Tutorial Sheet 2

Vectors

1. A ship is steaming due east at a speed of $12ms^{-1}$. A passenger runs across the deck at a speed of $5ms^{-1}$ toward north. What is the resultant velocity of the passenger relative to the sea?

2. A plane is travelling as fast as it can due north at 500 miles per hour (mph). There is a strong wind blowing to the east at 50 mph. What is the maximum speed of the plane when there is no wind?

3. You find yourself pacing, in a deep thought about a physics problem. First you walk 12 meters due east. Then, you walk 6 meters due north. Then you doze off and find yourself 50 meters from your starting place, 30° north of east. How far did you walk while you were not paying attention?

4. Vector $\vec{A}$ has a magnitude of 10 units and makes 60° with the positive x-axis. Vector $\vec{B}$ has a magnitude of 5 units and is directed along the negative *x*-axis. Find (a) the vector sum $\vec{A} + \vec{B}$ (b) the vector difference $\vec{A} - \vec{B}$

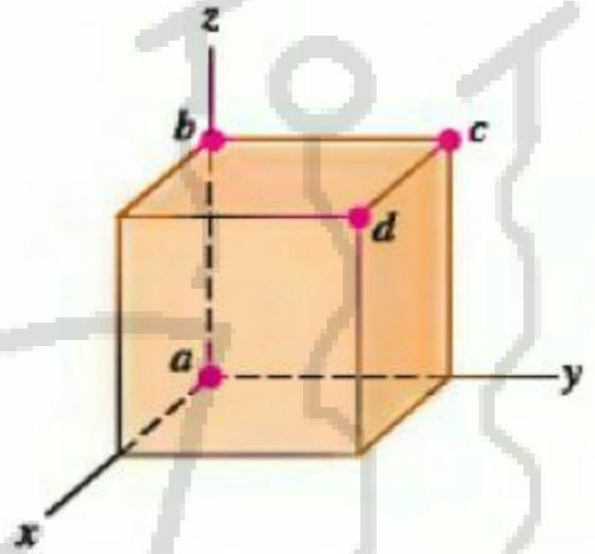
5. The $\vec{F} = q(\vec{v} \times \vec{B})$ equation gives the force on an electric point charge q moving with velocity $\vec{v}$ through a uniform magnetic field $\vec{B}$. Find the force on a proton of $q = 1.6 \times 10^{-19}$coulomb moving with velocity $\vec{v} = (2\vec{\imath} + 3\vec{\jmath} + 4\vec{k}) \times 10^5\ m/s$ in a magnetic field of $0.5\ \vec{k}$ tesla.

6. Physical quantities may be classified as scalars and vectors. Which of the following are scalars and which ones are vectors: Displacement, Momentum, Energy, Charge and half-life?

7. A cube is placed so that one corner is at the origin and the three edges are along the *x*-, *y*- and *z*-axes of a coordinate system. Use vectors to compute

(a) the angle between the edge along the *z*-axis (line *ab*) and the diagonal from the origin and to the opposite corner (line *ad*), and

(b) the angle between line *ac* (the diagonal of a face) and line *ad*.



8. A force $\vec{A}$ is added to another force $\vec{B}$ that has the x and y components equal to -10 N and 8 N respectively. The resultant of the two forces is in the positive x direction and has a magnitude of 12 N.

(i) Find the x and y components of the force $\vec{A}$, and

(ii) the angle it makes with respect to the positive x-axis.

9. A golfer takes three puts to get his ball into the hole once he is on the green. The first putt displaces the ball 12 ft north, the second 6.0 ft southeast, and the third 3.0 ft southwest. What displacement was needed to get the ball into the hole on the first putt?

10. Vector $\vec{A}$ has a magnitude of 5.0 units and is directed east. Vector $\vec{B}$ is directed 45° west of north and has a magnitude of 4.0 units.

(a) Construct vector diagrams for calculating

(i) $\vec{A} + \vec{B}$, and

(ii) $\vec{A} - \vec{B}$

(b) Estimate the magnitudes and directions of $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$ from your diagrams.

11. Two vectors $\vec{A}=4\hat{i}-3\hat{j}+\hat{k}$ and $\vec{B}=-\hat{i}+\hat{j}+4\hat{k}$. Find

(a) $\vec{A}+\vec{B}$

(b) $\vec{A}-\vec{B}$

(c) a vector $\vec{C}$ such that $\vec{A}-\vec{B}+\vec{C}=0$.

