Tutorial Sheet 10

Gravitation

1. The gravitational force that the sun exerts on the moon is perpendicular to the force that the Earth exerts on the moon. The masses are: Mass of sun $2 \times 10^{30} kg$, mass of Earth $6 \times 10^{24} kg$, mass of moon $7 \times 10^{22} kg$. The distance between the sun and the moon is 10^{11}m, and the distance between the moon and the Earth is 4×10^{8}m. Determine the magnitude of the net gravitational force on the moon

2. A body weighs 63 N on the earth. What is the gravitational force on it at a height equal to half the radius of the earth?

3. Calculate the height above the earth at which the geostationary satellite is orbiting the earth.

4. A satellite of mass 200 kg orbits the earth at a height of 400 km above the surface. How much energy must be expended to rocket the satellite out of earth's gravitational influence?

5. The escape velocity of a projectile on the earth's surface is 11.2 km/s. A body is projected out with twice its speed. What is the speed of the body far away from the earth (infinity)? Ignore the presence of sun and other planets

6. An artificial satellite circles Earth in a circular orbit at a location where the acceleration due to gravity is 9 m/s^2. Determine the orbital period of the satellite.

7. A satellite moves in a circular orbit around the Earth at a speed of 5000 m/s. Determine

(a) the satellite's altitude above the surface of the Earth and

(b) the period of the satellite's orbit.

8. A 1 000-kg satellite orbits the Earth at a constant altitude of 100 km. (a) How much energy must be added to the system to move the satellite into a circular orbit with altitude 200 km? What are the changes in the system's (b) kinetic energy and (c) potential energy?

9. Show that the speed of an Earth satellite in circular orbit is given by the expression

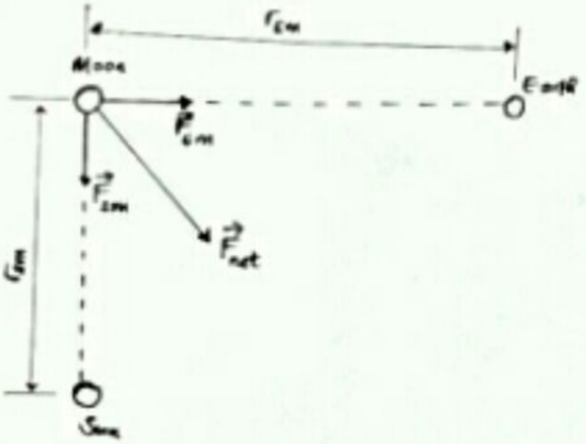
$$v = \sqrt{\frac{GM_E}{R_E + h}}$$

where h is the height of the satellite above the Earth's surface.

MR. AMON CHILESHE

Question ①

Diagram



Data

$M_s \equiv$ Mass of the Sun $= 2\times10^{30}$ kg

$M_e \equiv$ Mass of the earth $= 6\times10^{24}$ kg

$m_m \equiv$ Mass of the moon $= 7\times10^{22}$ kg

$r_{Em} \equiv$ Distance between the earth and the moon $= 4\times10^{8}$ m

$r_{sm} \equiv$ Distance between the sun and the moon $= 10^{11}$ m

- $\vec{F}_{sm}$ is the gravitational force that the Sun exerts on the moon, it's magnitude is;

$$F_{sm} = \frac{G M_s m_m}{r_{sm}^2}$$

$$= \frac{(6.67\times10^{-11})\times(7\times10^{22})\times(2\times10^{30})}{(10^{11})^2}$$

$$= \frac{9.34\times10^{42}}{1\times10^{22}}$$

$$\therefore F_{sm} = 9.34\times10^{20}\,N$$

- $\vec{F}_{em}$ is the gravitational force that the earth exerts on the moon, it's magnitude is;

$$F_{em} = \frac{G M_e m_m}{r_{em}^2}$$

$$= \frac{(6.67\times10^{-11})\times(6\times10^{24})\times(7\times10^{22})}{(4\times10^{8})^2}$$

$$F_{em} = \frac{2.80\times10^{37}}{1.6\times10^{17}}$$

$$\therefore F_{em} = 1.75\times10^{20}\,N$$

Therefore, the net gravitational force acting on the moon in it's vector form is;

$$\vec{F}_{net} = \vec{F}_{em} + \vec{F}_{sm}$$

And it's magnitude is

$$|\vec{F}_{net}| = \sqrt{F_{em}^2 + F_{sm}^2}$$

$$|\vec{F}_{net}| = \sqrt{(1.75\times10^{20})^2 + (9.34\times10^{20})^2}$$

$$\therefore |\vec{F}_{net}| = 9.50\times10^{20}\,N$$

Question ②

An object at a distance h above the earth's surface experiences a gravitational force of magnitude mg, where g is the free-fall acceleration at that height.

