



MA110 - MATHEMATICAL METHODS

Time allowed: Two hours (2:00 hours)

Instructions:

1. You must write your Name, Your Computer Number and programme of study on your answer sheet.
2. Calculators are not allowed in this paper.
3. There are three (3) questions in this paper. Attempt All questions and show detailed working for full credit

QUESTION ONE

- a) (i) If $C \subset D$, then simplify if possible
 $C' \cup D'$ (2.5 marks)
- b) (ii) Express $1.\overline{171717\dots}$ as a fraction $\frac{a}{b}$ in its simplest form where a and b are integers and $b \neq 0$. (2.5 marks)
- b) Consider the binary operation $a * b = a + b - 2ab$, where a and b are real numbers.
- (i) Is $*$ a binary operation on the set of real numbers? Give reason for your answer. (1) Mark
- (ii) Is the operation $*$ commutative? If not give a counter example. (1) Mark
- (iii) Find the value of $1 * (2 * 3)$ and $(1 * 2) * 3$ and state whether $*$ is associative (3) Marks
- c) Given the rational function $f(x) = \frac{x+2}{x-2}$. Sketch its graph indicating its domain and range, all the asymptotes and intercepts. (5 Marks)
- d) Prove that $\sqrt{2}$ is an irrational number (5 Marks)
- e) Let $f(x) = \frac{x+1}{x-1}$ and $g(x) = \sqrt{x}$. Find $(g \circ f)(x)$ and determine the domain (2, 2) (5 Marks)

$$\begin{aligned} & 3+2-2(-3)(2) \\ & 2-3-2(-3)(2) \end{aligned}$$

QUESTION TWO

- (4) a) Using the associative and distributive properties of union and intersection of sets. Show that

$$A \cup B = (A \cap B) \cup (A \cap B') \cup (A' \cap B) \quad (5 \text{ Marks})$$
- (5) b) Let α and β be the roots of the quadratic equation $3x^2 + 2x + 5 = 0$. Find a quadratic equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ without calculating α and β (5 Marks)
- (3) c) Solve the given radical function inequality $\sqrt{2} - \sqrt{x+6} \leq -\sqrt{x}$ (5 Marks)
- (5) d) Solve for x and y given that:
- $$\frac{x}{1+i} - \frac{y}{2-i} = \frac{1-5i}{3-2i} \quad (5 \text{ Marks})$$
- (5) e) Show that the function f defined by $f(x) = \frac{2x}{x-1}$, $x \in \mathbb{R}$, is a bijection on \mathbb{R} on to $\{y \in \mathbb{R}; y \neq 2\}$ (5 Marks)

QUESTION THREE

- (2) a) Use the Rational root theorem to solve $x^3 - 4x^2 + 8 = 0$ (5 Marks)
- (5) b) Rationalize the denominator $\frac{1}{(\sqrt{2}+1)(\sqrt{3}-1)}$ (5 Marks)
- (2.5) c) (i) Determine whether the function $f(x) = x^4 + x^2 + 1$ even, odd or neither. (2.5 marks)
- (2.5) (ii) Let $A = \{x \in \mathbb{R}; -4 \leq x < 2\}$ and $B = \{x \in \mathbb{R}; x \geq -1\}$.
~ Find a) $A \cap B$ b) A' (2.5 marks)
- (5) d) What are the dimensions of the largest rectangular field which can be enclosed by 1200 m of fencing? (5 Marks)
- (3) e) Sketch the graph of $f(x) = |2x+1|$. On the same diagram sketch also the graph of $g(x) = \sqrt{1-2x}$ and hence, find the values such that $\sqrt{1-2x} > |2x+1|$ (5 Marks)

$$\frac{x}{1200} : \frac{1-c}{c-c} \quad A = L \times B$$

$$1200 - c = m \times n$$

$$A = [m] [n]$$

$$A = [m]^{\frac{n+1}{2}}$$



The Copperbelt University

School of Mathematics And Natural Sciences

Department of Mathematics

MA 110 : (Mathematical Methods I) : Test One

February 23, 2022

Instructions

- (1). You must write your Name, Computer number and Programme of study on your answer sheet. Time allowed is 2 hours.
 - (2). Calculators and use of Cell phones are Not allowed in this paper.
 - (3). There are three (3) questions in this paper, attempt all the questions and show detailed working for full credit.
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QUESTION ONE

(a) Prove the De Morgan's law: $(A \cap B)' = A' \cup B'$.

(5 marks)

(b) Determine whether the function given is even, odd or neither

$$f(x) = x^6 + x^4 - 10x^2.$$

(5 marks)

(c) Given that $\sqrt{10}$ is irrational number, prove that $\sqrt{2} + \sqrt{5}$ is also an irrational number.

(5 marks)

(d) Graph the rational function : $f(x) = \frac{x^4 + 4}{(x - 3)(x + 2)}$.

(5 marks)

(e) Let $z = x + iy$ be a non-zero complex number. Given that $z + \frac{1}{z} = k$, where k is a real number, prove that either z is a real number or $|z|$ is a real number. prove that either z is a real number or $|z|$ is a real number.

(5 marks)

QUESTION TWO

- (a) Express $3.\overline{312}$ as a fraction $\frac{a}{b}$ in the simplest form where a and b are integers and $b \neq 0$. (5 marks)

- (b) Show that the two functions given below are inverses of each other:

$$f(x) = 4x - 3$$

and

$$g(x) = \frac{x+3}{4}$$

(5 marks)

- (c) If α and β are the roots of the quadratic equation $3x^2 + 6x - 15 = 0$, find the quadratic equation that has the roots α^2 and β^2 .

(5 marks)

- (d) Rationalize the denominator, expressing your answer in the form $a + b\sqrt{xy}$ where a and b are rational numbers

$$\frac{\sqrt{x} + \sqrt{y}}{\sqrt{x} - \sqrt{y}}$$

(5 marks)

- (e) Solve the polynomial equation given below using the rational root theorem

$$x^6 - x^5 - 4x^4 + 2x^3 + 5x^2 - x - 2 = 0.$$

(5 marks)

QUESTION THREE

- * (a) Define an operation * on the set of real numbers by $x * y = \sqrt[x]{y}$ where x is the index of the radical and y is the radicand.

