THE COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS AND NATURAL SCIENCES (NQ)

COURSE CODE: PH 110

TASK: TUTORIAL ASSIGNMENT

GROUP NO: E 11

LECTURER: MR MUMA

DUE DATE: 19/06/2023

$\frac{4.95}{5}$

GROUP MEMBERS:

KANSUMBA MWABA	22 11 0518
KANYANDA AKAPELWA	22 10 8473
KAONGA ENERST	2210 4666
KAPALU JOSEPH	2211 0416
KAPANGA CHITOTELE JONAS	2211 1573
KAPASA JOSHUA KONDWANI	2211 1278
KAPUNGWE ETHEL	22 11 3101
KAPUTA DIXON	2210 0367
KAPUTENI OBED	2210 3597

Q. 6. Data

Mass of a crate = 10kg

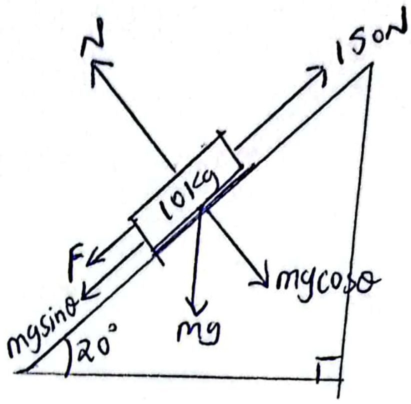
$V_0 = 2m/s$

$F = 150N$

$\theta = 20^\circ$

$\mu_K = 0.500$

$S = 6.00m$



(i). Work done by gravity

$W = F\cos\theta S$

$W = -mg\sin\theta S$

$W = -10\times 9.8\sin 20\times 6$

$W_g = -201.1078443J$

$\therefore W_g = -201.11J$

(ii) Energy lost due to Friction (work done by Friction)

$W = -F\times S$ ------ (i)

$F = \mu_K N$

$F = \mu_K mg\cos\theta$

$W = -\mu_K mg\cos\theta S$

$W = -0.5\times 10\times 9.8\cos 20\times 6$

$W = -276.27J$

$\therefore$ Energy lost due to friction = $-276.27J$

(iii) Work done by 150N

$W = FS$

$W = 150N\times 6.00m$

$W = 900J$

$\therefore$ work done by 150N is 900J

(iv)

Change in Kinetic Energy

$$W_{net} = \Delta KE$$

$$\Delta KE = 900J - 201.11J - 276.27J$$

$$\Delta KE = 422.62J$$

$\therefore$ Change in Kinetic Energy $\Delta KE = 422.62J$

(V) Speed of the crate after 6.00m.

$$\Delta KE = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

$$\Delta KE = \frac{1}{2}(mv^2 - mv_0^2)$$

$$2\Delta KE = mv^2 - mv_0^2$$

$$\frac{\cancel{m}v^2}{\cancel{m}} = \frac{2\Delta KE + mv_0^2}{m}$$

$$\sqrt{v^2} = \sqrt{\frac{2\Delta KE + mv_0^2}{m}}$$

$$v = \sqrt{\frac{2\Delta KE + mv_0^2}{m}}$$

$$v = \sqrt{\frac{(2 \times 422.62) + 10 \times 2^2}{10}}$$

$$v = \sqrt{88.524}$$

$$v = 9.4087193 6 \, m/s$$

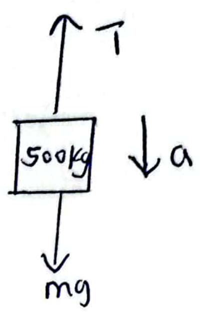
$$v = 9.4 \, m/s$$

$\therefore$ The speed of the crate after being pulled 6.00m is 9.4m/s

Data

- mass of the elevator cab = 500kg
- $V_0 = 4 m/s$
- $a = \frac{g}{5} m/s^2$
- $S = 10m$

FBD



ⓐ Work done by gravity

$W_g = F_g \cos\theta \, S$

$W_g = mg \times S$

$W_g = 500 \times 9.8 \times 10$

$W_g = 49000 J$

$\therefore$ Work done by Gravity $W_g = 49 KJ$

(b) Work done by Tension

$W_T = F_T S$

$W_T = T S$

$\sum F_y = ma$

$mg - T = ma$

$T = mg - ma$ - - - -

- - - - $W_T = (mg - ma) \times S$

$W_T = -m(g - a) \times S$

$W_T = -500\left(9.8 - \frac{9.8}{5}\right) \times 10$

$W_T = -500(7.84) \times 10$

$W_T = -39200 J$

$\therefore W_T = -39.2 KJ$

The work done is negative because the force acting was against ~~acting~~ the direction of the motion.

ⓒ Net work done on the cab

$W_{net} = W_g + W_T$

$W_{net} = 49KJ - 39.2KJ$

$\therefore W_{net} = 9.8 KJ$

ⓓ Cab's KE at the end of 10m fall.

$W_{net} = \Delta KE$

$W_{net} = KE_f - KE_i$

$W_{net} + KE_i = KE_f$

$KE_f = W_{net} + KE_i$

$KE_f = 9.8KJ + \frac{1}{2} m v_0^2$

$KE_f = 9800 + \frac{1}{2}(500)(4^2)$

$KE_f = 9800 + 4000$

$KE_f = 13800 J$

$KE_f = 13.8 KJ$

$\therefore$ the Cab's KE at the end of 10m Fall is $13.8 KJ$

8 **Data**

bullet $= 50g = 0.05\,kg$

$V_0 = 400\,m/s$

$V_f = 200\,m/s$

$S = -2mm = 0.002m$

$F = ?$

$W_{nc} = \Delta KE + \Delta U \qquad (\Delta U = 0)$

$W_{nc} = \Delta KE$

$-Fs = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$-Fs = \frac{1}{2}(mv_f^2 - mv_i^2)$

$\frac{-2Fs}{-2s} = \frac{m(v_f^2 - v_i^2)}{-2s}$

$F = \frac{m(v_f^2 - v_i^2)}{-2s}$

$F = \frac{0.05(200^2 - 400^2)}{-2(0.002)}$

$F = \frac{0.05(-120\,000)}{-0.004}$

$F = \frac{-6000}{-0.004}$

$F = 1500\,000\,N$

$F = 1.5 \times 10^6\,N$

$\therefore$ the ~~work~~ force applied on the bullet by the window pane is $\underline{1.5 \times 10^6\,N}$

