THE COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

PHYSICS DEPARTMENT

2016/2017 ACADEMIC YEAR

PH 110 TEST 1

TIME: TWO (2) HOURS

INSTRUCTIONS: THERE ARE FIVE (5) QUESTIONS IN THIS PAPER, ATTEMPT ANY FOUR (4) QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

MAXIMUM MARKS: **100**

USE THE FOLLOWING DATA WHERE NECESSARY:

Acceleration due to gravity, $g = 9.8\ m/s^2$

QUESTION 1

a. Two vectors are given by $\vec{A} = 3\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$ and $\vec{B} = 3\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

i. Define the cross [vector] product of these two vectors. [3]

ii. Let $\vec{C} = \vec{A} \times \vec{B}$ [a vector product], Find

a. the vector $\vec{C}$ **[3]**

b. the magnitude of vector $\vec{C}$ [2]

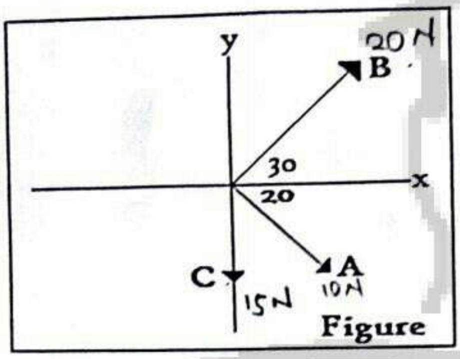
c. the unit vector ~~of~~ in the direction of vector $\vec{C}$. [2]

b. Find the components of a displacement which when added to a displacement of $[7\hat{\imath}-4\hat{\jmath}]$ will give a resultant displacement of $[5\hat{\imath} - 3\hat{\jmath}]$ [4]

c. A force $\vec{A}$ is added to a second force which has x and y components 3N and -5N. The resultant of the two forces is in the - x direction and has a magnitude of 4N. Find the x and y components of $\vec{A}$. **[4]**

d. Three vectors are oriented as shown in figure below, where $\vec{A}$ =10N, $\vec{B}$ = 20N and $\vec{C}$ =15N.

Find the x and y components of the resultant vector $\vec{D} = \vec{A} - \vec{B} + \vec{C}$ [3]



Figure

e. When vector $\vec{B}$ is added to vector $\vec{A}$ we get $[5\hat{\imath} - \hat{\jmath}]$ and when vector $\vec{B}$ is subtracted from $\vec{A}$ we get $[\hat{\imath} - 7\hat{\jmath}]$, what is the magnitude and direction of $\vec{A}$? [4]

QUESTION 2

A. A pendulum suspended from the roof of a bus moving along a horizontal track makes an angle of 5° with the vertical.

(i) What forces are acting on the pendulum bob? [4 marks]

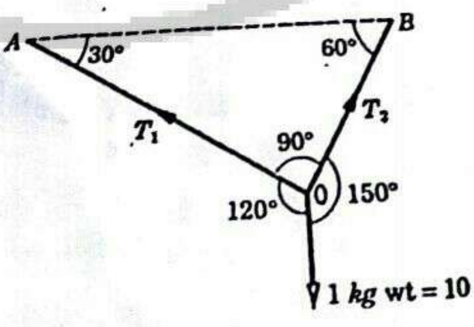
(ii) Determine the acceleration of the bus. [9 marks]

B. A ball of mass $1\,kg$ hangs in equilibrium from two strings OA and OB as shown in the figure below.

(i) Define concurrent forces? [2 marks]

(ii) Define moment of a force. [2 marks]

(iii) Determine ~~are~~ the tensions in the strings OA and OB? [8 marks]



$\frac{F}{\eta r v} = k$

$\frac{kg\ m/s^2}{kg\ m^{-1}s^{-1}\ m\ m/s}$

$\frac{MLT^{-2}}{M L^{-1}T^{-1}L^2T^{-1}}$

QUESTION 3

(a) (i) Explain the principle of homogeneity of dimensions [3]

(ii) State two limitations of dimensional analysis. [2]

(b) The velocity v of a particle varies with time t according to the relation $v = at^2 + bt + c$. Find the dimensions of a, b and c. [4]

