THE COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

PHYSICS DEPARTMENT

2018/2019 ACADEMIC YEAR

PH 110 TEST 1

INTRODUCTORY PHYSICS

INSTRUCTIONS:

1. THERE ARE FOUR (**4**) QUESTIONS IN THIS PAPER AND EACH QUESTION CARRIES **25** MARKS. ATTEMPT **ALL** QUESTIONS.
2. WRITE YOUR STUDENT IDENTIFICATION NUMBER **(SIN).**
3. INDICATE YOUR GROUP e.g GROUP **A**, GROUP **B**, GROUP **C**, e.t.c

DURATION: TWO (**2**) HOURS

USE THE FOLLOWING DATA WHERE NECESSARY:

Acceleration due to gravity, $g = 9.8\ m/s^2$

$\eta = Kgm^{-1}s^{-1}$

QUESTION 1

a) A dimension is a physical nature of a quantity.

(i) Give two (2) limitations of dimensional analysis. [2 Marks]

(ii) If velocity (**V**), time (T) and force (**F**) were chosen as basic quantities, find the dimensions of mass. [4 Marks]

b) If the units of force, energy and velocity in a new system are 10N, 5J and 0.5 m/s respectively, find the units of mass, length and time in that system. [8 Marks]

c) When a small sphere moves at low speed through a fluid, the viscous force **F** opposing the motion is found experimentally to depend on the radius r, velocity v and the viscosity η of the fluid. Find the force dimension of the force **F**. [5 Marks]

d) A cheap wrist watch loses time at the rate of 8.5s a day. How much time will the watch be off at the end of a month? Given that a month has 30 days. [3 Marks]

e) A solid cube of aluminum (density of 2.7 g/cm^3) has a volume of 0.9 cm^3. How many atoms are contained in the cube? [3 Marks]

QUESTION 2

(a) Given a vector $\vec{A} = 3\hat{i} + 6\hat{j} - 2\hat{k}$. Find another vector $\vec{B}$ which is parallel to vector $\vec{A}$ and has a magnitude of 17 units. [5 marks]

(b) The following instructions lead to a buried treasure: Go 75 m 30° west of south, turn northwest and walk 125 m, then travel 100 m 20° north of west. Determine the resultant displacement from the starting point. [7 marks]

(c) Which two of the following three vectors are parallel to each other:

$\vec{A} = 3\hat{i} + 6\hat{j} - 2\hat{k}$, $\vec{B} = \hat{i} + 5\hat{j} + 8\hat{k}$, and $\vec{C} = \frac{3}{2}\hat{i} + 3\hat{j} - \hat{k}$. [3 marks]

(d) Find the angles which the vector $\vec{A} = 3\hat{i} - 6\hat{j} + 2\hat{k}$ makes with the coordinate axes. [6 marks]

(e) Determine a vector perpendicular to the plane of $\vec{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\vec{B} = 4\hat{i} + 3\hat{j} - \hat{k}$. [4 marks]

QUESTION 3

(a) A stone thrown horizontally from the top of a 24 m tower hits the ground at a point 18 m from the base of the tower.

(i) Find the speed at which the stone was thrown. [4 marks]

(ii) Find the speed of the stone just before it hits the ground. [3 marks]

(b) A marble rolls horizontally off a table with a speed of 3.7 m/s. A second marble is dropped vertically from the table at the same instant. If the table is 1.20 m high,

(i) how far apart do the marbles land? [3 marks]

(ii) what difference is there in the times of impact of the two marbles? [3 marks]

(c) Just as a car starts to accelerate from rest at a constant acceleration of 2.44 m/s^2, a bus moving at a constant speed of 19.6 m/s passes the car in a parallel lane.

