With Solutions

A1 A3

B2 B4

C1 C3

THE COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

PHYSICS DEPARTMENT

2017/2018 ACADEMIC YEAR

SESSIONAL EXAMINATIONS

PH 110: INTRODUCTORY PHYSICS - I

TIME: THREE (3) HOURS

INSTRUCTIONS: ANSWER ANY FIVE (5) QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS. THE MARKS ARE SHOWN IN BRACKETS. CLEARLY INDICATE THE QUESTIONS YOU HAVE ATTEMPTED ON THE COVER PAGE OF YOUR ANSWER BOOKLET.

MAXIMUM MARKS: **100**

DO NOT FORGET TO WRITE YOUR STUDENT IDENTIFICATION NUMBER (S.I.N) ON THE ANSWER BOOKLET!

USE THE FOLLOWING DATA WHERE NECESSARY:

Acceleration due to gravity, $g = 9.8\ m/s^2$
Universal gravitational constant, $G = 6.67\times10^{-11}\ Nm^2Kg^{-2}$
Mass of electron, $m_e = 9.11\times10^{-31}$ kg; Mass of proton, $m_p = 1.67\times10^{-27}$ kg
$k = 9.0\times10^9\ Nm^2C^{-2}$

$$I_{sphere} = \frac{2}{5}mr^2$$

QUESTION ONE

(a) Dimensional analysis plays a vital role in the quantitative analysis of physical quantities.

(i) What is a physical quantity? [1 mark]

(ii) State two applications of dimensional analysis? [2 marks]

(iii) State any two limitations of dimensional analysis? [2 marks]

(b) A physics student conducting a laboratory experiment on kinematics derives a formula for velocity (v) of an object that varies with time expressed as $v = At^2 + Bt + C$, where velocity (v) and time (t) are expressed in terms of SI units. Determine the units of constants A, B, and C in the given equation. [3 marks]

(c) The centripetal force F acting on a particle moving uniformly in a circle depends on the mass of the particle m, the velocity of the particle v, and radius of the circle r. Use dimensional analysis to derive the formula for centripetal force. [3 marks]

(d) Find the angle θ, where θ is $0° < \theta < 180°$ between two force vectors of equal magnitude F, such that the resultant vector R is one-third of each of the original forces. [3 marks]

(e) The resultant displacement, $\vec{D}$, of three successive displacements is 100 m from the origin at an angle of 37° above the positive x-axis. If $\vec{d}_1$ is 100 m along the negative x-axis and $\vec{d}_2$ is 200 m at an angle of 150°, determine the magnitude and direction of the third vector $\vec{d}_3$. [6 marks]

QUESTION TWO

(a) An object moving along the x-axis in such a way that its displacement x is given by the expression

$$x = 30 + 20t - 15t^2$$

where x is in meters (m) and t is in seconds (s).

(i) Find the expressions for the velocity v and acceleration a. [2 marks]

(ii) Find the value of the initial position and the initial velocity of the object. [1 mark]

(iii) At what time and distance from the origin is the velocity zero? [2 marks]

(b) A motorcycle policeman hidden at an intersection observes a car that ignores a stop sign, crosses the intersection, and continues on at constant speed. The policeman starts off in pursuit 2.0 s after the car has passed the stop sign, accelerates at 6.2 m/s^2 until his speed is 110 km/h, and then continues at this speed until he catches the car. At that instant, the car is 1.4 km from the intersection. How fast was the car travelling? [5 marks]

(c) A projectile is shot from the edge of the building top 125 m above the ground level with an initial speed of 65 m/s at an angle of 37° with the horizontal as shown in Figure 2.1.