(c) Stoke's formula gives an expression for the viscous force F acting on a small sphere moving through a homogeneous viscous fluid. The magnitude of F depends on the viscosity η of the liquid, the radius r of the sphere and the velocity v of the sphere. Use dimensional analysis to derive Stoke's formula. (Units of η are kg m^{-1}s^{-1}) [7]

(d) Astronomical distances are sometimes described in terms of light-years. A light-year (ly) is the distance that light will travel in one year. How far in metres does light travel in a quarter year if the speed of light is 3×10^8 ms^{-1}? [4]

(e) Density is defined as mass per unit volume. A crude estimation of the average density of the earth was 5.5 g/cm^3. Express this density in kg/m^3. Hence calculate the mass of the earth in kilograms if the earth is considered to be a sphere of radius 6370 km. [5]

QUESTION 4

(a) You are standing on an observation deck 100 m above a city street and drop a rock from rest. A friend stands on the street directly below you and throws a rock vertically upward at the same instant that you drop the rock. The initial upward velocity of his rock is 50 m/s. Assuming that they are moving along the same vertical line, and that air resistance can be neglected, calculate:

(i) the height at which they will collide, [4]

(ii) when they will collide, and [2]

(iii) whether your friend's rock will be rising or descending when they collide. [2]

(b) A railcar is moving horizontally with a speed of 24 m/s and decelerating at 3.65 m/s^2 when a light bulb 2.55 m above the floor comes loose and drops. Where, relative to the point directly below its original position, will the bulb strike the floor? [9]

(c) (i) An elevator in which a woman is standing is moving upward with a constant speed of 3.35 m/s. The woman drops a coin from a height of 1.25 m above the elevator floor. How long does it take the coin to strike the elevator floor? [4]

(ii) If the elevator was at rest at the instant the coin was dropped, but is accelerating upward at 3.5 m/s^2, how long would it take the coin to strike floor? [4]

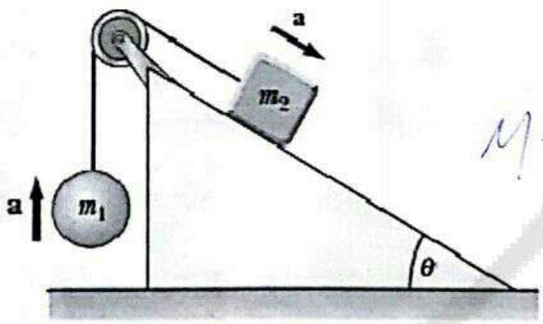
QUESTION FIVE

a. Explain the following terms: [2,2,2]
   i. Inertia
   ii. Weight
   iii. dynamics

b. Which of Newton's laws is referred to as the Principle of Inertia? What does it state? [3]

c. Consider a block of mass 10 kg placed on a rough surface inclined at 30 degrees. If the coefficient of static friction is 0.20, calculate its acceleration if let free. [5]

d. A ball of mass m_1 and a block of mass m_2 are attached by a lightweight cord that passes over a frictionless pulley of negligible mass, as in Figure 5.1. The block lies on a frictionless incline of angle θ. Find the magnitude of the acceleration of the two objects and the tension in the cord in terms of m_1, m_2 and θ. [7]



M.uK

Figure 5.1

e. A flat piece of polished wood is pulled by a 70 N force at an angle of 20 degrees above the horizontal. The polished wood is 25 kg and the surface is smooth. What acceleration does the piece of ~~would~~ Wood acquire and what velocity does it move with after 1 minute from the time the force begins to act on it? [4]

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Q1 (a)(i) $\vec{A} = 3\hat{i} - 2\hat{j} - 3\hat{k}$, $\vec{B} = 3\hat{i} - 2\hat{j} + 3\hat{k}$

(i) Cross product defn : $\vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\sin\theta\,\hat{n}$ [3]

(ii)(a) $\vec{C} = \vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & -3 \\ 3 & -2 & 3 \end{vmatrix} = -12\hat{i} - 18\hat{j}$ [3]

(b) $|\vec{C}| = \sqrt{C_x^2 + C_y^2 + C_z^2} = \sqrt{(-12)^2 + (-18)^2}$

$= \underline{21.6}$ [2]