$$g = \frac{GM_E}{r^2}$$

If $r = R_E + h$, then

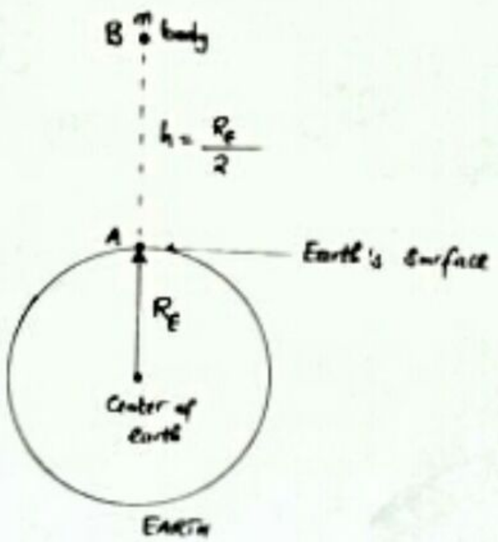
$$g = \frac{GM_E}{(R_E+h)^2}$$

where M_E = mass of the earth

R_E = Radius of the earth

Thus, the weight of an object or body decreases as the object moves away from the earth's surface.

Diagram



- The mass of the body at the surface of the earth or at point A is

$$F_g = mg$$

$$m = \frac{F_g}{g} \quad (\text{at point A, } g = 9.8\,N/Kg)$$

$$m = \frac{63\,N}{9.8\,N/Kg}$$

$$\therefore m = 6.43\,Kg$$

- The mass of the body does not change, this means that at point A and B or elsewhere, the mass is costant.

at point B, $m = 6.43 Kg$

- Therefore, the gravitational force acting on the body at a height equal to half the radius of the earth is:

$$F_g = \frac{G M_e m}{(R_e + h)^2}$$

where $h = \frac{R_e}{2}$

$$h = \frac{6.37 \times 10^6 m}{2}$$

$$\therefore h = 3.19 \times 10^6 m$$

$$F_g = \frac{(6.67 \times 10^{-11}) \times (5.98 \times 10^{24}) \times (6.43)}{(6.37 \times 10^6 + 3.19 \times 10^6)^2}$$

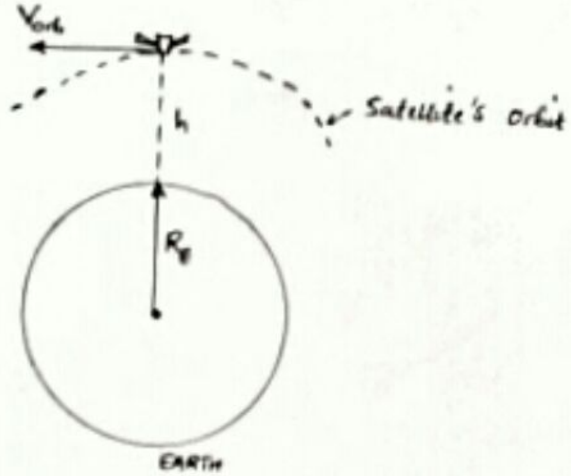
$$F_g = \frac{2.56 \times 10^{15}}{9.14 \times 10^{13}}$$

$$\therefore F_g = 28.01 N$$

Question ③

A Geostationary Orbit is when the Satellite remains Vertically above the Same point on the equator at all times and Consequently has an Orbital period of 24 hours.

Diagram



V_{orb} = Orbital speed of the Satellite

T_{orb} = 24 hours

$$T = 24\,\cancel{hours} \times \frac{60\,\cancel{min}}{1\,\cancel{hour}} \times \frac{60\,s}{1\,\cancel{min}} = 86400\,s$$

3^{rd} law of Kepler's laws of planetary motion, the Orbital period is:

$$T^2 = \frac{4\pi^2 r^3}{GM_e}$$

where

$$r = R_e + h$$

$$h = r - R_e \text{ ——— } ①$$

$$r^3 = \frac{T^2 G M_e}{4\pi^2}$$

$$r^3 = \frac{(86400)^2 \times (6.67\times10^{-11}) \times (5.98\times10^{24})}{4\pi^2}$$

$$r^3 = \frac{2.98 \times 10^{24}}{4\pi^2}$$

$$r = \sqrt[3]{7.54 \times 10^{22}}$$

$$\therefore r = 4.23 \times 10^7 m$$ (total distance from the earth's centre)