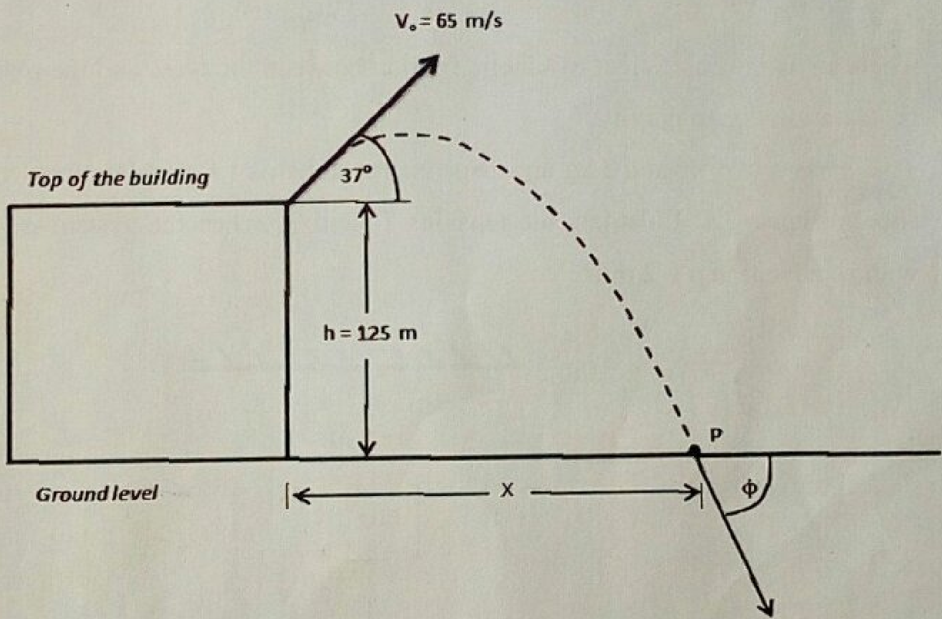

Figure 2.1

Determine

(i) The time taken by the projectile to hit point P at ground level. [2 marks]

(ii) Determine the range x of the projectile measured from the base of the building as shown in Figure 2.1. [2 marks]

(iii) The magnitude of velocity just before the particle reaches the ground. [2 marks]

(iv) The angle ϕ below the horizontal made by the velocity vector. [2 marks]

(v) The maximum height reached by the projectile from the top of the building. [2 marks]

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SAMPLE OF THE SOLUTIONS

Question one

Solutions

(a) (i) A physical quantity is a quantity that can be measured or a physical property that can be quantified.

(ii) two applications of dimensional analysis

- To Convert a physical quantity from one system of units to the Other.
- To check the dimensional correctness of a given equation

(iii) two Limitations of dimensional analysis

- Can not determine Value of dimensionless Costant
- Does not apply to equation involving exponential and trignometric functions.

(b)

$$v = At^2 + Bt + C$$

Applying the principle of homogeneity which states that we can only add/subtract physical quantities having the same dimensions or units.

L.H.S

$[v] = LT^{-1}$

$v = m/s$

R.H.S (Right Hand Side)

To determine the units of Costants A, B and C. Each term of the R.H.S is equated to the L.H.S

$$L.H.S = R.H.S \text{ of each term}$$

$$V = At^2$$

$$A = \frac{v}{t^2}$$

units of v and t are m/s and s

- For A

$$A = \frac{m/s}{s} = \frac{m}{s} \times \frac{1}{s^2}$$

$$\therefore A = m/s^3 = ms^{-3}$$

- For B

$$V = Bt$$

$$B = \frac{v}{t}$$

$$B = \frac{m/s}{s} = \frac{m}{s} \times \frac{1}{s}$$

$$\therefore B = m/s^2 = ms^{-2}$$

- For C

$$V = C$$

$$\therefore C = m/s = ms^{-1}$$

(c)

depends $= \propto$

$$F \propto m^a v^b r^c$$

$$F = K m^a v^b r^c \text{ ———— } (1)$$

where K is a non-dimensional Costant

$$F = m^a v^b r^c$$

L.H.S

F is measured in newtons (F = mass x acceleration)