(c) Unit vector in the direction of $\vec{C}$

$\hat{C} = \dfrac{\vec{C}}{|\vec{C}|} = \dfrac{1}{21.6}(-12\hat{i} - 18\hat{j}) = -0.55\hat{i} - 0.83\hat{j}$ [2]

(b) $(7\hat{i} - 4\hat{j}) + (A_x\hat{i} + A_y\hat{j}) = 5\hat{i} - 3\hat{j}$ [1]

$7 + A_x = 5 \Rightarrow A_x = 5 - 7 = -2$ [$1\frac{1}{2}$]

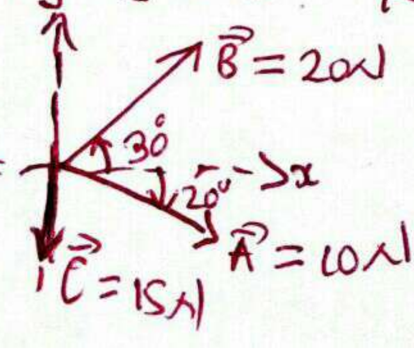
$-4 + A_y = -3 \Rightarrow A_y = -3 + 4 = \underline{1}$ [$1\frac{1}{2}$]

(c) $(A_x\hat{i} + A_y\hat{j}) + (3\hat{i} - 5\hat{j}) = -4\hat{i}$ [1]

$A_x + 3 = -4 \Rightarrow A_x = -4 - 3 = -7$ [$1\frac{1}{2}$]

$A_y - 5 = 0 \Rightarrow A_y = 5$ [$1\frac{1}{2}$]

(d)



$\vec{D} = \vec{A} - \vec{B} + \vec{C}$

~~$= (10\cos 20\hat{i} - 10\sin 20\hat{j}) - (20$~~

(P2)

$\vec{D} = \vec{A} - \vec{B} + \vec{C}$

$= (10\cos 20^\circ \hat{i} - 10\sin 20^\circ \hat{j}) - (20\cos 30^\circ \hat{i} + 20\sin 30^\circ \hat{j}) - 15\hat{j}$

$= (9.3969\hat{i} - 3.4202\hat{j}) - (17.3205\hat{i} + 10\hat{j}) - 15\hat{j}$

$= -7.9236\hat{i} - 28.42\hat{j}$ [3]

(e) $\vec{A} + \vec{B} = 5\hat{i} - \hat{j}$ —(i) Adding (i) and (ii)

$\vec{A} - \vec{B} = \hat{i} - 7\hat{j}$ —(ii)

$$\frac{2\vec{A} = 6\hat{i} - 8\hat{j}}{\vec{A} = 3\hat{i} - 4\hat{j}}$$

$\because |\vec{A}| = \sqrt{A_x^2 + A_y^2}$ [2]

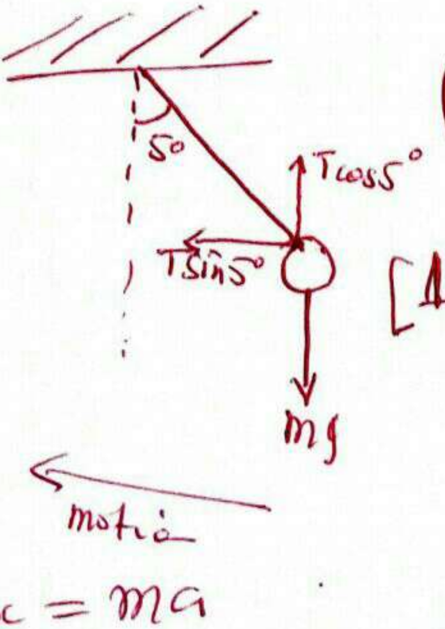
$= \sqrt{3^2 + (-4)^2}$

$= 5$

Directio

$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{-4}{3}\right) = -53^\circ$ [1]

Q2



[1]

(a)(i) Forces acting on pendulum

- gravitational force [2]
- tension [2]