Data

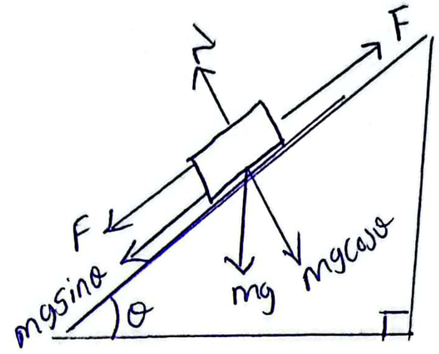
mass = 200tons = 200 000Kg

instant V = 72Km/h = 20m/s

P = 800KW = 800 000W

a = 0m/s^2

$\sin\theta = \frac{1}{50}$



Value of F

$\frac{P}{V} = \frac{FV}{V}$

$F = \frac{P}{V}$

$F = \frac{800\,000W}{20m/s}$

$F = 40\,000N$

$\sum F = ma$

$F - (f + mg\sin\theta) = ma$

$F - f - mg\sin\theta = ma$ - - - $a = 0m/s$

$F - f - mg\sin\theta = 0$

$f = F - mg\sin\theta$

$f = 40\,000N - 200\,000 \times 9.8 \left(\frac{1}{50}\right)$

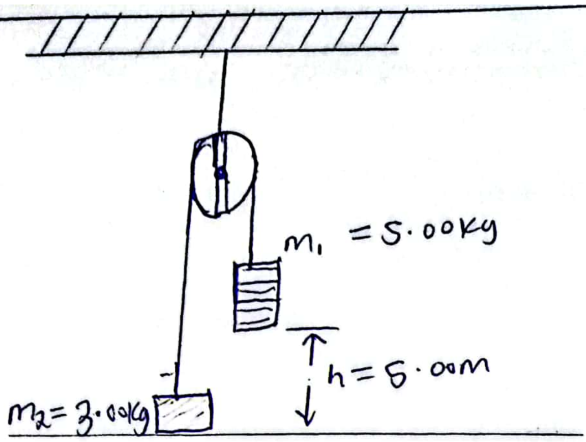
$f = 40\,000N - 39\,200N$

$f = 800N$

$\therefore$ the resistance to the motion of the train is

$f = 800N$

16.



The speed of the two bodies

$$KE_i + U_i = KE_F + U_F \qquad (KE_i = 0 \text{ since } v = 0)$$

$$U_i = KE_F + U_F$$

$$\therefore U_i = \frac{1}{2}(m_1+m_2)v^2 + m_2gh$$

$$\therefore m_1gh = \frac{1}{2}(m_1+m_2)v^2 + m_2gh$$

$$\therefore \frac{1}{2}(m_1+m_2)v^2 = m_1gh - m_2gh$$

$$\therefore \frac{\cancel{(m_1+m_2)}v^2}{\cancel{m_1+m_2}} = \frac{2(m_1gh - m_2gh)}{m_1+m_2}$$

$$\therefore \sqrt{v^2} = \sqrt{\frac{2(m_1gh - m_2gh)}{m_1+m_2}}$$

$$\therefore v = \sqrt{\frac{2(5\times9.8\times5 - 3\times9.8\times5)}{5+3}}$$

$$v = \sqrt{\frac{2(245-147)}{8}}$$

$$v = \sqrt{24.5}$$

$$\therefore \underline{v = 4.95m/s}$$

TUTORIAL SHEET 6 LINEAR MOMENTUM AND IMPULSE

Q. 3 Data:

(M) Cannon = 1.5 tons = 1500 Kg — initial velocities are 0

(m) Projectile = 100 Kg

Final velocity of the Projectile = 30 m/s

Recoil velocity = ?

$$mv_i + MV = mv_i' + MV_2'$$

$$0 = mv_i' + MV'$$

$$\frac{MV'}{M} = \frac{-mv_i'}{M}$$

$$V' = \frac{-mv_i'}{M}$$

$$V' = \frac{-(100 \times 30)\,\text{Kg m/s}}{1500\,\text{Kg}}$$

$$V' = \frac{-3000\,\text{m/s}}{1500}$$

$$V' = -2\,\text{m/s}$$

∴ the recoil velocity of the Cannon $V = -2\,\text{m/s}$

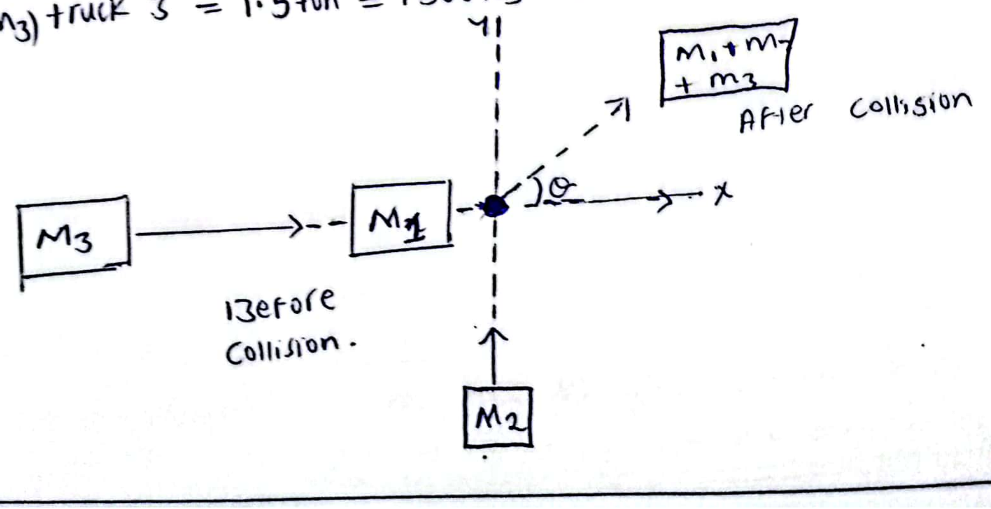
The negative sign denotes that the cannon was recoiling.

Q. 5. Data

(M_1) truck 1 = 2 ton = 2000 Kg → 2 m/s east

(M_2) truck 2 = 1 ton = 1000 Kg → 5 m/s North

(M_3) truck 3 = 1.5 ton = 1500 Kg → 10 m/s east



moment...

$$m_1v_1 + m_2v_2\cos 90 + m_3v_3 = (m_1+m_2+m_3)v\cos\theta$$

$$\frac{m_1v_1 + m_3v_3}{m_1+m_2+m_3} = \frac{\cancel{(m_1+m_2+m_3)}v\cos\theta}{\cancel{m_1+m_2+m_3}}$$

$$v\cos\theta = \frac{(2000\times2)+(1500\times10)}{2000+1000+1500}$$

$$v\cos\theta = \frac{19000}{4500}$$

$$v\cos\theta = 4.22\bar{2} \quad \text{---}(i)$$

momentum in y

$$m_1v_1 + m_2v_2\sin 90 + m_3v_3 = (m_1+m_2+m_3)v\sin\theta$$

$$\frac{m_2v_2\sin 90}{m_1+m_2+m_3} = \frac{\cancel{(m_1+m_2+m_3)}v\sin\theta}{\cancel{m_1+m_2+m_3}}$$

$$v\sin\theta = \frac{1000\times5\sin 90}{2000+1000+1500}$$

$$v\sin\theta = \frac{5000}{4500}$$

$$v\sin\theta = 1.11\bar{1} \quad \text{---}(ii)$$

$$\frac{\cancel{v}\sin\theta = 1.111}{\cancel{v}\cos\theta = 4.222}$$

$$\tan\theta = \frac{1.111}{4.222}$$

$$\theta = \tan^{-1}\left(\frac{1.111}{4.222}\right)$$

$$\theta = 14.74^\circ$$

$$\frac{v\cancel{\cos\theta}}{\cancel{\cos\theta}} = \frac{4.222}{\cos\theta}$$

$$v = \frac{4.222}{\cos 14.74}$$

$$v = 4.37\ m/s$$

$\therefore$ the velocity of the vehicles when they become entangle is $v = 4.37\ m/s$ in the direction $\theta = 14.74^\circ$

