THE COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS & NATURAL SCIENCES

DEPARTMENT OF PHYSICS

PH 110: INTRODUCTORY PHYSICS 2021/2022 TEST 2
DURATION: 2 HOURS **TOTAL MARKS:** 100 MARKS

INSTRUCTIONS:
1. Write your Names, **Student Identification Number and Lecture Group** on the front page of your answer booklet and possibly your ID on all your scripts.
2. There are **four (4)** questions in this test; **ANSWER ANY THREE (3).**
3. The marks for each question are shown in the square brackets [ ], show your working to avoid loss of marks.

CONSTANTS:
1. Acceleration due to gravity $g = 9.8\ m/s^2$
2. Gravitational constant $G = 6.673 \times 10^{-11}\ N.m^2.kg^{-2}$
3. Mass of the Earth $M_E = 5.98 \times 10^{24}$ kg

Where necessary, use

746 W = 1.0 horsepower, 1000 kg = 1.0 tonne.

QUESTION ONE.

(a) Starting from rest, a 5.00 kg block slides 2.50 m down a rough 30.0° incline. The coefficient of kinetic friction between the block and the incline is $\mu = 0.40$

Determine,

(i) the work done by the force of gravity, [3]

(ii) the work done by friction force between block and incline, [3]

(iii) the final velocity at the bottom of the incline [3]

(b) With the aid of an equation, state the:

(i) conservation of mechanical energy [2]

(ii) work – energy theorem

(c) Fig.1.1 shows a bead sliding on a wire. If the friction force are negl bead has a speed of 100 cm/s at A, what will be its speed in SI unit (i) point B? (ii) point C? [3, 3]

(i) conservation of mechanical energy [2]

(ii) work – energy theorem [2]

(c) Fig.1.1 shows a bead sliding on a wire. If the friction force are negligible and the bead has a speed of 100 cm/s at A, what will be its speed in SI units at,
(i) point B? (ii) point C? [3, 3]

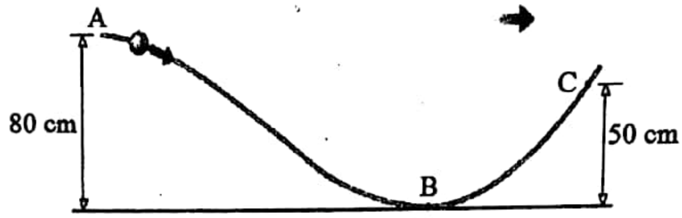


Fig.1.1

(d) At what average speed would a 70 kg student have to climb a 5 m rope to match with the power output of a 150 W light bulb? [3]

(e) A force $\mathbf{F} = (5\mathbf{i} + 2\mathbf{j} - 5\mathbf{k})N$ acts on a particle that undergoes displacement $\nabla \mathbf{r} = (2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})\,m$. Find the work done by the force on the particle. [3]

QUESTION TWO

(a) Define **linear momentum**, **elastic collision** and **inelastic collision**. [2, 2, 2]

(b) State the law of conservation of linear momentum. [2]

(c) A tennis ball of mass 40 g, moving to the right with a speed of 5 m/s, has an elastic collision with a target ball of mass 60 g that was at rest. What is the velocity of each ball after the collision? [7]

(d) A car of mass 800 kg travelling due north collides with a truck of mass 2000 kg travelling due east. After an investigation by the traffic police, it was determined that the truck was travelling at 36 km/h before the collision. After the collision the car and the truck stick together and move in the direction 53° North of East. Neglecting friction, determine

(i) their common speed after the impact [5]
(ii) the velocity of the car before the collision [5]

Neglecting friction, determine

(i) their common speed after the impact [5]
(ii) the velocity of the car before the collision [5]

QUESTION THREE

(a) A Physics lecturer swings a rubber stopper in a horizontal circle at the end of a string in front of his class. He tells Zuba, in the first row, that he is going to let the string go when the stopper is directly in front of her face. Will the stopper hit Zuba's face or not? [3]

(b) A one-ton car rounds an unbanked curve of radius 50 m at a speed of 50.4 km/h. Will the car follow the curve or skid in the following scenarios:
(i) Where the pavement is dry and the coefficient of the static friction is $\mu_s = 0.55$ [4]