Going back to eqn ①

$$h = r - R_e$$

$$h = (4.23 \times 10^7 m) - (6.37 \times 10^6 m)$$

$$\therefore h = 3.59 \times 10^7 m$$

Question (4)

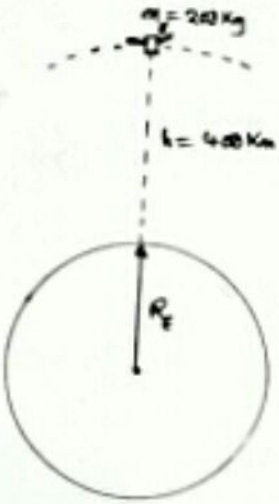
The gravitational potential energy associated with two particles seperated by a distance r is

$$U = -\frac{Gm_1m_2}{r}$$

where U is taken to be zero as $r \to \infty$. The total potential energy for a System of particles is the sum of energies for all pairs of particles, with each pair represented by term of the form given in the above equation.

If an Isolated System Consists of an Object of mass m moving with a Speed V in the Vicinity of a massive Object of mass M, the total energy E of the System is the sum of the Kinetic energy and the potential energy.

Diagram



Total energy of the satellite is ;

$$E_{tot} = KE + PE$$

where

$$KE = \frac{1}{2} m_s v_{orb}^2$$

And

$$PE = -\frac{GM_e m_s}{R_e + h} \quad \text{where } r = R_e + h$$

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- $KE = \frac{1}{2} m_s V_{orb}^2$

where

m_s = mass of the Satellite

V_{orb} = Orbital Speed of the Satellite

- the orbital speed of the Satellite is

$$V_{orb} = \sqrt{\frac{GM_e}{R_e + h}}$$

$$V_{orb} = \sqrt{\frac{(6.67 \times 10^{-11}) \times (5.98 \times 10^{24})}{(6.37 \times 10^6 + 400\,000)}}$$

$$V_{orb} = \sqrt{\frac{3.99 \times 10^{14}}{6.77 \times 10^6}}$$

$$\therefore V_{orb} = 7.68 \times 10^3 \, m/s$$

And the Kinetic energy is

$$KE = \frac{1}{2} m_s V_{orb}^2 = \frac{1}{2} \times (200) \times (7.68 \times 10^3)^2$$

$$\therefore KE = 5.90 \times 10^9 J$$

- $PE = -\frac{GM_e m_s}{R_e + h}$

$$= -\frac{(6.67 \times 10^{-11}) \times (5.98 \times 10^{24}) \times (200)}{(6.37 \times 10^6 + 400\,000)}$$

$$\therefore PE = -1.18 \times 10^{10} J$$

Therefore, the total energy of the Satellite at a height (h) is:

$$E_s = KE + PE$$

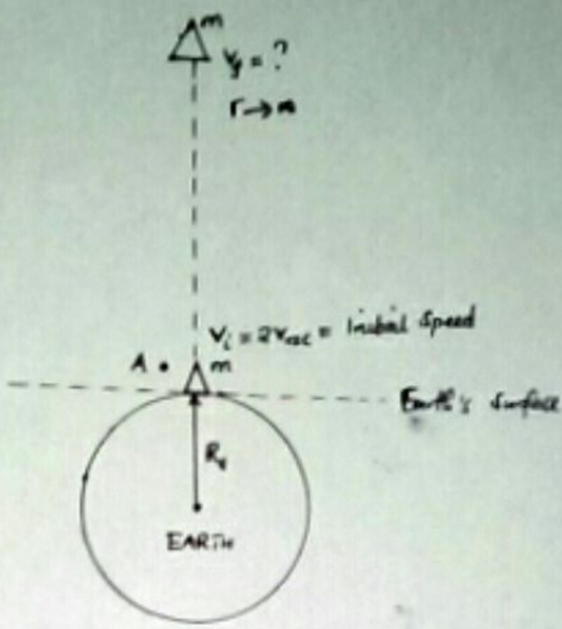
$$= (5.90 \times 10^9 J) - (1.18 \times 10^{10} J)$$

$$\therefore E_s = -5.90 \times 10^9 J$$

The negative Sign indicates that the Satellite is bound to the earth. This energy is called bound energy of the Satellite.