~~(b)(i) Concurrent forces are forces acting~~

$\Sigma F_x = ma$

[2] $T\sin 5^\circ = ma$ —(i) Dividing (ii) into (i)

[2] $T\cos 5^\circ = mg$ —(ii)

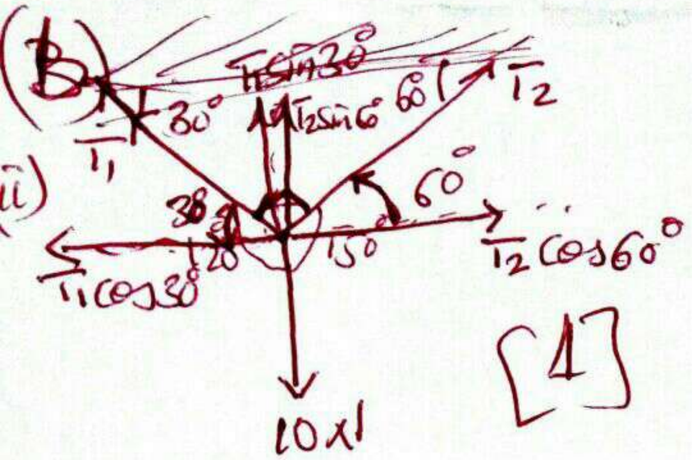
$\frac{T\sin 5^\circ}{T\cos 5^\circ} = \frac{ma}{mg}$ [2]

$\tan 5^\circ = \frac{a}{g} \Rightarrow a = g\tan 5^\circ$

$= 9.8\tan 5^\circ = 0.86\,m/s^2$

[2]

(B)



[1]

(P3)

$\Sigma F_y = 0$ [2]

$T_1 \sin 30° + T_2 \sin 60° - 10 = 0$ (i)

$\Sigma F_x = 0$

$-T_1 \cos 30° + T_2 \cos 60° = 0$

$T_2 \cos 60° = T_1 \cos 30°$ [2]

$T_2 = T_1 \left(\frac{\cos 30°}{\cos 60°}\right) = 1.73 T_1$

Substituting for T_2 in eqn (i)

$T_1 \sin 30° + 1.73\, T_1 \sin 60° - 10 = 0$

$0.5 T_1 + (0.5)(1.73) T_1 = 10$

$1.998 T_1 = 10$

$T_1 = \frac{10}{1.998} = 4.9 \simeq 5.0$ N [2]

and $T_2 = 1.73 T_1 = 1.73(4.9) = 8.6$ N [1]

b)(i) Concurrent forces: forces acting through the same point [2]

(ii) Moment of a force: product of force and lever arm [2]

Q3 Q3 (P4)

(a)(i) Statement of principle of homogeneity
- dimensions on all sides of the eqn must be equal (same) [3]

(ii) Limitations of dimensional analysis (any two) [2]

(b) $V = at^2 + bt + c$

$[LT^{-1}] = [a][T^{+2}] + [b][T] + [C]$

$[a][T^2] = [LT^{-1}]$

$[a] = \frac{LT^{-1}}{T^{+2}} = [LT]^{-3} [1\frac{1}{2}]$

$[b][T] = [LT^{-1}]$

$[b] = \frac{LT^{-1}}{T} = [LT^{-2}] [1\frac{1}{2}]$

$[C] = [LT^{-1}]$ 1

(C) $F = K\eta^{x} r^{y} v^{z}$ [1]

$[MLT^{-2}] = [ML^{-1}T^{-1}]^{x}[L]^{y}[LT^{-1}]^{z}$

$= [M]^{x}[L]^{-x+y+z}[T]^{-x-z}$ [3]

for M: $1 = x$

T: $-2 = -x - z \Rightarrow z = 2 - x = 2 - 1 = +1$

L: $1 = -x + y + z \Rightarrow y = 1 + x - z = 1 + 1 - 1 = 1$

VERTEX

$\therefore F = K\eta r v$ as $x = y = z = 1$ [3] PS

(d) $t = \frac{1}{4}$ year $= \frac{1}{4} \times 365$ days $\times 24$ hrs/d $\times 60$ min/h $\times 60$ sec/min