- (i) Is * is a binary operation on the set of real numbers? justify your answer. (1 mark)

- (ii) Evaluate $3 * (-64)$. (1 mark)

- (iii) Solve the equation : $3 * (2x - 3) = 3$. (3 marks)

- (b) Given the quadratic function $f(x) = -3x^2 + 6x - 5$. Express the function $f(x) = -3x^2 + 6x - 5$ in the vertex form $f(x) = a(x + p)^2 + q$ and sketch its graph. (5 marks)

- * (c) Determine the domain of the function, expressing the answer in interval notation

$$f(x) = 3x + \sqrt{x^2 + 4x - 12}$$

(5 marks)

- (d) Solve the radical equation : $\sqrt{\sqrt{2y} - \sqrt{y-1}} = 1$. (d) Solve the radical equation : $\sqrt{\sqrt{2y} - \sqrt{y-1}} = 1$. (5 marks)

- * (e) Sketch the graph of $f(x) = |2x + 1| - |x - 2|$. (e) Sketch the graph of $f(x) = |2x + 1| - |x - 2|$. (5 marks)

THE END OF TEST



MA110 – Mathematical Method

Time allowed: Two hours (2:00 hours)

Instructions:

1. You must write your Name, your Computer Number and programme of study on your answer sheet.
 2. Calculators are not allowed in this paper.
 3. There are three (3) questions in this paper, Attempt All questions and show detailed working for full credit
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QUESTION ONE

- a) Express $2.0\overline{72}$ as a fraction $\frac{a}{b}$ in its simplest form where a and b are integers and $b \neq 0$. (5 Marks)
- b) Given the rational function $f(x) = \frac{x^2 + 2}{x - 1}$. Sketch its graph indicating its domain and range, all the asymptotes and intercepts. (5 Marks)
- c) Given that $\sqrt{7}$ is an irrational number, Show that $2 + \sqrt{7}$ is also an irrational number (5 Marks)
- d) Verify that the two given functions are inverses of each other

$$f(x) = x^3 + 1 \text{ and } g(x) = \sqrt[3]{x - 1} \quad (5 \text{ Marks})$$

- e) Define an operation * on the set of real numbers by $a * b = a^b$
 - i). Is * a binary operation on the set of real numbers ? Give reason for your answer. (1 Mark)
 - ii). Is the operation commutative? (2 Marks)
 - iii). Evaluate $(3 * 2) * -2$ (2 Marks)

QUESTION TWO

- a) Given that A and B are sets, simplify the following if possible

$$[(A \cap B)' \cap (A' \cup B)]'$$
 (5 Marks)

- b) Determine the domain of the following function:

$$f(x) = \sqrt{\frac{x+1}{x-1}}$$
 (5 Marks)

- c) Let α and β be the roots of the quadratic equation $4x^2 + 3x - 2 = 0$

Find a quadratic equation whose roots are α^2 and β^2 (5 Marks)

- d) Solve the given inequality $10 - \sqrt{2x+7} \leq 3$ (5 Marks)

- e) Solve for x and y given that $\frac{1}{x+iy} + \frac{1}{1+3i} = 1$ (5 Marks)

QUESTION THREE

- a) Sketch the graph of the function $k(x) = |2x - 1| - |x + 2|$ (5 Marks)

- b) Using synthetic division find the quotient and the remainder when

$f(x) = x^3 + 2x^2 + x - 2$ is divided by $x - (1 + i)$. (5 Marks)

- c) Let \mathbf{R} , the set of real numbers be the universal set. If

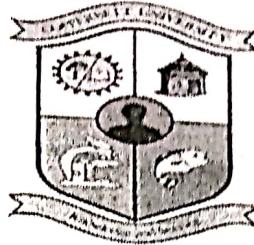
$A = [-7, 8] \cup [11, \infty)$ and $B = [0, 20]$, find the following sets and display them on the number line:

(i) A' . (2.5 Marks) (ii) $A \cap B$. (2.5 Marks)

- d) Express $\frac{\sqrt{3}+1}{\sqrt{3}-1} + \sqrt{3} - 1$ in the form $a + b\sqrt{3}$ where a and b are rational numbers. (5 Marks)

- e) Determine whether the function $f(x) = x^5 + x^3 + x$ is even, odd or neither. (5 Marks)

COPPERBELT UNIVERSITY



SCHOOL OF MATHEMATICS AND NATURAL SCIENCES DEPT OF PURE & APPLIED MATHEMATICS MA 110 - MATHEMATICAL METHODS I | TEST 1

INSTRUCTIONS; 1. Attempt all Questions in this Paper without Using a Calculator.
 2. Indicate clearly your Names, SIN and the Group you belong to.
 3. Duration is 3 Hours Only.

1. a. i.) Let $A = \{1, 2, 3\}$ and $B = \{2, 4\}$, Find $A \times B$. 1 Mk
 ii.) Prove that $(A^c)^c = A$ by Arbitrary Elementary Method. 4 Mks
- b. If the Operation * is defined as, "add the first number to 8 times the second number"
 Find $(2 * 3) * 5$ 2 Mks
- c. Find the value of k given that when $2x^3 - 2kx^2 - 3x - 2$ is divided by $x - 2$,
 the Remainder is 40. 3 Mks
- d. If $gof(x) = x$ and $g(x) = \frac{x+1}{x-1}$,
 i.) Find $f(x)$ ii.) Sketch the Graph of $f(x)$ and Find the Range of $f(x)$ 3 Mks, 4 Mks

2. a. Prove the De Morgan's Law: $A^c \cup B^c = (A \cap B)^c$ 5 Mks
- b. Solve the following Equations involving the Absolute value functions:

$$|8x + 3| = |2x - 21| \quad \text{3 Mks}$$

c. Solve the following inequation:

$$\frac{x-2}{x+1} \geq \frac{x-6}{x-2} \quad 5 \text{ Mks}$$

d. Using Synthetic Division, show that both $x-2$ and $x+3$ are Factors of:

$$f(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18.$$

Hence, or otherwise Factorize $f(x)$ completely 4 Mks

3. a. Express the following in the form $\frac{a}{b}$, where $a, b \in \mathbb{Z}$ and $b \neq 0$.

i.) $0.\overline{121212\dots}$ ii.) $1.\overline{3121212\dots}$ 1.5 Mks, 1.5 Mks

b. Use the fact that $\sqrt{6}$ is Irrational to prove that $\sqrt{2} + \sqrt{3}$ is Irrational. 4 Mks

c. Sketch the graphs of:

i.) $f(x) = -|x+3| - 4$ ii.) $f(x) = 3 + \sqrt{3-x}$ 2.5 Mks, 2.5 Mks

d. Solve the Polynomial Equation $x^4 - 6x^3 + 22x^2 - 30x + 13 = 0$ 5 Mks

4. a. Determine the vertex and Intercepts for the following Quadratic function:

$$f(x) = x^2 - 6x - 16 \quad 2 \text{ Mks}$$

b. Sketch the graph of the Polynomial given by;

$$f(x) = (x-1)^2(x-3)^3(x+4) \quad 5 \text{ Mks}$$

c. Given that the roots of $x^2 + 3x + 17 = 0$ are α and β respectively. Find a Quadratic Function

whose roots are $\alpha^3 + \beta^3$ and $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$. 5 Mks

d. Given that set $A = \{1, 2, 3\}$ and set $B = \{2, 4, 6\}$, Determine whether the Operation;

$$A \bullet B = P(A) - P(B).$$

is Binary on the Universal Power set, $P(E)$. 5 Mks

5. a. If $x = \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}}$, find the value of $8x - x^2$. 4 Mks

b. Given the Functions, $f(x) = x^2 + 4$ and $g(x) = x - 9$.