(ii) Where the pavement is icy and the coefficient of static friction is $\mu_s = 0.24$. [3]

(c) The mass of the Earth is 5.97 x 10^{24} kg, the mass of the Moon is 7.35 x 10^{22} kg, and the mean distance of the Moon from the center of the Earth is 3.84 x 10^5 km. Use these data to calculate the magnitude of the gravitational force exerted by Earth on the Moon. [3]

(d) State Keplers's three Laws of Planetary motion [6]

(e) (i) Derive the relationship between the period of the planet, the radius of its orbit and the mass of the Sun. [3]

(ii) The planet Mercury travels around the Sun with a mean orbital radius of 5.8 x 10^{10} m. The mass of the Sun is 1.99 x 10^{30} kg. How long does it take Mercury to orbit the Sun. [3]

QUESTION FOUR

(a) The angular velocity of a rotating disk with a radius of 2 m decreases from 6 rads per second to 3 rads per second in 2 seconds. What is the linear acceleration of a point on the edge of the disk during this time interval? [4]

(b) A solid sphere of radius 0.2 m and mass 2 kg is at rest at a height 7 m at the top of an inclined plane making an angle 60° with the horizontal. Assuming no slipping, what is the speed of the solid sphere at the bottom of the incline? Take Moment of Inertia of a solid sphere to be $\frac{2}{5}mr^2$. [7]

(c) A solid, uniform, frictionless cylindrical **disc** ($I_{disc} = \frac{1}{2}mr^2$) of mass M = 3.00 kg and radius R = 0.40 m is used to draw water from a well as shown in Figure 4.1. A bucket of mass m = 2.00 kg is attached to a cord that is wrapped around the disc.

(i) Sketch free body diagrams for the disc and for the bucket, [2]
(ii) Find the acceleration of the bucket. [4]

km. Use these data to calculate the magnitude of the gravitational force exerted by Earth on the Moon. [3]

(d) State Keplers's three Laws of Planetary motion [6]

(e) (i) Derive the relationship between the period of the planet, the radius of its orbit and the mass of the Sun. [3]

(ii) The planet Mercury travels around the Sun with a mean orbital radius of 5.8×10^{10} m. The mass of the Sun is 1.99×10^{30} kg. How long does it take Mercury to orbit the Sun. [3]

QUESTION FOUR

(a) The angular velocity of a rotating disk with a radius of 2 m decreases from 6 rads per second to 3 rads per second in 2 seconds. What is the linear acceleration of a point on the edge of the disk during this time interval? [4]

(b) A solid sphere of radius 0.2 m and mass 2 kg is at rest at a height 7 m at the top of an inclined plane making an angle 60° with the horizontal. Assuming no slipping, what is the speed of the solid sphere at the bottom of the incline? Take Moment of Inertia of a solid sphere to be $\frac{2}{5}mr^2$. [7]

(c) A solid, uniform, frictionless cylindrical **disc** ($I_{disc} = \frac{1}{2}mr^2$) of mass M = 3.00 kg and radius R = 0.40 m is used to draw water from a well as shown in Figure 4.1. A bucket of mass m = 2.00 kg is attached to a cord that is wrapped around the disc.

(i) Sketch free body diagrams for the disc and for the bucket, [2]

(ii) Find the acceleration of the bucket. [4]

(iii) Find the tension *(T)* in the cord. [5]

(iv) If the bucket starts from rest at the top of the well and falls for 3.00 s before hitting the water, how far does it fall? [3]

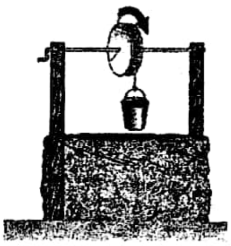
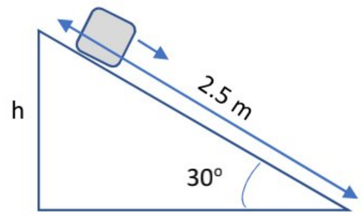


Figure 4.1: Cylindrical Disc and bucket for drawing water.

