

THE COPPERBELT UNIVERSITY
SCHOOL OF MATHEMATICS AND NATURAL SCIENCES
DEPARTMENT OF PHYSICS

PH 110 INTRODUCTORY PHYSICS

TUTORIAL SHEET 1 2023: Units and Measurements

1. A carpet is to be installed in a room whose length is measured to be 13.71 m and whose width is measured to be 4.46 m. Find the area of the room.
2. A worker is to paint the walls of a square room 8.00 ft high and 12 ft along each side. What surface area in square meters must she cover?
3. An object in the shape of rectangular parallelepiped measures 2.0 in x 3.5 in x 6.5 in. Determine the volume of the object in m^3 .
4. One gallon of paint (volume = $3.78 \times 10^{-3} \text{ m}^3$) covers an area of 25.0 m^2 . What is the thickness of the paint on the wall?
5. A rectangular plate has a length of $(21.3 \pm 0.2) \text{ cm}$ and a width of $(9.8 \pm 0.1) \text{ cm}$. Calculate the area of the plate including its uncertainty.
6. How many significant figures are in the following numbers? (a) 78.9 ± 0.2 (b) 3.788×10^9 (c) 2.26×10^{-6} (d) 0.0053
7. The radius of a solid sphere is measured to be $(6.50 \pm 0.20) \text{ cm}$, and its mass is measured to be $(1.85 \pm 0.02) \text{ kg}$. Determine the density of the sphere in kg/m^3 and the uncertainty in the density.
8. Compute the value of $\sum_{i=1}^4 x_i$ if $x_i = (2i + 1)$.
9. The square of the speed of an object undergoing a uniform acceleration a is some function of a and the displacement s , according to the expression given by:
$$v^2 = ka^m s^n$$
where k is a dimensionless constant. Show by dimensional analysis that this expression is satisfied only if $m = n = 1$.
10. The period T of a simple pendulum is measured in time units and is given by:

$$T = 2\pi \sqrt{\frac{l}{g}}$$

where l is the length of the pendulum and g is the acceleration due to gravity in units of length divided by the square of time. Show that this equation is dimensionally consistent.

11. Express the following in terms of prefixes
 - (a) $0.00085 \text{ } l$ (b) $5.44 \times 10^{-11} \text{ } g$ (c) $73,000,000 \text{ m}$ (d) 9.450 s
12. Express the following
 - a) $100\text{L}/\text{hour}$ to cubic meters per second
 - b) 1 atmosphere to pascal
 - c) 100 Dyne to Newtons
13. Estimate how many people if they stood on top of each other would reach the moon.
14. If the unit of force is 100 N, unit of length is 10 m and unit of time is 100 s. What is the unit of mass in this system of units?
15. A furlong is 220 yards, a mile is 1760 yards or 1609 meters, and a fortnight is 14 days. In 1991, the Zambian athlete, Samuel Matete won an Olympic gold medal, in Zurich, Switzerland, when he represented Zambia in the 400 m hurdles. His average speed was 8.5 meters per second. Express his speed in
 - (a) kilometer per minute
 - (b) mile per hour
 - (c) furlong per fortnight
16. A unit of area, often used in measuring land areas, is the *hectare*, defined as 10^4 m^2 . A new open-pit copper mine in North-Western Province plans to excavate 75 hectares of land, down to a depth of 26 m, each year. What volume of earth, in cubic kilometers, would be removed during this time?
17. A given formula is derived for the velocity of an object v that varies with time expressed as $v = at^2 + bt + c$ where velocity v and time t are expressed in terms of SI units; determine the units of constants a , b and c in the given equation.
18. The centripetal force F acting on a particle depends on the mass m of the particle, its velocity v and radius r of the circle. Derive dimensionally the formula for force F .
19. In a particle accelerator operating at power P , a charged sphere of radius r and density ρ has its speed increased from u to $2u$ over a distance of x . A first year CBU student derives the formula:

$$x = \frac{2\pi r^3 u^2 \rho}{9P}$$

- i) Show that this formula is dimensionally not correct.
 - ii) Given that there is only one error in this formula for x , obtain the correct formula.
20. The speed of sound v might plausibly depend on the pressure p , the density ρ , and the volume V of the gas. Use dimensional analysis to determine the exponents x , y and z in the formula:

$$v = Cp^x \rho^y V^z$$

where C is a dimensionless constant. Hence write down the relationship between the said quantities based on the derived exponents.