Find the value of x for which $g[f(x)] = f[g(x)]$ 4 Mks

c. Write the Expression $f(x) = 2x^2 + 12x + 14$ in the form $f(x) = a(x + h)^2 + k$ where $a, h, k \in \mathbb{R}$.

Hence, state the turning point of $f(x)$. 4 Mks

d. Calculate the value(s) of x that are valid for the Equation below.

$$\left| \frac{x-2}{x+3} \right| = 4 \quad 5 \text{ Mks}$$

6. a. Simplify $-\frac{25}{2} \left[\frac{1+2i}{3+4i} - \frac{2-5i}{-i} \right]$ 3 Mks

b. Solve for x and y given that;

$$\frac{x}{1+i} - \frac{y}{2-i} = \frac{1-5i}{3-2i} \quad 5 \text{ Mks}$$

c. Solve the Inequality below and present your answer in Interval Notation:

$$3x^2 + 2x + 2 < 2x^2 + x + 4 \quad 4 \text{ Mks}$$

d. Graph the Rational Functional by finding the Asymptotes and Intercept:

$$f(x) = \frac{5x^2 - 2}{1-x} \quad 5 \text{ Mks}$$

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THE COPPERBELT UNIVERSITY
SCHOOL OF MATHEMATICS AND NATURAL SCIENCES
DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MA 110 - MATHEMATICAL METHODS I

TEST 1

DATE: 8th December 2017
DURATION: 3 Hours
MARKS: 100

READ THE FOLLOWING INSTRUCTIONS

1. Write your NAME, PROGRAM, COMPUTER NUMBER AND GROUP on the cover of your answer sheet.
2. This is a THREE Hours test. Cell phones are NOT allowed.
3. Attempt ALL questions. Answers to questions should fully be explained. A correct but unclear answer will not get full marks.
4. No pencil work (except for graph sketching) or any work in red ink will be marked.
5. Use of correction fluid or "Tip-Ex" and calculators are NOT allowed.

Question 1

- ✓ (a) Let $E = \{3, 4, 5, 6\}$, $F = \{0, 2, 4, 6, 8\}$ and the universal set $X = \{0, 1, 2, \dots, 10\}$. Find $E \cap F'$. [1 mark]
- ✓ (b) Let $f(x) = \frac{2}{x^2 - 2}$, and $g(x) = \frac{1}{\sqrt{x+1}}$
Find $(g \circ f)(-2)$. [4 marks]
- ✓ (c) Use long division to divide $4x^3 - 7x^2 - 11x + 5$ by $4x + 5$. [3 marks]
- ✓ (d) If A and B are subsets of the universal set U such that $A \subset B$. Simplify
(i) $A' \cup B'$ (ii) $A - B$. [2+2 marks]
- ✓ (e) Find the possible values of λ and k if the expression $3x^4 + \lambda x^3 + kx + 4$ is exactly divisible by $x - 1$ and leaves a remainder of 18 when divided by $x + 2$. [5 marks]

Question 2

- ✓(a) Prove the De-Morgan's law; $(A \cap B)' = A' \cup B'$. [5 marks]
- ✓(b) Is the function $f(x) = |x| + x^2$ even, odd or neither? Justify your answer. [3 marks]
- ✗(c) Solve the inequality below and write the solution set in interval notation

$$\frac{x+4}{x+1} \leq \frac{x-2}{x-4}$$
 [5 marks]
- ✓(d) Use Factor Theorem and synthetic division to factorise
 $f(x) = 6x^4 - 19x^3 + 17x^2 - x - 3$ completely. [4 marks]

Question 3

- ✓(a) Sketch the following piecewise defined function

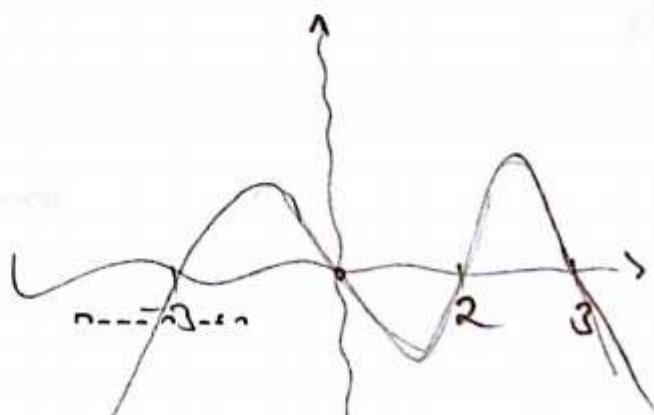
$$f(x) = \begin{cases} |x+2| & \text{if } -4 \leq x < 0 \\ x^2 & \text{if } 0 \leq x \leq 2 \\ 3 & \text{if } 2 < x \leq 3 \end{cases}$$
 [4 marks]

- ✓(b) Prove that $\sqrt{2} + \sqrt{3}$ is irrational. [4 marks]
- ✓(c) Solve the polynomial equation $2x^5 - 5x^4 + x^3 + x^2 - x + 6 = 0$. [5 marks]
- ✓(d) Solve the following inequality and write the solution in interval notation
 $|2x+1| + 1 \leq 7$. [4 marks]

Question 4

- ✓(a) Let the universal set be the set of real numbers, with $A = (3, 8]$, $B = (2, 7)$, $C = [1, 5]$ and $D = [6, \infty)$. Find (i) $(A \cup C)$ (ii) $(B \cap D)$ (iii) $(A \cup C) - (B \cap D)$ [1+1+2 marks]
- ✓(b) Is the binary operation $*$ defined by $a * b = a + b - ab$ both commutative and associative. Justify your answer. [3 marks]
- ✓(c) Sketch the graph of the polynomial $f(x) = -(x-3)(x-2)^3(x+1)^2$. [5 marks]
- ✓(d) Solve the equation $|2x+1| = 7$. [2 marks]
- ✓(e) The roots of the equation $x^2 - 9x + K = 0$ are α and $\alpha + 1$. Find the value of K . [3 marks]

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Question 5

- (a) Express

$$\frac{\sqrt{3} + 1}{\sqrt{3} - 1} + \sqrt{3} - 1$$

in the form $a + b\sqrt{3}$ where a and b are rational numbers.