PH 11O TEST 2 2022 SOLUTIONS

QUESTION ONE (1)

(a) Data: u = 0 m/s, v = ?, m = 5 kg, g = 9.8 m/s^2



(i) $\sin(30^0) = \frac{h}{2.5}$;

$h = 2.4(0.5) = 1.25\ m$ **[1 mark]**

$W_g = F_g h = mgh = 5(9.8)(1.25) = 61.25\ J$ **[2 marks]**

(ii) $f = \mu \boldsymbol{N}$

$N = mgCos(30^0) = 5(9.8)(0.866025) = 42.435\ N$

$f = 0.4(42.435) = 16.974\ N$ **[1 mark]**

$W_f = -f(x) = -16.974(2.5) = -42.44\ J$ **[2 marks]**

(iii) $W_f = \Delta PE + \Delta KE = PE_f - PE_i + KE_f - KE_i$ [1]

$PE_f = 0;\ KE_i = 0$

$W_f = -PE_i + KE_f$

$-42.44 = -5(9.8)1.25 + \frac{1}{2}(5)v^2$

$v^2 = 7.526; v = 2.74\ m/s$ **[2 marks]**

(b) (i) In an isolated system, the initial ME is equal to the final ME.

$PE_i + KE_i = PE_f + KE_f$ **[2 marks]**

(ii) Any one of these three: **[2 marks]**

Work done is equal to the change in kinetic energy, $W = \Delta KE$

Work done is equal to the change in potential energy, $W = \Delta PE$

Work done by non-conservative forces is equal to change in mechanical energy, $W_{nc} = \Delta KE + \Delta PE$

c) By conservation of energy

i)
$$E_i = E_F$$
$$E_A = E_B$$

$$\therefore \quad KE_A + PE_A = KE_B + PE_B$$
$$\frac{1}{2}mv_A^2 + mgh_A = \frac{1}{2}mv_B^2 + mgh_B$$

dividing through by m and taking note that $h_B = 0$ we have;

$$\frac{1}{2}v_A^2 + gh_A = \frac{1}{2}v_B^2$$
$$v_B^2 = v_A^2 + 2gh_A$$
$$v_B = \sqrt{v_A^2 + 2gh_A}$$

[3 marks]

$$v_B = \sqrt{1^2 + 2*9.8*0.8}$$
$$v_B = \sqrt{16.68}$$
$$v_B = 4.08\,m/s$$

ii)
$$E_B = E_C$$

$$\therefore \quad KE_B + PE_B = KE_C + PE_C$$
$$\frac{1}{2}mv_B^2 + mgh_B = \frac{1}{2}mv_C^2 + mgh_C$$

dividing through by m and taking note that $h_B = 0$ we get;

$$\frac{1}{2}v_B^2 = \frac{1}{2}v_C^2 + gh_C$$
$$v_c^2 = v_B^2 - 2gh_C$$
$$v_c = \sqrt{v_B^2 - 2gh_C}$$
$$v_c = \sqrt{(4.08)^2 - 2*9.8*0.5}$$

[3 marks]

$$v_c = \sqrt{16.68 - 9.8}$$
$$v_c = \sqrt{6.8}$$
$$v_c = 2.62\ m/s$$

d)

$$Power = Fv$$
$$P = mg * v$$ **[3 marks]**
$$v = \frac{p}{mg} = \frac{150W}{70kg * 9.8m/s^2} = 0.22m/s$$

e)

$$w = \vec{F} * \nabla \mathbf{r}$$
$$w = (5\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}) * (2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$$ **[3 marks]**
$$= 10 + 6 - 20$$
$$w = -4\ J$$

QUESTION TWO (2)

(a) -Linear momentum is defined as the product of a system's mass and its linear velocity. It is given by:

$$p = mv$$ **[2 marks]**

-Elastic collision is type of collision in which both momentum and kinetic energy of a system are conserved. **[2 marks]**

-Inelastic collision is type of collision in which only the momentum of a system is conserved, but not kinetic energy. **[2 marks]**

(b) If no net external force acts on a system, the total linear momentum of an isolated system remains constant. **[2 marks]**