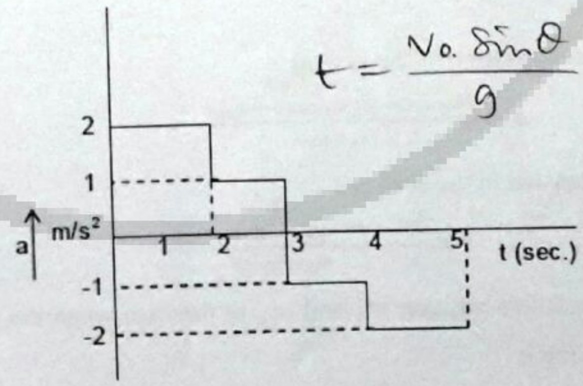
(i) How long does it take the car to overtake the bus? [3 marks]

(ii) How fast is the car going?. [2 marks]

(iii) How far has the car gone at that point? [2 marks]

(d) A car starting from rest moving on a straight line has acceleration – time graph as shown in the figure below. Draw the velocity – time graph. [5 marks]

$t = \frac{V_0 . \sin\theta}{g}$



QUESTION 4

(a) Explain briefly why

(i) action and reaction (equal and opposite) forces do not cancel each other out, resulting in zero net force. [2 marks]

(ii) a man standing in an elevator going down with uniform acceleration experiences loss of weight [2 marks]

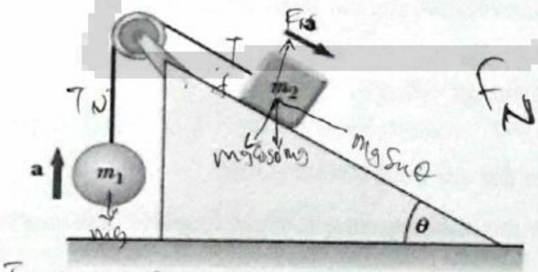
(b) An object of mass 10 g is moving in a plane. Its position as a function of time is given by

$$\vec{r} = (2t^3 - t^2)\vec{i} + (4t^3 + 2t)\vec{j}$$

Find the magnitude and direction of resultant force acting on the object at $t = 2$ s.

[5 marks]

(c) A ball of mass m_1 hanging vertically is connected to a block of mass m_2 lying on the rough inclined surface by means of a light non-stretchable cord over a massless, frictionless pulley as shown in the figure below. The surface is inclined at an angle θ



$f_N - mg = ma$

$N - T - mg = ma$

Show that:

(i) the acceleration of the system is

$$a = \frac{(m_2 \sin\theta - \mu_k m_2 \cos\theta - m_1)g}{m_1 + m_2}$$ [5 marks]

$+ m_2 \cos\theta$

(ii) the tension in the cord is

$$T = \frac{(1 + \sin\theta - \mu_k \cos\theta) m_1 m_2 g}{m_1 + m_2}$$ [3 marks]

(iii) the relation between m_1 and m_2 in the case when the system is on the verge of slipping is

$$m_1 = (\sin\theta - \mu_s \cos\theta) m_2$$ [3 marks]

where μ_s, is the coefficient of static friction, and μ_k is the coefficient of kinetic friction between the block and the surface.

(c) A CBU physics student shopping at Mukuba mall pushes a trolley of mass 18 kg along a rough horizontal surface at a constant speed. To do this requires a force of 80 N directed along the handle, which is at an angle of 40 degrees below the horizontal. Calculate the coefficient of kinetic friction between the wheels of trolley and the surface. [5 marks]

QUESTION 2

[a] $\vec{A} = 3\hat{i} + 6\hat{j} - 2\hat{k}$

→ Find a unit vector in the direction of vector $\vec{A}$.