[4 marks]

- (b) Verify that

$$f(x) = 4x - 5 \text{ and } g(x) = \frac{x+5}{4}$$

are inverse functions of each other.

[4 marks]

- ✓(c) Express $f(x) = 1 - 6x - x^2$ in the form $f(x) = a(x + h)^2 + k$ where a , h and k are rational numbers. Hence, write down the coordinates of the turning point of the graph $f(x) = 1 - 6x - x^2$.

[5 marks]

- ✓(d) Solve the equation $2x - 11\sqrt{x} + 12 = 0$.

[4 marks]

Question 6

- ✓(a) Solve for x and y where x and y are real numbers

$$(x + yi) - i = i(x + yi) + 5.$$

[4 Marks]

- ✗(b) The equation $Kx^2 - 2Kx + 2K = 1$ where K is a constant has two real solutions.

- (i) Show that K satisfies the inequality

$$K^2 - K \leq 0.$$

[2 Marks]

- (ii) Hence, find the set of all possible values of K .

[3 Marks]

- ✓(c) For the following rational function

$$f(x) = \frac{2x^2 - 2}{x^2 - 4},$$

- (i) Determine the x -intercepts and the y -intercept.

[3 Marks]

- (ii) Find the horizontal and the vertical asymptotes.

[2 Marks]

- (iii) Sketch the graph of f .

[3 Marks]



THE COPPERBELT UNIVERSITY
SCHOOL OF MATHEMATICS AND NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS

2015/2016 Academic Year
MA110 - Mathematical Methods

Test I

5th September, 2015.

Time Allowed : Three hours (3:00 hrs)

Instructions:

1. You must write your Name, your Computer Number and programme of study on your answer sheet.
2. Calculators are not allowed in this paper.
3. There are five (5) questions in this paper, Attempt All questions and show detailed working for full credit.

Question 1

- a) If C and D are disjoint, simplify if possible $(C \cup D)^c$ ✓
- b) Express $2.\overline{143}$ in the form of $\frac{a}{b}$ where a and b are integers, $b \neq 0$. ✓
- c) Rationalize the denominator and express the final answer in simplest radical form for $\frac{5\sqrt[3]{y^2}}{4\sqrt[4]{x}}$ ✓
- d) Sketch and determine the domain and the range of the function $f(x) = \frac{2}{x^2+4}$
- e) Prove that if $a + c = b + c$ then $a = b$ when $a, b, c \in R$. ✓

Question 2

- a) Let binary operation * defined $a * b = a - b + ab$ where a and $b \in R$, solve $|x * 2| = 1$. ✓
- b) Rationalize the denominator of $\frac{1}{(\sqrt{2}+1)(\sqrt{3}-1)}$ ✓
- c) Determine whether $f(x) = x^2 + 1$ is even, odd or neither.
- d) Solve the equation $\sqrt{-2x-7} + \sqrt{x+9} = \sqrt{8-x}$.
- e) Solve $x^4 + 3x - 2 = 0$.

Question 3

- a) Solve for x and y given that $\frac{x}{1+i} - \frac{y}{2-i} = \frac{1-5i}{3-2i}$.
- b) Sketch $f(x) = 2 + 3\sqrt{-x+1}$ and determine its range and domain.
- c) The roots of the equation $2x^2 + 6x - 15 = 0$ are α and β . Find the value of $(\alpha - \beta)$.
- d) Prove that $\sqrt{3}$ is an irrational number.
- e) Solve $\frac{3x+2}{x-1} > 0$ expressing the set of solution sets in interval notation.

Question 4

- a) Verify that the two given functions are inverses of each other $f(x) = x^3 + 1$ and $g(x) = \sqrt[3]{x-1}$.
- b) Express $5 - x - 2x^2$ in the form $a - b(x+c)^2$ and hence or otherwise find its maximum value and the value of x where this occurs.
- c) Using the associative and distributive properties of union and intersection of sets. Show that
- $$A \cup B = (A \cap B) \cup (A \cap B') \cup (A' \cap B)$$
- d) Solve for x given $|3x+1| < |4-2x|$.
- e) What type of roots does the equation $5x^2 - 3x + 1 = 0$ have?

Question 5

- a) Determine whether $f(x) = x^2 - 2$ is one-to-one. If it is, find the inverse and graph both the function and its inverse.
- b) Given that $z + \frac{1}{z} = k$, where k is a real number, prove that either z is real or $|z| = 1$.
- c) Given the set $X = \{0, 1, 2, 3\}$. Determine whether the operations $+$, $-$, \times are binary operations on X .
- d) Determine whether or not $x + 3i$ is a factor of $f(x) = x^4 + 14x^2 + 45$.
- e) Solve $\frac{4}{x-2} + \frac{x}{x+1} = \frac{x^2-2}{x^2-x-2}$.
- f) Sketch $f(x) = |x^2 + 5x + 4| - 2$ and determine its domain and range.

END

LAW COLLEGE UNIVERSITY
SCHOOL OF MATHEMATICS AND NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS
2015/16 ACADEMIC YEAR
SESSIONAL EXAM
MA110 - MATHEMATICS

TIME ALLOWED: Three (3) Hours.

18th JUNE, 2016

INSTRUCTIONS:

- (1) You must write your Computer number and program on each answer booklet you have used.
- (2) There are Seven (7) questions in this paper. Attempt Any Five (5). Each question consists of a,b,c,d,e. All questions carry equal marks.
- (3) Calculators are NOT allowed in this paper.
- (4) Should you have any problem or if you are in need of more answer booklets, put up your hand, an invigilator will come and attend to you.