(c) Let $m_1 = 40g$, $m_2 = 60g$, $u_1 = 5\,m/s$, and $u_2 = 0$
By the law of conservation of linear momentum

$$\sum p_i = \sum p_f$$
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$40 \times 5 + 0 = 40v_1 + 60v_2$$
$$40v_1 + 60v_2 = 200$$
$$2v_1 + 3v_2 = 10 \ldots\ldots (1)$$ **[2 marks]**

Since the collision is elastic, the coefficient of restitution $e = 1$. That is

$$\frac{v_2 - v_1}{u_2 - u_1} = -e$$
$$\frac{v_2 - v_1}{0 - 5} = -1$$
$$v_2 - v_1 = 5 \ldots\ldots (2)$$ **[2 marks]**

Solving equations (1) and (2) simultaneously yields,

$$v_2 = 5 + v_1$$

$$2v_1 + 3(5 + v_1) = 10$$

$$5v_1 = -5$$

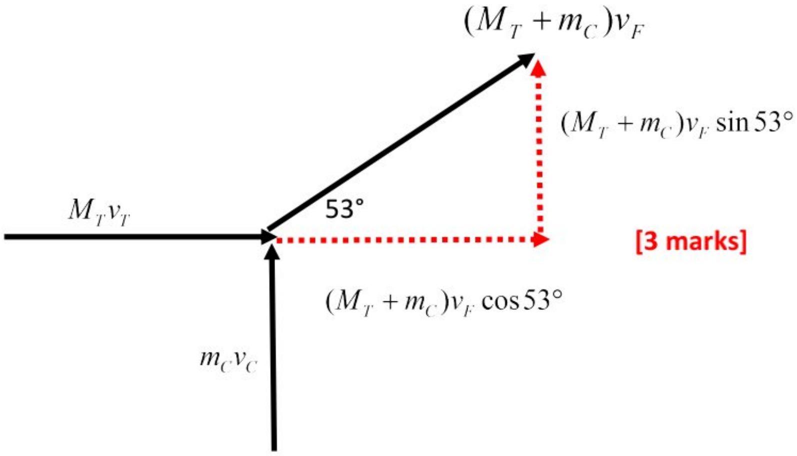
$$v_1 = -1\, m/s$$

$$v_2 = 5 - 1$$

$$v_2 = 4\, m/s$$ **[3 marks]**

After collision the tennis ball reverses direction and moves to the left with speed 1 m/s, while the target ball moves to the right with speed 4 m/s.

d)



[3 marks]

By conservation of linear momentum

$$M_T v_T + m_c v_c = (M_T + m_c)v_F$$

Along the x-axis we have:

$$M_T v_T = (M_T + m_c)v_F \cos 53°$$

$$v_F = \frac{M_T v_T}{(M_T + m_c)\cos 53°}$$

$$v_F = \frac{2000\,kg * 10\,m/s}{(2000\,kg + 800\,kg)\cos 53°}$$

$$v_F = 11.87\,m/s$$ **[4 marks]**

Along the y-axis we have:

$$M_C v_C = (M_T + m_c) v_F \sin 53°$$

$$v_C = \frac{(M_T + m_c) v_F \sin 53°}{M_C}$$

$$v_C = \frac{2800\,kg \times 11.87\,m/s \times \sin 53°}{800\,kg}$$

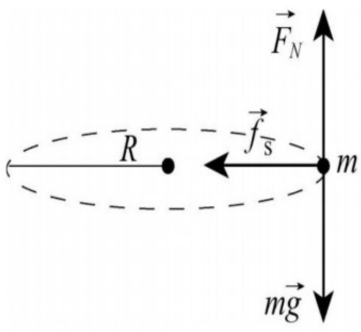
$$v_C = 33.18\,m/s$$ **[3 marks]**

QUESTION THREE (3)

a) **NO**. The velocity of the stopper is most likely to hit Barbara because its velocity is **tangential** to the path of the circular motion. So when the stopper passes through near Zuba's face, the direction of the velocity will be tangential to the circular path. That is parallel to Zuba's face. **[3 marks]**

b) The curve is unbanked (flat) and this means that friction is what provides the centripetal force needed for this circular motion. The free body diagram is:

i)



[1 mark]