Q. 6 **Data**

Gas molecule 1

$m_1 = m$

$v_1 = 300\,m/s$

$v_1' = ?$

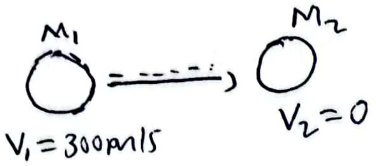
Gas molecule 2

$m_2 = m$

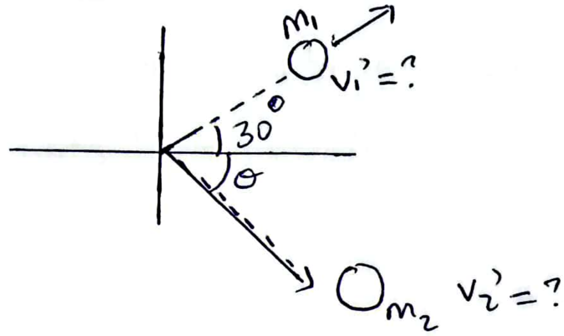
$v_2 = 0$

$v_2' = ?$

Before Collision



After Collision



momentum in x

$$m_1v_1 + \underset{0}{m_2v_2} = m_1v_1'\cos 30 + m_2v_2'\cos\theta$$

$$m_1v_1 = m_1v_1'\cos 30 + m_2v_2'\cos\theta$$

$$\cancel{m}v_1 = \cancel{m}v_1'\cos 30 + \cancel{m}v_2'\cos\theta$$

$$v_1 = v_1'\cos 30 + v_2'\cos\theta$$

$$v_1 - v_1'\cos 30 = v_2'\cos\theta \text{ ---------- (i)}$$

momentum in y

$$m_1v_1 + m_2v_2 = m_1v_1'\sin 30 + m_2v_2'\sin\theta$$

$$0 + 0 = m_1v_1'\sin 30 + m_2v_2'\sin\theta$$

$$-\cancel{m}v_1'\sin 30 = \cancel{m}v_2'\sin\theta$$

$$-v_1'\sin 30 = v_2'\sin\theta \text{ ------ (ii)}$$

Conservation of KE

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$$

$$\frac{1}{\cancel{2}}\cancel{m}v_1^2 + \frac{1}{2}m(0) = \frac{1}{\cancel{2}}\cancel{m}v_1'^2 + \frac{1}{\cancel{2}}\cancel{m}v_2'^2$$

$$v_1^2 = v_1'^2 + v_2'^2 \text{ ---------- (iii)}$$

Square and add equations (i) and (ii)

$(v_1 - v_1' \cos 30)^2 = (v_2' \cos\theta)^2$ Squaring (i)

$v_1^2 - 2v_1v_1'\cos 30 + v_1'^2\cos^2 30 = v_2'^2\cos^2\theta$ - - - - - - -

$(-v_1'\sin 30)^2 = (v_2'\sin\theta)^2$ squaring (ii)

$v_1'^2\sin^2 30 = v_2'^2\sin^2\theta$ - - - - - - -

<u>Add (i) and (ii)</u>

$v_1^2 - 2v_1v_1'\cos 30 + v_1'^2\cos^2 30 + v_1'^2\sin^2 30 = v_2'^2\cos^2\theta + v_2'^2\sin^2\theta$

$v_1^2 - 2v_1v_1'\cos 30 + v_1'^2(\cos^2 30 + \sin^2 30) = v_2'^2(\cos^2\theta + \sin^2\theta)$

$v_1^2 - 2v_1v_1'\cos 30 + v_1'^2 = v_2'^2$ - - - - - - (iv)

<u>Solving (iii) and (iv) simultaneously</u>

$= v_1^2 = v_1'^2 + v_2'^2$ - - - - (iii)

$= v_1^2 - 2v_1v_1'\cos 30 + v_1'^2 = v_2'^2$ - - - (iv)

$= v_1'^2 + \cancel{v_2'^2} - 2v_1v_1'\cos 30 + v_1'^2 = \cancel{v_2'^2}$

$= v_1'^2 - 2v_1v_1'\cos 30 + v_1'^2 = 0$

$= 2v_1'^2 - 2v_1v_1'\cos 30 = 0$

$= 2v_1'(v_1' - v_1\cos 30) = 0$

$= 2v_1' = 0$ or $v_1' - v_1\cos 30 = 0$

$= v_1' - v_1\cos 30 = 0$

$v_1' = v_1\cos 30$ $(v_1 = 300\,m/s)$

$v_1' = 300\cos 30$

$v_1' = \cancel{8}\,259.8076211\,m/s$

$\underline{v_1' = 260\,m/s}$

$$v_1'^2 + v_2'^2 = v_1^2$$

$$\sqrt{v_2'^2} = \sqrt{v_1^2 - v_1'^2}$$

$$v_2' = \sqrt{v_1^2 - v_1'^2}$$

$$v_2' = \sqrt{300^2 - 260^2}$$

$$v_2' = \sqrt{22400}$$

$$v_2' = 149.6662955 \text{ m/s}$$

$$\underline{v_2' = 150 \text{ m/s}}$$

<u>Value of θ</u>

$$v_1 = v_1' \cos 30 + v_2' \cos\theta$$

$$\frac{\cancel{v_2'} \cos\theta}{\cancel{v_2'}} = \frac{v_1 - v_1' \cos 30}{v_2'}$$

$$\cos\theta = \frac{300 - 260\cos 30}{150}$$

$$\theta = \cos^{-1}\left(\frac{300 - 260\cos 30}{150}\right)$$

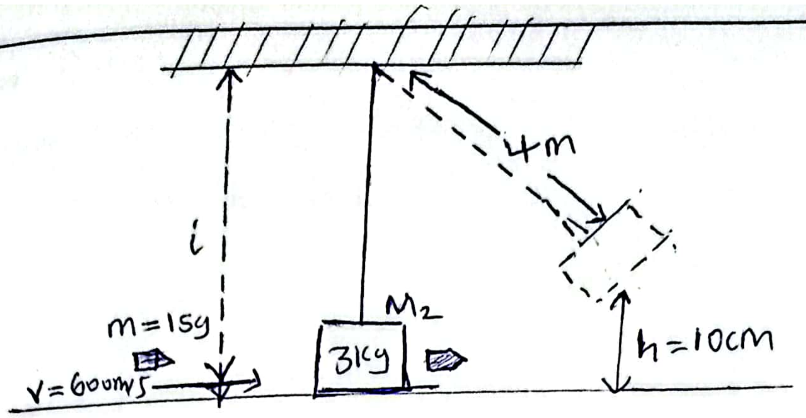
$$\theta = \cos^{-1}(0.4988893)$$

$$\theta = 60.07345615^\circ$$

$$\underline{\theta = 60^\circ}$$

$\therefore$ the first molecule has final velocity $\underline{v_1' = 260 \text{ m/s}}$, ~~and~~ the second molecule has final velocity $v_2' = 150 \text{ m/s}$ in the direction $\theta = 60^\circ$ under the positive x-axis.