1. a) i) Prove the De Morgan's laws

2x2 = 4
4x8

$$(A \cup B)' = A' \cap B'.$$

ii) Show that

$$X = (X \cap Y) \cup (X \cap Y')$$

b) i) Express the following in the form of $\frac{a}{b}$ where a and b are integers, $b \neq 0$.

$$0.533333\dots$$

ii) Given that $\sqrt{5}$ is an irrational number, prove that $\sqrt{5} + 1$ is an irrational number.

c) Determine whether the function f is even, odd or neither.

$$f(x) = x^3 + x$$

2. a) Express $\frac{\sqrt{x}}{2\sqrt{x}-1}$ in the form $a + b\sqrt{x}$, where a and b are real numbers.

b) i) Express $1 + 4i + \frac{5}{2-i}$ in form $a + bi$ and find its absolute value(modulus).

ii) Solve for x and y given that:

$$\frac{x+iy}{2+i} = 5 - i$$

c) Let '*' be a binary operation on the set of real numbers defined by $a * b = 2^{b-a}$, where a and b are real numbers.

i). Is '*' commutative on real numbers? Justify your answer.

ii). Find $-1 * (4 * 9)$.

d) Sketch the graph of the following radical function:

$$f(x) = 2 + \sqrt{-x+2}$$

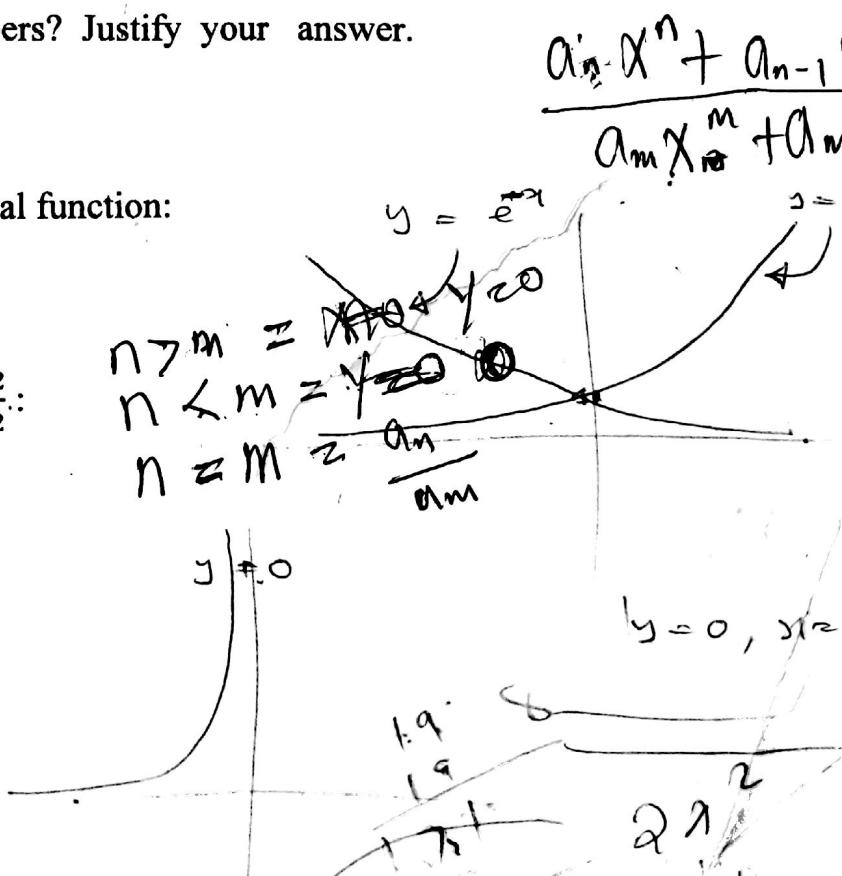
3. a) Given the rational function $f(x) = \frac{x+2}{x-2}$:

i). Find the domain of $f(x)$

ii). Find the vertical asymptotes

iii). Find the horizontal asymptotes

iv). Sketch the graph of $f(x)$.



b) Given the universal set $U = [1, 12]$ where $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $B = [1, 5)$. Find
i) $A \cap B$ ii) $A - B$.

c) i) Verify that the two given functions are inverses of each other

$$f(x) = x^3 + 1 \text{ and } g(x) = \sqrt[3]{x - 1}.$$

ii) Determine the domain of functions:

$$f(x) = x + \sqrt{x^2 + 4x - 12}.$$

4. a) Use the Rational root theorem to solve the following equation:

$$3x^4 + 5x^3 - 5x^2 - 5x + 2 = 0.$$

b). i) Solve each of the following equation

$$\left| \frac{x+1}{x-4} \right| \leq 3$$

ii). Redefine $k(x) = |2x - 1| - |x + 2|$ by removing the modulus, hence sketch the graph of function:

c) Let $f(x) = 3x^2 + 12x + 5$ be a quadratic function.

i) By completing the square, express $f(x)$ in the form $f(x) = a(x + p)^2 + q$ where a, p and q are constants.

ii) State the maximum or the minimum point of the function f .

iii) Sketch the graph of the function $f(x) = 3x^2 + 12x + 5$

d) i) Find the values of k if the equation $x^2 + (k - 2)x + 10 - k = 0$ has equal roots.

ii) The roots of the equation $2x^2 + 6x - 15 = 0$ are α and β . Find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$. Hence find the quadratic equation whose roots are

$$\frac{1}{\alpha^2} \text{ and } \frac{1}{\beta^2}.$$

$$\frac{\beta^2 + \alpha^2}{(\alpha^2 \beta^2)}$$



THE COPPERBELT UNIVERSITY
SCHOOL OF MEDICINE
DEPARTMENT OF MATHEMATICS
TEST ONE 2018/19 ACADEMIC YEAR
MA110 - Mathematical methods

INSTRUCTIONS

- (1) You must write your NAME, PROGRAM, COMPUTER NUMBER AND GROUP on the cover of your answer sheet.
- (2) There are five questions in this paper. Attempt all Questions.
- (3) Show all necessary working and number the pages in your answer sheet
- (4) Calculators are NOT allowed in this paper

QUESTION ONE

(a) Given that A, B and C are sets, simplify the following if possible
$$[(A \cap B)' \cap (A' \cup B)]'$$

(b) Rationalize the denominator of $\frac{1}{(\sqrt{2}+1)(\sqrt{3}-1)}$

(c) Sketch the graph of $f(x) = \begin{cases} 2x + 3 & \text{if } x < 0 \\ x^2 & \text{if } 0 \leq x < 2 \\ 1 & \text{if } x \geq 2 \end{cases}$

(d) Prove that $\sqrt{2}$ is an irrational number.
$$\sqrt{2} = \sqrt{1+1} = \sqrt{1+\frac{1}{1}} = \sqrt{1+\frac{1}{1+\frac{1}{2}}} = \sqrt{1+\frac{1}{1+\frac{1}{1+\frac{1}{2}}}}$$

(e) Using synthetic division find the quotient and the remainder when $f(x) = x^3 + 2x^2 + x - 2$ is divided by $x - (1+i)$.