$= \frac{1}{4} \times 365 \times 24 \times 60 \times 60$ s

$= 7{,}889{,}400$ sec [2]

$d = vt = ct = 3 \times 10^8$ m/s $\times 7{,}889{,}400$ s

$= 2.37 \times 10^{15}$ m [2]

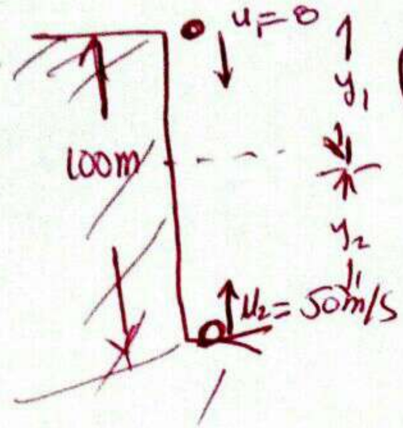
(e) $5.5\,g/cm^3 = \frac{5.5 \times 10^{-3}\,kg}{(10^{-2}\,m)^3} = 5.5 \times 10^3\,kg/m^3$ [2]

Volume of earth is

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6.37 \times 10^6)^3$ [1]

$\rho = \frac{m}{V} = \frac{\cancel{5.5 \times 10^3}}{4/3\,\pi(6.37 \times 10^6)^3}$ $5.5 \times 10^3 \times 4/3\,\pi(6.37 \times 10^6)^3$ $= 5.95 \times 10^{24}$ kg [2]

$M = \rho V$

Q4 

(i) $y_1 = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2y_1}{g}}$ [1] (P6)

$y_2 = 100 - y_1 = u_2t - \frac{1}{2}gt^2$ [1]

$\cancel{100 - y_1 = 50t - \frac{1}{2}(9.8)t^2}$

$100 - y_1 = 50\sqrt{\frac{2y_1}{g}} - \frac{1}{2}g\left(\frac{2y_1}{g}\right)$

$100 - y_1 = 50\sqrt{\frac{2y_1}{g}} - y_1$

$100 = 50\sqrt{\frac{2y_1}{g}}$ or $2 = \sqrt{\frac{2y_1}{g}}$ [2]

Squaring

$4 = \frac{2y_1}{g} \Rightarrow y_1 = \frac{4g}{2} = 2g$

$= 2 \times 9.8 = 19.6\,m$

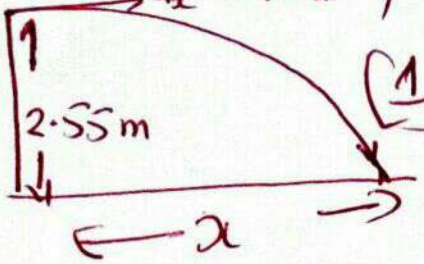
$y_2 = 100 - y_1 = 100 - 19.6 = 80.4m$

They will meet at 80.4m from the bottom or 19.6m from the top.

(ii) $t = \sqrt{\frac{2y_1}{g}} = \sqrt{\frac{2 \times 19.6}{9.8}} = 2s$ [2]

(iii) $v = u - gt = 50 - 9.8 \times 2 = 30.4m/s$

v is +v so the rock is still going up [2]

(b) 

(P6)

[1] $y = u_y t + \frac{1}{2} g t^2$ (free fall)

$2.55 = 0 + 4.9t^2$

$t^2 = \frac{2.55}{4.9} \Rightarrow t = \pm\sqrt{\frac{2.55}{4.9}}$

$= \pm 0.72\,s$ [3]

$x = u_x t = 24 \times 0.72$

$= 17.31\,m$ [1]

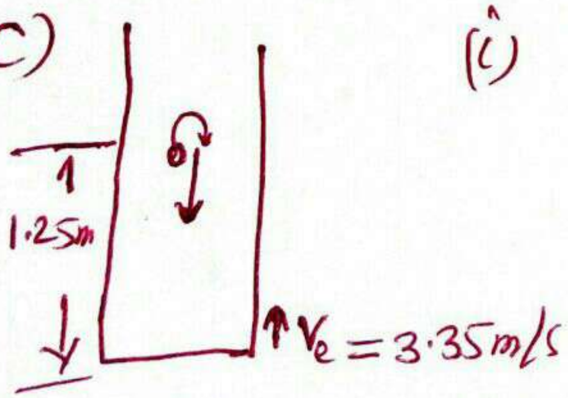
Position on floor directly below the bulb is