$$\hat{n} = \frac{\vec{A}}{\|\vec{A}\|}$$

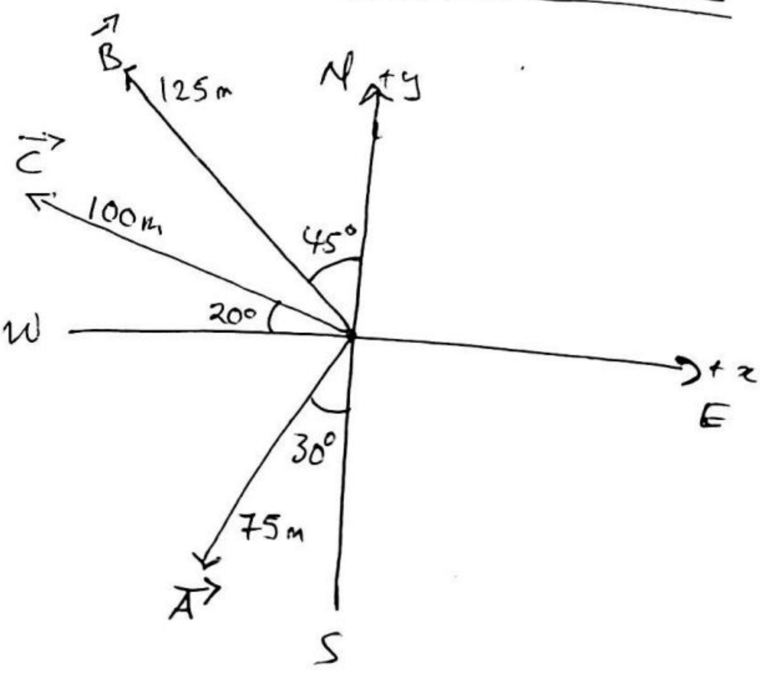
$$\|\vec{A}\| = \sqrt{3^2 + 6^2 + 2^2} = 7$$

$$\therefore \quad \hat{n} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{7}$$

→ The magnitude of a unit vector is one. In order to get a vector with a magnitude of 17 units in the direction of $\vec{A}$, we have to multiply the unit vector in the direction of $\vec{A}$ by 17.

$$\therefore \quad \vec{B} = 17\hat{n} = \frac{17}{7}\left(3\hat{i} + 6\hat{j} - 2\hat{k}\right) = \frac{51}{7}\hat{i} + \frac{102}{7}\hat{j} - \frac{34}{7}\hat{k}$$

[b]



$\vec{A} \equiv r = 75, \ \theta = 240°$

$\vec{B} \equiv r = 125, \ \theta = 135°$

$\vec{C} \equiv r = 100, \ \theta = 160°$

$x = r\cos\theta$ and $y = r\sin\theta$

$\vec{A} = -37.5\hat{i} - 65\hat{j}$

$\vec{B} = -88.4\hat{i} + 88.4\hat{j}$

$\vec{C} = -94\hat{i} + 34.2\hat{j}$

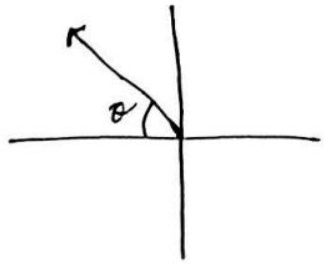
$$\therefore \quad \vec{R} = \vec{A} + \vec{B} + \vec{C} = -220\hat{i} + 57.6\hat{j}$$

QUESTION 2

PAGE

$$\|\vec{R}\| = \sqrt{220^2 + 57.6^2} = \underline{\underline{227.4\ m}}$$

$$\theta = \tan^{-1}\left(\frac{57.6}{220}\right) = \underline{\underline{14.7^\circ}}$$



~~The displacement is 2~~

$\therefore$ The resultant displacement is $227.4\ m$, 14.7° north of west.

[C] Note: The vector product of two parallel vectors is zero.

$$\vec{A} \wedge \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 6 & -2 \\ 1 & 5 & 8 \end{vmatrix} = \hat{i}\begin{vmatrix} 6 & -2 \\ 5 & 8 \end{vmatrix} - \hat{j}\begin{vmatrix} 3 & -2 \\ 1 & 8 \end{vmatrix} + \hat{k}\begin{vmatrix} 3 & 6 \\ 1 & 5 \end{vmatrix}$$

$$\vec{A} \wedge \vec{B} = \hat{i}\left[(6\times 8) - (-2\times 5)\right] - \hat{j}\left[(3\times 8) - (-2\times 1)\right] + \hat{k}\left[15 - 6\right]$$

$$\vec{A} \wedge \vec{B} = 58\hat{i} - 26\hat{j} + 9\hat{k}$$

$\therefore$ $\vec{A}$ is not parallel to $\vec{B}$

$$\vec{A} \wedge \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 6 & -2 \\ 3/2 & 3 & -1 \end{vmatrix} = \hat{i}\left(-6-(-6)\right) - \hat{j}\left(-3-(-3)\right) + \hat{k}(9-9)$$

$$\vec{A} \wedge \vec{C} = 0$$

$\therefore$ $\vec{A}$ is parallel to $\vec{C}$. This ~~shows~~ means that $\vec{B}$ is not parallel to $\vec{C}$.