9.



Data

bullet $= 15g = 0.015kg$
$v_1 = 600 m/s$

block $= 3kg$
~~$v_1 = 0 m/s$~~

Length of the string $= 4m$

hieght $h = 10cm$

\# Final velocity of the block (by Conservation of mechanical energy)

$$\frac{1}{2}m_2 v^2 + \underset{0}{mgh} = \frac{1}{2}\underset{0}{m_2 v^2} + mgh$$

$$\frac{1}{2}m_2 v^2 = mgh$$

$$\cancel{m_2} v^2 = 2\cancel{m}gh$$

$$\sqrt{v^2} = \sqrt{2gh}$$

$$V = \sqrt{2gh}$$

$$V = \sqrt{2 \times 9.8 \times 0.1}$$

$$V = \sqrt{1.96}$$

$$\therefore V' = 1.4 m/s$$

\# Final velocity of the bullet

$$m_1 v_1 + \underset{0}{m_2 v_2} = m_1 v_1' + m_2 v_2'$$

$$= m_1 v_1 = m_1 v_1' + m_2 v_2'$$

$$= \frac{\cancel{m_1} v_1'}{\cancel{m_1}} = \frac{m_1 v_1 - m_2 v_2'}{m_1} \qquad (v_2' = 1.4 m/s)$$

$$= v_1' = \frac{0.015 \times 600 - 3 \times 1.4}{0.015}$$

$$v_1' = \frac{4.8}{0.015}$$

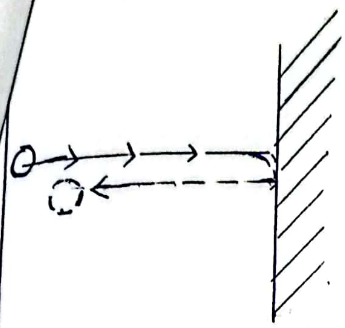
$$v_1' = 320 m/s$$

$\therefore$ The mass of the bullet after emerging from the block is $V = 320 m/s$

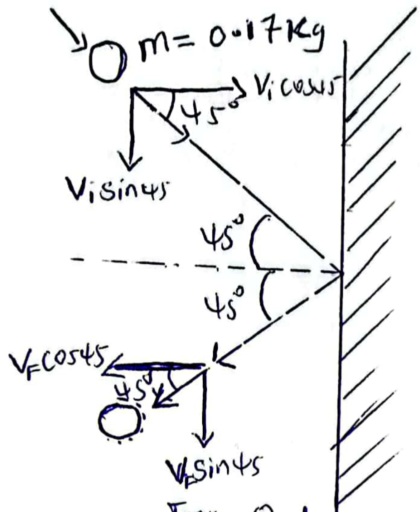
11 Data

billiard ball $= 170g = 0.17 Kg$ Impulse = ?

speed $= 4\sqrt{2}$ m/s.



For Q a



For Q b

(a) Impulse when the ball strikes the wall normally.

$J = \Delta P$

$J = mv_F - mv_i$

$J = m(v_F - v_i)$

$J = 0.17(-4\sqrt{2} - 4\sqrt{2})$

$J = 0.17\,(-8\sqrt{2})$

$J = -1.923330445$ Kgm/s

$\therefore$ $J = -1.92$ Kgm/s

(b) ~~Change~~ Impulse in x

$J_x = mv_F - mv_i$

$J_x = -mv\cos 45 - mv\cos 45$

$J_x = -2mv\cos 45$

$J_x = -2 \times 0.17 \times 4\sqrt{2}\cos 45$

$J_x = -1.36$ Kgm/s

Impulse in y

$J_y = mv_F - mv_i$

$J_y = mV_F \sin 45 - (-v_i \sin 45)$

$J_y = mV_F \sin 45 + mv_i \sin 45$

$J_y = (-4\sqrt{2} \times 0.17 \sin 45) + (0.17 \times 4\sqrt{2} \sin 45)$

$J_y = 0$ Kgm/s

$\vec{J} = \sqrt{J_x^2 + J_y^2}$

$J = \sqrt{(-1.36)^2}$ $\therefore$ impulse $J = 1.36$ Kgm/s

TUTORIAL

CIRCULAR MOTION AND GRAVITATIONAL

CIRCULAR MOTION

Q.2.

Data

$w_0 = 20\,rev/min$

$w = 30\,rev/min$

$t = 20\,sec$

Conversions

$w_0 \Rightarrow \frac{20\,rev}{min} \times \frac{2\pi\,rad}{1\,rev} \times \frac{1\,min}{60\,sec} = \frac{2}{3}\pi\,rad/s$

$w_f \Rightarrow \frac{30\,rev}{min} \times \frac{2\pi\,rad}{1\,rev} \times \frac{1\,min}{60\,sec} = \pi\,rad/s$

(a) Angular acceleration in (rad/s^2)

$W = W_0 + \alpha t$

$\alpha = \frac{W - W_0}{t}$

$\alpha = \frac{\left(\pi - \frac{2\pi}{3}\right)rad/s}{20s}$

$\alpha = \frac{\left(\frac{3\pi - 2\pi}{3}\right)rad/s}{20s}$

$\alpha = \left(\frac{\pi}{3} \times \frac{1}{20}\right) rad/s^2$

$\alpha = 0.052359877\,rad/s^2$

$\therefore$ Angular ~~a~~cceleration $\alpha = 0.052\,rad/s^2$

(b) numbers of degrees

$\theta = w_0 t + \frac{1}{2}\alpha t^2$

$\theta = \left(\frac{2\pi}{3} \times 20\right) + \frac{1}{2}(0.052)(20)^2$

$\theta = 52.36790205\,rad$

$\theta = 52.\cancel{3}4\,rad$

π — 180°

52.4 — x

$\frac{\pi x}{\pi} = \frac{9432}{\pi}$

$x = 3002.298845^\circ$

$\therefore$ the number of degrees the wheel turned in 20 seconds is

$\theta = 3002.3^\circ$

4 Data

$r = 20m$ $\quad a_t = 0.6 m/s^2$

$v = 4m/s$

(a) Tangential acceleration

Tangential acceleration is given by $a_t = \frac{\Delta v}{\Delta t}$, However in this question we are told that the linear speed increases at a rate of $0.6 m/s^2$

$\therefore \underline{\underline{a_t = 0.6 m/s^2}}$

(b) Centripetal acceleration.

$a_c = \frac{v^2}{r}$

$a_c = \frac{(4m/s)^2}{20m}$

$\therefore \underline{\underline{a_c = 0.8 m/s^2}}$

(c) magnitude and direction of the total acceleration.