- y-components

$$\Sigma F_y = 0$$

$$F_N - mg = 0$$

$$F_N = m$$

$$m = 1ton = 1000kg; g = 9.8\, m/s^2$$
$$F_N = 9800N$$ **[2 marks]**

- x-components

$$\Sigma F_x = ma_x (a_x = a_c)$$
$$f_s = F_c = \frac{mv^2}{r}$$
$$(m = 1ton = 1000kg; v = 50.4\, km/h = 14\, m/s\,; r = 50m)$$
$$F_c = \frac{(1000kg)\left(\frac{14m}{s}\right)^2}{50m}$$

$$F_c = 3920N$$ **[2 Marks]**

The maximum friction force attainable is:
$$f_s = \mu_s F_N$$
$$\mu_s = 0.55; F_N = 9800N$$
$$f_s = (0.55 \times 9800N) = 5390N$$
Since $F_c < f_s$ then the car will follow the curve and there is no skidding.

ii) If the pavement was icy and $\mu_s = 0.24$, then the maximum static friction force possible.
$$f_s = \mu_s F_N$$
$$f_s = (0.24)(9800N)$$
$$f_s = 2352N$$ **[2 Marks]**

Since $F_c > f_s$ then the car will not follow the curve and it will skid **[1 mark]**

3 **(c)**

$m_E = 5.97x10^{24}\ kg$

$m_M = 7.35x10^{22}\ kg$

$r = 3.84x10^5\ km = 3.84x10^8\ m$

$G = 6.673x10^{-11}\ N \cdot m^2/kg^2$

$F_{ME} = ?$

$$F_{ME} = G\frac{m_E m_M}{r^2}$$

$$F_{ME} = (6.673x10^{-11}\ N\cdot m^2/kg^2)\left[\frac{(5.97x10^{24}\ kg)(7.35x10^{22}\ kg)}{(3.84x10^8\ m)^2}\right]$$

$$F_{ME} = (6.673x10^{-11}\ N\cdot m^2/kg^2)\left(\frac{4.39x10^{47}\ kg^2}{1.47x10^{17}\ m^2}\right)$$

[3 marks]

$$F_{ME} = (6.673x10^{-11}\ N\cdot m^2/kg^2)(2.98x10^{30}\ kg^2/m^2)$$

$$F_{ME} = 1.99x10^{20}\ N$$

(d)

- All planets move in elliptical orbits with the Sun at one focus. **[2 marks]**
- The radius vector drawn from the Sun to a planet sweeps out equal areas in equal time intervals. **[2 marks]**
- The square of the orbital period of any planet is proportional to the cube of the semi major axis of the elliptical orbit. **[2 marks]**

(e)

i) For a planet going the Sun, the centripetal force is equal to the gravitational force between the planet and the Sun. Thus;

$$\frac{m_p v^2}{r} = \frac{GM_S m_p}{r^2} \qquad \text{Eqn. 1} \qquad \textbf{[1mark]}$$

$$\Rightarrow \qquad v^2 = \frac{GM_S}{r^2} \qquad \text{Eqn. 2}$$

but $v = \frac{2\pi r}{T} \Rightarrow v^2 = \frac{4\pi^2 r^2}{T^2}$ **[1mark]** put this in Eqn.2 we get;

$$T^2 = \frac{4\pi^2 r^3}{GM_S} \qquad \textbf{[1mark]}$$

ii) $r_M = 5.810x10^{10}\ m$

$m_S = 1.99x10^{30}\ kg$

$T_M = ?$

$$T_M{}^2 = \left(\frac{4\pi^2}{Gm_S}\right)r^3$$

$$T_M{}^2 = \left[\frac{39.5}{(6.673x10^{-11}\ N\cdot m^2/kg^2)(1.99x10^{30}\ kg)}\right](5.810x10^{10}\ m)^3$$

$$T_M{}^2 = \left[\frac{39.5}{1.33x10^{20}\ N\cdot m^2/kg}\right](1.96x10^{32}\ m^3)$$ **[3marks]**

$$T_M{}^2 = (2.96x10^{-19}\ s^2/m^3)(1.96x10^{32}\ m^3)$$

$$T_M{}^2 = 5.82x10^{13}\ s^2$$

$$T_M = \sqrt{5.82x10^{13}\ s^2}$$

QUESTION FOUR (4)

