THE COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS

TUTORIAL SHEET (STANDARD QUESTIONS) : MA110 - Mathematical Methods
2023/2024

1) Express $2.5\overline{90}$ in the form $\frac{a}{b}$ where a and b are integers and $\frac{a}{b}$ is in its lowest terms.

2) Express $\frac{\sqrt{3}+1}{\sqrt{3}-1}+\sqrt{3}-1$ in the form $a+b\sqrt{3}$ where a and b are rational numbers.

3) Express $\frac{3-i}{4i}$ in the form $a+ib$ where a and b are rational numbers.

4) Let $f(x)=\frac{x-3}{x+2}$ be a function.

(i) Find the domain and the range of $f(x)$.

(ii) Find also the inverse $f^{-1}(x)$ of $f(x)$.

(iii) Given also that $g(x)=\frac{1}{x+1}$, solve the equation $f(g(x))=1$.

5) A binary operation $*$ is defined on the set of real numbers as follows:

$a*b=2^{-a}+b$, $a, b\in \mathbf{R}$

(i) Is the operation $*$ commutative? If not give a counterexample.

(ii) Find the value of $-1*(0*1)$ and $(-1*0)*1$, and state whether $*$ is associative.

6) The function $f(x)=\frac{a}{x}+b$ is such that $f(-1)=\frac{3}{2}$ and $f(2)=9$.

(i) Find the values of a and b.

(ii) State the domain of f.

(iii) Determine whether $f(x)$ a one – to – one function is.

7) Let $\mathbf{R}$, the set of real numbers be the universal set. If $A=[-7, 8)\cup[11,\infty)$ and $B=[0, 20]$, find the following sets and display them on the number line:

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(i) A'.

(ii) $A \cap B$.

(iii) $(A \cup B)'$.

8. Sketch the graph of the function $g(x) = -\sqrt{1-2x}$. Hence find the solution set of the inequality $\sqrt{1-2x} > 1$.

9. Solve the following inequality $\frac{2x}{x+1} \leq \frac{1}{2}$.

10. Given the quadratic function $f(x) = 6x^2 + x - 2$

(i) Find the y – intercept and the x – intercept.

(ii) Determine the maximum or the minimum point of the function.

(iii) Sketch the graph of $f(x) = 6x^2 + x - 2$ and the graph of $g(x) = |6x^2 + x - 2|$.

11) The remainder when the polynomial $P(x) = x^3 - px + q$ is divided by $x^2 - 3x + 2$ is $4x - 1$.

(i) Find the constants p and q.

(ii) Hence express the polynomial in the form $P(x) = (2x-1)Q(x) + R$ where $Q(x)$ is the quotient and R is the remainder when $P(x)$ is divided by $2x - 1$.

12. Given that the roots of the equation $x^2 - 21x + 4 = 0$ are α^2 and β^2, where both α and β are positive, find

(i) $\alpha\beta$.

(ii) $\alpha + \beta$.

(iii) an equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

13. Let $f(x) = x^3 - 5x^2 + 2x + 8$ be a polynomial.

(i) Show that $x + 1$ is a factor of the polynomial $f(x)$

(ii) Factorize the polynomial $f(x)$ completely.

(iii) Calculate the range of values of x for which $2x + 8 > 5x^2 - x^3$.

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14. (a) Let $f(x) = |2x-3| - 1$ be a function.

(i) Is the function f, even, odd or neither? Justify your answer.

(ii) State the range of $f(x)$.

(iii) Sketch the graph of $f(x)$.

(b) Given that X and Y are subsets of some universal set U, simplify the following:

(i) $X \cap (X' \cup Y)$.

(ii) $[(X \cap Y)' \cap (X' \cup Y)]'$.

15 a) (i) Express the complex numbers $\dfrac{x+iy}{y+ix}$ in the form $a+ib$.

(ii) Find p and q given that $2p + 3iq + \dfrac{1}{1+i} = (2+i)^2$.

(iii) Solve the inequality $|x+2| > 3$.

16 (a) Solve each of the equations below for real values of x.

(i) $x = \sqrt{\dfrac{1}{2}(5x+3)}$.

(ii) $\dfrac{x-3}{x^2+2} = \dfrac{1}{x+1}$.

17. The roots of the equation $2x^2 + 6x - 15 = 0$ are α and β. Find the value of:

(a) $(\alpha+1)(\beta+1)$ (b) $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2}$ (c) $\alpha^2\beta + \alpha\beta^2$ (d) $(\alpha-\beta)^2$ (e) $\dfrac{1}{2\alpha+\beta} + \dfrac{1}{\alpha+2\beta}$ (f) $\dfrac{1}{\alpha^2+1} + \dfrac{1}{\beta^2+1}$

18. (a) If the roots of the equation $-2x^2 + 2x - 7 = 0$ are α and β, write down an equation whose roots are $\dfrac{2}{\alpha}$ and $\dfrac{2}{\beta}$.

(b) The roots of the quadratic equation $x^2 - px + q = 0$ are α and β, write down in terms of p and q an equation whose roots are $\alpha^2 + p\alpha^2$ and $\beta^2 + p\beta^2$.

(c) Form an equation with roots which exceed by 2 the roots of the quadratic equation $2x^2 - (p-4)x - (2p+1) = 0$

19. (a) Find the value of k or the range of values of k in each case below:

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(i) The roots of $3x^2 - 5x + k = 0$ differ by two.
(ii) The roots of $3x^2 + (k-1)x - 2 = 0$ are equal and opposite
(iii) The roots of $x^2 + (2-k)x + 1 - 2k = 0$ are real and distinct

(b) (i) If α is a positive constant, find the set of values of x for which $\alpha(x^2 + 2x - 8)$ is negative. Find the value of α if this function has minimum value of -27.

(ii) Find two quadratic functions in x which are zero at $x = 1$, which take the value 10 when $x = 0$ and which have a maximum value of 18. Sketch the graphs of these two functions.

(c) (i) The equation $P(x) = -2x^2 + 280x - 1000$, where x represents the number of items sold, describes the profit function for a certain business. How many items should be sold to maximize the profit?

(ii) Two hundred and forty meters of fencing is available to enclose a rectangular playground. What should be the dimensions of the ground to maximize the area?

20. (a) Solve the following inequalities:

(i) $2x - 1 < 4(x-3)$ (ii) $\frac{3}{x-1} > 1$ (iii) $\frac{x-1}{x+1} > 2$

(iv) $\frac{x+3}{x-1} \le \frac{x-3}{x+1}$ (v) $|x-2| \ge 2$ (vi) $\left|\frac{x-1}{x+2}\right| < 2$

(b) (i) Find the set of values of x for which $-1 < \frac{2-x}{2+x} \le 1$

(ii) The function f is defined for all real x by $f(x) = \begin{cases} \frac{1}{x} & \text{for } x > 0 \\ |x| & \text{for } x \le 0 \end{cases}$

Find the set of values of x for which $f(x) \le 4$

21. (a) Sketch the graphs of the following functions:

(i) $f(x) = \sqrt{3+x}$ (ii) $f(x) = 1 - 3\sqrt{x - \frac{1}{2}}$

(iii) $f(x) = \sqrt{2x-1} - 3$

(b) Redefine each of the following modulus functions by removing the modulus, hence sketch the graph of each function:

(i) $g(x) = |2x+3|$ (ii) $f(x) = -2|5x-4|$

(iii) $h(x) = |3x+1| + |2x-3|$ (iv) $k(x) = |2x-1| - |x+2|$

(v) $p(x) = |x^2 - 3x - 4|$

22. Find the remainder when $x^3 + 2x^2 - x - 1$ is divided by

(a) $x-1$ (b) $x+1$ (c) $2x-1$ (d) $3x+2$
(e) $x-3$ (f) $x+4$

23. Express each of the following polynomials in the form $P(x) = (ax+b)Q(x) + R$ where $Q(x)$ is the quotient and R is the remainder when the polynomial $P(x)$ is divided by the given linear function given:

(a) $6x^3 + 7x^2 - 15x + 4$ is divided by $x - 1$

(b) $5 + 6x + 7x^2 - x^3$ is divided by $x + 1$

(c) $x^4 - 3x^2 + x + 1$ is divided by $x - 1$

(d) $9x^3 + 4$ is divided by $3x + 2$

(e) $8x^3 - 10x^2 + 7x + 3$ is divided by $2x - 1$

24. Factorize each of the following polynomials:

(a) $x^3 - 2x^2 - 5x + 6$ (b) $3x^3 + 2x^2 - 3x - 2$

(c) $6x^3 - 13x^2 + 9x - 2$ (c) $x^4 - 1$ (d) $x^4 - 2x^3 + x - 2$

25 (a) The polynomial $6x^3 - 23x^2 + ax + b$ gives a remainder of 11 when divided by $x - 3$ and a remainder of -21 when divided by $x+1$. Find the values of a and b.

(b) Find the remainder in terms of p when $2x^3 + px^2 - x - 2$ is divided by $x+3$.

(c) The expression $x^2 - 4x - 2$ has the same remainder when it is divided by either $x - a$ or $x - b$ $(a \neq b)$. Show that $a + b = 4$

Given that the remainder is 10 when the expression is divided by $x - 2a$, find the values of a and b.

(d) Given that $2x^3 - 7x^2 + 7x - 5 = A(x-1)^3 + Bx(x-1) + C$ for all values of x, find the values of A, B and C.

26. Using synthetic division find the quotient and the remainder when

(a) $x^3 - 2x^2 + 9$ is divided by $x + 2$

(b) $x^4 - 2x^3 - 3x^2 - 4x - 8$ is divided by (i) $x - 2$ (ii) $x+1$

(c) $8x^3 - 10x^2 + 7x + 3$ is divided by $2x - 1$

27. Find a relationship between a, b and c such that the equations $x^2 - ax + b = 0$ and $ax^2 + x - c = 0$ have a common root.

28. (a) Factorize the polynomial $x^3 - 3x^2 - 4x + 12$. Hence calculate the range of values of x such that $x^3 - 4x > 3x^2 - 12$.

(b) Solve the equation $x^3 - 7x + 6 = 0$. Hence state the solution of the equation $(x-2)^2 - 7(x-2) + 6 = 0$

(c) Find the x – coordinates of the points where the line $y = 5x - 1$ meets the curve $y = 2x^3 + x^2 + 1$

29. Let $X = (-10, 10)$ be the universal set and $A = (-2, 6]$, $B = [-5, 3]$ and $C = [-1, 8)$. Find each of the following sets and display it on the number line:

(i) A' (ii) $X - A$ (iii) $(A \cap C)'$ (iv) $(B - A) \cap C$

30. Given that $\mathbb{R}$, the set of real numbers is the universal set, $A = (-8, 6]$ and $B = [5, \infty)$, find

(i) A' (ii) B' (iii) $A - B$ (iv) $B - A$

31. Using the associative and distributive properties of union and intersection of sets, show that

(a) $X = (X \cap Y) \cup (X \cap Y')$ (b) $X \cup (X' \cap Y) = X \cup Y$

(c) $X \cup Y = (X \cap Y) \cup (X \cap Y') \cup (X' \cap Y)$

32. Rationalize the numerator in each of the following:

(i) $\dfrac{\sqrt{5+h}-3}{h}$ (ii) $\dfrac{\sqrt{3}+\sqrt{5}}{7}$ (iii) $\dfrac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}$

(iv) $\sqrt{x^2+1}-x$

33. Determine whether each of the given relation below is a function or not:

(i) $y = 3x - 1$ (ii) $y = \begin{cases} 2x+3, & x \le 1 \\ 6-x^2, & x \ge 1 \end{cases}$

(iii) $y = \begin{cases} x, & x \le 3 \\ -x, & x \ge 3 \end{cases}$ (iv) $y = \begin{cases} x^3, & x < 2 \\ \dfrac{1}{x}, & x \ge 2 \end{cases}$

34. Determine whether the function f is even, odd or neither.

(i) $f(x) = 1 - x^4$ (ii) $f(x) = x^2 - x$

(iii) $f(x) = 3x^3 + 2x - 1$ (iv) $f(x) = x + \dfrac{1}{x}$

35. (a) (i) Express $3.12\overline{12}$ in the form $\dfrac{a}{b}$ where a and b are integers and $b \neq 0$.

(ii) Express $\dfrac{5}{3-2\sqrt{3}}$ in the form $a + b\sqrt{3}$ where a and b are rational numbers.

(b) Let $f(x) = \dfrac{x+1}{2x-1}$ and $g(x) = \dfrac{1}{x}$ be two functions.

(i) Find $f\left(\dfrac{1}{4}\right)$

(ii) Find the **domain** and the **range** of the function f.

(iii) Show that the function f is a one – to – one function.

(iv) Find also the composition $(fog)(x)$

(c) Given that $\sqrt{3}$ is an irrational number, show that $1+\sqrt{3}$ is an irrational number.

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AACHOO!
bless you
WE'VE TALKED ABOUT THIS
sorry
THIS COMIC MADE POSSIBLE THANKS TO LALIT VARADPANDE
MRLOVENSTEIN.COM
I'LL LET IT SLIDE... THIS TIME
oh thank god
ifunny.co

1 $2.5\overline{90}$

$x = 2.5\overline{90}$

$10x = 25.\overline{90}$

$1000x = 2590.\overline{90}$

$1000x - 10x = 2590.\overline{90} - 25.\overline{90}$

$$\frac{990x}{990} = \frac{2565}{990}$$

$$x = \frac{2565}{990}$$

2. $\dfrac{\sqrt{3}+1}{\sqrt{3}-1} + \sqrt{3} - 1$

$$\frac{\sqrt{3}+1}{\sqrt{3}-1}\left(\frac{\sqrt{3}+1}{\sqrt{3}+1}\right)$$

$$\frac{(\sqrt{3})^2 + 2(\sqrt{3})(1) + (1)^2}{(\sqrt{3})^2 - (1)^2}$$

$$\frac{3 + 2\sqrt{3} + 1}{3 - 1}$$

$$\frac{4 + 2\sqrt{3}}{2}$$

$2 + \sqrt{3} + \sqrt{3} - 1$

$1 + 2\sqrt{3}$

3 $\frac{3-i}{4i}\left(\frac{-4i}{-4i}\right)$

$$\frac{-12i+4(-1)}{16}$$

$$\frac{-4-12i}{16}$$

$$\frac{-4}{16}-\frac{12i}{16}$$

$$-\frac{1}{4}-\frac{3}{4}i$$

4. $f(x)=\frac{x-3}{x+2}$

Domain;

$x+2=0$

$x=-2$

Domain is all ~~real~~ numbere except -2

$(-\infty,-2)\cup(-2,\infty)$

Range;

$$y=\frac{x-3}{x+2}$$

$xy+2y=x-3$

$xy-x=-3-2y$

$$\frac{x(y-1)}{y-1}=\frac{-3-2y}{y-1}$$

$$x=\frac{-3-2y}{y-1}$$

$$y - 1 = 0$$
$$y = 1$$

$\therefore$ Range is all real numbers but 1

$(-\infty, 1) \cup (1, \infty)$

(ii) # from (i) above

$$x = \frac{-3 - 2y}{y - 1}$$

$$y = \frac{-3 - 2x}{x - 1}$$

$$\therefore F^{-1}(x) = \frac{-3 - 2x}{x - 1}$$

(iii) $g(x) = \frac{1}{x+1}$

lets start with $f(g(x))$. $f(x) = \frac{x - 3}{x + 2}$

$$f(g(x)) = \frac{\frac{1}{x+1} - 3}{\frac{1}{x+1} + 2}$$

$$= \left(\frac{1}{x+1} - \frac{3}{1}\right) \div \left(\frac{1}{x+1} + \frac{2}{1}\right)$$

$$= \left(\frac{1 - 3x - 3}{x+1}\right) \div \left(\frac{1 + 2x + 2}{x+1}\right)$$

$$\left(\frac{-2-3x}{x+1}\right) \div \left(\frac{3+2x}{x+1}\right)$$

$$\left(\frac{-2-3x}{x+1}\right) \times \left(\frac{x+1}{3+2x}\right)$$

$$\frac{-2-3x}{3+2x} = f(g(x))$$

$\#$ $f(g(x)) = 1$

$$\frac{-2-3x}{3+2x} = \frac{1}{1}$$

$$-2-3x = 3+2x$$

$$-2-3 = 3x+2x$$

$$\frac{-5}{5} = \frac{5x}{5}$$

$$\underline{\underline{x = -1}}$$

(5.) $a * b = 2^{-a} + b$, $a, b \in R$

$$a * b = b * a$$

$$1 * 2 = 2 * 1$$

$$2^{-1} + 2 = 2^{-2} + 1$$

$$\frac{1}{2} + 2 = \frac{1}{2^2} + 1$$

$$2\tfrac{1}{2} = 1\tfrac{1}{4}$$

$$\frac{5}{2} = \frac{5}{4}$$

$a * b \neq b * a$, it is not commutative.