[4] $\vec{A} = 3\hat{i} - 6\hat{j} + 2\hat{k}$

$\|\vec{A}\| = \sqrt{3^2 + 6^2 + 2^2} = \underline{7}$

(i) <u>Angle between $\vec{A}$ and the x-axis</u>

$\vec{A} \cdot \vec{i} = \|\vec{A}\| \|\vec{i}\| \cos\theta$

$\vec{A} \cdot \vec{i} = (3\hat{i} - 6\hat{j} + 2\hat{k}) \cdot \vec{i} = \underline{3}$

$3 = 7 \times 1 \times \cos\theta$

$\frac{7\cos\theta}{7} = \frac{3}{7}$

$\cos\theta = \frac{3}{7}$

$\theta = \cos^{-1}\left(\frac{3}{7}\right) = \underline{64.6^\circ}$

$\therefore$ The angle between $\vec{A}$ and the x-axis is $\underline{64.6^\circ}$

(ii) <u>Angle between $\vec{A}$ and the y-axis</u>

$\vec{A} \cdot \vec{j} = (3\hat{i} - 6\hat{j} + 2\hat{k}) \cdot \vec{j} = \underline{-6}$

$\vec{A} \cdot \vec{j} = \|\vec{A}\| \|\vec{j}\| \cos\theta$

$-6 = 7\cos\theta \iff \cos\theta = -\frac{6}{7}$

$\theta = \cos^{-1}\left(-\frac{6}{7}\right) = \underline{149^\circ}$

$\therefore$ The angle between $\vec{A}$ and the y-axis is $\underline{149^\circ}$

(iii) Angle between $\vec{A}$ and the z-axis

$\vec{A} \cdot \hat{k} = (3\hat{i} - 6\hat{j} + 2\hat{k}) \cdot \hat{k} = \underline{2}$

$\vec{A} \cdot \vec{k} = \|\vec{A}\| \|\vec{k}\| \cos\theta$

$2 = 7\cos\theta \iff \cos\theta = \frac{2}{7}$

$\theta = \cos^{-1}\left(\frac{2}{7}\right) = \underline{73.4^\circ}$

$\therefore$ The angle between $\vec{A}$ and the z-axis is $\underline{73.4^\circ}$

e]

$$\vec{A} \wedge \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix} = \hat{i}\left[6-(-9)\right] - \hat{j}\left[-2-(-12)\right] + \hat{k}\left(6-(-24)\right)$$

$$\vec{A} \wedge \vec{B} = 15\hat{i} - 10\hat{j} + 30\hat{k}$$

$$\therefore \quad \underline{\underline{\vec{A} \wedge \vec{B} = 15\hat{i} - 10\hat{j} + 30\hat{k}}}$$

We can go beyond this point if we wish to, by determining the unit vector.

$$\|\vec{A} \wedge \vec{B}\| = \sqrt{15^2 + 10^2 + 30^2} = 35$$

$$\hat{n} = \frac{\vec{A} \wedge \vec{B}}{\|\vec{A} \wedge \vec{B}\|} = \frac{1}{35}\left(15\hat{i} - 10\hat{j} + 30\hat{k}\right) = \frac{15}{35}\hat{i} - \frac{10}{35}\hat{j} + \frac{30}{35}\hat{k}$$

$$\underline{\underline{\hat{n} = \frac{\vec{A} \wedge \vec{B}}{\|\vec{A} \wedge \vec{B}\|} = \frac{3}{7}\hat{i} - \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k}}}$$