QUESTION TWO

(a) Express $2.07\bar{2}$ as a fraction $\frac{a}{b}$ in its simplest form where a and b are integers and $b \neq 0$.
$$2.07\bar{2} = 2 + 0.07\bar{2} = 2 + \frac{7}{99} = \frac{187}{99}$$

(b) Express $f(x) = 2x^2 - 3x - 4$ in the form $f(x) = a(x+p)^2 + q$ where a, p and q are constants indicating the axis of symmetry and the coordinates of its maximum or the minimum point.

- (c) Define an operation * on the set of real numbers by $a * b = a + b - 2\sqrt{ab}$
- i). Is * a binary operation on the set of real numbers? Give reason for your answer.
 - ii). Evaluate $(1 * -1) * 2$ and $1 * (-1 * 2)$ and state whether * is associative.

(d) Let $f(x) = \frac{7}{x-4}$ and $g(x) = \frac{2}{x}$, find $(f \circ g)\left(-\frac{1}{2}\right)$ and determine its domain of $(f \circ g)$

(e) Determine whether the function $f(x) = \frac{x}{\sqrt{x^2 + 4}}$ is one-to-one. If it is, find the inverse and graph both the function and its inverse

QUESTION THREE

(a) State and prove one of de-Morgan's laws.

(b) Let \mathbf{R} , the set of real numbers be the universal set. If $A = [-7, 8] \cup [1, \infty)$ and $B = [0, 20]$, find the following sets and display them on the number line:

(i) A' . (ii) $A \cap B$.

(c) Let α and β be the roots of the quadratic equation $4x^2 + 3x - 2 = 0$

Find the sum $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

(d) Sketch the graph of the function $k(x) = |2x - 1| - |x + 2|$

QUESTION FOUR

$\frac{4+6}{1} - 2x + 1 + x + 2$

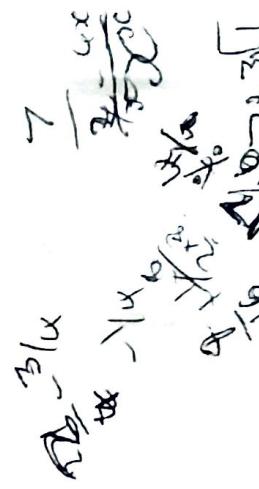
(a) Solve the inequality $|x - 1| > 1 - x^2$

(b) Given the rational function $f(x) = \frac{x^2 + 2}{x - 1}$. Sketch its graph indicating its domain and range, all the asymptotes and intercepts.

(c) Determine the domain of the given function: $f(x) = \sqrt{\frac{x+1}{x-1}}$

(d) Let $z = x + iy$ be a non zero complex number. Given that $z + \frac{1}{z} = k$, where k is a real number, prove that $|z| = 1$

(e) Solve for x and y given that $\frac{1}{x+iy} + \frac{1}{1+3i} = 1$



QUESTION FIVE

(a) Simplify $\frac{2^n - 6^n}{1 - 3^n}$

(b) Sketch $f(x) = 2 + 3\sqrt{-x + 1}$ and determine its range and domain

(c) Rationalize the denominator $\frac{5\sqrt[3]{x^2}}{4\sqrt{x}}$ and express the final answer in simplest radical form.

(d) If the equation $x^2 - (p - 2)x + 1 = p(x - 2)$ is satisfied by only one value of x , what are the possible values of p

(e) Use the definition of the absolute value, evaluate $\left|x - \frac{2}{3}\right| = \frac{3}{4}$



The Copperbelt University
School of Mathematics And Natural Sciences

Department of Mathematics

MA 110 : (Mathematical Methods I) : Deferred Test

August 15, 2022

Instructions

- (1). You must write your Name, Student Identification Number (SIN) and Programme of study on your answer sheet. Calculators are not allowed. Time allowed is 1hr:30 minutes
- (2). There are Four (4) questions in this paper, for deferred test 1, attempt questions 1 and 2 and deferred test 2, attempt questions 3 and 4.
-

QUESTION ONE

(a). Express $2.0\overline{72}$ as a fraction $\frac{a}{b}$ in its simplest form where a and b are integers and $b \neq 0$ (5 marks)

(b). Evaluate and Simplify $\frac{2^{n-1} - 8^n}{\frac{1}{2} - 4^n}$. (5 marks)

(c). Rationalize the denominator of $\frac{1}{(\sqrt{2} + 1)(\sqrt{3} - 1)}$. (5 marks)

(d). Determine the domain of the given function $f(x) = \sqrt{\frac{x+1}{x-1}}$. (5 marks)

(e). Solve for x and y given that $\frac{1}{x+iy} + \frac{1}{1+3i} = 1$. (5 marks)

QUESTION TWO

(a). Prove the De Morgans law $(A \cap B)' = A' \cup B'$. (5 marks)

(b). Determine whether the given function is odd, even or neither

$$f(x) = \frac{x^3 + 2x}{2}.$$

(5 marks)

(c). Prove that $\sqrt{2}+\sqrt{3}$ is an irrational number using the fact that $\sqrt{6}$ is an irrational number.

(5 marks)

(d). Find the square root of a complex number $15 + 8i$.

(5 marks)

(e). Sketch the graph of

$$f(x) = \begin{cases} 2x + 3 & x < 0 \\ x^2 & 0 \leq x < 2 \end{cases}$$

(5 marks)

(Total Marks: 25)

QUESTION THREE

(a). Change the repeating decimal $5.\overline{7}$ to its reduced form $\frac{a}{b}$, where a and b are integers and $b \neq 0$ using sum to infinity of a geometric series.

(5 marks)

(b). Express $\frac{3x^2 + 2x - 9}{(x^2 - 1)^2}$ into a partial fraction.

$$(x^2 - 1)(x^2 - 1)$$

(5 marks)

(c). Use Mathematical Induction to prove that $3^{2n} - 1$ is divisible by 8 for all $n \in \mathbb{N}$.

3.4
(5 marks)

(d). Find the first term and the general expansion of $\frac{1}{(2 - 6x)^5}$ in ascending power of x . State the range of value of x for which this expansion is valid.

(5 marks)

(e). Graph the function of $f(x) = 2^{(x-3)} + 2$ and obtain its inverse on the same graph. (3 marks)

the same axis.
(5 marks)

(Total Marks: 25)

QUESTION FOUR

(a) Prove that $\log_a(A^C) = C \log_a(A)$.