(ii) $-1 * (0 * 1)$

$0 * 1$

$0 * 1 = 2^{-0} + 1$
$= 1 + 1$
$0 * 1 = 2$

$-1 * 2 = 2^{-(-1)} + 2$
$= 2 + 2$
$-1 * 2 = 4$

$\therefore -1 * (0 * 1) = 4$

$(-1 * 0) * 1$

$-1 * 0$

$-1 * 0 = 2^{-(-1)} + 0$
$-1 * 0 = 2$

$2 * 1 = 2^{-2} + 1$
$= \frac{1}{4} + 1$

$2 * 1 = \frac{5}{4}$

$(-1 * 0) * 1 = \frac{5}{4}$

$-1 * (0 * 1) \neq (-1 * 0) * 1$

$\therefore$ It is not associative

6 $f(x) = \frac{a}{x} + b$

$F(-1) = \frac{a}{-1} + b = \frac{3}{2}$
$= -a + b = \frac{3}{2}$ ...(i)

$F(2) = \frac{a}{2} + b = 9$
$\frac{a}{2} + b = 9$ ...(ii)

$$-\left|\begin{array}{l} -a + b = \frac{3}{2} \\ \frac{a}{2} + b = 9 \end{array}\right.$$

$$\frac{-3a}{2} = \frac{3}{2} - \frac{9}{1}$$

$$= \frac{3-18}{2}$$

$$-\frac{2}{3} \times \left[\frac{-3}{2}a = \frac{-15}{2} \right]$$

$$a = 5$$

$$-a + b = \frac{3}{2}$$

$$b = \frac{3}{2} + a$$

$$b = \frac{3}{2} + \frac{5}{1}$$

$$b = \frac{3+10}{2}$$

$$b = \frac{13}{2}$$

$$a = 5 \quad , \quad b = \frac{13}{2}$$

(ii) $f(x) = \frac{a}{x} + b$

$x = 0$, Domain is all real numbers but 0

Domain ; $\{x : x \in R, x \neq 0\}$.

$\frac{a}{1} + \frac{9}{2}$

$\frac{2a + a^2}{2}$

(iii) ~~f(a)~~ f

$$f(c) = f(e)$$

$$\frac{a}{c} + b = \frac{a}{e} + b$$

$$\frac{a}{c} = \frac{a}{e} + b - b$$

$$\frac{a}{c} = \frac{a}{e}.$$

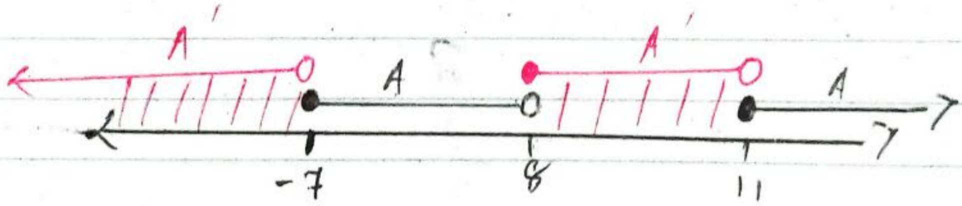
$$\frac{ae}{a} = \frac{ac}{a}$$

$$e = c$$

$\therefore$ it is a one to one function

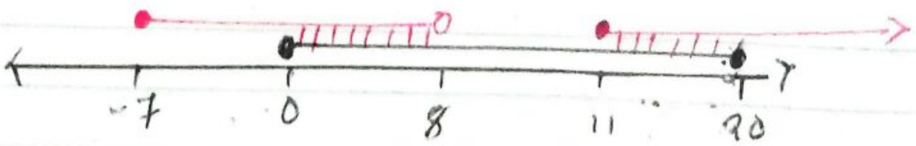
Q.7 $A = [-7, 8) \cup [11, \infty)$ $\quad B = [0, 20]$

(i) A'



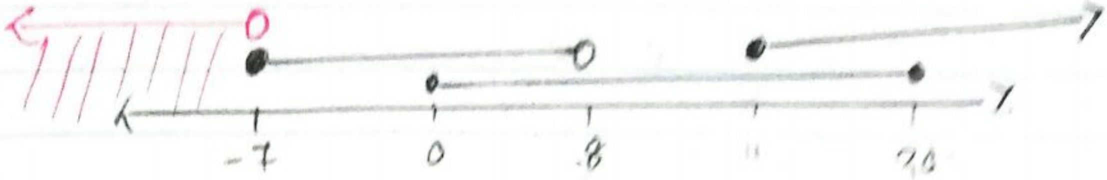
$A' = (-\infty, -7) \cup [8, 11)$

(ii) $A \cap B$



$A \cap B = [0, 8) \cup [11, 20]$

(iii) $(A \cup B)'$

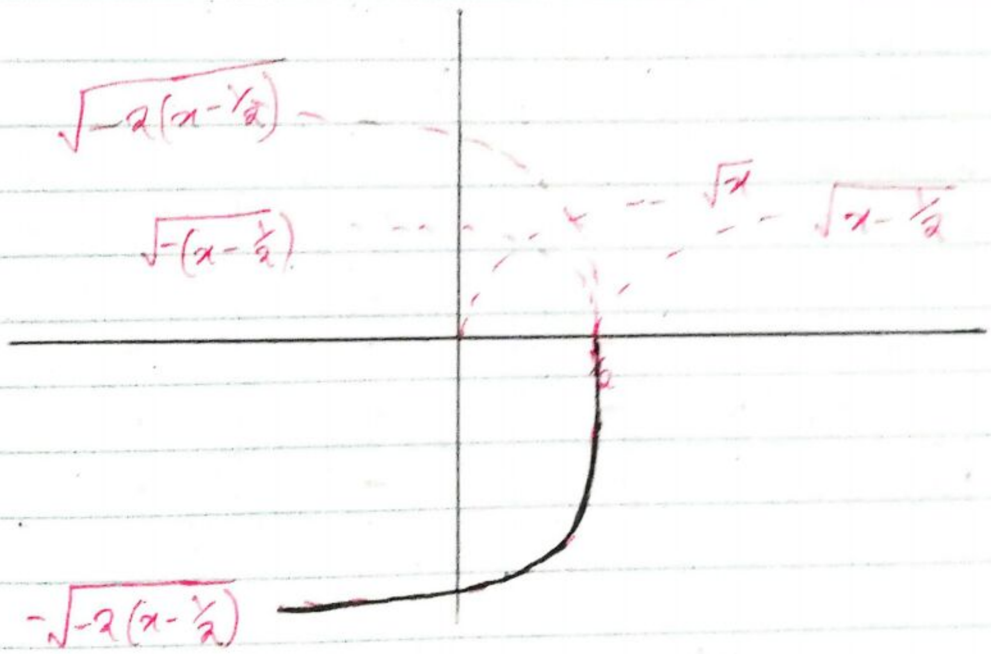


$(A \cup B)' = (-\infty, -7)$

8 $g(x) = -\sqrt{1-2x}$

$= -\sqrt{-2x+1}$

$g(x) = -\sqrt{-2(x-\frac{1}{2})}$



$(\sqrt{1-2x})^2 > (1)^2$

$1-2x > 1$

$-2x > 0$

$\frac{-2x}{-2} \quad \frac{0}{-2}$

$x < 0$

$(-\infty, 0)$

9 $\frac{2x}{x+1} \le \frac{1}{2}$

$\frac{2x}{x+1} - \frac{1}{2} \le 0$

$\frac{4x-(x+1)}{2(x+1)} \le 0$

$\frac{4x-x-1}{2x+2}$

$\frac{3x-1}{2x+2} \le 0$

$3x-1=0$ or $2x+2=0$

$x=\frac{1}{3}$ $\quad$ $x=-1$

Range	$x < -1$	$-1 < x < \frac{1}{3}$	$x > \frac{1}{3}$
Test value	-2	0	1
$3x-1$	$-$	$-$	$+$
$2x+2$	$-$	$+$	$+$
Overal	$+$	$-$	$+$

$-1 < x \le \frac{1}{3}$

$\left(-1, \frac{1}{3}\right]$

Q.10

(i) $f(x) = 6x^2 + x - 2$

y - intercept [x = 0]

$y = 6(0)^2 + (0) - 2$

$y = -2$

$\underline{\underline{(0, -2)}}$

x - intercept [y = 0]

$0 = 6x^2 + x - 2$

$p = -12 \quad , \quad s = 1 \quad f = 4, -3$

$6x^2 - 3x + 4x - 2$

$3x(2x - 1) + 2(2x - 1) = 0$

$(3x + 2)(2x - 1) = 0$

$3x + 2 = 0 \quad \text{or} \quad 2x - 1 = 0$

$x = -\frac{2}{3} \qquad x = \frac{1}{2}$

$\underline{\underline{\left(-\frac{2}{3}, 0\right)}} \quad \text{or} \quad \underline{\underline{\left(\frac{1}{2}, 0\right)}}$

(ii) # minimum, apa ni minimum kaili a is (+)

$6x^2 + x - 2$

$6\left(x^2 + \frac{x}{6} - \frac{1}{3}\right)$

$6\left[x^2 + \frac{x}{6} + \left(\frac{1}{12}\right)^2 - \left(\frac{1}{12}\right)^2 - \frac{1}{3}\right]$

$$6\left[\left(x+\frac{1}{12}\right)^2-\frac{1}{144}-\frac{1}{3}\right]$$

$$6\left[\left(x+\frac{1}{12}\right)^2-\frac{1-48}{144}\right]$$

$$6\left[\left(x+\frac{1}{12}\right)^2-\frac{49}{144}\right]$$

$$6\left(x+\frac{1}{12}\right)^2-\frac{49}{24}$$

$\therefore$ minimum value $= -\frac{49}{24}$

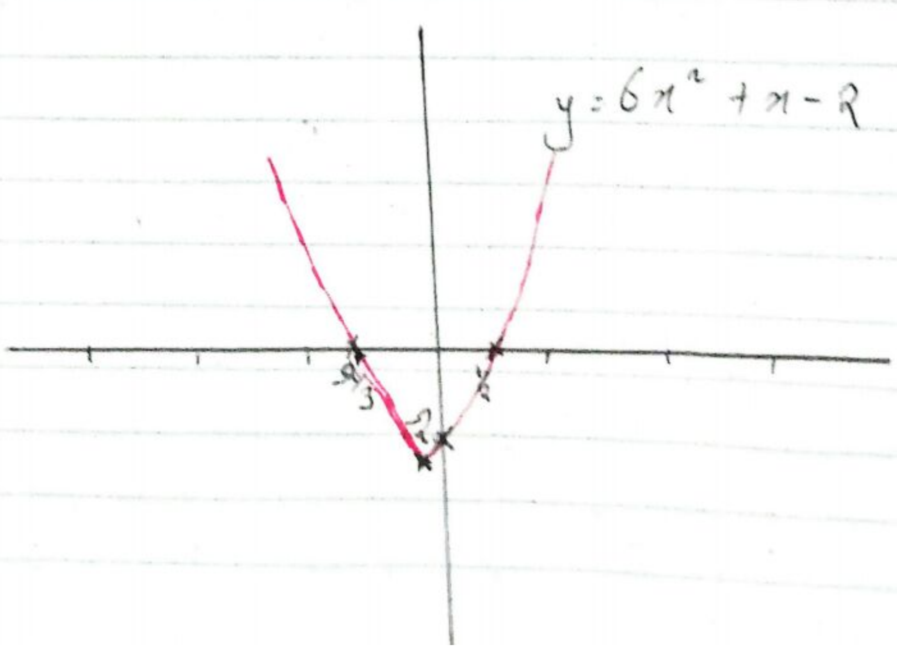
(iii) # To sketch we need turning point.