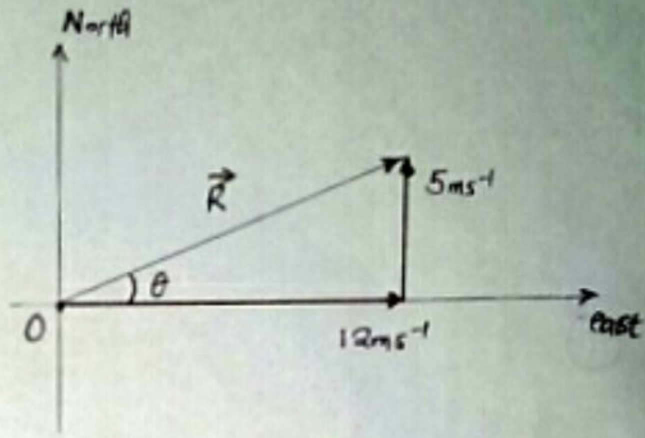
(d) a unit vector perpendicular to both $\vec{A}$ and $\vec{B}$.

(e) the angle between vectors $\vec{A}$ and $\vec{B}$.

TUTORIAL SHEET 2

CHAPTER 2 : VECTORS (MR - AMON CHILESHE)

Question 1



Let the Original = the sea

Data

$\vec{A} = 12ms^{-1}$ and $\vec{B} = 5ms^{-1}$

$$\vec{R} = 12\hat{i} + 5\hat{j} \quad \text{—— (1)}$$

the resultant velocity is the magnitude of vector $\vec{R}$

$$|\vec{R}| = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{(12)^2 + (5)^2}$$

$$= \sqrt{144 + 25}$$

$$= \sqrt{169}$$

$$= 13m/s$$

$\therefore$ **the resultant velocity is 13m/s** the direction is

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

$$= \tan^{-1}\left(\frac{5}{12}\right)$$

$$= \tan^{-1}(0.417)$$

$\therefore \theta = 22.6^\circ$ North of east

Question 2

for below will help us solve the maximum speed of the plane where there is no wind.

Vector	x-component	y-component
$\vec{A}$	$A_x = 50\cos 0^\circ$	$A_y = 50\sin 0^\circ$
$\vec{B}$	B_x	B_y
$\vec{R}$	$R_x = 500\cos 90^\circ$	$R_y = 500\sin 90^\circ$

x-Component (→)

$$A_x = 50\cos 0^\circ = 50$$

$$B_x = ?$$

$$R_x = 500\cos 90^\circ = 0$$

Y-Component (↑)

$$A_y = 50\sin 0^\circ = 0$$

$$B_y = ?$$

$$R_y = 500$$

we can find the x- Component for vector $\vec{B}$ as

$$R_x = A_x + B_x$$

$$B_x = R_x - A_x$$

$$B_x = 0 - 50 = -50\hat{i}$$

and the y-Component for vector $\vec{B}$ as

$$R_y = A_y + B_y$$

$$B_y = R_y - A_y$$

$$B_y = 500 - 0 = 500\hat{j}$$

then

$$\vec{B} = -50\hat{i} + 500\hat{j}$$

the magnitude of vector $\vec{B}$ will be the maximum Speed of the plane.

$$|\vec{B}| = \sqrt{(-50)^2 + (500)^2}$$

$$= \sqrt{252500}$$

$$|\vec{B}| = 502.5 \text{ mph}$$

Question 3

Vector	x- Component	y- Component
$\vec{A}$	$A_x = 12\cos 0^\circ = 12$	$A_y = 12\sin 0^\circ = 0$
$\vec{B}$	$B_x = 6\cos 90^\circ = 0$	$B_y = 6\sin 90^\circ = 6$
$\vec{C}$	$C_x = ?$	$C_y = ?$
$\vec{R}$	$R_x = 50\cos 30^\circ = 43.30$	$R_y = 50\sin 30^\circ = 25$

we can find the x-component for vector $\vec{C}$ as

$$R_x = A_x + B_x + C_x$$

$$C_x = R_x - A_x - B_x$$

$$C_x = 43.30 - 12 - 0$$

$$C_x = 31.3\hat{i}$$

and the y-component for vector $\vec{C}$ as

$$R_y = A_y + B_y + C_y$$

$$C_y = R_y - A_y - B_y$$

$$C_y = 25 - 0 - 6$$

$$C_y = 19\hat{j}$$

then

$$\vec{C} = 31.3\hat{i} + 19\hat{j}$$

$$|\vec{C}| = \sqrt{(31.3)^2 + (19)^2}$$

$$= \sqrt{979.69 + 361}$$

$$\therefore\ |\vec{C}| = 36.6\,m \text{ due north of east}$$

$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right)$$

$$= \tan^{-1}\left(\frac{19}{31.3}\right)$$

$$\theta = 31.3^\circ$$

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Question (4)

Vector	x - Component	y - Component
$\vec{A}$	$A_x = 10\cos 60° = 5$	$A_y = 10\sin 60° = 8.66$
$\vec{B}$	$B_x = 5\cos 180° = -5$	$B_y = 5\cos 180° = 0$
$\vec{A}+\vec{B}$	?	?
$\vec{A}-\vec{B}$	?	?