Question ⑤

Diagram



If $r \to \infty$, the gravitational potential energy of the projected body far away from the earth is zero. And the total energy of the projectile far away from the earth is

$$E_f = \frac{1}{2} m v_f^2$$

where m is the mass of the projectile and v_f is the final velocity of the projectile body far away from the earth. Now if we ignore atmospheric friction and the rotation of the earth or if we ignore the presence of the sun and other planets. From the law of conservation of energy;

$$E_i = E_f \quad \text{where } E_i \equiv \text{total initial energy at point A}$$

$$KE_i + U_i = K_f + U_f \qquad E_f \equiv \text{total final energy at point B}$$

where $U_f = 0$ since $r \to \infty$

$$KE_i + U_i = K_f$$

$$\frac{1}{2}\cancel{m}v_i^2 - \frac{GM_e\cancel{m}}{R_e} = \frac{1}{2}\cancel{m}v_f^2$$

$$\frac{V_i^2}{2} - \frac{GM_e}{R_e} = \frac{V_f^2}{2} \quad \text{where } V_i = 2V_{esc} = 2 \times 11200 = 22400\,m/s$$

and V_{esc} is the escape speed

DATA

$G = 6.67 \times 10^{-11}\ N.m^2/Kg^2$

$M_E = 5.98 \times 10^{24}$

$R_E = 6.37 \times 10^6$

$$\frac{(22400)^2}{2} - \frac{(6.67 \times 10^{-11}) \times (5.98 \times 10^{24})}{(6.37 \times 10^6)} = \frac{V_f^2}{2}$$

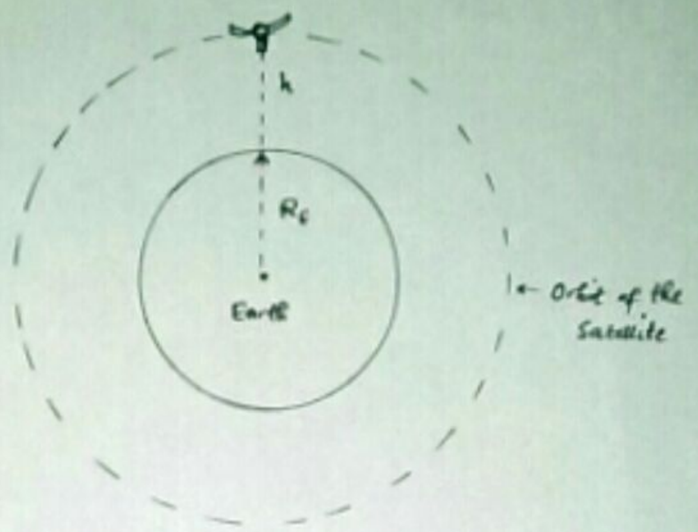
$$(2.51 \times 10^8) - (6.26 \times 10^7) = \frac{V_f^2}{2}$$

$$V_f = 19411.34 m/s$$

$$\therefore V_f = 19.41\ Km/s$$

Question ⑥

Diagram



$r = R_E + h$

We know that the acceleration due to gravity of an artificial satellite is

$$g = \frac{GM_E}{(R_E+h)^2}$$

$$g = \frac{GM_E}{r^2}$$

$$r = \sqrt{\frac{GM_E}{g}}$$

$$r = \sqrt{\frac{(6.67\times10^{-11})\times(5.98\times10^{24})}{9}}$$

$$r = 6.66\times10^{6}\,m$$

According to Kepler's third law of planetary motion which states that the square of the orbital period of any planet is proportional to the cube of the semimajor axis of the elliptical orbit.

$$T^2 = \left(\frac{4\pi^2}{GM_E}\right)a^3$$

In general

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$= \frac{4\pi^2 r^3}{GM_E}$$

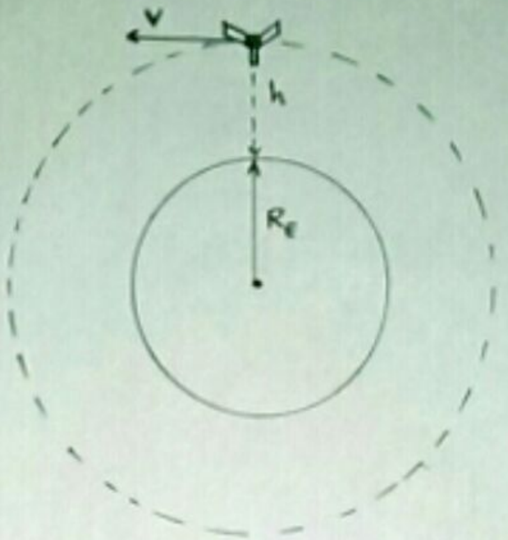
$$= \frac{4\pi^2 \times (6.66\times10^6)^3}{(6.67\times10^{-11})\times(5.98\times10^{24})}$$