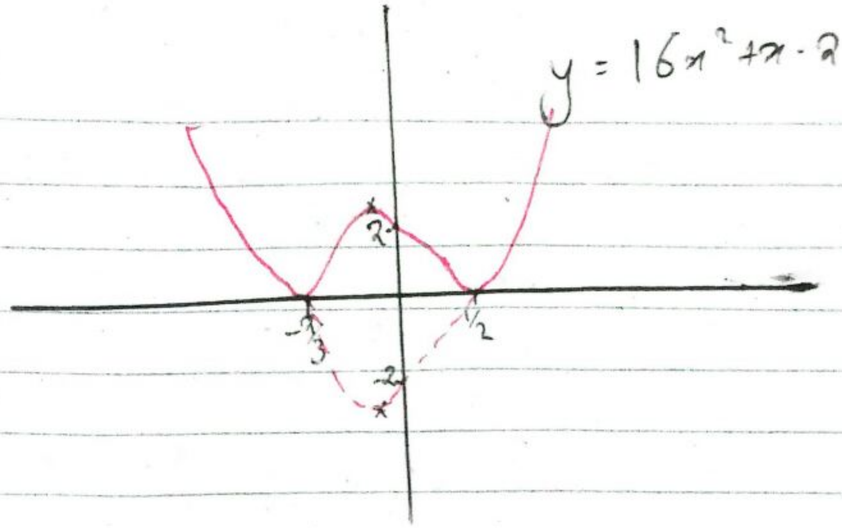
$$6\left(x+\frac{1}{12}\right)^2-\frac{49}{24}$$

$x+\frac{1}{12}=0$

$x=-\frac{1}{12}$ $\quad y=-\frac{49}{24}$

$\therefore$ T.p $\left(-\frac{1}{12}, -\frac{49}{24}\right)$





Q11 $p(x) = x^3 - px + q$

get $x^2 - 3x + 2$ and factor it

$$x^2 - 3x + 2$$
$$x^2 - x - 2x + 2$$
$$x(x-1) - 2(x-1) = 0$$
$$(x-2)(x-1) = 0$$
$$x = 2 \quad x = 1$$

when $x = 2$, mul $p(x)$

$$(2)^3 - p(2) + q = 4(2) - 1$$

$$8 - 2p + q = 8 - 1$$
$$-2p + q = 7 - 8$$
$$-2p + q = -1 \quad \ldots (i)$$

When $x = 1$

$$(1)^3 - p(1) + q = 4(1) - 1$$
$$1 - p + q = 4 - 1$$
$$-p + q = 2 \quad \ldots (ii)$$

$$-\left|\begin{array}{l} -2p + q = -1 \\ -p + q = 2 \end{array}\right.$$

$$-p = -3$$
$$p = 3$$

$$-p + q = 2$$
$$q = 2 + p$$
$$q = 2 + 3$$
$$q = 5$$

$$\therefore p = 3 \quad \text{and} \quad q = 5$$

(ii) $\quad -3 \quad \frac{1}{4} \quad -\frac{13}{4}$

$$2x - 1 = 0$$
$$x = \frac{1}{2}$$

where $p(x) = x^3 - 3x + 5$

$$\begin{array}{c|cccc} \frac{1}{2} & 1 & 0 & -3 & 5 \\ & & \frac{1}{2} & \frac{1}{4} & -\frac{13}{8} \\ \hline & 1 & \frac{1}{2} & -\frac{13}{4} & \frac{27}{8} \end{array}$$

$$Q(x) = x^2 + \frac{1}{2}x - \frac{13}{4} \qquad R = \frac{27}{8}$$

$$P(x) = (2x-1)\left(x^2 + \frac{1}{2}x - \frac{13}{4}\right) + \frac{27}{8}$$

Q12 $\quad x^2 - 21x + 4$

$$a = 1 \quad b = -21 \quad c = 4$$

(i) $\alpha\beta = \frac{c}{a}$ $\qquad$ (ii) $\alpha + \beta = -\frac{b}{a}$

$$= \frac{4}{1} \qquad\qquad = -\frac{(-21)}{1}$$

$$\alpha\beta = 4 \qquad\qquad \alpha + \beta = 21$$

(iii) roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$

sum of roots;

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$$

$$\frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$\frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$\frac{(21)^2 - 2(4)}{4}$$

$$\frac{433}{4}$$

Product of roots;

$$\frac{\alpha}{\beta} \times \frac{\beta}{\alpha} = \frac{\alpha\beta}{\alpha\beta} = 1$$

new equation is expressed as;

$x^2 - (\text{sum of roots})x + (\text{product of roots})$

$$x^2 - \left(\frac{433}{4}\right)x + 1$$

$$\left[x^2 - \frac{433}{4}x + 1\right] \times 4$$

$$4x^2 - 433x + 4$$

Q13 $f(x) = x^3 - 5x^2 + 2x + 8$

$x + 1 = 0$

$x = -1$

$(-1)^3 - 5(-1)^2 + 2(-1) + 8$

$-1 - 5 - 2 + 8$

$\underline{\underline{0}}$

<u>Hence shown</u>

(ii) $f(x) = x^3 - 5x^2 + 2x + 8$

$x + 1 = 0$

$x = -1$

-1	1	-5	2	8
		-1	6	-8
	1	-6	8	0

ah wait this is a quadratic

$x^2 - 6x + 8$

$p = 8 \quad s = -6 \quad f = -2, -4$

$\to x^2 - 2x - 4x + 8$

$\to x(x-2) - 4(x-2) = 0$

$(x-4)(x-2) = 0$

~~$x-4=0$ or $x-2=0$~~

$x =$

$\therefore f(x) = x^3 - 5x^2 + 2x + 8 = (x+1)(x-4)(x-2)$

iii $\quad 2x + 8 > 5x^2 - x^3$

$$x^3 - 5x^2 + 2x + 8 > 0$$

Secret here is to factor the polynomial.

$$\begin{array}{r|rrrr} -1 & 1 & -5 & 2 & 8 \\ & & -1 & 6 & -8 \\ \hline & 1 & -6 & 8 & 0 \end{array}$$

$$x^2 - 6x + 8$$

Oh we did this before, let me just write down the factors.

$$(x+1)(x-4)(x-2)$$

Critical points monja apa,

$x = -1 \quad , \quad x = 4 \quad , \quad x = 2$

Range / Test value	$x < -1$ / -2	$-1 < x < 2$ / 0	$2 < x < 4$ / 3	$x > 4$ / 5
$x + 1$	$-$	$+$	$+$	$+$
$x - 4$	$-$	$-$	$-$	$+$
$x - 2$	$-$	$-$	$+$	$+$
	$-$	$+$	$-$	$+$

$-1 < x < 2 \quad$ or $\quad x > 4$

$$(-1, 2) \cup (4, \infty)$$

14

(a) $f(x) = |2x-3| - 1$

$f(x) = f(-x)$

$f(-x) = |2(-x) - 3| - 1$

$f(-x) = |-2x-3| - 1$

$f(-x) = |-(2x+3)| - 1$

$f(x) \neq f(-x)$, hence not even.

$-f(x) = -(|2x-3| - 1)$

$-f(x) = -|2x-3| + 1$

$f(-x) \neq -f(x)$, hence not odd too

$\therefore$ it neithers

(b) Range: $\{y : y \in \mathbb{R}, y \geq -1\}$

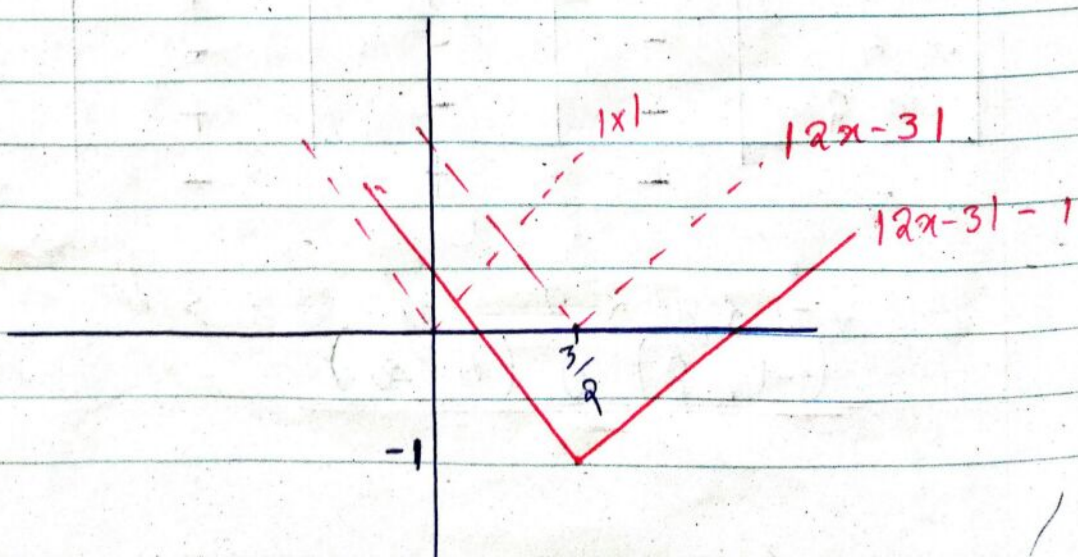
or

$[-1, \infty)$

$2x-3=0$

$x = \frac{3}{2}$

C



(b) $X \cap (X' \cup Y)$

$\rightarrow (X \cap X') \cup (X \cap Y)$

$\rightarrow \emptyset \cup (X \cap Y)$

$\rightarrow X \cap Y$

(ii) $[(X \cap Y)' \cap (X' \cup Y)]'$

$\rightarrow [(X' \cup Y') \cap (X' \cup Y)]'$

$\rightarrow [X' \cup (Y' \cap Y)]'$

$\rightarrow [X' \cup \emptyset]'$

$\rightarrow [X']'$

X

(15)

a, i $\dfrac{x+iy}{y+ix}$

$\rightarrow \dfrac{x+iy}{y+ix}\left(\dfrac{y-ix}{y-ix}\right)$

$\rightarrow \dfrac{xy - ix^2 + iy^2 + xy}{y^2-(ix)^2}$

$\rightarrow \dfrac{2xy - ix^2 + iy^2}{y^2+x^2}$

$\rightarrow \dfrac{2xy}{y^2+x^2} + \dfrac{(-x^2+y^2)i}{y^2+x^2}$

(ii) $2p + 3iq + \frac{1}{1+i} = (2+i)^2$

$\Rightarrow 2p + 3iq + \frac{1}{1+i}\left(\frac{1-i}{1-i}\right) = 4 + 4i + (i)^2$

$\Rightarrow 2p + 3iq + \frac{1-i}{(1)^2-(i)^2} = 4 - 1 + 4i$

$\Rightarrow 2p + 3iq + \frac{1-i}{2} = 3 + 4i$

$\Rightarrow 2p + 3iq + \frac{1}{2} - \frac{i}{2} = 3 + 4i$

$\Rightarrow 2p + \frac{1}{2} + 3iq - \frac{i}{2} = 3 + 4i$

equat real to real & fake to fake

$2p + \frac{1}{2} = 3$ $\qquad$ $3iq - \frac{i}{2} = 4i$

$2p = 3 - \frac{1}{2}$ $\qquad$ $3q - \frac{1}{2} = 4$

$\frac{1}{2} \times \left[2p = \frac{5}{2}\right]$ $\qquad$ $3q = 4 + \frac{1}{2}$

$p = \frac{5}{4}$ $\qquad$ $\cancel{3q = \frac{5}{2}}$

$\frac{1}{3}\left[3q = \frac{9}{2}\right]$

$q = \frac{3}{2}$

$\therefore \underline{p = \frac{5}{4}}, \ \underline{q = \frac{3}{2}}$

iii $|x+2| > 3$

$\rightarrow -3 > x + 2 > 3$

$\rightarrow -3 - 2 > x + 2 - 2 > 3 - 2$

$\rightarrow -5 > x > 1$

$x < -5$ or $x > 1$

16. (a)

(i) $(x)^2 = \left(\sqrt{\frac{1}{2}(5x+3)}\right)^2$

$\rightarrow \frac{x^2}{1} = \frac{1}{2}(5x+3)$

$\rightarrow 2x^2 = 5x + 3$

$\rightarrow 2x^2 - 5x - 3 = 0$

$\rightarrow 2x^2 + x - 6x - 3 = 0$

$\rightarrow x(2x+1) - 3(2x+1) = 0$

$\rightarrow (2x+1)(x-3) = 0$

$2x + 1 = 0$ or $x - 3 = 0$

$x = -\frac{1}{2}$ $\quad$ $x = 3$

(ii) $\frac{x-3}{x^2+2} = \frac{1}{x+1}$

$\rightarrow (x-3)(x+1) = x^2 + 2$

$\rightarrow x^2 + x - 3x - 3 = x^2 + 2$

$$\frac{-2x}{-2} = \frac{5}{-2}$$

$$\underline{x = -\frac{5}{2}}$$

17. $2x^2 + 6x - 15 = 0$, $a = 2$, $b = 6$, $c = -15$

$\alpha + \beta = -\frac{b}{a}$ $\qquad$ $\alpha\beta = \frac{c}{a}$

$= -\frac{6}{2}$ $\qquad$ $\alpha\beta = -\frac{15}{2}$

$\boxed{\alpha + \beta = -3}$ $\qquad$ $\boxed{\alpha\beta = -\frac{15}{2}}$

(a) $(\alpha + 1)(\beta + 1)$

$\rightarrow \alpha\beta + \alpha + \beta + 1$

$\rightarrow \left(-\frac{15}{2}\right) + (-3) + 1$

$\rightarrow -\frac{15}{2} - \frac{2}{1}$

$\rightarrow \frac{-15 - 4}{2}$

$\rightarrow \underline{-\frac{19}{2}}$

(b) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$\rightarrow \frac{\beta^2 + \alpha^2}{\alpha^2\beta^2}$

$\rightarrow \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2}$

$\rightarrow \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$

$\rightarrow \frac{(-3)^2 - 2\left(-\frac{15}{2}\right)}{\left(-\frac{15}{2}\right)^2}$

$\rightarrow (9 + 15) \div \frac{225}{4}$

$\rightarrow 24 \times \frac{4}{225}$

$\rightarrow \frac{96}{225}$

(c) $\alpha^2\beta + \alpha\beta^2$

$\rightarrow \alpha\beta(\alpha+\beta)$

$\rightarrow -\frac{15}{2}(-3)$

$\rightarrow \frac{45}{2}$

(d) $(\alpha-\beta)^2$

$\rightarrow \alpha^2 - 2\alpha\beta + \beta^2$

$\rightarrow \alpha^2 + \beta^2 - 2\alpha\beta$

$\rightarrow (\alpha+\beta)^2 - 2\alpha\beta - 2\alpha\beta$

$\rightarrow (\alpha+\beta)^2 - 4\alpha\beta$

$\rightarrow (-3)^2 - 4\left(-\frac{15}{2}\right)$

$\rightarrow 9 + 30$

$\rightarrow 39$

e $\frac{1}{2\alpha+\beta} + \frac{1}{\alpha+2\beta}$

$\rightarrow \frac{\alpha+2\beta + 2\alpha+\beta}{(2\alpha+\beta)(\alpha+2\beta)}$

$\rightarrow \frac{3\alpha+3\beta}{2\alpha^2+4\alpha\beta+\alpha\beta+2\beta^2}$

$\rightarrow \frac{3(\alpha+\beta)}{2\alpha^2+2\beta^2+5\alpha\beta}$

$\rightarrow \frac{3(\alpha+\beta)}{2(\alpha^2+\beta^2)+5\alpha\beta}$

$\rightarrow \frac{3(\alpha+\beta)}{2[(\alpha+\beta)^2-2\alpha\beta]+5\alpha\beta}$

$\rightarrow \frac{3(\alpha+\beta)}{2(\alpha+\beta)^2-4\alpha\beta+5\alpha\beta}$