$|a| = \sqrt{a_t^2 + a_c^2}$

$= \sqrt{(0.6)^2 + 0.8^2}$

$= \sqrt{1}$

$\underline{\underline{|a| = 1 m/s^2}}$

$\theta = \tan^{-1}\left(\frac{a_c}{a_t}\right)$

$\theta = \tan^{-1}\left(\frac{0.8}{0.6}\right)$

$\theta = 53.13010235°$

$\underline{\underline{\theta = 53.1°}}$

$\therefore$ magnitude of the total acceleration is $1 m/s^2$ with the angle $\theta = 53.1°$ measured from the direction tangent to the circle.

When the Coefficient of Static friction on the icy pavement is 0.25

We find the maximum speed for $\mu_s = 0.25$

$F = F_c$ $\qquad \sum F_y = 0$

$F = \frac{mv^2}{r}$ $\qquad N = mg$

$\mu_s N = \frac{mv^2}{r}$

$\mu_s \cancel{m} g = \frac{\cancel{m} v^2}{r}$

$\sqrt{v^2} = \sqrt{\mu_s g r}$

$v = \sqrt{\mu_s g r}$

$v = \sqrt{0.25 \times 9.8 \times 50}$

$v = \sqrt{122.5}$

$v = 11.06797181 \, m/s$

$V_{max} = 11.1 \, m/s$

∴ In this Case the Car will Skid because it is moving at the speed ($v = 14 m/s$) which greater than the maximum Speed ($V_{max} = 11.1 m/s$) which a Car can move with on the icy Pavement without skidding.

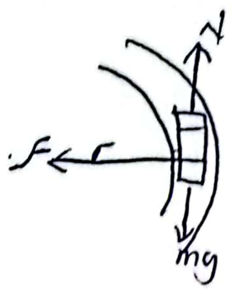
6 **Data**

$m = 1000\,kg$ $\mu_s = 0.6$

$r = 50m$ $\mu_s = 0.25$

$V = 14m/s$

FBD



(a). When A Static Coefficient Friction is $\mu_s = 0.6$

\# We Find the maximum speed that a car can have on the dry Pavement ($\mu_s = 0.6$)

$f = F_c$

$f = \frac{mv^2}{r}$

$\mu_s N = \frac{mv^2}{r}$

$\mu_s \cancel{m} g = \frac{\cancel{m} v^2}{r}$

$\mu_s g = \frac{v^2}{r}$

$\sqrt{v^2} = \sqrt{\mu_s g r}$

$V = \sqrt{\mu_s g r}$

$V = \sqrt{0.6 \times 9.8 \times 50}$

$V = 17.1464282$

$V_{max} = 17.15\,m/s$

$\sum F_y = 0$

$N = mg$ - - - -

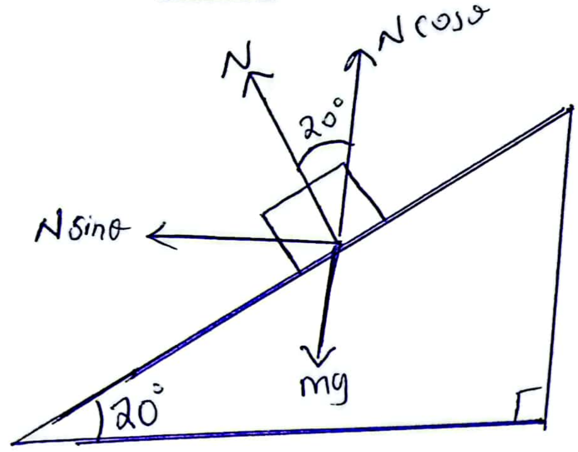
∴ The car will follow the curve because the speed at which it is moving (i.e 14m/s) is less than the maximum speed ($V = 17.15\,m/s$) for this given coefficient of friction.

.7 **Data**

$r = 500m$

$\theta = 20^\circ$ (angle of banking)

FBD



$N\sin\theta = F_c$

$N\sin\theta = \frac{mv^2}{r}$ - - - (i)

$\sum F_y = 0$

$\frac{N\cancel{\cos\theta}}{\cancel{\cos\theta}} = \frac{mg}{\cos\theta}$

$N = \frac{mg}{\cos\theta}$ - - - (ii)

replace (ii) into (i)

$\left(\frac{\cancel{m}g}{\cos\theta}\right)\sin\theta = \frac{\cancel{m}v^2}{r}$

$g\tan\theta = \frac{v^2}{r}$

$\sqrt{v^2} = \sqrt{g\tan\theta\, r}$

$v = \sqrt{9.8 \times \tan 20 \times 500}$

$v = 42.23096196\ m/s$

$v = 42.23\ m/s$

$\therefore$ the maximum permissible speed to avoid slipping is $V_{max} = 42.23\ m/s$

9

Data

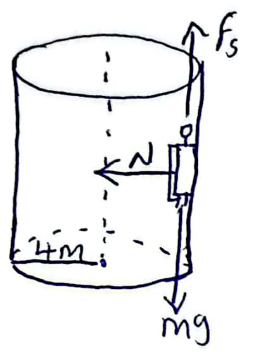
$r = 4.0m$

$w = 0.8\ rev/s$

Conversion

$$\frac{0.8 rev}{s} \times \frac{2\pi\ rad}{1 rev} = 1.6\pi\ rad/s.$$

FBD



$\sum F_y = 0$

$f_s = mg$

$\mu_s N = mg$

$\mu_s = \frac{mg}{N}$ --- (i)

$N = F_c$

$N = \frac{mv^2}{r}$ --- (ii)

replace equation (ii) into (i)

$$\mu_s = \frac{\cancel{m}g}{\frac{\cancel{m}v^2}{r}}$$

$$\mu_s = g \times \frac{r}{v^2}$$

$$\mu_s = \frac{gr}{v^2} \quad \text{where} \quad V = wr$$

$$\mu_s = \frac{gr}{(wr)^2}$$

$$\mu_s = \frac{g}{w^2 r}$$

$$\mu_s = \frac{9.8}{(1.6\pi)^2 \times 4}$$

$$\mu_s = \frac{9.8}{101.0647}$$

$$\mu_s = 0.0969675 39$$

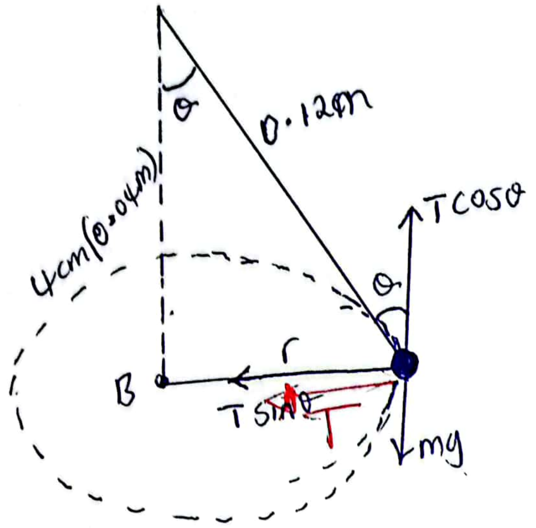
$$\mu_s = 0.1$$

$\therefore$ the required coefficient of static friction to keep the man from slipping down is $\mu_s = 0.1$