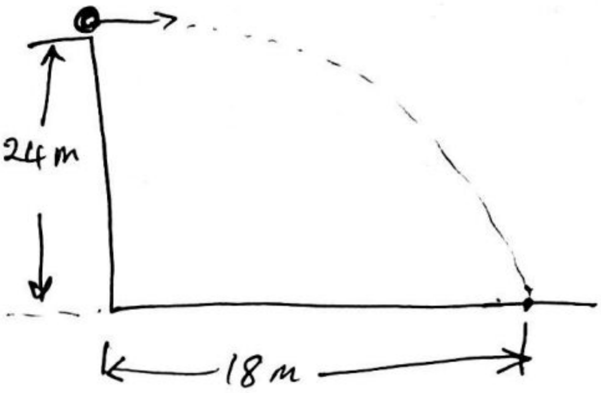
QUESTION THREE

[a]

(i) Y-axis

$X = 0\,m$
$X_0 = 24\,m$
$a = -9.81$
$t = ?$
$u = 0\,m/s$

$$X - X_0 = ut + \frac{1}{2}at^2$$
$$0 - 24 = 0t + \frac{1}{2}(-9.81)t^2$$
$$\frac{-24}{-4.9} = \frac{-4.9t^2}{-4.9}$$
$$t^2 = 4.898$$
$$t = 2.2\ sec$$



X-axis

$X = 18$
$X_0 = 0\,m$
$t = 2.2\ sec$
$u = ?$
$a = 0\,m/s^2$

$$X - X_0 = ut + \frac{1}{2}at^2$$
$$18 - 0 = u(2.2) + \frac{1}{2}(0)(2.2)^2$$
$$\frac{18}{2.2} = \frac{2.2u}{2.2}$$
$$u = 8.2\,m/s$$

$\therefore$ The Stone was thrown at a speed of 8.2 m/s.

(ii)
$$V_y = U_y + at$$
$$V_y = 0 + (-9.81)(2.2)$$
$$\cancel{V_y = -19.6}$$
$$V_y = -21.6\,m/s$$

Note: $V_x = 8.2\,m/s$

$$V = \sqrt{V_y^2 + V_x^2} = \sqrt{(21.6)^2 + (8.2)^2}$$
$$V = 23.1\,m/s$$

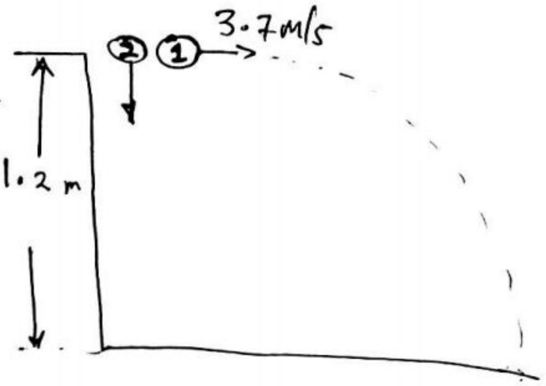
$\therefore$ It's speed just before it hits the ground is 23.1 m/s

[b]

(i) Y-axis

$x = 0m$
$x_0 = 1.2$
$u = 0m$
$a = -9.81$
$t = ?$

$$x - x_0 = ut + \frac{1}{2}at^2$$
$$0 - 1.2 = 0t + \frac{1}{2}(-9.81)t^2$$
$$-1.2 = -4.9t^2$$
$$\frac{-1.2}{-4.9} = \frac{-4.9t^2}{-4.9}$$
$$t^2 = 0.2449$$
$$t = 0.49\,sec$$



X-axis

$x_0 = 0$
$x = x$
$u = 3.7$
$t = 0.49$
$a = 0$

$$x - x_0 = ut + \frac{1}{2}at^2$$
$$x - 0 = 3.7(0.49) + \frac{1}{2}(0)t^2$$
$$x = 1.813$$
$$x = 1.8\,m$$

∴ The Marbles land 1.8 m apart.