$\rightarrow \frac{3(\alpha+\beta)}{2(\alpha+\beta)^2+\alpha\beta}$

$\rightarrow \frac{3(-3)}{2(-3)^2+\left(-\frac{15}{2}\right)}$

$\rightarrow \frac{-9}{18-\frac{15}{2}}$

$\rightarrow -9 \div \frac{21}{2}$

$\rightarrow -\cancel{9}^{3} \times \frac{2}{\cancel{21}_{7}}$

$\rightarrow -\frac{6}{7}$

(f) $\dfrac{1}{\alpha^2+1} + \dfrac{1}{\beta^2+1}$

$\rightarrow \dfrac{\beta^2+1+\alpha^2+1}{(\alpha^2+1)(\beta^2+1)}$

$\rightarrow \dfrac{\alpha^2+\beta^2+2}{\alpha^2\beta^2+\alpha^2+\beta^2+1}$

$\rightarrow \dfrac{(\alpha+\beta)^2-2\alpha\beta+2}{(\alpha\beta)^2+(\alpha+\beta)^2-2\alpha\beta+1}$

$\rightarrow \dfrac{(-3)^2-2\left(-\frac{15}{2}\right)+2}{\left(-\frac{15}{2}\right)^2+(-3)^2-2\left(-\frac{15}{2}\right)+1}$

$\rightarrow \dfrac{9+15+2}{\frac{225}{4}+9+15+1}$

$\rightarrow \dfrac{26}{\frac{225}{4}+25}$

$\rightarrow 26 \div \dfrac{325}{4}$

$\rightarrow \overset{2}{\cancel{26}} \times \dfrac{4}{\cancel{325}_{25}}$

$\rightarrow \dfrac{8}{25}$

18

(a) $-2x^2 + 2x - 7 = 0$

$a = -2 \quad b = 2 \quad c = -7$

$\alpha + \beta = -\frac{b}{a} \qquad \alpha\beta = \frac{c}{a}$

$= -\frac{2}{-2} \qquad = \frac{-7}{-2}$

$\boxed{\alpha + \beta = 1} \qquad \boxed{\alpha\beta = \frac{7}{2}}$

* Sum of roots

$\frac{2}{\alpha} + \frac{2}{\beta}$

$\frac{2\beta + 2\alpha}{\alpha\beta}$

$\frac{2\alpha + 2\beta}{\alpha\beta}$

$\frac{2(\alpha+\beta)}{\alpha\beta}$

$2(1) \div \frac{7}{2}$

$2 \times \frac{2}{7}$

$\frac{4}{7}$

* Product of roots

$\frac{2}{\alpha} \times \frac{2}{\beta} = \frac{4}{\alpha\beta}$

$= \frac{4}{\frac{7}{2}}$

$= 4 \div \frac{7}{2}$

$= 4 \times \frac{2}{7}$

$= \frac{8}{7}$

$x^2 - (\text{sum of roots})x + (\text{product of roots})$

$x^2 - \frac{4}{7}x + \frac{8}{7}$

$7x^2 - 4x + 8$

b. $x^2 - px + q = 0$

$a = 1 \quad b = -p \quad c = q$

$\alpha + \beta = -\frac{b}{a} \qquad \alpha\beta = \frac{c}{a}$

$= -\frac{(-p)}{1} \qquad \alpha\beta = \frac{q}{1}$

$\alpha + \beta = p \qquad \alpha\beta = q$

sum of roots

$\alpha^2 + p\alpha^2 + \beta^2 + p\beta^2$

$\alpha^2 + \beta^2 + p(\alpha^2 + \beta^2)$

$(\alpha+\beta)^2 - 2\alpha\beta + p[(\alpha+\beta)^2 - 2\alpha\beta]$

$(p)^2 - 2(q) + p[(p)^2 - 2(q)]$

$p^2 - 2q + p(p^2 - 2q)$

$p^2 - 2q + p^3 - 2pq$

Product of roots

$(\alpha^2 + p\alpha^2)(\beta^2 + p\beta^2)$

$\alpha^2\beta^2 + p\alpha^2\beta^2 + p\alpha^2\beta^2 + p^2\alpha^2\beta^2$

$(\alpha\beta)^2 + 2p(\alpha\beta)^2 + p^2(\alpha\beta)^2$

$(q)^2 + 2p(q)^2 + p^2(q)^2$

$q^2 + 2pq^2 + p^2q^2$

$x^2 - (\text{sum of roots})x + (\text{product of roots})$

$$x^2 - (p^2 - 2p + p^3 - 2pq)x + (q^2 + 2pq^2 + p^2q^2)$$

c $2x^2 - (p-4)x - (2p+1) = 0$

$a = 2$, $b = -(p-4)$, $c = -(2p+1)$

let the roots of the equation above be α & β

$$\alpha + \beta = \frac{-b}{a} = \frac{-(-(p-4))}{2}$$

$$\alpha + \beta = \frac{p-4}{2}$$

$$\alpha\beta = \frac{c}{a} = \frac{-(2p+1)}{2}$$

$$\alpha\beta = \frac{-2p-1}{2}$$

now the roots of the other equation exceed by 2 the previous α & β

old roots : α And β

new roots ; $\alpha + 2$ and $\beta + 2$

sum of roots ; $(\alpha + 2) + (\beta + 2)$

$\rightarrow \alpha + 2 + \beta + 2$

$\rightarrow \alpha + \beta + 4$

$\rightarrow \dfrac{p-4}{2} + \dfrac{4}{1}$

$\rightarrow \dfrac{p-4+8}{2}$

$\rightarrow \dfrac{p+4}{2}$

product of roots; $(\alpha + 2)(\beta + 2)$

$\rightarrow \alpha\beta + 2\alpha + 2\beta + 4$

$\rightarrow \alpha\beta + 2(\alpha + \beta) + 4$

$\rightarrow \left(\dfrac{-2p-1}{2}\right) + 2\left(\dfrac{p-4}{2}\right) + 4$

$\rightarrow \dfrac{-2p-1}{2} + p - 4 + 4$

$\rightarrow -\dfrac{1}{2}$

Finally !

$x^2 - (\text{sum of roots})x + (\text{product of roots})$

$$\left[x^2 - \left(\frac{p+r}{2}\right)x + \left(-\frac{1}{2}\right) \right] \times 2$$

$$\underline{\underline{2x^2 - (p+r)x - 1}}$$

19.(a)

(i) $3x^2 - 5x + k$, $a = 3$, $b = -5$, $c = k$

let the roots be α and $\alpha + 2$

$$\alpha + (\alpha + 2) = -\frac{b}{a}$$

$$\alpha + \alpha + 2 = -\frac{(-5)}{3}$$

$$2\alpha + 2 = \frac{5}{3}$$

$$2\alpha = \frac{5}{3} - 2$$

$$2\alpha = -\frac{1}{3}$$

$$\alpha = -\frac{1}{6}$$

$$\alpha \times (\alpha + 2) = \frac{c}{a}$$

$$\alpha^2 + 2\alpha = \frac{k}{3}$$

but $\alpha = -\frac{1}{6}$

$$\left(-\frac{1}{6}\right)^2 + 2\left(-\frac{1}{6}\right) = \frac{k}{3}$$

$$\frac{1}{36} - \frac{1}{3} = \frac{k}{3}$$

$$-\frac{11}{36} = \frac{k}{3}$$

$$-\frac{11}{12} = k$$

$$k = -\frac{11}{12}$$

ii $3x^2 + (k-1)x - 2 = 0$

roots are equal & opposite

$$D > 0$$

$$b^2 - 4ac > 0$$

$$b^2 > 4ac$$

$$(k-1)^2 > 4(3)(-2)$$

$$k^2 - 2k + 1 > -24$$

$$k^2 - 2k + 25 > 0$$

19. c i) $p(x) = -2x^2 + 280x - 1000$

$$p(x) = -2(x^2 - 140x + 500)$$
$$= -2(x^2 - 140x + (-70)^2 - (-70)^2 + 500)$$
$$= -2[(x-70)^2 - 4900 + 500]$$
$$= -2[(x-70)^2 - 4400]$$
$$p(x) = -2(x-70)^2 + 8800$$

$$x - 70 = 0 \Rightarrow x = 70$$

$\therefore$ 70 items need to be sold to gain the max profit of 8800.

ii

$$p = 2(l + b)$$
$$240 = 2(l + b)$$
$$140 = l + b$$
$$l = 140 - b$$

$$A = lb$$
$$A = (140 - b) b^2$$
$$A = 140b - b^2$$
$$A = -b^2 + 140b$$
$$A = -1(b^2 - 140b)$$
$$A = -1(b^2 - 140b + (-70)^2 - (70)^2)$$
$$A = -1[(b-70)^2 - 4900]$$
$$A = -(b-70)^2 + 4900$$

$$b - 70 = 0 \Rightarrow b = 70$$

$l = 140 - b$

$l = 140 - 70$

$l = 70$

$\therefore$ $l = 70m$ & $b = 70m$ to attain the maximum area of 4900 m

19/9

(i) $\cancel{3x^2 - 5x + k = 0}$

11

20

a

(i) $2x - 1 < 4(x - 3)$

$2x - 1 < 4x - 12$

$\frac{-2x}{-2} < \frac{-11}{-2}$

$x > \frac{11}{2}$

(ii) $\frac{3}{x-1} > 1$

$\frac{3}{x-1} - \frac{1}{1} > 0$

$\frac{3 - (x-1)}{x-1}$

$\frac{3 - x + 1}{x-1} > 0$

$\frac{4 - x}{x-1} > 0$

Critical points

$4 - x = 0$
$x = 4$

$x - 1 = 0$
$x = 1$

Range	$x < 1$	$1 < x < 4$	$x > 4$
Test value	0	2	5
$4 - x$	+	+	−
$x - 1$	−	+	+
	−	+	−

$1 < x < 4$

When your mom is angry
But you are a good Photographer 📸

(iii) $\dfrac{x-1}{x+1} > 2$

$\rightarrow \dfrac{x-1}{x+1} - \dfrac{2}{1} > 0$

$\rightarrow \dfrac{x-1-2(x+1)}{x+1} > 0$

$\rightarrow \dfrac{x-1-2x-2}{x+1} > 0$

$\rightarrow \dfrac{-x-3}{x+1} > 0$

\# Critical pimo pimo

$-x-3=0$ $\qquad$ $x+1=0$

$x=-3$ $\qquad$ $x=-1$

Range	$x<-3$	$-3<x<-1$	$x>-1$
Test value	-4	-2	0
$-x-3$	$+$	$-$	$-$
$x+1$	$-$	$-$	$+$
overall	$-$	$+$	$-$
		✓	

$$-3 < x < -1$$

(ii) $\dfrac{x+3}{x-1} \leq \dfrac{x-3}{x+1}$

$\dfrac{x+3}{x-1} - \dfrac{x-3}{x+1}$

$\dfrac{(x+1)(x+3) - (x-1)(x-3)}{(x-1)(x+1)} \leq 0$

$\dfrac{(x^2+3x+x+3) - (x^2-3x-x+3)}{(x-1)(x+1)} \leq 0$

$\dfrac{x^2+4x+3-x^2+4x-3}{(x-1)(x+1)} \leq 0$

$\dfrac{8x}{(x-1)(x+1)} \leq 0$

$8x = 0$	$x-1=0$	$x+1=0$
$x=0$	$x=1$	$x=-1$

Range	$x<-1$	$-1<x<0$	$0<x<1$	$x>1$
Test value	-2	$-\frac{1}{2}$	$\frac{1}{2}$	2
$8x$	$-$	$-$	$+$	$+$
$x-1$	$-$	$-$	$-$	$+$
$x+1$	$-$	$+$	$+$	$+$
overall	$-$	$+$	$-$	$+$

$x < -1$ or $0 \leq x < 1$

$x = (-\infty, -1) \cup [0, 1)$

(v) $|x-2| \geq 2$

$x-2 \geq 2$ or $x-2 \leq -2$

$x \geq 2+2$ $\quad$ $x \leq -2+2$

$x \geq 4$ $\quad$ $x \leq 0$

$x = [-\infty, 0] \cup [4, \infty)$

(vi) $\left|\dfrac{x-1}{x+2}\right| < 2$

$\dfrac{x-1}{x+2} < 2$ or $\dfrac{x-1}{x+2} > -2$

$\dfrac{x-1}{x+2} - \dfrac{2}{1} < 0$ $\quad$ $\dfrac{x-1}{x+2} + \dfrac{2}{1} > 0$

$\dfrac{x-1-2(x+2)}{x+2} < 0$ $\quad$ $\dfrac{x-1+2(x+2)}{x+2} > 0$

$\dfrac{x-1-2x-4}{x+2} < 0$ $\quad$ $\dfrac{x-1+2x+4}{x+2} > 0$

$\dfrac{-x-5}{x+2} < 0$ $\quad$ $\dfrac{3x+3}{x+2} > 0$

kilitiko $\quad$ Kilitiko

$-x-5=0$ $\quad$ $x+2=0$ $\quad$ $3x+3=0$ $\quad$ $x+2=0$

$x=-5$ $\quad$ $x=-2$ $\quad$ $x=-1$ $\quad$ $x=-2$

	$x<-5$	$-5<x<-2$	$x>-2$
$-x-5$	+	−	−
$x+2$	−	−	+
	−	+	−

	$x<-2$	$-2<x<-1$	$x>-1$
$3x+3$	−	−	+
$x+2$	−	+	+
	+	−	+

$x<-5$ or $x>-2$ $\quad$ $x<-2$ $\quad$ $x>-1$

Put the solutions on a number line and find the intersection!