$x_f = ut + \frac{1}{2} a t^2 = 24 \times 0.72 + \frac{1}{2}(-3.65)(0.72)^2$

$= 17.28 - 0.95 = 16.33\,m$ [3]

The bulb strikes the floor at

$x - x_f = 17.31 - 16.33 = 0.98\,m$ [1]

(C) 

(i) $x_f = ut = 3.35t$ — (i)

$x_c = ut - \frac{1}{2} g t^2$ — (ii)

$x_c = 1.25 - x_f$

~~$\therefore 1.25 - 3.35t - 4.9t^2 = 3.35t$~~

~~$\therefore 4.9t^2 + 6.7t - 1.25$~~ $= 0$

$x_c = 1.25 - x_f = 1.25 - 3.35t$

$3.35t - 4.9t^2 = 1.25 - 3.35t$

$4.9t^2 + 6.7t - 1.25 = 0$

PH110 TEST 1 (2016)

P1

Q5

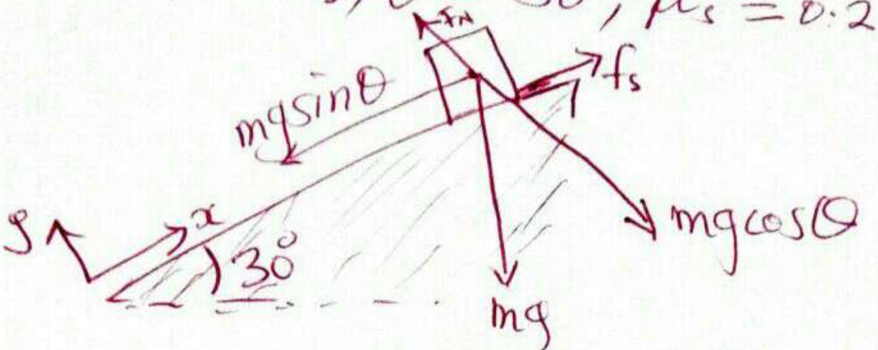
(a) (i) Inertia: tendency of a body to resist change in its state of motion [2]

(ii) Weight: gravitational pull on an object. $W = mg$ [2]

(iii) Dynamics: Study of motion using concepts of space and time as well as force which causes that motion. [2]

(b) Newton's first law is referred to as the 'Principle of Inertia'. It states that [1] 'an object at resist will remain at rest and an object in motion will remain in motion with constant velocity unless a net external force acts on it to change its state of motion". [2]

(c) $m = 10\,kg,\ \theta = 30^\circ,\ \mu_s = 0.2$



[1]

$\Sigma F_y = 0$ (No motion along the y-direction)

$F_N - mg\cos 30^\circ = 0$

$F_N = mg\cos 30^\circ$ —— (i) [1]

$f_s = \mu_s F_N = \mu_s mg\cos 30^\circ$ — (ii) [1]

$\Sigma F_x = ma$

$mg\sin 30^\circ - f_s = ma$

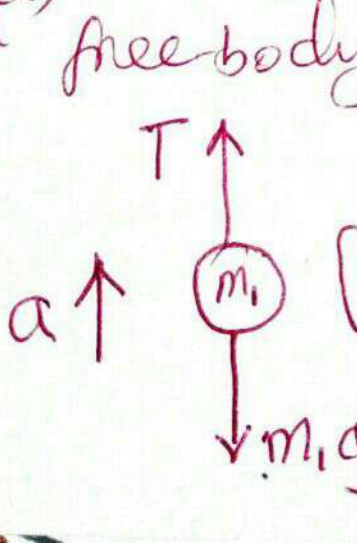
$\cancel{m}g\sin 30^\circ - \mu_s \cancel{m}g\cos 30^\circ = \cancel{m}a$