(5 marks)

(b) Find the center and radius of the circle whose equation is

$$x^2 + y^2 + 8x - 2y + 13 = 0.$$

(5 marks)

(c) Solve the equation $\log_3 x - 4 \log_x 3 + 3 = 0$.

(5 marks)

(d) Write down the constant term in the expansion of $\left(x - \frac{1}{2x^2}\right)^9$.

(5 marks)

(e) Use crammer's method to solve the linear system of equation

$$x + 2z = 9$$

$$2y + z = 8$$

$$4x - 3y = -2$$

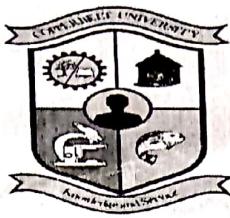
(5 marks)

(Total Marks: 25)

THE END OF TEST

$$\begin{array}{r} 5.2 \\ 0.7 \\ \hline 5.19 \end{array} \quad \begin{array}{r} 57 \\ 0.007 \\ \hline 5.707 \end{array}$$





THE COPPERBELT UNIVERSITY
SCHOOL OF MATHEMATICS AND NATURAL SCIENCES
DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MA 110 - MATHEMATICAL METHODS I

TEST 1

DATE: 8th December 2017

DURATION: 3 Hours

MARKS: 100

READ THE FOLLOWING INSTRUCTIONS

1. Write your **NAME, PROGRAM, COMPUTER NUMBER AND GROUP** on the cover of your answer sheet.
2. This is a **THREE Hours** test. Cell phones are **NOT** allowed.
3. Attempt **ALL** questions. Answers to questions should fully be explained. A correct but unclear answer will not get full marks.
4. No pencil work (except for graph sketching) or any work in red ink will be marked.
5. Use of correction fluid or "Tip-Ex" and **calculators** are **NOT** allowed.

Question 1

- (a) Let $E = \{3, 4, 5, 6\}$, $F = \{0, 2, 4, 6, 8\}$ and the universal set $X = \{0, 1, 2, \dots, 10\}$. Find $E \cap F'$. [1 mark]

(b) Let

$$f(x) = \frac{2}{x^2 - 2}, \text{ and } g(x) = \frac{1}{\sqrt{x+1}}$$

Find $(g \circ f)(-2)$.

[4 marks]

- (c) Use long division to divide $4x^3 - 7x^2 - 11x + 5$ by $4x + 5$.

[3 marks]

- (d) If A and B are subsets of the universal set U such that $A \subset B$. Simplify

(i) $A' \cup B'$ (ii) $A - B$.

[2+2 marks]

- (e) Find the possible values of λ and k if the expression $3x^4 + \lambda x^3 + kx + 4$ is exactly divisible by $x - 1$ and leaves a remainder of 18 when divided by $x + 2$. [5 marks]

$x = 1$

$x = -2$

Question 2

- (a) Prove the De-Morgan's law; $(A \cap B)' = A' \cup B'$. [5 marks]
- (b) Is the function $f(x) = |x| + x^2$ even, odd or neither? Justify your answer. [3 marks]
- (c) Solve the inequality below and write the solution set in interval notation [5 marks]
- $$\frac{x+4}{x+1} \leq \frac{x-2}{x-4}$$
- (d) Use Factor Theorem and synthetic division to factorise $f(x) = 6x^4 - 19x^3 + 17x^2 - x - 3$ completely. [4 marks]

Question 3

- (a) Sketch the following piecewise defined function

$$f(x) = \begin{cases} |x+2| & \text{if } -4 \leq x < 0 \\ x^2 & \text{if } 0 \leq x \leq 2 \\ 3 & \text{if } 2 < x \leq 3 \end{cases}$$

$$K = \frac{8t - t(1)(e)}{2}$$

$$= \frac{\cancel{7}6}{\cancel{7}6}$$

[4 marks]

- (b) Prove that $\sqrt{2} + \sqrt{3}$ is irrational.

$$K = 38c$$

[4 marks]

- (c) Solve the polynomial equation $2x^5 - 5x^4 + x^3 + x^2 - x + 6 = 0$.

[5 marks]

- (d) Solve the following inequality and write the solution in interval notation

$$3|2x+1| + 1 \leq 7.$$

[4 marks]

Question 4

- (a) Let the universal set be the set of real numbers, with $A = (3, 8]$, $B = (2, 7)$, $C = [1, 5]$ and $D = [6, \infty)$. Find (i) $(A \cup C)$ (ii) $(B \cap D)$ (iii) $(A \cup C) - (B \cap D)$ [1+1+2 marks]

- (b) Is the binary operation $*$ defined by $a * b = a + b - ab$ both commutative and associative. Justify your answer. [3 marks]

- (c) Sketch the graph of the polynomial $f(x) = -(x-3)(x-2)^3(x+1)^2$. [5 marks]

- (d) Solve the equation $|2x+11|=7$. [2 marks]

- (e) The roots of the equation $x^2 - 9x + K = 0$ are α and $\alpha + 1$. Find the value of K .

[3 marks]

$$\begin{aligned} -x^2 + 2x + 3 &= 0 \\ -x^2 &= -3 \\ x^2 &= 3 \\ x &= \pm\sqrt{3} \end{aligned}$$

$$\begin{aligned} x^2 - 6x - 3x + K &= 0 \\ x(x-6) - 3(x-K) &= 0 \\ x^2 - 9x + K &= 0 \end{aligned}$$

$$\begin{array}{cccccc} 2x^5 & + & 5x^4 & + & x^3 & + x^2 - x + 6 \\ 5 & & 8 & & 3 & & 3 & - & 9 \end{array}$$

$$\begin{array}{r} x+2 \\ \hline x+1 & 2x^5 + 5x^4 + x^3 + \\ & - 2x^5 - 5x^4 - x^3 \\ & \hline & 2 & 3 & - 2 \\ & & 2 & 3 & 5 \end{array}$$

Question 5

(a) Express

$$\frac{\sqrt{3} + 1}{\sqrt{3} - 1} + \sqrt{3} - 1$$

in the form $a + b\sqrt{3}$ where a and b are rational numbers.

(b) Verify that

[4 marks]

$$f(x) = 4x - 5 \text{ and } g(x) = \frac{x+5}{4}$$

are inverse functions of each other.