$$T^2 = \frac{1.17\times10^{22}}{3.99\times10^{14}}$$

$\therefore$ T = 5415.10 s

Question ⑦

ⓐ **Diagram**



h= altitude (the height of an object in relation to ground level

Let $r = R_E + h$, $v = 5000 m/s$

The acceleration of the Satellite toward the center of the earth earth is

$$a_r = \frac{v^2}{r} \quad \text{——— ①}$$

where r is its total orbital radius. This acceleration must be provided by the acceleration

$$g = \frac{GM_E}{r^2} \quad \text{——— ②}$$

due to the earth's gravitational attraction. Hence equating equation ① and ②

$$\frac{v^2}{\cancel{r}} = \frac{GM_E}{r^{\cancel{2}}}$$

$$r = \frac{GM_E}{v^2}$$

$$r = \frac{(6.67 \times 10^{-11}) \times (5.98 \times 10^{24})}{(5000)^2}$$

$$r = \frac{3.99 \times 10^{14}}{25\,000\,000}$$

$$r = 1.60 \times 10^{7} m$$

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If $r = R_E + h$

$h = r - R_E$

$h = (1.60 \times 10^7 m) - (6.37 \times 10^6 m)$

$\therefore h = 9.63 \times 10^6 m$

(b) The satellite's orbital period is simply

$$T = \frac{2\pi r}{v}$$

$$= \frac{2\pi \times (1.60 \times 10^7)}{5000}$$

$\therefore T = 20106.19s$ or 5.59 hours

Question ⑧

Page 465, 6^{th} edition Serway text book

13.7 ENERGY CONSIDERATION IN PLANETARY AND SATELLITE MOTION

ⓐ

$$E_{tot} = -\frac{GMm}{2r}$$

Initial total energy

$$E_i = -\frac{GM_e M_s}{2r} \qquad \text{where} \quad r = R_E + h_i$$

$$r = 6.37\times10^6 + 100\,000\,m$$

$$r = 6.47\times10^6\,m$$

$$= -\frac{(6.67\times10^{-11})\times(5.98\times10^{24})\times(1000)}{2\times(6.47\times10^6)}$$

$$= -\frac{3.99\times10^{17}}{1.29\times10^7}$$

$$\therefore E_i = -3.09\times10^{10}\,J$$

Final total energy

$$E_f = -\frac{GM_e M_s}{2r} \qquad \text{where} \quad r = R_E + h_f$$

$$r = (6.37\times10^6) + (200\,000)$$

$$r = 6.57\times10^6\,m$$

$$= -\frac{(6.67\times10^{-11})\times(5.98\times10^{24})\times(1000)}{2\times(6.57\times10^6)}$$

$$= -\frac{3.99\times10^{17}}{1.31\times10^7}$$

$$\therefore E_f = -3.05\times10^{10}\,J$$

Therefore, the difference in total energies is the energy that must be added to the system

$$\Delta E = E_f - E_i$$

$$= (-3.05\times10^{10}) - (-3.09\times10^{10})$$

$$\underline{\Delta E = 4\times10^8\,J}$$

ⓑ Change in Kinetic energy

$$\Delta KE = KE_f - KE_i$$

$$= \frac{1}{2}m_s v_f^2 - \frac{1}{2}m_s v_i^2$$

$$= \frac{1}{2}m_s\left(v_f^2 - v_i^2\right) \text{ ——— } ①$$

$$V_f = \sqrt{\frac{GM_E}{R_E + h_f}} \quad \text{at } h_f = 202\,022\text{m}$$

$$= \sqrt{\frac{(6.67\times10^{-11})\times(5.98\times10^{24})}{(6.37\times10^{6})+(202\,022)}}$$

$$= \sqrt{\frac{3.99\times10^{14}}{6.57\times10^{6}}}$$

$$\therefore V_f = 7793\text{ m/s}$$

$$V_i = \sqrt{\frac{GM_E}{R_E + h_i}} \quad \text{at } h_i = 102\,022\text{m}$$

$$= \sqrt{\frac{(6.67\times10^{-11})\times(5.98\times10^{24})}{(6.37\times10^{6})+(102\,022)}}$$