(ii) Note: The time of Impact for the first Marble is 0.49 sec

~~x = 0m~~ Y-axis

$x = 0m$
$x_0 = 1.2\,m$
$u = 0\,m/s$
$a = -9.81\,m/s^2$
$t = ?$

$$x - x_0 = ut + \frac{1}{2}at^2$$
$$0 - 1.2 = 0t + \frac{1}{2}(-9.81)t^2$$
$$-1.2 = -4.9t^2$$
$$\frac{-1.2}{-4.9} = \frac{-4.9t^2}{-4.9}$$
$$t^2 = 0.2449$$
$$t = 0.49\,sec$$

The time of Impact ~~of~~ for the second marble is 0.49 sec

∴ There is no difference in the time of impact ~~of~~ of the two Marbles

[c]

(i) <u>Car :</u>

$a = 2.44 m/s^2$

$u = 0$

$x - x_0 = ut + \frac{1}{2}at^2$

$x = 0t + \frac{1}{2}(2.44)t^2$

$x = 1.22t^2 \quad \ldots \text{eq (i)}$

<u>bus</u>

$a = 0 m/s^2$

$u = 19.6 m/s$

$x - x_0 = ut + \frac{1}{2}at^2$

$x = 19.6t + \frac{1}{2}(0)t^2$

$x = 19.6t \quad \ldots \text{eq (ii)}$

<u>eq (i) into eq (ii) :</u>

$$\frac{1.22t^2}{1.22t} = \frac{19.6t}{1.22t}$$

$$t = 16 \text{ sec} \quad \ldots (1)$$

[ii]

$V = u + at$

$V = 0 + 2.44 m/s^2 \times 16s$

$V = 39 m/s$

#

From eq (ii)

$x = 19.6t$

$x = 19.6 m/s \times 16s$

$= 313.6m$

#

[d] → at $t = 0s$, $v = 0\,m/s$.

→ at $t = 2s$: $V = u + at$

$V = 0 + (2)(2)$

$V = 4\,m/s$

→ at $t = 3sec$: $V = u + at$

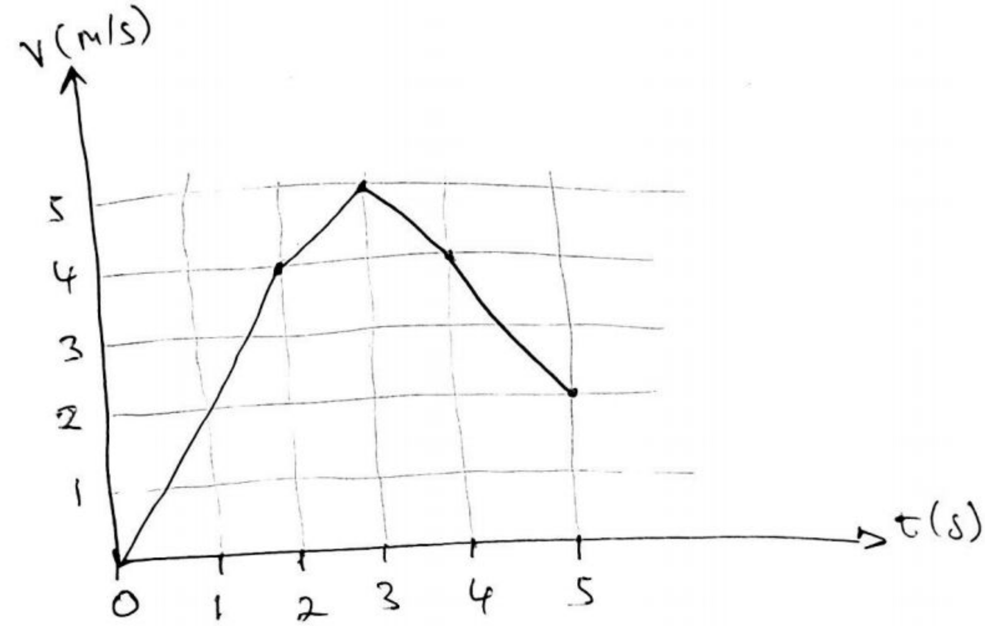
$V = 4 + (1)(1) = 5\,m/s$

→ at $t = 4s$; $V = u + at$

$V = 5 + (-1)(1) = 4\,m/s$

→ at $t = 5\,sec$; $V = u + at$

$V = 4 + (-2)(1) = 2\,m/s$



QUESTION 4

1 [a] (i) They act on different objects

(ii) He does not exert a force on the floor of the elevator.