$x < -5$ or $x > -1$

$x = (-\infty, -5) \cup (-1, \infty)$

(b) $-1 < \dfrac{2-x}{2+x} \leq 1$

$\dfrac{2-x}{2+x} > -1$ or $\dfrac{2-x}{2+x} \leq 1$

$\dfrac{2-x}{2+x} + \dfrac{1}{1} > 0$

$\dfrac{2-x+2+x}{2+x} > 0$

$\dfrac{4}{2+x} > 0$

$2+x = 0$

$x = -2$

$\dfrac{2-x}{2+x} - \dfrac{1}{1} \leq 0$

$\dfrac{2-x-1(2+x)}{2+x} \leq 0$

$\dfrac{2-x-2-x}{2+x} \leq 0$

$\dfrac{-2x}{2+x} \leq 0$

	$x < -2$	$x > -2$
$2+x$	$-$	$+$
overall	$-$	$+$

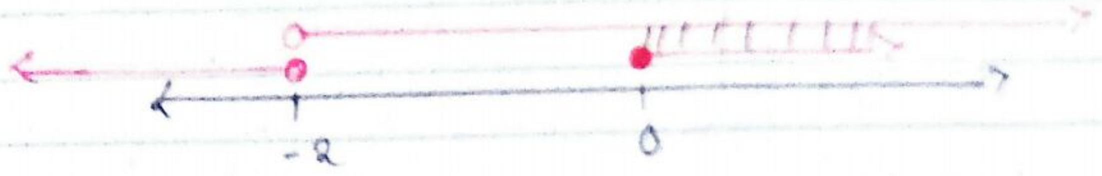
$x > -2$

$-2x = 0$, $x = 0$

$2+x = 0$, $x = -2$

	$x < -2$	$-2 < x < 0$	$x > 0$
$-2x$	$+$	$+$	$-$
$2+x$	$-$	$+$	$+$
	$-$	$+$	$-$

$x < -2$ or $x \geq 0$



$\cancel{x \geq}\ x \geq 0$

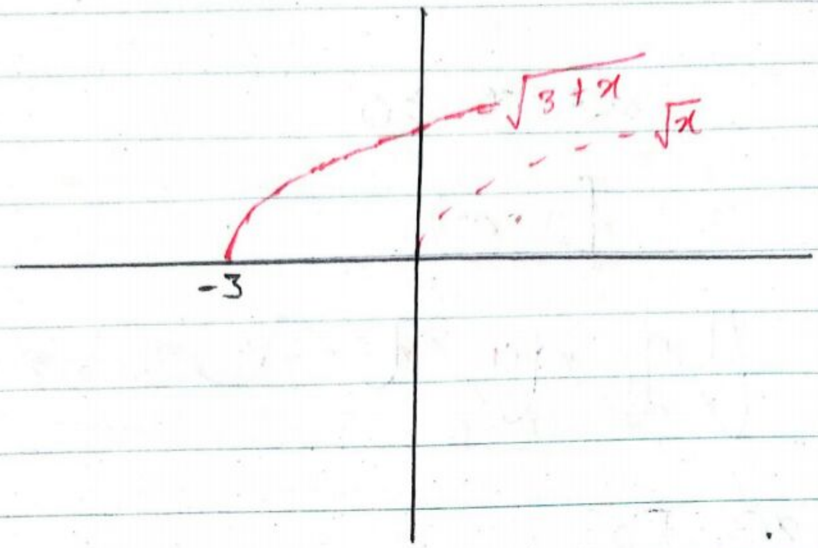
$x = [0, \infty)$

~~but when $x = -2$ the function is underfined~~

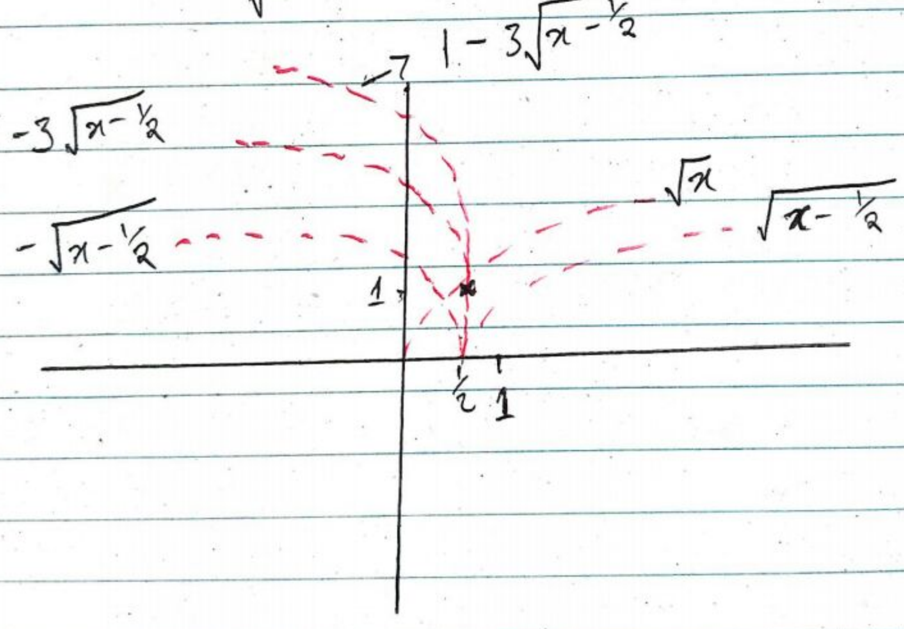
$x = [0,$

(21, a)

(i) $f(x) = \sqrt{3+x}$

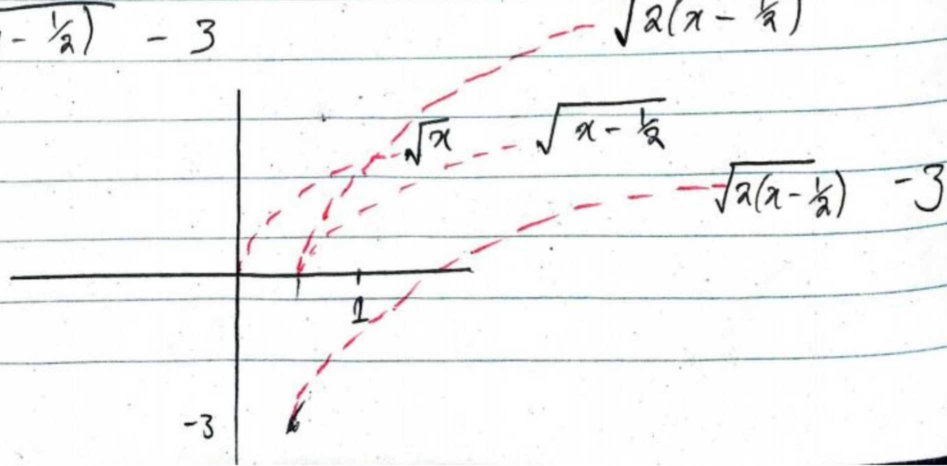


(ii) $f(x) = 1 - 3\sqrt{x-\frac{1}{2}}$



(iii) $f(x) = \sqrt{2x-1} - 3$

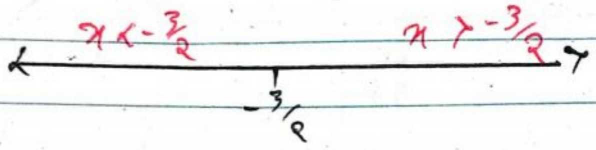
$= \sqrt{2(x-\frac{1}{2})} - 3$



(b.)

(i) $g(x) = |2x + 3|$

$$2x + 3 = 0$$
$$2x = -3$$
$$x = \frac{-3}{2}$$ Break point



$$g(x) = \begin{cases} |2x+3| & \text{for } x < -\frac{3}{2} \\ |2x+3| & x > -\frac{3}{2} \end{cases}$$

$$g(x) = \begin{cases} -(2x+3) & x < -\frac{3}{2} \\ 2x+3 & x > -\frac{3}{2} \end{cases}$$

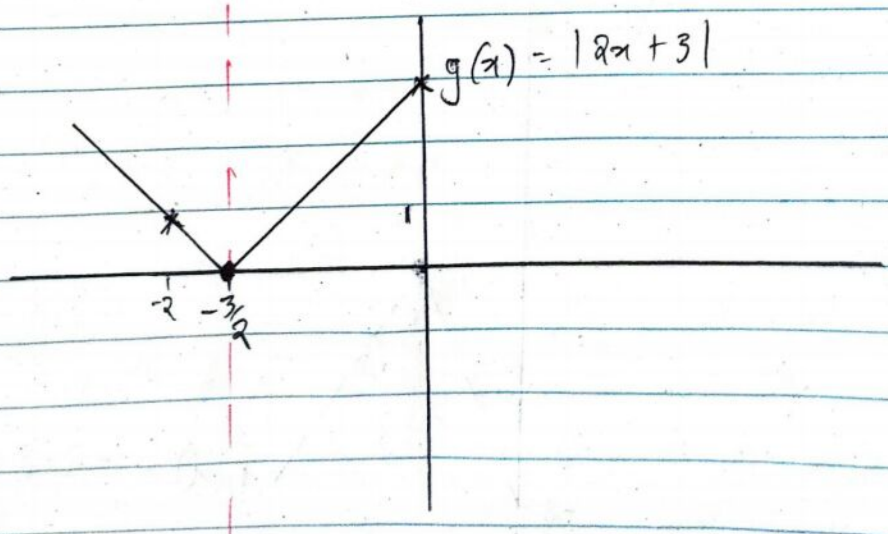
$$g(x) = \begin{cases} -2x-3 & x < -\frac{3}{2} \\ 2x+3 & x > -\frac{3}{2} \end{cases}$$

$y = -2x - 3$

x	-2	$-\frac{3}{2}$
y	1	0

$y = 2x + 3$

x	$-\frac{3}{2}$	0
y	0	3



(ii) $f(x) = -2|5x-4|$

$5x - 4 = 0$

$5x = 4$

$x = \frac{4}{5}$ → Break point

$x < \frac{4}{5}$ $\qquad$ $x > \frac{4}{5}$

$\frac{4}{5}$

$$F(x) = \begin{cases} -2|5x-4| & \text{if } x < \frac{4}{5} \\ -2|5x-4| & x \geq \frac{4}{5} \end{cases}$$

$$F(x) = \begin{cases} 2(5x-4) & x < \frac{4}{5} \\ -2(5x-4) & x \geq \frac{4}{5} \end{cases}$$

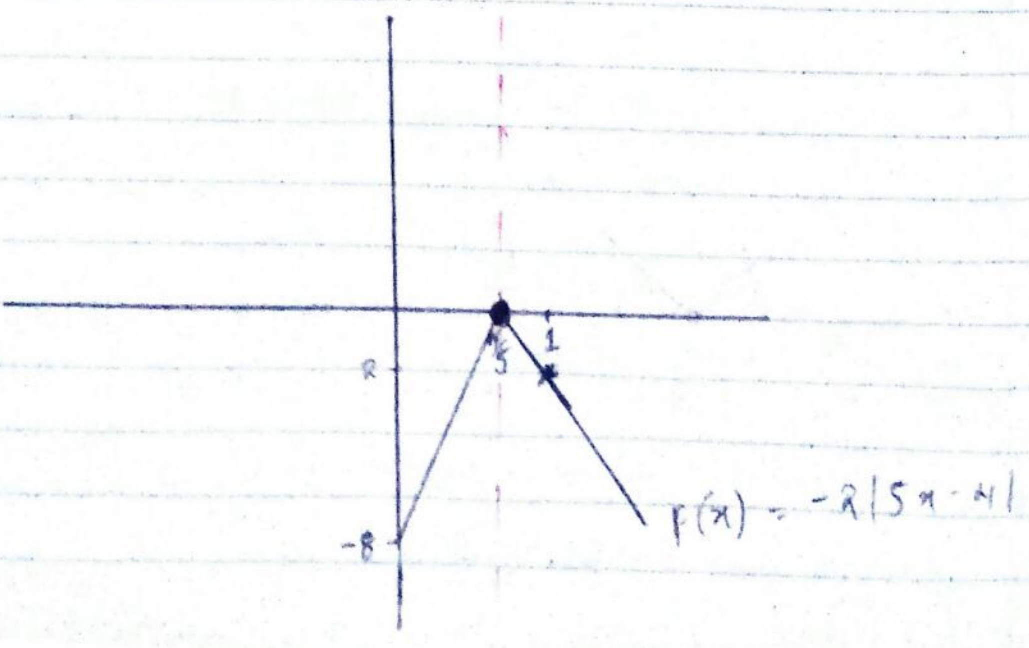
$$F(x) = \begin{cases} 10x - 8 & x < \frac{4}{5} \\ -10x + 8 & x \geq \frac{4}{5} \end{cases}$$

$y = 10x - 8$

x	0	$\frac{4}{5}$
y	-8	0

$y = -10x + 8$

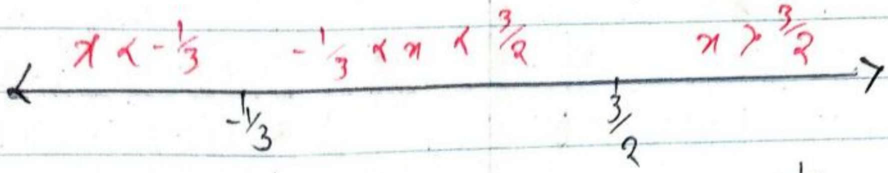
x	0	1
y	0	-2



(iii) $h(x) = |3x+1| + |2x-3|$

$3x+1 = 0 \qquad\qquad 2x-3=0$

$x = -\frac{1}{3} \qquad\qquad x = \frac{3}{2}$



$$h(x) = \begin{cases} |3x+1| + |2x-3| & \text{if } x < -\frac{1}{3} \\ |3x+1| + |2x-3| & \text{if } -\frac{1}{3} \leq x \leq \frac{3}{2} \\ |3x+1| + |2x-3| & \text{if } x > \frac{3}{2} \end{cases}$$

$$h(x) = \begin{cases} -(3x+1) - (2x-3) & \text{for } x < -\frac{1}{3} \\ (3x+1) - (2x-3) & \text{for } -\frac{1}{3} \leq x \leq \frac{3}{2} \\ (3x+1) + (2x-3) & \text{for } x > \frac{3}{2} \end{cases}$$

$$h(x) = \begin{cases} -3x-1-2x+3 & \text{for } x < -\frac{1}{3} \\ 3x+1-2x+3 & \text{for } -\frac{1}{3} \leq x \leq \frac{3}{2} \\ 3x+1+2x-3 & \text{for } x > \frac{3}{2} \end{cases}$$

$$h(x) = \begin{cases} -5x+2 & \text{for } x < -\frac{1}{3} \\ x+4 & \text{for } -\frac{1}{3} \leq x \leq \frac{3}{2} \\ 5x-2 & \text{for } x > \frac{3}{2} \end{cases}$$