$\therefore a = g\sin 30^\circ - \mu_s g\cos 30^\circ$

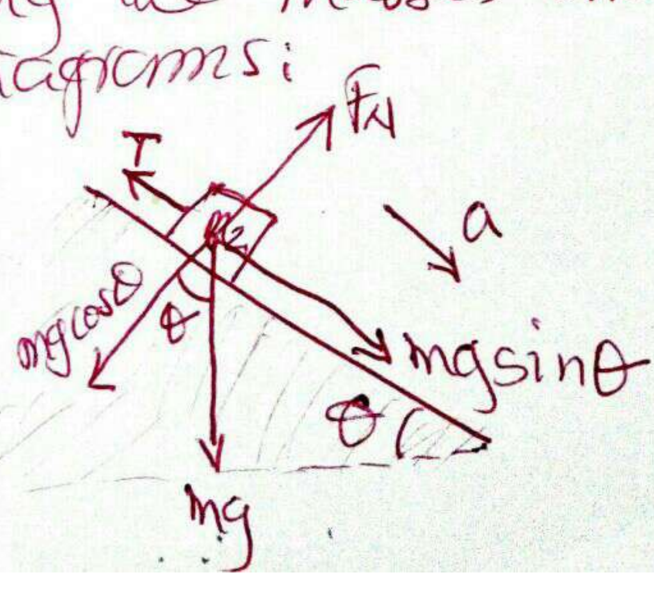
$= 9.8 \times 0.5 - 0.2 \times 9.8\cos 30^\circ$

$= \underline{3.2\ m/s^2}$ [2]

(d) Isolating the masses and drawing free-body diagrams:



[1]



[1]

For m_1,

$\Sigma F_y = m_1 a$ (P3)

$T - m_1 g = m_1 a$ — (i) [1]

For m_2,

$\Sigma F_x = m_2 a$

$m_2 g \sin\theta - T = m_2 a$ — (ii) [1]

Adding (i) and (ii),

$m_2 g \sin\theta - m_1 g = (m_1 + m_2)a$

$\therefore a = \dfrac{m_2 g \sin\theta - m_1 g}{m_1 + m_2}$ [1]

From eqn (i)

$T = m_1 g + m_1 a$, substituting for a,

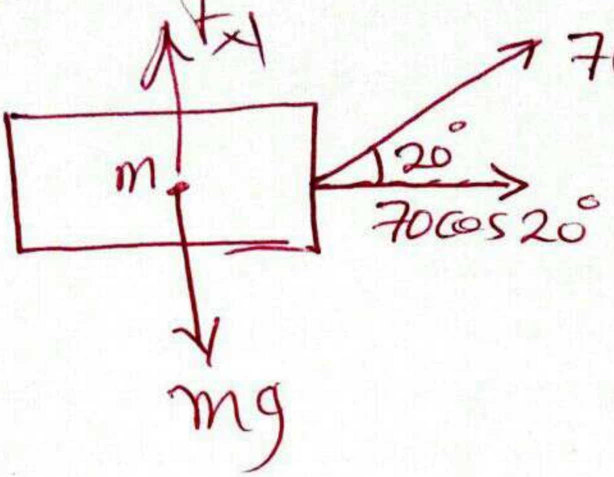
$= m_1 g + m_1 \left(\dfrac{m_2 g \sin\theta - m_1 g}{m_1 + m_2}\right)$

$= \dfrac{m_1 g (m_1 + m_2) + m_1 (m_2 g \sin\theta - m_1 g)}{m_1 + m_2}$

$= \dfrac{\cancel{m_1^2 g} + m_1 m_2 g + m_1 m_2 g \sin\theta - \cancel{m_1^2 g}}{m_1 + m_2}$

$$T = \frac{m_1 m_2 g (1 + \sin\theta)}{m_1 + m_2}$$ [2]

(e)



[1]

$m = 25kg$
Negligible friction since the surfaces are smooth.

$$\sum F_x = ma$$

$$70\cos 20° = ma$$

$$a = \frac{70\cos 30°}{25}, \quad m = 25kg$$

$$= 2.63\ m/s^2$$ [2]

For $t = 1min = 60s$, $u = 0$

$$v = u + at = 0 + 2.63 + 60$$

$$= 157.8\ m/s$$ [1]