b) Data

$m = 4kg$

length of the String $= 12cm\ (0.12m)$



radius

$$|r| = \sqrt{(0.12)^2 - (0.04)^2}$$

$$|r| = \sqrt{0.0128}$$

$$r = 0.113137085$$

$$r = 0.113m$$

Angle θ

$$\cos\theta = \frac{Adj}{Hyp}$$

$$\cos\theta = \frac{0.04}{0.12}$$

$$\theta = \cos^{-1}\left(\frac{0.04}{0.12}\right)$$

$$\theta = 70.53^\circ$$

i) Angular speed

$$T\sin\theta = F_c = \frac{mv^2}{r}$$

$$T\sin\theta = \frac{mv^2}{r} \quad \text{.... (i)}$$

$$\sum F_y = 0$$

$$\frac{T\cancel{\cos\theta}}{\cancel{\cos\theta}} = \frac{mg}{\cos\theta}$$

$$T = \frac{mg}{\cos\theta} \quad \text{------ (ii)}$$

$$\left(\frac{\cancel{m}g}{\cos\theta}\right)\sin\theta = \frac{\cancel{m}v^2}{r}$$

$$g\tan\theta = \frac{v^2}{r}$$

$$v^2 = rg\tan\theta \quad \text{where } (v = wr)$$

$$(rw)^2 = rg\tan\theta$$

$$\frac{\cancel{r^2}w^2}{\cancel{r^2}} = \frac{rg\tan\theta}{r^2}$$

$$w^2 = \frac{g\tan\theta}{r}$$

$$\omega^2 = \sqrt{\frac{g\tan\theta}{r}}$$

$$w = \sqrt{\frac{9.8\tan 70.53^\circ}{0.1131}}$$

$$w = 15.65557247$$

$$w = 15.66 \text{ rad/s}$$

$\therefore$ Angular speed $w = 15.66 \text{ rad/s}$

(ii) rotational Frequency

$$f = \frac{v}{2\pi r} \quad \text{where } v = wr$$

$$f = \frac{wr}{2\pi r}$$

$$F = \frac{w}{2\pi}$$

$$F = \frac{15.66 \text{ rad/s}}{2\pi}$$

$$F = 2.49166 1745 \, s^{-1}$$

$$F = 2.5 \text{ Hz}$$

$\therefore$ the Rotational Frequency $F = 2.5 \text{ Hz}$

(iii) The Tension in the String.

$$T = \frac{mg}{\cos\theta} \quad \text{from equation (ii)}$$

$$T = \frac{4 \times 9.8}{\cos 70.53}$$

$$T = 117.6070867 N$$

$$T = 117.6 N$$

$\therefore$ The Tension in the String is $T = 117.6 N$

B. GRAVITATION

Q. 13. Data

Period $T = 90$ minutes $= 5400s$

Hieght of the satellite above the earth $= ?$

$$T = \frac{2\pi r}{v}$$

$$v = \frac{2\pi r}{T} \text{ ------ (i)}$$

$$F_G = \frac{GmM}{r^2} = F_c = \frac{mv^2}{r}$$

$$\frac{G\cancel{m}M}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}}$$

$$v^2 = \frac{GM}{r} \text{ --- (ii)}$$

replace (i) into (ii)

$$\left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r}$$

$$\frac{4\pi^2 r^2}{T^2} \times \frac{GM}{r}$$

$$\frac{\cancel{4\pi^2} r^3}{\cancel{4\pi^2}} = \frac{T^2 GM}{4\pi^2}$$

$$\sqrt[3]{r^3} = \sqrt[3]{\frac{T^2 GM}{4\pi^2}}$$

$$r = \sqrt[3]{\frac{(5400)^2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{4\pi^2}}$$

$$r = \underline{6661442.585\,m}$$

$r = R + h$

$h = r - R$

$h = 6661442.585 - 6400000$

$h = 261442.5853\,m = \underline{261.44\,Km}$

$\therefore$ the hieght of the satellite with period 90 minutes above the earth is 261442.6m or 261.44Km

Q.16 **Data**

Period $T = 24h = 86400s$

geosynchronous orbit

mean distance $r = ?$

(a) Distance from centre of the Earth

\# We use the formula $T = 2\pi\sqrt{\frac{r^3}{GM}}$

$$(T)^2 = \left(2\pi\sqrt{\frac{r^3}{GM}}\right)^2$$

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$\sqrt[3]{r^3} = \sqrt[3]{\frac{T^2 GM}{4\pi^2}}$$

$$r = \sqrt{\frac{(86400)^2 \times 6.67\times10^{-11} \times 6\times10^{24}}{4\pi^2}}$$

$$r = 42297523.87\,m$$

$$r = 4.23\times10^{7}\,m$$

$\therefore$ the distance from the centre of the Earth were the geosynchronous orbit can be found is $r = 4.23\times10^7\,m$.

(b) orbital speed of the satellite.

\# It is given by the formula $V = \sqrt{\frac{GM}{r}}$

$$V = \sqrt{\frac{GM}{r}}$$

$$V = \sqrt{\frac{6.67\times10^{-11} \times 6\times10^{24}}{42297523.87}}$$

$$V = 3075.962737\,m/s$$

$$V = 3075.96\,m/s$$

$\therefore$ The orbital speed $V = 3075.96\,m/s$

1b.

Mass of Jupiter $= 1.9 \times 10^{27}$ Kg

mass of Sun $= 1.99 \times 10^{30}$ Kg.

Mean distance $= 7.8 \times 10^{11}$ m

$$F_G = \frac{G\,mM}{r^2}$$

$$= \frac{6.67 \times 10^{-11} \times 1.9 \times 10^{27} \times 1.99 \times 10^{30}}{(7.8 \times 10^{11})^2}$$

$$= 4.1451791 58 \times 10^{23}\ N$$

$$F_G = 4.15 \times 10^{23}\ N$$

$\therefore$ the gravitational Force of the Sun exerted on Jupiter is $\underline{\underline{F_G = 4.15 \times 10^{23}\ N}}$

Speed of Jupiter.

$$F_G = F_c = \frac{mv^2}{r}$$

$$F_G = \frac{mv^2}{r}$$ ... where m is the mass of Jupiter.

$$\frac{\cancel{m}v^2}{\cancel{m}} = \frac{rF_G}{m}$$

$$\sqrt{v^2} = \sqrt{\frac{rF_G}{M}}$$

$$V = \sqrt{\frac{7.8 \times 10^{11} \times 4.15 \times 10^{23}}{1.9 \times 10^{27}}}$$

$$V = \sqrt{170368421.1}$$

$$V = 13052.52547\ m/s$$

$$V = 13052.5\ m/s$$

$\therefore$ The speed of Jupiter around the Sun is $\underline{\underline{V = 13052.5\ m/s \text{ or } 13.05\ Km/s}}$