(c) Express $f(x) = 1 - 6x - x^2$ in the form $f(x) = a(x + h)^2 + k$ where a, h and k are rational numbers. Hence, write down the coordinates of the turning point of the graph $f(x) = 1 - 6x - x^2$.

[4 marks]

(d) Solve the equation $2x - 11\sqrt{x} + 12 = 0$.

[5 marks]

[4 marks]

Question 6

(a) Solve for x and y where x and y are real numbers

$$(x + yi) - i = i(x + yi) + 5.$$

[4 Marks]

(b) The equation $Kx^2 - 2Kx + 2K = 1$ where K is a constant has two real solutions.

(i) Show that K satisfies the inequality

$$K^2 - K \leq 0.$$

[2 Marks]

(ii) Hence, find the set of all possible values of K .

[3 Marks]

(c) For the following rational function

$$f(x) = \frac{2x^2 - 2}{x^2 - 4},$$

(i) Determine the x -intercepts and the y -intercept.

[3 Marks]

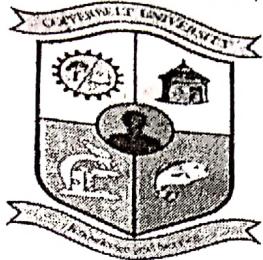
(ii) Find the horizontal and the vertical asymptotes.

[2 Marks]

(iii) Sketch the graph of f .

[3 Marks]

COPPERBELT UNIVERSITY



SCHOOL OF MATHEMATICS AND NATURAL SCIENCES DEPT OF PURE & APPLIED MATHEMATICS MA 110 - MATHEMATICAL METHODS I | TEST 1 2018

INSTRUCTIONS; 1. Attempt all Questions in this Paper without Using a Calculator.
 2. Indicate clearly your Names, SIN and the Group you belong to.
 3. Duration is 3 Hours Only.

1.
 - a. i.) Let $A = \{1, 2, 3\}$ and $B = \{2, 4\}$, Find $A \times B$. 1 Mks
 - ii.) Prove that $(A^c)^c = A$ by Arbitrary Elementary Method. 4 Mks
- * b. If the Operation $*$ is defined as, "add the first number to 8 times the second number"
 Find $(2 * 3) * 5$ $2 + 8 \times 5$
2 Mks
- c. Find the value of k given that when $2x^3 - 2kx^2 - 3x - 2$ is divided by $x - 2$,
 the Remainder is 40. 3 Mks
- * d. If $gof(x) = x$ and $g(x) = \frac{x+1}{x-1}$,
- * i.) Find $f(x)$ ii.) Sketch the Graph of $f(x)$ and Find the Range of $f(x)$ 3 Mks, 4 Mks

2. a. Prove the De Morgan's Law: $A^c \cup B^c = (A \cap B)^c$ 5 Mks
- b. Solve the following Equations involving the Absolute value functions:

$$|8x + 3| = |2x - 21| 3 Mks$$

c. Solve the following inequation:

$$\frac{x-2}{x+1} \geq \frac{x-6}{x-2} \quad 5 \text{ Mks}$$

d. Using Synthetic Division, show that both $x-2$ and $x+3$ are Factors of:

$$f(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18.$$

Hence, or otherwise Factorize $f(x)$ completely 4 Mks

3. a. Express the following in the form $\frac{a}{b}$, where $a, b \in \mathbb{Z}$ and $b \neq 0$.

i.) $0.\overline{121212\dots}$ ii.) $1.\overline{3121212\dots}$ 1.5 Mks, 1.5 Mks

* * b. Use the fact that $\sqrt{6}$ is Irrational to prove that $\sqrt{2} + \sqrt{3}$ is Irrational. 4 Mks

c. Sketch the graphs of:

i.) $f(x) = -|x+3| - 4$ ii.) $f(x) = 3 + \sqrt{3-x}$ 2.5 Mks, 2.5 Mks

d. Solve the Polynomial Equation $x^4 - 6x^3 + 22x^2 - 30x + 13 = 0$ 5 Mks

4. a. Determine the vertex and Intercepts for the following Quadratic function:

$$f(x) = x^2 - 6x - 16 \quad 2 \text{ Mks}$$

Vertex $\left[\frac{-b}{2a}, \dots \right]$

b. Sketch the graph of the Polynomial given by;

$$f(x) = (x-1)^2(x-3)^3(x+4) \quad 5 \text{ Mks}$$

c. Given that the roots of $x^2 + 3x + 17 = 0$ are α and β respectively. Find a Quadratic Function

whose roots are $\alpha^3 + \beta^3$ and $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

roots
sum $\alpha + \beta = -1$
product $\alpha\beta = \frac{c}{a}$

* d. Given that set $A = \{1, 2, 3\}$ and set $B = \{2, 4, 6\}$, Determine whether the Operation;

$$A \circ B = P(A) - P(B).$$

- 5(2)
10+

is Binary on the Universal Power set, $P(E)$. 5 Mks

Hence if is Binary on the

$$P(E) \text{ since } P(E) = P(A) \cup P(B)$$

$$5+17-13$$

$$1-6+22-30+\beta$$

$$5+4$$

$$11-17$$

intersection the complement.

$$2+11-18+9$$

$$9-9=0$$

$$11-2$$

$$9$$

$$18-9$$

$$-5(-1)+16 \\ 5+16=21$$

5. a. If $x = \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}}$, find the value of $8x - x^2$. 4 Mks
- ✓ b. Given the Functions, $f(x) = x^2 + 4$ and $g(x) = x - 9$.
 Find the value of x for which $g[f(x)] = f[g(x)]$ 4 Mks
- ✓ c. Write the Expression $f(x) = 2x^2 + 12x + 14$ in the form $f(x) = a(x + h)^2 + k$
 where $a, h, k \in \mathbb{R}$.
Hence, state the turning point of $f(x)$. 4 Mks
- d. Calculate the value(s) of x that are valid for the Equation below.

$$\left| \frac{x-2}{x+3} \right| = 4$$
 5 Mks
6. a. Simplify $-\frac{25}{2} \left[\frac{1+2i}{3+4i} - \frac{2-5i}{-i} \right]$ 3 Mks
- ✓ b. Solve for x and y given that;

$$\frac{x}{1+i} - \frac{y}{2-i} = \frac{1-5i}{3-2i}$$
 5 Mks
- c. Solve the Inequality below and present your answer in Interval Notation:
 $3x^2 + 2x + 2 < 2x^2 + x + 4$ 4 Mks
- d. Graph the Rational Functional by finding the Asymptotes and Intercept:

$$f(x) = \frac{5x^2 - 2}{1-x}$$
 5 Mks