$1N = 1 Kg.ms^{-2}$

$1N = 1 Kg.m/s^2$

And the dimensions are

$[F] = MLT^{-2}$ ———— L.H.S

R.H.S

$m^a v^b r^c$

$[m]^a = (M)^a = M^a$

$[v]^b = (LT^{-1})^b = L^b T^{-b}$

$[r]^c = (L)^c = L^c$

So that

$[m^a v^b r^c] = M^a L^b T^{-b} L^c$

$[m^a v^b r^c] = M^a T^{-b} L^{b+c}$ ———— R.H.S

Now equating the L.H.S and the R.H.S dimensionally

$$M^1 L^1 T^{-2} = M^a T^{-b} L^{b+c}$$

- For M

  $a = 1$

- For T

  $-b = -2$

  $b = 2$

• For L

$$b+c=1 \quad (b=2)$$

$$2+c=1$$

$$c=1-2$$

$$c=-1$$

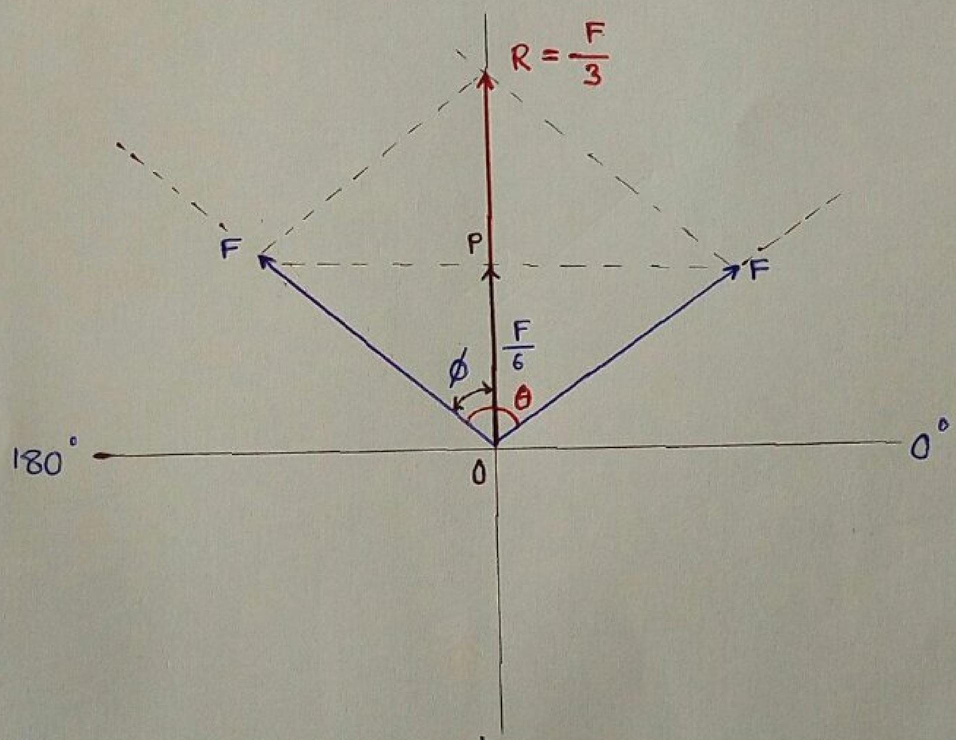
So that equation ① becomes

$$F=Km^a v^b r^c$$

$$F=Km^1 v^2 r^{-1}$$

$$\therefore F=\frac{Kmv^2}{r}$$

ⓓ <u>Diagram</u>



from O to P the magnitude is

$$OP=\frac{F}{3}\div 2$$

$$OP=\frac{F}{3}\times\frac{1}{2}$$

$$OP=\frac{F}{6}$$

Applying SOHCAHTOA to find ϕ

$$\cos\phi = \frac{Adj}{Hyp}$$

$$\cos\phi = \frac{F/6}{F}$$

$$\cos\phi = \frac{F}{6} \div F$$

$$\cos\phi = \frac{\cancel{F}}{6} \times \frac{1}{\cancel{F}}$$

$$\phi = \cos^{-1}\left(\frac{1}{6}\right)$$

$$\phi = 80.41^\circ$$

therefore, the angle θ between $0^\circ < \theta < 180^\circ$ is

$$\theta = \phi + \phi$$

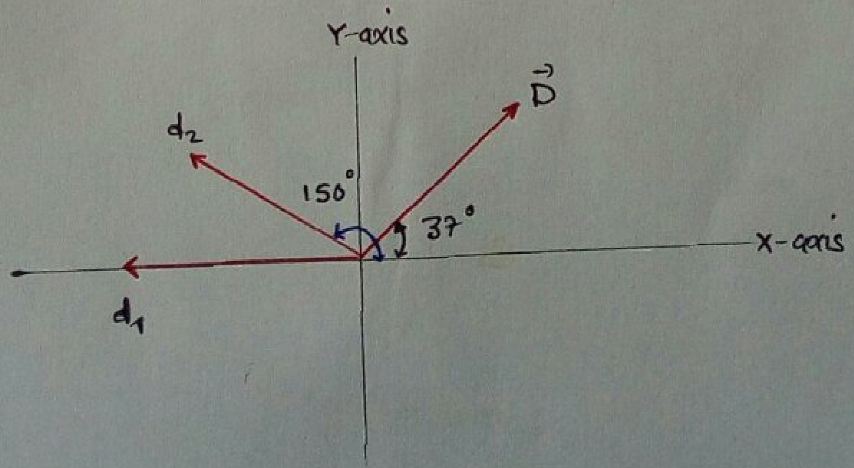
$$\theta = 80.41^\circ + 80.41^\circ$$

$$\therefore \theta = 160.82^\circ$$

(e) $\vec{D} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3$

Data

- $\vec{D}$ = 100m at 37° above the positive x-axis
- $\vec{d}_1$ = 100m along the negative x-axis
- $\vec{d}_2$ = 200m at an angle of 150°
- $\vec{d}_3$ = ?



Vector - Component table

Vector	X - Component	Y - Component
d_1	$100 \cos 180°$	$100 \sin 180°$
d_2	$200 \cos 150°$	$200 \sin 150°$
d_3	?	?
D	$100 \cos 37°$	$100 \sin 37°$

$\cancel{\vec{d_3} = d_{3x}}$

$$\vec{d_3} = d_{3x}\hat{i} + d_{3y}\hat{j} \quad \text{——— (1)}$$

$\cancel{d_{3x} = d_{1x} + d}$

$$D_x = d_{1x} + d_{2x} + d_{3x}$$

$$d_{3x} = D_x - d_{1x} - d_{2x}$$

$$d_{3x} = 100 \cos 37° - 100 \cos 180° - 200 \cos 150°$$