$y = -5x+2$

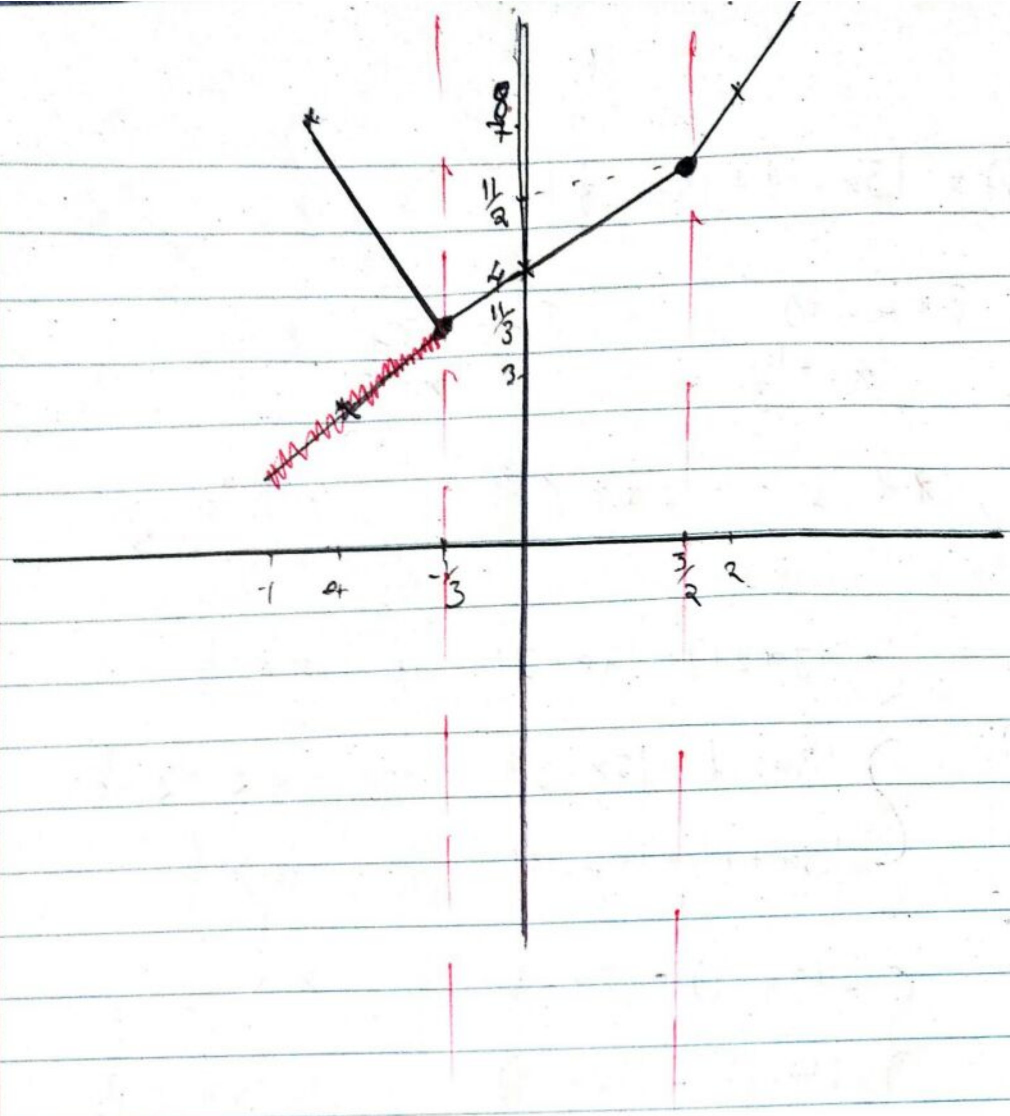
x	-1	$-\frac{1}{3}$
y	7	$\frac{11}{3}$

$y = x+4$

x	$-\frac{1}{3}$	0	$\frac{3}{2}$
y	$\frac{11}{3}$	4	$\frac{11}{2}$

$y = 5x-2$

x	$\frac{3}{2}$	2
y	$\frac{11}{2}$	8



(iv) $k(x) = |2x-1| - |x+2|$

$2x - 1 = 0 \qquad x + 2 = 0$

$x = \frac{1}{2} \qquad x = -2$

$x < -2 \qquad -2 < x < \frac{1}{2} \qquad x > \frac{1}{2}$

$-2 \qquad \frac{1}{2}$

$$K(x) = \begin{cases} |2x-1| - |x+2| & \text{for } x \leq -2 \\ |2x-1| - |x+2| & \text{for } -2 < x < \frac{1}{2} \\ |2x-1| - |x+2| & \text{for } x \geq \frac{1}{2} \end{cases}$$

$$K(x) = \begin{cases} -(2x-1) + (x+2) & \text{for } x \leq -2 \\ -(2x-1) - (x+2) & \text{for } -2 < x < \frac{1}{2} \\ (2x-1) - (x+2) & \text{for } x \geq \frac{1}{2} \end{cases}$$

$$K(x) = \begin{cases} -2x+1+x+2 & \text{por } x \leq -2 \\ -2x+1-x-2 & \text{por } -2 < x < \frac{1}{2} \\ 2x-1-x-2 & \text{por } x \geq \frac{1}{2} \end{cases}$$

$$K(x) = \begin{cases} -x+3 & \text{por } x \leq -2 \\ -3x-1 & \text{por } -2 < x < \frac{1}{2} \\ x-3 & \text{por } x \geq \frac{1}{2} \end{cases}$$

$y = -x + 3$

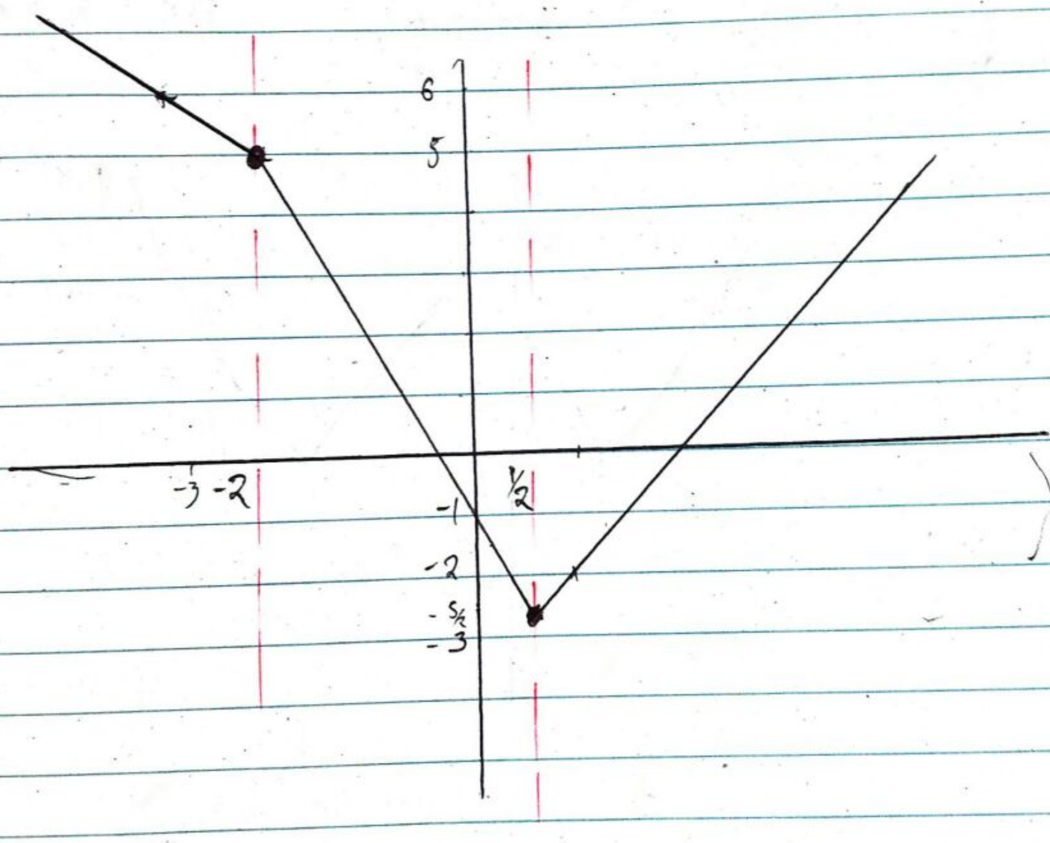
x	-3	-2
y	6	5

$y = -3x - 1$

x	-2	0	$\frac{1}{2}$
y	5	-1	$-\frac{5}{2}$

$y = x - 3$

x	$\frac{1}{2}$	1
y	$-\frac{5}{2}$	-2



(v) $p(x) = |x^2 - 3x - 4|$

$x^2 - 3x - 4 = 0$

$x^2 - 4x + x - 4 = 0$

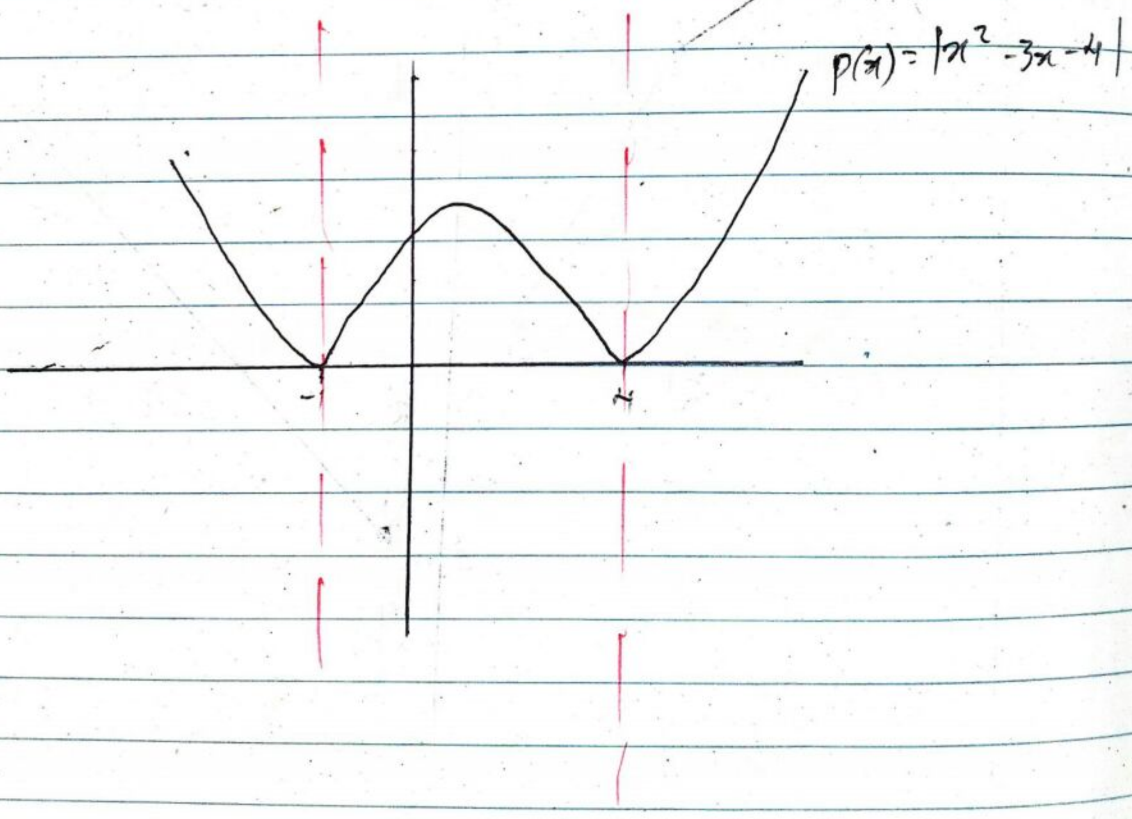
$x(x-4) + 1(x-4) = 0$

$(x+1)(x-4) = 0 \Rightarrow x = -1 \quad x = 4$

$$p(x) = \begin{cases} |x^2 - 3x - 4| & \text{for } x < -1 \\ |x^2 - 3x - 4| & -1 \le x \le 4 \\ |x^2 - 3x - 4| & x > 4 \end{cases}$$

$$p(x) = \begin{cases} x^2 - 3x - 4 & \text{for } x < -1 \\ -(x^2 - 3x - 4) & \text{for } -1 \le x \le 4 \\ x^2 - 3x - 4 & \text{for } x > 4 \end{cases}$$

$$p(x) = \begin{cases} x^2 - 3x - 4 & \text{for } x < -1 \\ -x^2 + 3x + 4 & \text{for } -1 \le x \le 4 \\ x^2 - 3x - 4 & \text{for } x > 4 \end{cases}$$



22

(a) Here just equat what you've been given to zero solve for x the replace the x in the whole polynomial

(a) $x - 1 = 0 \Rightarrow x = 1$

$x^3 + 2x^2 - x - 1$

$(1)^3 + 2(1)^2 - (1) - 1$

$1 + 2 - 1 - 1$

$\underline{1}$

$\therefore$ remainder is 1

you can do the rest !!

Q3

(a) $6x^3 + 7x^2 - 15x + 4$ by $x - 1$

$x - 1 = 0$

$x = 1$

1	6	7	-15	4
		6	13	-2
	6	13	-2	2

$6x^2 + 13x - 2$ no remainder 2

$p(x) = (x-1)(6x^2 + 13x - 2) + 2$

(b) $5 + 6x + 7x^2 - x^3$ by $x + 1$

$x + 1 = 0$

$x = -1$

$-x^2 + 7x^2 + 6x + 5$

$$
\begin{array}{r|rrrr}
-1 & -1 & 7 & 6 & 5 \\
 & & 1 & -8 & 2 \\
\hline
 & -1 & 8 & -2 & 7
\end{array}
$$

$-x^2 + 8x - 2$ with 7 as remainder

$$5 + 6x + 7x^2 - x^3 = (x+1)(-x^2 + 8x - 2) + 7$$

(c) Aah h

(d) $9x^3 + 4$ by $3x + 2$

$$
\begin{array}{r|l}
 & 3x^2 - 2x + \frac{4}{3} \\
\hline
3x+2 & 9x^3 + 0x^2 + 0x + 4 \\
 & -(9x^3 + 6x^2) \\
 & \quad -6x^2 + 0x \\
 & \quad -(-6x^2 - 4x) \\
 & \qquad 4x + 4 \\
 & \qquad -(4x + \frac{8}{3}) \\
\hline
 & \qquad\quad \frac{4}{3}
\end{array}
$$

$$9x^3 + 4 = (3x+2)\left(3x^2 - 2x + \tfrac{4}{3}\right) + \frac{4}{3}$$

(e.) With all that data you also want verification to this one too... lol, okay nga shaku kalipa call / text me 0965091047 (solved!!)