4 (a) We will use the two relationships:

$a_T = \alpha \cdot r$ and $\alpha = \frac{\omega - \omega_0}{t}$

$\alpha = \frac{\omega - \omega_0}{t} = \frac{3\frac{rad}{s} - 6\frac{rad}{s}}{2\,s} = \frac{-3}{2} = -1.5\ \text{rad/s}^2$

$a_T = \alpha \cdot r = (-1.5\ \text{rad/s}^2)(2\ \text{m}) = \boxed{-3\ \text{m/s}^2}$

[4 marks]

4 (b)

$E_{in} = E_{out}$

$GPE_{in} = TKE_{out} + RKE_{out}$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\cdot(\frac{2}{5}\boldsymbol{m \cdot r^2})\left(\frac{v}{r}\right)^2$

$gh = \frac{1}{2}v^2 + \frac{1}{2}\cdot(\frac{2}{5}\boldsymbol{\cdot r^2})\frac{v^2}{r^2}$

$g \cdot h = \frac{1}{2}v^2 + \frac{1}{5}v^2$

$g \cdot h = \frac{7}{10}v^2$

$\sqrt{\frac{10}{7}gh} = v$

$\sqrt{\frac{10}{7}(10\frac{m}{s^2})(7\ m)} = v$

$\sqrt{100\ m} = v$

$\boxed{10\ m/s = v}$

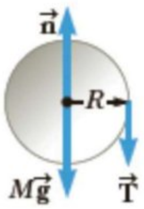
[7marks]

4 c option 1

(c) Data: m_{disc} = M = 3 kg, m_{bucket} = m = 2 kg, R = 0.40 m, g = 9.8 m/s^2

(i)

[1mark]



[1 mark]

(ii) We have three equations:

$ma = T - mg$ (1) **[1mark]**

$T = \frac{MR\alpha}{2}$ (2) **[1 mark]**

$\alpha = \frac{a}{R}$ (3) **[1 mark]**

Substituting (3) into (2) we get

$T = -\frac{Ma}{2}$ (4) **[1 mark]**

Substituting (4) into (1) we get

$ma = -mg - \frac{Ma}{2} \quad => \quad a = -\frac{mg}{(m+\frac{M}{2})}$

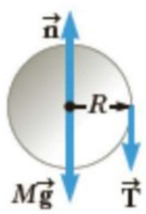
$a = -\frac{2*9.8}{\left(2+\frac{3}{2}\right)} = \frac{19.6}{3.5} = -5.6\ m/m^2$ **[2 marks]**

(iii) $T = -\frac{Ma}{2} = -\frac{3(-5.6)}{2} = 8.4\ N$ **[3 marks]**

(iv) Data: u = 0; t = 3s; a = -5.6 m/s^2;

$y = ut + \frac{1}{2}at^2 => y = 0 + \frac{1}{2}(-5.6)3^2 = -25.2m$ **[3 marks]**

4 c option 2



[1 mark]

$\tau = I\alpha$ **[1 mark]** Eqn. 1

$TR = I\alpha = \frac{1}{2}MR^2\alpha$ **[1 mark]**

$T = \frac{\frac{1}{2}MR^2\alpha}{R} = \frac{1}{2}MR\alpha$ **[1 mark]**

but $a = \alpha R \quad \therefore T = \frac{1}{2}Ma$ **[1 mark]** Eqn.2

[1 mark]

From the free body diagram for the bucket the Newton's second law of motion become;

$mg - T = ma$ Eqn. 3

Putting Eqn. 2 into Eqn. 3 we get:

$$mg - \frac{1}{2}Ma = ma$$

$$2mg - Ma = 2ma$$

$$a = \frac{2mg}{2m + M} = \frac{2 \times 2 \times 9.8}{(2 \times 2) + 3} = \frac{39.2}{7}$$ **[3 marks]**

$$a = 5.6\ m/s^2$$

iii) $T = \frac{1}{2}Ma = \frac{1}{2}(3)(5.6) = 8.4\,N$ **[2 marks]**

iv) $s = ut + \frac{1}{2}at^2$ **[1 mark]**

$s = \frac{1}{2}(5.6)(3^2) = 25.2\,m$ **[2 marks]**