Q. 19

Data

Escape velocity = 11.2 Km/s (V_{esc})

$$V_0 = 2V_{esc} = 22.4 Km/s \ (22400 m/s)$$

$$V_F = ?$$

We apply the laws of conservation of mechanical energy

$$E_i = E_F$$

$$\frac{1}{2}mv_0^2 + mgh_0 = \frac{1}{2}mv_F^2 + mgh_F \quad \left(\text{where } mgh = -\frac{GmM_e}{R_e}\right)$$

$$\frac{1}{2}mv_0^2 - \frac{GmM_e}{R_e}{}_0 = \frac{1}{2}mv_F^2 - \frac{GmM_e}{R_e}{}_F \quad (\text{at infinite hieght } PE = 0)$$

$$\frac{1}{2}\cancel{m}v_0^2 - \frac{G\cancel{m}M_e}{R_e} = \frac{1}{2}\cancel{m}v_F^2$$

$$\left(\frac{1}{2}v_F^2 = \frac{1}{2}v_0^2 - \frac{GM_e}{R_e}\right) \times 2$$

$$\sqrt{V_F^2} = \sqrt{V_0^2 - \frac{2GM_e}{R_e}}$$

$$V_F = \sqrt{V_0^2 - \frac{2GM_e}{R_e}}$$

$$V_F = \sqrt{(22400)^2 - \frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{6400000}}$$

$$V_F = \sqrt{376697500}$$

$$V_F = 19408.6965 \ m/s$$

$$\underline{V_F = 19.41 \ Km/s}$$

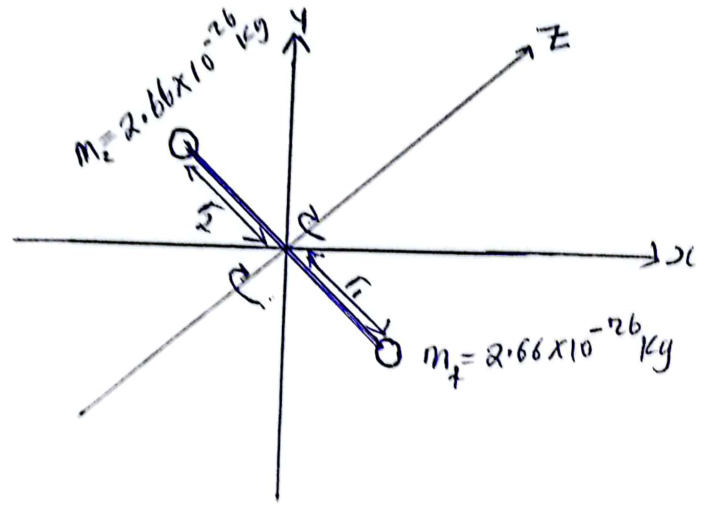
∴ the speed of the body ~~about~~ at infinite distance from the earth is $V = 19.41 \ Km/s$

CamScanner

TUTORIAL SHEET 9 ROTATIONAL MOTION

Q. 1. **Data**

- mass of each oxygen atom = 2.66×10^{-26} Kg
- distance between the atoms $d = 1.21 \times 10^{-10}$ m
- rotating about the Z-axis.



Since the axis of rotation passes through the centre of the molecule $r_1 = r_2 = \frac{d}{2}$

$$= \frac{1.21 \times 10^{-10}\ m}{2}$$

$$\underline{r_1 = r_2 = 6.05 \times 10^{-11}\ m}$$

(a) moment of Inertia about the Z-axis

$$I = \sum_{i=1}^{n} m_i r_i^2$$

$$I = m_1 r_1^2 + m_2 r_2^2 \quad \text{where } m_1 = m_2,\ r_1 = r_2$$

$$I = 2mr^2$$

$$I = 2 \times 2.66 \times 10^{-26} \times (6.05 \times 10^{-11})^2$$

$$I = 1.947253 \times 10^{-46}\ Kgm^2$$

$$\underline{I = 1.947 \times 10^{-46}\ Kgm^2}$$

∴ <u>The moment of Inertia of the molecule about the Z-axis</u>

$$\underline{\underline{I = 1.947 \times 10^{-46}\ Kgm^2}}$$

b) Rotational Kinetic Energy.

given the angular speed of the molecule about the z-axis $\omega = 4.60 \times 10^{12}$ rad/s

$$KE = \frac{1}{2} I \omega^2$$

$$K.E = \frac{1}{2}(1.947 \times 10^{-46}) \times (4.60 \times 10^{12})^2$$

$$K.E = \frac{4.120387348 \times 10^{-21}}{2}$$

$$K.E = 2.060193674 \times 10^{-21} \text{ J}$$

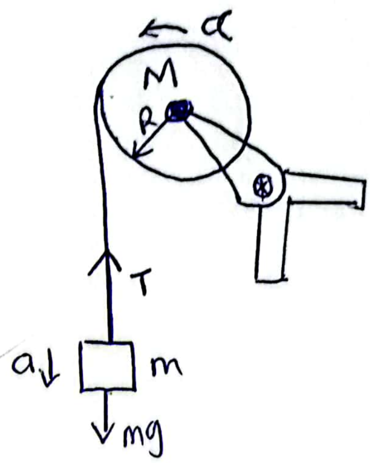
$$K.E = 2.06 \times 10^{-21} \text{ J}$$

$\therefore$ the Rotational Kinetic Energy of the molecule is

$$K.E = 2.06 \times 10^{-21} \text{ J}$$

3.
- Mass of the Pulley $M = 15\,Kg$
- $R = 20m$
- mass of the block $m = 5Kg$

F.B.D



(i) Linear acceleration of the block

pulley has Moment of Inertia $I = \frac{1}{2}MR^2$

$\sum F_y = ma$

$mg - T = ma$ - - - (i)

$\sum \tau = I\alpha$ $\qquad I = \frac{1}{2}MR^2$ - - - (ii)

$TR = I\alpha$ - - - - where $\alpha = \frac{a}{r}$

$TR = \frac{MR^2}{2} \times \frac{a}{R}$

$T\cancel{R} = \frac{MaR^{\cancel{2}}}{2\cancel{R}}$

$T = \frac{Ma}{2}$ - - - - - - (iii)

replace (iii) into (i)