24

(a) $x^3 - 2x^2 - 5x + 6$, let $x = 1 \Rightarrow x - 1$

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -5 & 6 \\ & & 1 & -1 & -6 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$

$x^2 - x - 6$

$p = -6 \quad s = -1 \quad f = 2, -3$

$x^2 - 3x + 2x - 6$

$x(x-3) + 2(x-3) = 0$

$(x-3)(x+2)$

$\therefore \underline{x^3 - 2x^2 - 5x + 6 = (x-1)(x-3)(x+2)}$

(b.) $3x^3 + 2x^2 - 3x - 2$

let $x = 1 \Rightarrow (x - 1)$ is the factor

$$\begin{array}{r|rrrr} 1 & 3 & 2 & -3 & -2 \\ & & 3 & 5 & 2 \\ \hline & 3 & 5 & 2 & 0 \end{array}$$

$3x^2 + 5x + 2$

$p = 6 \quad s = 5 \quad f = 2\ 3$

$3x^2 + 3x + 2x + 2$

$3x(x+1) + 2(x+1) = 0$

$(3x+2)(x+1)$

$\therefore \underline{3x^3 + 2x^2 - 3x - 2 = (x-1)(3x+2)(x+1)}$

(c) $6x^3 - 13x^2 + 9x - 2$

$$
\begin{array}{r|rrrr}
1 & 6 & -13 & 9 & -2 \\
  &   & 6 & -7 & 2 \\
\hline
  & 6 & -7 & 2 & 0
\end{array}
$$

$6x^2 - 7x + 2$

$p = 12 \quad s = -7 \quad f = -4; 3$

$6x^2 - 3x - 4x + 2$

$3x(2x-1) - 2(2x-1) = 0$

$(2x-1)(3x-2) = 0$

$\underline{6x^3 - 13x^2 + 9x - 2 = (x-1)(2x-1)(3x-2)}$

(d.) $x^4 - 1 \Rightarrow x^4 + 0x^3 + 0x^2 + 0x - 1$

$$
\begin{array}{r|rrrrr}
1 & 1 & 0 & 0 & 0 & -1 \\
  &   & 1 & 1 & 1 & 1 \\
\hline
-1 & 1 & 1 & 1 & 1 & 0 \\
  &   & -1 & 0 & -1 & \\
\hline
  & 1 & 0 & 1 & 0 &
\end{array}
$$

$x^2 + 0x + 1 = 0$

$x^2 + 1 \rightarrow$ this can't be factored

$\underline{x^4 - 1 = (x-1)(x+1)(x^2+1)}$

(e) $x^4 - 2x^3 + x - 2 \Rightarrow x^4 - 2x^3 + 0x^2 + x - 2$

$$
\begin{array}{r|rrrrr}
-1 & 1 & -2 & 0 & 1 & -2 \\
  &   & -1 & 3 & -3 & 2 \\
\hline
  & 1 & -3 & 3 & -2 & 0
\end{array}
$$

$$
\begin{array}{c|cccc}
2 & 1 & -3 & 3 & -2 \\
  &   & 2 & -2 & 2 \\
\hline
  & 1 & -1 & 1 & 0
\end{array}
$$

$x^2 - x + 1 \rightarrow$ this can't be factored

$x^4 - 2x^3 + x - 2 = (x+1)(x-2)(x^2-x+1)$

25

(a) $6x^3 - 23x^2 + ax + b$

$x - 3 = 0$

$x = 3$

$6(3)^3 - 23(3)^2 + a(3) + b = 11$

$162 - 207 + 3a + b = 11$

$3a + b = 56 \quad \ldots (i)$

$x + 1 = 0$

$x = -1$

$6(-1)^3 - 23(-1)^2 + a(-1) + b = -21$

$-6 - 23 - a + b = -21$

$-a + b = 8 \quad \ldots (ii)$

$$
- \left| \begin{array}{l} 3a + b = 56 \\ -a + b = 8 \end{array} \right.
$$

$4a = 48$

$a = 12$

$\therefore \quad a = 12, \; b = 20$

$-a + b = 8$

$-12 + b = 8 \rightarrow b = 20$

0

b $2x^3 + px^2 - x - 2$ by $x + 3 = 0$

$x = -3$

$2(-3)^3 + p(-3)^2 - (-3) - 2$

$-54 + 9p + 3 - 2$

$R = 9p - 53$

c $x^2 - 4x - 2$ by $x - a$ & $x - b$

$(a)^2 - 4(a) - 2 = (b)^2 - 4(b) - 2$

$a^2 - 4a - 2 = b^2 - 4b - 2$

$a^2 - 4a = b^2 - 4b$

$a^2 - 4a = b^2 - 4b$

$a^2 - b^2 = 4a - 4b$

$$\frac{(a+b)\cancel{(a-b)}}{\cancel{a-b}} = \frac{4\cancel{(a-b)}}{\cancel{a-b}}$$

$a + b = 4$ hence shown !!

$x - 2a = 0$

$x = 2a$

$x^2 - 4x - 2$

$(2a)^2 - 4(2a) - 2 = 10$

$4a^2 - 8a - 2 = 10$

$4a^2 - 8a - 12 = 0$

$$4a^2 - 8a - 12 = 0$$
$$a^2 - 2a - 3 = 0$$

$p = -3 \quad s = -2 \quad f = 1, -3$

$$a^2 - 3a + a - 3$$
$$a(a - 3) + 1(a - 3) = 0$$
$$(a - 3)(a + 1) = 0$$

$a = 3$ or $a = -1$

$a + b = 4$	$a + b = 4$
$3 + b = 4$	$-1 + b = 4$
$b = 4 - 3$	$b = 4 + 1$
$b = 1$	$b = 5$
$\therefore \underline{a = 3}$	$\underline{a = -1}$
$\underline{b = 1}$	$\underline{b = 5}$

d $\quad 2x^3 - 7x^2 + 7x - 5 = A(x - 1)^3 + Bx(x - 1) + C$

first let us expand the Right side

$$A(x-1)^3 + Bx(x-1) + C$$
$$A(x-1)(x-1)^2 + Bx(x-1) + C$$
$$A(x-1)(x^2 - 2x + 1) + Bx(x-1) + C$$
$$A(x^3 - 2x^2 - x^2 + x + 2x - 1) + Bx(x-1) + C$$
$$A(x^3 - 3x^2 + 3x - 1) + Bx(x-1) + C$$
$$Ax^3 - 3Ax^2 + 3Ax - A + Bx^2 - Bx + C$$
$$Ax^3 - 3Ax^2 + Bx^2 + 3Ax - Bx - A + C$$
$$Ax^3 + (-3A + B)x^2 + (3A - B)x - A + C$$

now equat to the other side [left side]

$Ax^3 = 2x^3$

$\underline{A = 2}$

$-7x^2 = (-3A + B)x^2$

$-7 = -3A + B$

$A = 2$

$-7 = -3(2) + B$

$-7 = -6 + B$

$B = -7 + 6$

$\underline{B = -1}$

$-5 = -A + C$

$A = 2$

$-5 = -2 + C$

$C = -5 + 2$

$\underline{C = -3}$

$\therefore \underline{A = 2, \ B = -1, \ C = -3}$

Q6

(a) mmmmm... ba jobi

(b) $x^4 - 2x^3 - 3x^2 - 4x - 8$ by $x + 1 = 0$

$x = -1$

-1	1	-2	-3	-4	-8
		-1	3	0	4
	1	-3	0	-4	-4

$\underline{\text{Quotient} = x^3 - 3x^2 - 4}$

$\underline{\text{Remainder} = \frac{-4}{x+1}}$

c $8x^3 - 10x^2 + 7x + 3$ by $2x - 1$

$2x - 1 = 0 \Rightarrow x = \frac{1}{2}$

$\frac{1}{2}$	8	-10	7	3
		4	-3	2
	8	-6	4	5

quotient - $8x^2 - 6x + 4$ na remainder $\frac{5}{2x-1}$

28

(a) $x^3 - 3x^2 - 4x + 12$

-2	1	-3	-4	12
		-2	10	-12
	1	-5	6	0

$x^2 - 5x + 6$

$p = 6 \quad s = -5 \quad f = -2, -3$

$x^2 - 2x - 3x + 6$

$x(x-2) - 3(x-2)$

$(x-3)(x-2)$

$\therefore x^3 - 3x^2 - 4x + 12 = (x+2)(x-3)(x-2)$

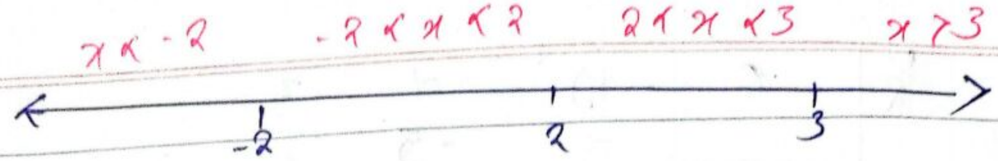
$x^3 - 4x > 3x^2 - 12$

$x^3 - 3x^2 - 4x + 12 > 0$

$(x+2)(x-3)(x-2) > 0$

$x = -2, \quad x = 3 \quad x = 2$

0



Range	$x < -2$	$-2 < x < 2$	$2 < x < 3$	$x > 3$
Test value	-3	0	2.5	4
$x+2$	$-$	$+$	$+$	$+$
$x-3$	$-$	$-$	$-$	$+$
$x-2$	$-$	$-$	$+$	$+$
Overall	$-$	$+$	$-$	$+$

$-2 < x < 2$ or $x > 3$

$x = (-2, 2) \cup (3, \infty)$

(b) $x^3 - 7x + 6 = 0 \Rightarrow x^3 + 0x^2 - 7x + 6$

$$\begin{array}{c|cccc} 1 & 1 & 0 & -7 & 6 \\ & & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$x^2 + x - 6$

$p = -6 \quad s = 1, \quad f = -2, 3$

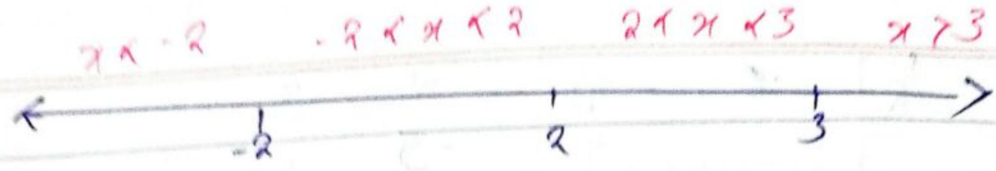
$x^2 - 2x + 3x - 6$

$x(x-2) + 3(x-2) = 0$

$(x+3)(x-2) = 0$

$x = -3, \quad x = 2$

$\therefore \; x = 1, \quad x = 2, \quad x = -3$



Range	$x<-2$	$-2<x<2$	$2<x<3$	$x>3$
Test value	-3	0	2.5	4
$x+2$	$-$	$+$	$+$	$+$
$x-3$	$-$	$-$	$-$	$+$
$x-2$	$-$	$-$	$+$	$+$
Overall	$-$	$+$	$-$	$+$

$-2<x<2$ or $x>3$

$x = (-2, 2) \cup (3, \infty)$

(b) $x^3 - 7x + 6 = 0 \Rightarrow x^3 + 0x^2 - 7x + 6$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -7 & 6 \\ & & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$x^2 + x - 6$

$p = -6 \quad s = 1, \quad f = -2, 3$

$x^2 - 2x + 3x - 6$

$x(x-2) + 3(x-2) = 0$

$(x+3)(x-2) = 0$

$x = -3, \quad x = 2$

$\therefore \; x = 1, \quad x = 2, \quad x = -3$

$$(x-2)^2 - 7(x-2) + 6 = 0$$
$$x^2 - 4x + 4 - 7x + 14 + 6 = 0$$
$$x^2 - 11x + 24 = 0$$
$$p = 24 \quad s = -11 \quad f = -3, -8$$
$$x^2 - 8x - 3x + 24$$
$$x(x-8) - 3(x-8) = 0$$
$$(x-3)(x-8) = 0$$
$$\underline{x = 3}, \quad \underline{x = 8}$$

(c) $y = 5x - 1$ $\qquad y = 2x^3 + x^2 + 1$

$$5x - 1 = 2x^3 + x^2 + 1$$
$$2x^3 + x^2 - 5x + 2$$

$$\begin{array}{c|cccc} 1 & 2 & 1 & -5 & 2 \\ & & 2 & 3 & -2 \\ \hline & 2 & 3 & -2 & 0 \end{array}$$

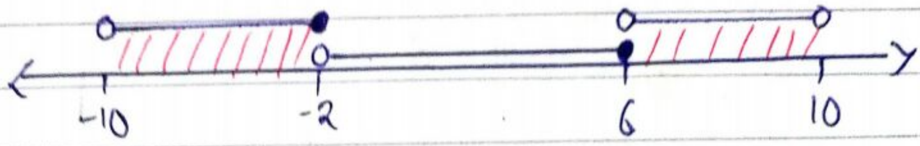
$$2x^2 + 3x - 2 = 0$$
$$p = -4 \quad s = 3 \quad f = -1, 4$$
$$2x^2 + 4x - x - 2$$
$$2x(x+2) - 1(x+2)$$
$$(x+2)(2x-1)$$
$$x = -2 \qquad x = \tfrac{1}{2}$$

∴ The x-cordinates where the line touches the curve is $\underline{x = 1}$, $\underline{x = -2}$, $\underline{x = \tfrac{1}{2}}$

Q9 $X = (-10, 10)$ $A = (-2, 6]$ $B = [-5, 3]$

$C = [-1, 8)$

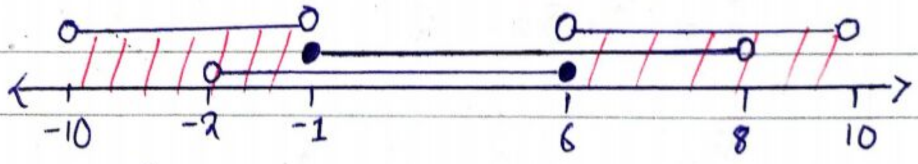
(i) A'



$A' = (-10, -2] \cup (6, 10)$

(ii) $X - A \Rightarrow X \cap A' \Rightarrow A'$

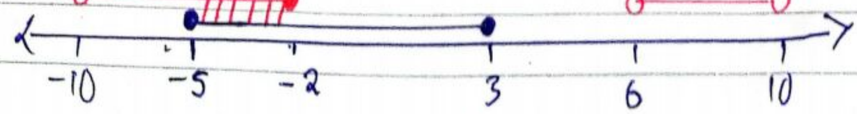
(iii) $(A \cap C)'$



$(A \cap C)' = (-10, -1) \cup (6, 10)$

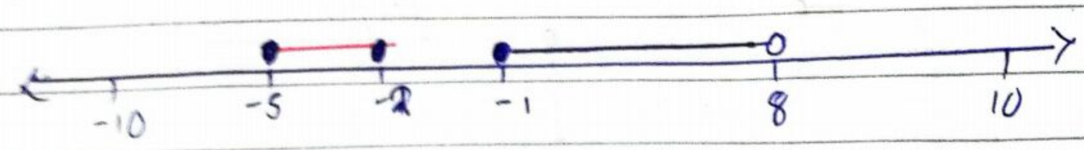
(iv) $(B - A) \cap C \Rightarrow (B \cap A') \cap C$

$B \cap A'$



$B \cap A' = [-5, -2]$

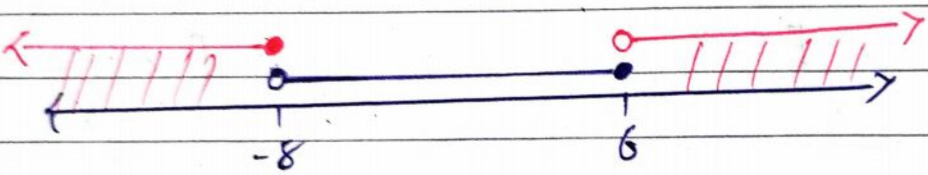
$(B \cap A') \cap C$



$(B \cap A') \cap C = \phi$

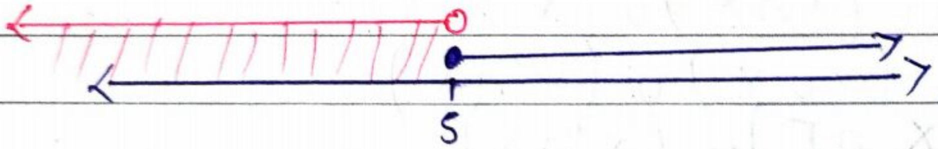
30 $A = (-8, 6]$ $B = [5, \infty)$

(i) A'