$$d_{3x} = 79.9 + 100 + 173.2$$

$$d_{3x} = 353.1$$

$$D_y = d_{1y} + d_{2y} + d_{3y}$$

$$d_{3y} = D_y - d_{1y} - d_{2y}$$

$$d_{3y} = 100 \sin 37° - 100 \sin 180° - 200 \sin 150°$$

$$d_{3y} = 60.2 - 0 - 100$$

$$d_{3y} = -39.8$$

then (1) becomes

$$\vec{d_3} = 353.1\hat{i} - 39.8\hat{j}$$

And the magnitude is

$$|\vec{d_3}| = \sqrt{(353.1)^2 + (-39.8)^2}$$

$$\therefore |\vec{d_3}| = 355.3\,m$$

the direction is

$$\tan\theta = \frac{d_{3y}}{d_{3x}}$$

$$\theta = \tan^{-1}\left(\frac{-39.8}{353.1}\right)$$

$$\therefore \theta = -6.4°$$

θ is 6.4° below the x-axis and 355.3m from the origin

Question two

Solutions

(a)

$$x = 30 + 20t - 15t^2$$

where x is in meters (m) and t is in seconds (s)

(i) the first derivative of x or the first differentiation of x is the velocity (v) of an object

$$v = \frac{dx}{dt}$$

$$v = \frac{d}{dt}(30 + 20t - 15t^2) = 0 \times 30t^{0-1} + 1 \times 20t^{1-1} - 2 \times 15t^{2-1}$$

$$v = 0 + 20 - 30t$$

$$\therefore \quad v = (20 - 30t)\,m/s$$

the second derivative of x or the second differentiation of x and also the first differentiation of velocity is the acceleration of an object

$$a = \frac{d^2x}{dt^2} \quad \text{or} \quad a = \frac{dv}{dt}$$

$$a = \frac{dv}{dt}$$

$$a = \frac{d}{dt}(20 - 30t) = 0 \times 20t^{0-1} - 1 \times 30t^{1-1}$$

$$\therefore \quad a = -30\,m/s^2$$

(ii) initial means $t=0$

- initial position

$$x(t) = 30 + 20t - 15t^2$$

$$x(0) = 30 + 20(0) - 15(0)^2$$

$$\therefore x(0) = 30\,m$$

- initial velocity

$$v(t) = 20 - 30t$$

$$v(0) = 20 - 30(0)$$

$$\therefore v(0) = 20\,m/s$$

(iii) At what time and distance from the origin is the velocity zero?

from the velocity equation

$$v = 20 - 30t$$

if $v=0$

$$0 = 20 - 30t$$

$$\frac{-20}{-30} = \frac{-\cancel{30}t}{-\cancel{30}}$$

$$\therefore t = 0.67\,s$$

Substituting t into displacement equation to find the position

$$x = 30 + 20t - 15t^2$$

$$x = 30 + 20(0.67) - 15(0.67)^2$$

$$\therefore x = 36.67\,m$$