(a) the vector Sum $\vec{A}+\vec{B}$

$$\vec{A} = 5\hat{i} + 8.66\hat{j}$$

and

$$\vec{B} = -5\hat{i} + 0\hat{j}$$

$$\therefore \vec{A}+\vec{B} = (5\hat{i} + 8.66\hat{j}) + (-5\hat{i} + 0\hat{j})$$

$$\vec{A}+\vec{B} = 0\hat{i} + 8.66\hat{j}$$

$$|\vec{A}+\vec{B}| = \sqrt{(0)^2 + (8.66)^2}$$

$$= \sqrt{(8.66)^2}$$

$$= 8.66 \text{ units}$$

$$\theta = \tan^{-1}\left(\frac{8.66}{0}\right)$$

$$\theta = 90°$$

$\therefore$ A+B = 866 units at 90°

(b) the vector difference $\vec{A}-\vec{B}$

$$\vec{A}-\vec{B} = (5\hat{i} + 8.66\hat{j}) - (-5\hat{i} + 0\hat{j})$$

$$\therefore \vec{A}-\vec{B} = 10\hat{i} + 8.66\hat{j}$$

$$|\vec{A}-\vec{B}| = \sqrt{10^2 + 8.66^2}$$

$$= \sqrt{174.9956}$$

$$\theta = \tan^{-1}\left(\frac{8.66}{10}\right)$$

$$\theta = 40.9°$$

A - B = 13.23 units

$\therefore$ A-B = 13.23 units at 40.9°

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Question 5

$$\vec{F} = q(\vec{v} \times \vec{B})$$

Data

$q = 1.6 \times 10^{-19}$ coulomb

$\vec{v} = (2\hat{i} + 3\hat{j} + 4\hat{k}) \times 10^5 m/s = (2 \times 10^5 m/s)\hat{i} + (3 \times 10^5 m/s)\hat{j} + (4 \times 10^5 m/s)\hat{k}$

$= 200{,}000\hat{i} + 300{,}000\hat{j} + 400{,}000\hat{k}$

$\vec{B} = 0\hat{i} + 0\hat{j} + 0.5\hat{k}$

$\vec{F} = ?$

Let's find the vector product or Cross product $\vec{v} \times \vec{B}$

$$\vec{v} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 \times 10^5 & 3 \times 10^5 & 4 \times 10^5 \\ 0 & 0 & 0.5 \end{vmatrix}$$

$$= \hat{i} \begin{vmatrix} 3 \times 10^5 & 4 \times 10^5 \\ 0 & 0.5 \end{vmatrix} - \hat{j} \begin{vmatrix} 2 \times 10^5 & 4 \times 10^5 \\ 0 & 0.5 \end{vmatrix} + \hat{k} \begin{vmatrix} 2 \times 10^5 & 3 \times 10^5 \\ 0 & 0 \end{vmatrix}$$

$$\therefore \vec{v} \times \vec{B} = (1.5 \times 10^5 m/s)\hat{i} - (1 \times 10^5 m/s)\hat{j} + 0\hat{k}$$

$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$= [(1.6 \times 10^{-19}) \times (1.5 \times 10^{5} m/s)]\hat{i} - [(1.6 \times 10^{-19}) \times (1 \times 10^{5})]\hat{j} + 0\hat{k}$$

$$= (1.6 \times 10^{-19} \times 10^{5})[1.5\hat{i} - \hat{j}]$$

$$\therefore \vec{F} = 1.6 \times 10^{-14}(1.5\hat{i} - \hat{j})N$$

Question 6

VECTORS ~~Scalars~~	SCALAR ~~Vectors~~
Displacement	half-life
Momentum	Energy
	Charge