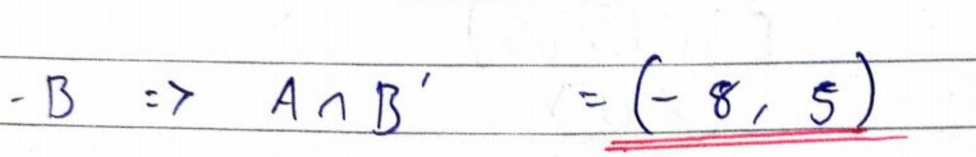
$A' = (-\infty, -8] \cup (6, \infty)$

(ii) B'

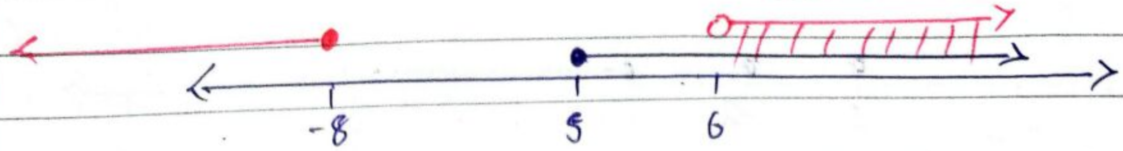


$B' = (-\infty, 5)$

(iii) $A - B \Rightarrow A \cap B' = (-8, 5)$



(iv) $B - A \Rightarrow B \cap A' = (6, \infty)$



31

(a.) $X = (X \cap Y) \cup (X \cap Y')$

R.H.S

$(X \cap Y) \cup (X \cap Y')$

$X \cap (Y \cup Y')$

$X \cap E$

X

(b) $X \cup (X' \cap Y) = X \cup Y$

L.H.S

$X \cup (X' \cap Y)$

$(X \cup X') \cap (X \cup Y)$

$E \cap (X \cup Y)$

$X \cup Y$

c $X \cup Y = (X \cap Y) \cup (X \cap Y') \cup (X' \cap Y)$

R.H.S

$(X \cap Y) \cup (X \cap Y') \cup (X' \cap Y)$

$X \cap (Y \cup Y') \cup (X' \cap Y)$

$(X \cap E) \cup (X' \cap Y)$

$X \cup (X' \cap Y)$

$(X \cup X') \cap (X \cup Y)$

$E \cap (X \cup Y)$

$X \cup Y$

$$\frac{h - x}{h\sqrt{5+h} + 3h}$$

(ii) $\frac{\sqrt{3} + \sqrt{5}}{7}$

$$\frac{\sqrt{3} + \sqrt{5}}{7}\left(\frac{\sqrt{3} - \sqrt{5}}{\sqrt{3} - \sqrt{5}}\right)$$

$$\frac{(\sqrt{3})^2 - (\sqrt{5})^2}{7(\sqrt{3} - \sqrt{5})}$$

$$\frac{3 - 5}{7\sqrt{3} - 7\sqrt{5}}$$

$$\frac{-2}{7\sqrt{3} - 7\sqrt{5}}$$

(iii) $\frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x} + \sqrt{x+h}}$

$$\frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x} + \sqrt{x+h}}\left(\frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}}\right)$$

$$\frac{(\sqrt{x})^2 - (\sqrt{x+h})^2}{h(x) + h(\sqrt{x})(\sqrt{x+h}) + (\sqrt{x})(\sqrt{x+h}) + x + h}$$

$$\frac{x - (x+h)}{xh + h\sqrt{x^2+xh} + \sqrt{x^2+xh} + x + h}$$

$$\frac{x - x - h}{xh + h\sqrt{x^2+xh} + \sqrt{x^2+xh} + x + h}$$

(c) $\frac{\sqrt{5+h} - 3}{h}$

$$\frac{\sqrt{5+h} - 3}{h}\left(\frac{\sqrt{5+h} + 3}{\sqrt{5+h} + 3}\right)$$

$$\frac{(\sqrt{5+h})^2 - (3)^2}{h(\sqrt{5+h} + 3)}$$

$$\frac{5 + h - 9}{h\sqrt{5+h} + h3}$$

#

$$\frac{-h}{xh + h\sqrt{x^2+xh} + \sqrt{x^2+xh} + x + h}$$

If Moses had a smartphone

(iv) $\sqrt{x^2+1} - x$

$$\rightarrow \frac{\left(\sqrt{x^2+1} - x\right)}{1} \left(\frac{\sqrt{x^2+1} + x}{\sqrt{x^2+1} + x}\right)$$

$$\rightarrow \frac{\left(\sqrt{x^2+1}\right)^2 - (x)^2}{\sqrt{x^2+1} + x}$$

$$\rightarrow \frac{x^2 + 1 - x^2}{\sqrt{x^2+1} + x}$$

$$\rightarrow \frac{1}{\sqrt{x^2+1} + x}$$

33

(i) $y = 3x - 1$

the equation is in the form $y = mx + c$, this is a straight line my friend. Hence this is a function.

(ii) $$y = \begin{cases} 2x + 3 & x \leq 1 \\ 6 - x^2 & x \geq 1 \end{cases}$$

This guy is a function becoz for every x value we have one y value therefore this is a function.

(iii)

$$y = \begin{cases} x, & x \leq 3 \\ -x, & x \geq 3 \end{cases}$$

This one ain't a function for one x value we have two y values

$$y = \begin{cases} x, & x \leq 3 \\ -x, & x \geq 3 \end{cases}$$

$x = 3$

$y = x$ or $y = -x$

$y = 3$ $\quad$ $y = -3$

Hence this is not a function

(iv)

$$y = \begin{cases} x^3, & x < 2 \\ 1/x, & x \geq 2 \end{cases}$$

for every x value we will only have one y value

34

(i) $f(x) = 1 - x^4$

$F(x) = f(-x)$

$f(-x) = 1 - (-x)^4$

$f(-x) = 1 - x^4 \rightarrow$ Even

$f(-x) = -f(x)$

$-f(x) = -(1 - x^4)$

$-f(x) = -1 + x^4$

$f(-x) \neq -f(x) \rightarrow$ not odd

(ii) $f(x) = x^2 - x$

$f(-x) = (-x)^2 - (-x)$

$f(-x) = x^2 + x$

$f(x) \neq f(-x) \rightarrow$ not Even

$-F(x) = -(x^2 - x)$

$-F(x) = -x^2 + x$

$F(-x) \neq -f(x) \rightarrow$ not odd

(iii) $f(x) = 3x^3 + 2x - 1$

$F(-x) = 3(-x)^3 + 2(-x) - 1$

$f(-x) = -3x^3 - 2x - 1$

$F(x) \neq F(-x) \rightarrow$ not even

$-f(x) = -(3x^3 + 2x - 1$

$-f(x) = -3x^3 - 2x + 1$

$f(-x) \neq -f(x) \rightarrow$ not odd

(iv) $f(x) = x + \frac{1}{x}$

$f(-x) = (-x) + \frac{1}{(-x)}$

$f(-x) = -x - \frac{1}{x}$

$f(x) \neq f(-x) \rightarrow$ not even

$-f(x) = -\left(x + \frac{1}{x}\right)$

$-f(x) = -x - \frac{1}{x}$

$f(-x) = -f(x) \rightarrow$ it is odd

35 a

(i) $3.12\overline{12} \Rightarrow 3.\overline{12}$

$x = 3.\overline{12} \quad \ldots (i)$

$100x = 312.\overline{12} \quad \ldots (ii)$

$100x - x = 312.\overline{12} - 3.\overline{12}$

$\frac{99x}{99} = \frac{309}{99}$

$x = \frac{103}{33}$

(ii) $\dfrac{5}{3-2\sqrt{3}}$

$$\frac{5}{3-2\sqrt{3}}\left(\frac{3+2\sqrt{3}}{3+2\sqrt{3}}\right)$$

$$\frac{15+10\sqrt{3}}{(3)^2-(-2\sqrt{3})^2}$$

$$\frac{15+10\sqrt{3}}{9-4(3)}$$

$$\frac{15+10\sqrt{3}}{9-12}$$

$$\frac{15+10\sqrt{3}}{-3}$$

$$\frac{15}{-3}+\frac{10\sqrt{3}}{-3}$$

$$-5-\frac{10\sqrt{3}}{3}$$

b

(i) $F(x) = \dfrac{x+1}{2x-1}$ $\qquad g(x) = \dfrac{1}{x}$

$$F\left(\tfrac{1}{4}\right) = \frac{\frac{1}{4}+1}{2\left(\frac{1}{4}\right)-1}$$

$$= \frac{\frac{5}{4}}{-\frac{1}{2}}$$

$$= \frac{5}{4} \div -\frac{1}{2}$$

$$= \frac{5}{4} \times -\frac{2}{1}$$

$$F\left(\tfrac{1}{4}\right) = -\frac{5}{2}$$

(ii) $F(x) = \dfrac{x+1}{2x-1}$

Domain

$2x - 1 = 0$

$2x = 1$

$x = \frac{1}{2}$

$\therefore$ Domain is all real numbers except $\frac{1}{2}$

$(-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$

Range

$$y = \frac{x+1}{2x-1}$$

$$2xy - y = x + 1$$

$$2xy - x = y + 1$$

$$\frac{x(2y-1)}{2y-1} = \frac{y+1}{2y-1}$$

$$x = \frac{y+1}{2y-1}$$

$$2y - 1 = 0$$

$$2y = 1$$

$$y = \frac{1}{2}$$

$$\text{Range} = \left(-\infty, \tfrac{1}{2}\right) \cup \left(\tfrac{1}{2}, \infty\right)$$

(iii) $f(x) = \dfrac{x+1}{2x-1}$

$$f(a) = f(b)$$

$$\frac{a+1}{2a-1} = \frac{b+1}{2b-1}$$

$$(a+1)(2b-1) = (2a-1)(b+1)$$

$$\cancel{2ab} - a + 2b - \cancel{1} = \cancel{2ab} + 2a - b - \cancel{1}$$

$$-a + 2b = 2a - b$$

$$2b + b = a + 2a$$

$$3b = 3a$$

$$b = a$$

Hence shown!

(iv) $(f \circ g)(x) = \dfrac{\frac{1}{x} + 1}{2\left(\frac{1}{x}\right) - 1}$

$$= \left(\frac{1}{x} + \frac{1}{1}\right) \div \left(\frac{2}{x} - \frac{1}{1}\right)$$

$$= \left(\frac{1 + x}{x}\right) \div \left(\frac{2 - x}{x}\right)$$

$$= \left(\frac{1 + x}{x}\right) \times \left(\frac{x}{2 - x}\right)$$

$$(f \circ g)(x) = \frac{1 + x}{2 - x}$$

C $1 + \sqrt{3}$

Proof by Contradiction

Assume that $1 + \sqrt{3}$ is rational such that it can be expressed in the form a/b such that $a, b \in \mathbb{Z}$ and $b \neq 0$.

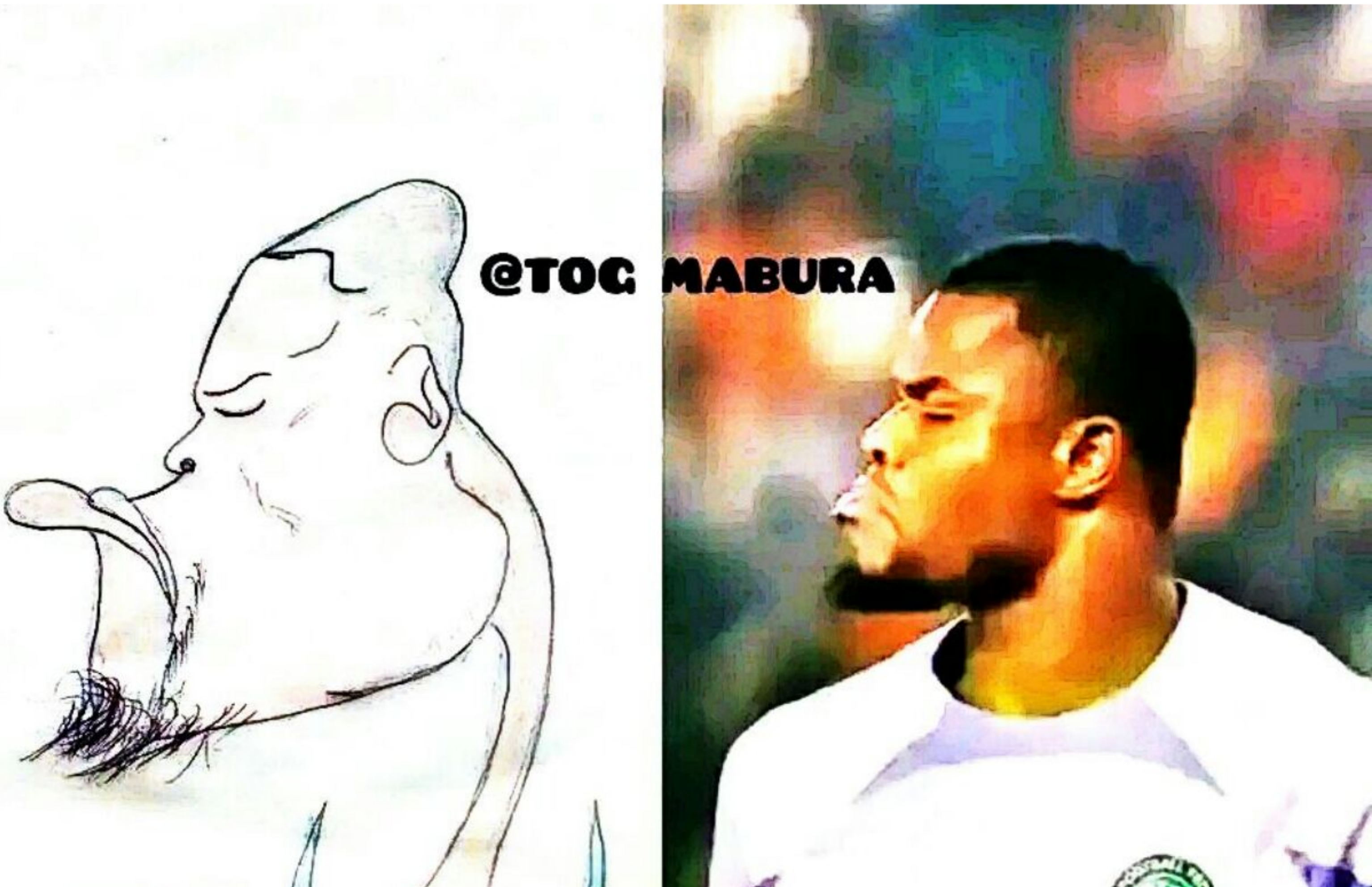
$$1 + \sqrt{3} = a/b$$

$$\sqrt{3} = \frac{a}{b} - 1$$

$$\sqrt{3} = \frac{a - b}{b}$$

$\therefore$ since a, b are integers it means $\dfrac{a - b}{b}$

is rational. but $\sqrt{3}$ is irrational and a rational number can't be equal to an irrational number, therefore by method of contradiction $1 + \sqrt{3}$ is irrational.


@TOG MABURA
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