[b] $\vec{r} = (2t^3 - t^2)\hat{i} + (4t^3 + 2t)\hat{j}$

$\vec{v} = \frac{d}{dt}\vec{r} = (6t^2 - 2t)\hat{i} + (12t^2 + 2)\hat{j}$

$\vec{a} = \frac{d}{dt}\vec{v} = (12t - 2)\hat{i} + (24t)\hat{j}$

at $\cancel{t=2}$ $t=2$ sec, the acceleration is:

$\vec{a} = [12(2) - 2]\hat{i} + (24(2))\hat{j}$

$\vec{a} = 10\hat{i} + 48\hat{j}$

$\cancel{\vec{a}}$ $\vec{F} = m\vec{a}$

$m = 10g = 0.01\ kg$

$\vec{F} = 0.01\vec{a} = 0.01(10\hat{i} + 48\hat{j})$

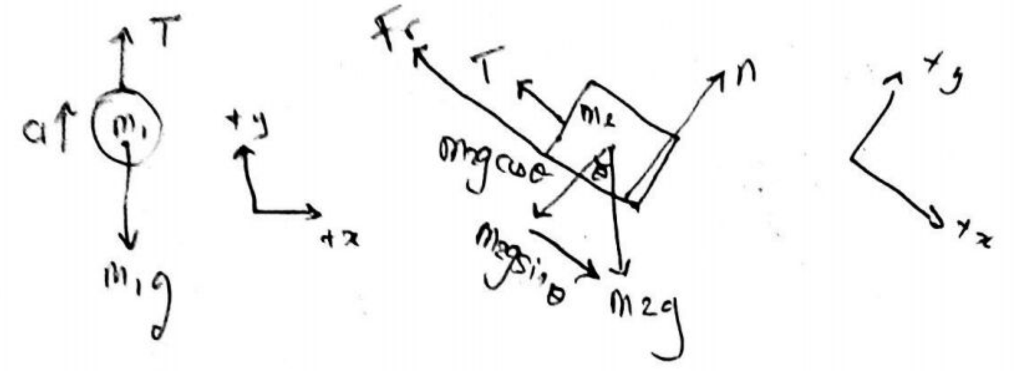
$\vec{F} = 0.1\hat{i} + 0.48\hat{j}$

$\|\vec{F}\| = \sqrt{(0.1)^2 + (0.48)^2} = \underline{\underline{0.49\ N}}$

$\theta = \tan^{-1}\left(\frac{0.48}{0.1}\right) = \underline{\underline{78.2^\circ}}$

The magnitude 0.49N, 78.2° with respect the positive x-axis.

[c]



$\Sigma F_y = m a_y$

$T - m_1 g = m_1 a$

$T = m_1 g + m_1 a$ --- eq (i)

$\Sigma F_x = m a_x$

$m_2 g\sin\theta - T - F_r = m_2 a$ ---- eq (ii)

$\Sigma F_y = m a_y$

$n - m_2 g\cos\theta = 0$

$n = m_2 g\cos\theta$ ---- eq (iii)

$F_r = \mu n$ ---- eq (iv)

eq (iii) into eq (iv)

$F_r = \mu m_2 g\cos\theta$ --- eq (v)

eq (i) and eq (v) into eq (ii)

$m_2 g\sin\theta - m_1 g - m_1 a - \mu m_2 g\cos\theta = m_2 a$

$m_1 a + m_2 a = m_2 g\sin\theta - \mu m_2 g\cos\theta - m_1 g$

$a(m_1 + m_2) = (m_2\sin\theta - \mu m_2\cos\theta - m_1)g$

$$a = \frac{(m_2\sin\theta - \mu m_2\cos\theta - m_1)g}{m_1 + m_2}$$

Since the system is in motion, $\mu = \mu_K$

$$\boxed{a = \frac{(m_2\sin\theta - \mu_K m_2\cos\theta - m_1)g}{(m_1 + m_2)}}$$ shown. ---eq (vi)