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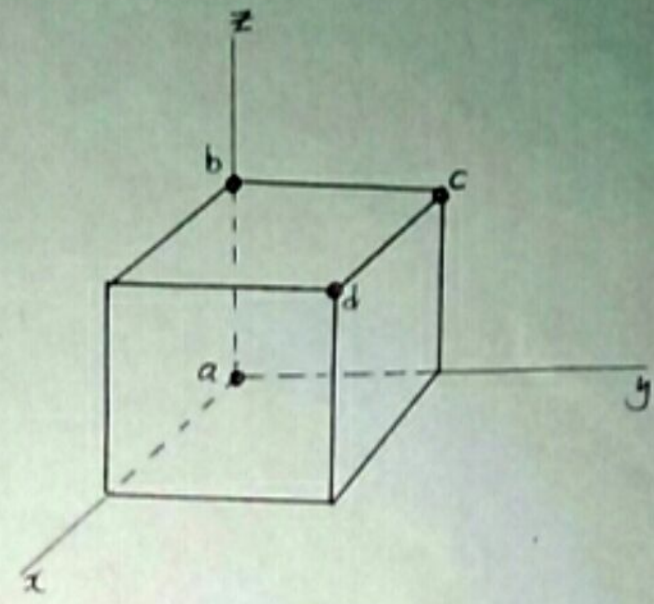
Question (7)



(a) let ℓ be the length of side of the cube and θ be the angle between ab and ad. then

$$\vec{ab} = \ell\hat{k} \quad \text{and} \quad \vec{ad} = \ell\hat{i} + \ell\hat{j} + \ell\hat{k}$$

the magnitude of $\vec{ab}$ is

$$|\vec{ab}| = \ell$$

and the magnitude of $\vec{ad}$ is

$$|\vec{ad}| = \sqrt{\ell^2 + \ell^2 + \ell^2} = \sqrt{3\ell^2} = \ell\sqrt{3}$$

then

$$\vec{ab} \cdot \vec{ad} = (0\hat{i} + 0\hat{j} + \ell\hat{k}) \cdot (\ell\hat{i} + \ell\hat{j} + \ell\hat{k}) = \ell^2$$

And use 1st
we can find the angle θ by using the dot-product (scalar product)

$$\vec{ab} \cdot \vec{ad} = |ab| \cdot |ad| \cos\theta$$

$$\cos\theta = \frac{\vec{ab}\cdot\vec{ad}}{|\vec{ab}|\cdot|\vec{ad}|} = \frac{\ell^2}{\ell(\ell\sqrt{3})} = \frac{\ell^2}{\ell^2\sqrt{3}} = \frac{1}{\sqrt{3}} = 0.5774$$

$$\theta = \cos^{-1}\left(\frac{\vec{ab}\cdot\vec{ad}}{|\vec{ab}|\cdot|\vec{ad}|}\right)$$

$$\therefore\ \underline{\theta = \cos^{-1}(0.5774) = 57.7^\circ \simeq 55^\circ}$$

(b) Let ℓ be the length of the side of the cube and θ be the angle betw lines ac and ad. then

$$\vec{ac} = \ell\hat{j} + \ell\hat{k} \quad \text{and} \quad \vec{ad} = \ell\hat{i} + \ell\hat{j} + \ell\hat{k}$$

the magnitude for $\vec{ac}$ is

$$|\vec{ac}| = \sqrt{\ell^2+\ell^2} = \sqrt{2\ell^2} = \ell\sqrt{2}$$

and the magnitude for ~~$|\vec{ad}|$~~ $\vec{ad}$ is

$$|\vec{ad}| = \sqrt{\ell^2+\ell^2+\ell^2} = \sqrt{3\ell^2} = \ell\sqrt{3}$$

therefore, the dot-product of $\vec{ac}$ and $\vec{ad}$ is

$$\vec{ac}\cdot\vec{ad} = (0\hat{i} + \ell\hat{j} + \ell\hat{k})\cdot(\ell\hat{i} + \ell\hat{j} + \ell\hat{k}) = 2\ell^2$$

and

$$\cos\theta = \frac{\vec{ac}\cdot\vec{ad}}{|\vec{ac}|\cdot|\vec{ad}|} = \frac{2\ell^2}{(\ell\sqrt{2})(\ell\sqrt{3})} = \frac{2\ell^2}{(\sqrt{6})\ell^2} = \frac{2}{\sqrt{6}} = 0.8165$$

$$\therefore\ \underline{\theta = \cos^{-1}(0.8165) = 35.3^\circ}$$

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Question ⑧

Vector	x- component	y- component	
$\vec{A}$	$A_x = ?$	$A_y = ?$	
$\vec{B}$	$B_x = -10$	$B_y = 8$	
$\vec{R}$	$R_x = 12\cos 0^\circ = 12$	$R_y = 12\sin 0^\circ = 0$	