$mg - \frac{Ma}{2} = ma$

$mg = \frac{ma}{1} + \frac{Ma}{2}$

$mg = \frac{2ma + Ma}{2}$

$2mg = 2ma + Ma$

$\frac{2mg}{2m+M} = \frac{a\cancel{(2m+M)}}{\cancel{2m+M}}$

$a = \frac{2mg}{2m+M}$

$a = \frac{2 \times 5 \times 9.8}{(2 \times 5) + 15}$

$a = 3.92 m/s^2$

∴ The Linear acceleration of the block is $a = 3.92 m/s^2$

Angular acceleration is related to linear acceleration by;

$$\frac{a}{r} = \frac{r\alpha}{r}$$

$$\alpha = \frac{a}{R}$$

$$\alpha = \frac{3.92\,m/s^2}{20m}$$

$$\underline{\alpha = 0.196\,rad/s^2}$$

$\therefore$ the angular acceleration of the pulley is

$$\underline{\alpha = 0.196\,rad/s^2}$$

(iii) Tension in the Cord

Using equation (iii)

$$T = \frac{Ma}{2}$$

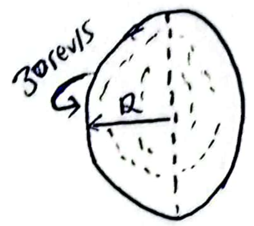
$$T = \frac{15 \times 3.92}{2}$$

$$T = \frac{58.8}{2}$$

$$\underline{T = 29.4N}$$

$\therefore$ the Tension in the cord is $T = 29.4N$

FBD



...m Sphere $= 500g = 0.5Kg$

...adius $R = 7.0cm = 0.07m$

Angular velocity $\omega = 30rev/s \left(\frac{30 rev}{s} \times \frac{2\pi rad}{rev}\right)$

$= \underline{60\pi \, rad/s}$

(a) Rotational Kinetic energy

K:E is given by

$KE = \frac{1}{2} I\omega^2$

Moment of Inertia of the sphere is given by $I = \frac{2}{5}MR^2$

$KE = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2$

$K:E = \frac{MR^2\omega^2}{5}$

$K:E = \frac{0.5 \times (0.07^2)(60\pi)^2}{5}$

$K:E = \frac{87.04991082}{5}$

$K:E = 17.40998216 \, J$

$\underline{K:E = 17.4 \, J}$

∴ The rotational kinetic energy $K:E = 17.4J$

(b) Angular momentum

Angular momentum is given by $L = I\omega$

$L = I\omega$

$L = \frac{2}{5}MR^2\omega$

$L = \frac{2}{5} \times 0.5 \times (0.07)^2 (60\pi)$

$L = \frac{0.92362824}{5}$

$L = 0.184725648 \, Kgm^2/s$

$L = 0.185 \, Kgm^2/s$

∴ Angular momentum $L = 0.185 \, Kgm^2/s$

radius of gyration.

is given by $K = \sqrt{\frac{I}{M}}$

$$K = \sqrt{\frac{I}{M}}$$

$$K = \sqrt{\left(\frac{2}{5}MR^2\right) \div M}$$

$$K = \sqrt{\frac{2\cancel{M}R^2}{5} \times \frac{1}{\cancel{M}}}$$

$$K = \sqrt{\frac{2R^2}{5}}$$

$$K = \sqrt{\frac{2 \times 0.07^2}{5}}$$

$$K = \sqrt{1.96 \times 10^{3}}$$

$$K = 0.044271887$$

$$\underline{K = 0.044 \text{ m}}$$

$\therefore$ radius of gyration $K = 0.044\,m$

Q. 5.

Data

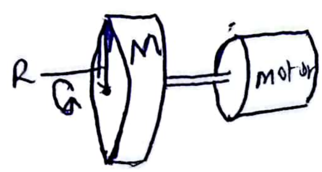
- Disk $m = 0.2\,kg$
  - $R = 5\,cm = 0.05\,m$
- motor Power $P = 5\,hp\,(5 \times 746\,W)$

Angular Velocity $W = 900\,rpm$

$$= \frac{\cancel{900}^{3}\,rev}{1\,min} \times \frac{2\pi\,rad}{1\,reve} \times \frac{\cancel{60}\,1\,min}{\cancel{60}_{3}\,s}$$

$$W = \underline{30\pi\ rad/s}$$

FBD



Angular speed of the disk in SI units.

$w = 900 rpm$

$$w = \frac{90\cancel{0} \text{ rev}}{1 \text{ min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{ min}}{6\cancel{0} s}$$

$w = 30\pi \text{ rad/s}$

∴ Angular Speed in SI is $w = 30\pi \text{ rad/s}$

(b) Rotational Kinetic Energy.

$I = \frac{1}{2}MR^2$ - - - -

$K.E = \frac{1}{2} I w^2$

$K.E = \frac{1}{2}\left(\frac{1}{2}MR^2\right)w^2$

$$K.E = \frac{MR^2w^2}{4} = \frac{0.2 \times 0.05^2 \times (30\pi)^2}{4}$$

$KE = 1.110330495 \text{ J}$

$KE = 1.11 \text{ J}.$

∴ The Rotational Kinetic Energy of the disk

$KE = 1.11 \text{ J}$

(c) The torque of the motor.

We use the Power and the angular velocity

$P_{ower} = \tau w$

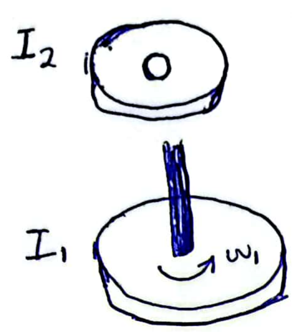
$\tau = \frac{P}{w}$

$$\tau = \frac{3730 \text{ watts}}{30\pi \text{ rad/s}}$$

$\tau = 39.57652918 \text{ Nm}$

$\tau = 39.58 \text{ Nm}$

∴ The torque of the motor $\tau = 39.58 \text{ Nm}$



ⓐ Angular velocity of the combination if the upper disk had angular velocity zero (0).

Conservation of Linear momentum

$$L_i = L_f$$

$$I_1 w_1 + I_2 w_2 = (I_1 + I_2) w \quad \text{where } w_2 = 0$$

$$\frac{I_1 w_1}{I_1 + I_2} = \frac{(\cancel{I_1 + I_2}) w}{\cancel{I_1 + I_2}}$$

$$w = \frac{I_1 w_1}{I_1 + I_2} \text{ rad/s}$$

∴ the combined angular velocity $w = \frac{I_1 w_1}{I_1 + I_2}$ rad/s

b) Combined angular velocity when w_2 is in the same direction as w_1

$$L_i = L_f$$

$$\frac{I_1 w_1 + I_2 w_2}{I_1 + I_2} = \frac{(\cancel{I_1 + I_2}) w}{\cancel{I_1 + I_2}}$$

$$w = \frac{I_1 w_1 + I_2 w_2}{I_1 + I_2} \text{ rad/s}$$

∴ The combined angular velocity when w_2 and w_1 are in the same direction is $w = \frac{I_1 w_1 + I_2 w_2}{I_1 + I_2}$ rad/s

When w_2 is in the opposite direction as w_1

$$L_1 = L_f$$

$$I_1 w_1 - I_2 w_2 = (I_1 + I_2) w$$

$$\frac{\cancel{(I_1+I_2)} w}{\cancel{I_1+I_2}} = \frac{I_1 w_1 - I_2 w_2}{I_1 + I_2}$$

$$w = \frac{I_1 w_1 - I_2 w_2}{I_1 + I_2} \text{ rad/s}$$

$\therefore$ The combined velocity when w_2 is in the opposite direction as w_1 is

$$w = \frac{I_1 w_1 - I_2 w_2}{I_1 + I_2} \text{ rad/s}$$