(iv) eq (vi) into eq (i)

$$T = m_1(g+a) = m_1\left[g + \frac{(\mu_2 \sin\theta - \mu_k m_2 \cos\theta - m_1)g}{(m_1+m_2)}\right]$$

$$T = m_1 g\left[\frac{(m_1 + m_2 + \cancel{M_2}\sin\theta - \cancel{\mu_k M_2} - m_1)}{(m_1+m_2)}\right] \quad \mu_k m_2 \cos\theta$$

$$T = m_1 g\left[\frac{m_2 + \cancel{M_2}\sin\theta - \cancel{\mu_k M_2}}{m_1+m_2}\right] \quad \mu_k m_2 \cos\theta \quad = \cancel{m_1 g\, m_2\left[1 + M_2 \sin\right.}$$

$$T = m_1 g\left[\frac{m_2 + m_2 \sin\theta - \mu_k m_2 \cos\theta}{(m_1+m_2)}\right] = m_1 g m_2\left[\frac{1+\sin\theta - \mu_k\cos\theta}{(m_1+m_2)}\right]$$

$$\boxed{T = \left[\frac{1+\sin\theta - \mu_k\cos\theta}{m_1+m_2}\right] m_1 m_2 g} \rightarrow \text{hence shown}$$

(ii) In the case when $a = 0$, $\mu = \mu_s$

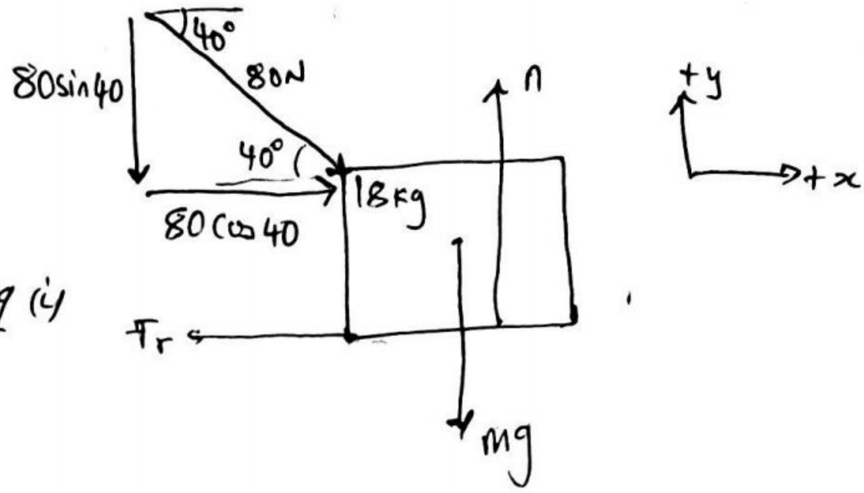
From eq (vi):

$$\left[0 = \frac{[m_2 \sin\theta - \mu_k m_2 \cos\theta - m_1]g}{m_1+m_2}\right] \times \frac{m_1+m_2}{g}$$

$$0 = m_2 \sin\theta - \mu_k m_2 \cos\theta - m_1$$

$$\boxed{m_1 = m_2[\sin\theta - \mu_k \cos\theta]}$$

hence shown.



$\sum F_y = ma_y$

$n - mg - 80\sin 40 = 0$

$n = mg + 80\sin 40 \quad \cdots \text{eq (i)}$

$\sum F_x = ma_x$

$80\cos 40 - F_r = 0$

$F_r = 80\cos 40 \quad \cdots \text{eq (ii)}$

$F_r = \mu_k n \quad \cdots \text{eq (iii)}$

~~eq (iii) into eq (ii) a~~

eq (ii) and eq (i) into eq (iii)

$$\frac{80\cos 40}{mg + 80\sin 40} = \frac{\mu_k (mg + 80\sin 40)}{mg + 80\sin 40}$$

$$\mu_k = \frac{80\cos 40}{mg + 80\sin 40} = 0.269$$

$\therefore \mu_k = 0.3$