}}$

Intergers are numbers expressed as;

$1, 2, 3, 4, 5, 6, \ldots\ldots, n$

they have a common difference of $\underline{1}$ and from here the first term is $\underline{1}$.

now if you have an ~~number~~ interger a_1 the ~~n~~ sum of the next terms would be

$$S_n = a_1 + (a+d) + \ldots\ldots a_1 + (n-2)d + a_1 + (n-1)d \ldots (i)$$

$$S_n = [a_1 + (n-1)d] + [a + d(n-2)d] + \ldots\ldots + \ldots (a+d) + a_1 \ldots (ii)$$

adding up equ (i) & (ii);

$$\frac{\cancel{2}S_n}{\cancel{2}} = \frac{[2a_1 + (n-1)d] \times n}{2}$$

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

where $a_1 = 1$, $d = 1$

$$S_n = \frac{n}{2}[2(1) + (n-1)1]$$

$$= \frac{n}{2}(2 + n - 1)$$

$$= \frac{n}{2}(n+1)$$

$$S_n = \frac{1}{2}n(n+1)$$

hence shown

5