$$= \sqrt{\frac{3.99\times10^{14}}{6.47\times10^{6}}}$$

$$\therefore V_i = 7853\text{ m/s}$$

Therefore, the change in Kinetic energy is

$$\Delta KE = \frac{1}{2}\times(1022)\times\left[(7793)^2 - (7853)^2\right]$$

$$\therefore \Delta KE = -4.69\times10^{8}\text{ J}$$

ⓒ potential energy

Initial GPE
final GPE $\quad U = -\dfrac{GMm}{R_e + h_f}$

$$U_f = -\frac{(6.67\times10^{-11})\times(5.98\times10^{24})\times(1000)}{(6.37\times10^{6}) + (202\,000)} \quad \text{at } h_f = 202\,000\text{m}$$

$$= -\frac{3.99\times10^{17}}{6.57\times10^{6}}$$

$$\therefore U_f = -6.07\times10^{10}\text{J}$$

for
Initial GPE

$$U_i = -\frac{GM_em_s}{R_e + h_i}$$

$$= -\frac{(6.67\times10^{-11})\times(5.98\times10^{-11})\times(1000)}{(6.37\times10^{6}) + (102000)}$$

$$= -\frac{3.99\times10^{17}}{6.47\times10^{6}}$$

$$\therefore U_i = -6.17\times10^{10}\text{J}$$

Therefore, the change in potential energy will be;

$$\Delta U = U_f - U_i$$

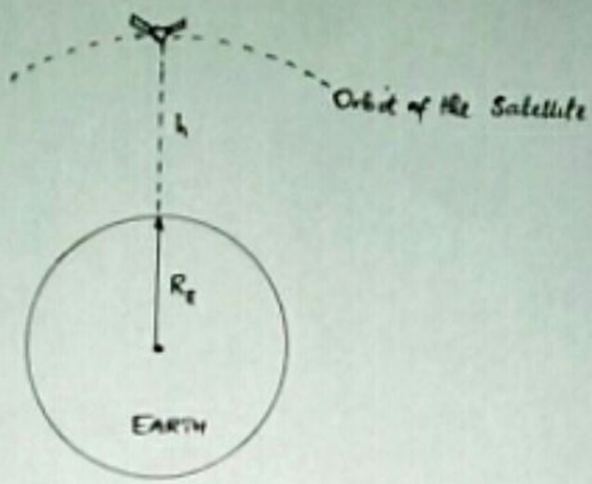
$$= -6.07\times10^{10}\text{J} - (-6.17\times10^{10})$$

$$= (6.17\times10^{10}) - (6.07\times10^{10})$$

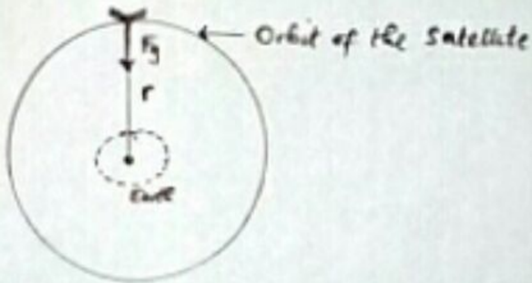
$$\therefore \underline{\Delta U = 1\times10^{9}\text{J}}$$

Question 9

Diagram



Let $r = R_E + h$, the total distance from the center of the earth to where the satellite is orbiting the earth.



$F_g \equiv$ the gravitational force directed toward the center of the earth or the centripetal force

$$F_g = \frac{mv^2}{r} \quad \text{where } g = \frac{v^2}{r} \text{ and } r = R_E + h$$

$$F_g = \frac{mv^2}{r} \quad \text{——— (1)}$$

$F_g \equiv$ is also the gravitational force the earth exerts on the satellite.

$$F_g = \frac{GM_e m}{r^2} \quad \text{where } M_e \equiv \text{mass of the earth}, \; m \equiv \text{mass of the satellite}$$

$$F_g = \frac{GM_e m}{r^2} \quad \text{——— (2)}$$

Equating equation (1) and (2)

$$\frac{\cancel{m}v^2}{\cancel{r}} = \frac{GM_e\cancel{m}}{\cancel{r^2}_r}$$

$$V^2 = \frac{GM_e}{r} \quad \text{where } r = R_e + h$$

$$\therefore V_{orb} = \sqrt{\frac{GM_e}{R_e + h}} \quad \text{Hence Shown}$$

V_{orb} is the orbital speed of the Satellite

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