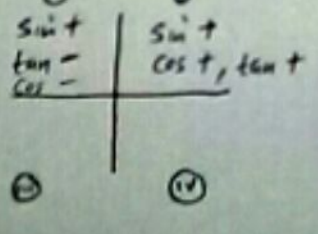
(i) the x- component for vector $\vec{A}$ is

$$R_x = A_x + B_x$$
$$A_x = R_x - B_x$$
$$A_x = 12 - (-10)$$
$$A_x = 12 + 10 = 22\,\hat{i}$$

and the y- component for vector $\vec{A}$ is

$$R_y = A_y + B_y$$
$$A_y = R_y - B_y$$
$$= 0 - 8$$
$$A_y = -8\,\hat{j}$$

(ii) the angle it makes with respect to the positive x-axis is



$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$
$$= \tan^{-1}\left(\frac{-8}{22}\right)$$

$360^\circ - 20^\circ = 340^\circ$.

$\therefore \theta = -19.98^\circ = \cancel{180 - 19.98 = 200}$

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Question (11)

$\vec{A} = 4\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{B} = -\hat{i} + \hat{j} + 4\hat{k}$

(a) $\vec{A} + \vec{B}$

$$\vec{A} + \vec{B} = \left(4\hat{i} - 3\hat{j} + \hat{k}\right) + \left(-\hat{i} + \hat{j} + 4\hat{k}\right)$$

$$= 3\hat{i} - 2\hat{j} + 5\hat{k}$$

$$\therefore \underline{\vec{A} + \vec{B} = 3\hat{i} - 2\hat{j} + 5\hat{k}}$$

(b) $\vec{A} - \vec{B}$

$$\vec{A} - \vec{B} = \left(4\hat{i} - 3\hat{j} + \hat{k}\right) - \left(-\hat{i} + \hat{j} + 4\hat{k}\right)$$

$$= 5\hat{i} - 4\hat{j} - 3\hat{k}$$

$$\therefore \underline{\vec{A} - \vec{B} = 5\hat{i} - 4\hat{j} - 3\hat{k}}$$

(c) a vector $\vec{C}$ such that $\vec{A} - \vec{B} + \vec{C} = 0$

$$\vec{C} = \vec{B} - \vec{A}$$

$$\vec{C} = \left(-\hat{i} + \hat{j} + 4\hat{k}\right) - \left(4\hat{i} - 3\hat{j} + \hat{k}\right)$$

$$= -5\hat{i} + 4\hat{j} + 3\hat{k}$$

$$\therefore \underline{\vec{C} = -5\hat{i} + 4\hat{j} + 3\hat{k}}$$

(d) a unit Vector perpendicular to both $\vec{A}$ and $\vec{B}$

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

- $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -3 & 1 \\ -1 & 1 & 4 \end{vmatrix}$

$$= \begin{vmatrix} -3 & 1 \\ 1 & 4 \end{vmatrix} \hat{i} - \begin{vmatrix} 4 & 1 \\ -1 & 4 \end{vmatrix} \hat{j} + \begin{vmatrix} 4 & -3 \\ -1 & 1 \end{vmatrix} \hat{k}$$

$$= (-12-1)\hat{i} - (16+1)\hat{j} + (4-3)\hat{k}$$

$\therefore$ $\underline{\vec{A} \times \vec{B} = -13\hat{i} - 17\hat{j} + \hat{k}}$

the magnitude for $\vec{A} \times \vec{B}$ is

$$|\vec{A} \times \vec{B}| = \sqrt{(-13)^2 + (-17)^2 + (1)^2}$$

$$= \sqrt{169 + 289 + 1}$$

$\therefore$ $\underline{|\vec{A} \times \vec{B}| = 21.4 \text{ units}}$

the unit vector for $\vec{A}$ and $\vec{B}$ is

$\therefore$ $\underline{\hat{n} = \frac{1}{21.4}(-13\hat{i} - 17\hat{j} + \hat{k})}$

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(e) the angle between vectors $\vec{A}$ and $\vec{B}$

$$\vec{A} \cdot \vec{B} = |A|\,|B| \cos\theta$$

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}|\,|\vec{B}|}$$

- $\vec{A} \cdot \vec{B} = (4\hat{i} - 3\hat{j} + \hat{k}) \cdot (-\hat{i} + \hat{j} + 4\hat{k})$

$= -4 - 3 + 4$

$= -3$

- $|A| = \sqrt{4^2 + (-3)^2 + (1)^2}$

$= \sqrt{16 + 9 + 1}$

$= 5.1$

- $|B| = \sqrt{(-1)^2 + (1)^2 + (4)^2}$

$= \sqrt{1 + 1 + 16}$

$= \sqrt{18}$

$= 4.2$

$$\cos\theta = \frac{-3}{21.42} = -0.1400 \simeq 82^\circ$$

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