21. A Copperbelt University railways engineering student, uses dimensional analysis to find the distance d over which a signal can be seen clearly in foggy conditions. The student assumes that the distance depends on the frequency f of the signal, the density ρ of the fog, and intensity of light (power/area) I from the signal. Show that

$$d = k \frac{1}{f} \left(\sqrt[3]{\frac{I}{\rho}} \right)$$

where k , is a dimensionless constant of proportionality

22. In the gas equation $\left(p + \frac{a}{V^2} \right)(V - b) = RT$, where p is the pressure, V is the volume R is the universal gas constant and T the temperature, what are the dimensions of a and b ?
23. The viscosity η of a gas depends on the mass m , the effective diameter d and the mean speed of the molecules v . Use dimensional analysis to find an expression for η .

TUTORIAL SHEET 1 2022: UNITS & MEASUREMENTS
SOLUTION MANUAL.

①

① Finding the area in m^2
Data given

$$L = 13.71 \text{ m}$$

$$W = 4.46 \text{ m}$$

$$A = m^2 = ?$$

Solution

from the given data, Area in m^2 is found as follows.

$$\text{Area } (A) = \text{length } (L) \times \text{width } (W)$$

$$A = L \times W$$

$$= 13.71 \text{ m} \times 4.46 \text{ m}$$

$$= (13.71 \times 4.46) \text{ m}^2$$

$$= 61.1466 \text{ m}^2$$

$$= \underline{\underline{61.1 \text{ m}^2}}$$

our ans remains correct
to 3 sif according to
multiplication rule

(2) Finding the surface area in square meters of the walls of the room.

Data given

$$h = 8.00 \text{ ft}$$

$$L = 12 \text{ ft}$$

$$A = \text{ft}^2 = ?$$

$$A = m^2 = ?$$

Solution

We must convert the area from feet^2 to meters^2 . Then in this case we have 4 sides of equal measurements so we multiply our answer by 4 sides.

$$\text{Area } (A) = \text{height } (h) \times \text{length } (L)$$

$$A = hL$$

$$A = 8.00 \text{ ft} \times 12 \text{ ft}$$

$$A = 96.00 \text{ ft}^2 = * \text{Area for one side}$$

$$A (4 \text{ sides}) = 4 \times 96.00 \text{ ft}^2$$

$$= 384.00 \text{ ft}^2$$

$$A (m^2) = 384.00 \text{ ft}^2 \times \left(\frac{1 \text{ m}}{3.28084 \text{ ft}} \right)^2$$

$$= 35.674676508 \text{ m}^2$$

$$= 35.675 \text{ m}^2 //$$

③ Finding the volume of the given object in m^3 .

Data given:

$$l = 2.0 \text{ in}$$

$$b = 3.5 \text{ in}$$

$$h = 6.5 \text{ in}$$

$$V = \text{in}^3 = ?$$

$$V = m^3 = ?$$

Solution.

here we first find the volume in inches and convert it to meters as required.

$$\text{So Volume } V = \text{length } (l) \times \text{breadth } (b) \times \text{height } (h)$$

$$V = l b h$$

$$= 2.0 \text{ in} \times 3.5 \text{ in} \times 6.5 \text{ in}$$

$$= (2.0 \times 3.5 \times 6.5) \text{ in}^3$$

$$= 45.5 \text{ in}^3$$

$$V (m^3) = 45.5 \text{ in}^3 \times \left(\frac{0.0254 \text{ m}}{1 \text{ in}} \right)^3$$

$$V = 0.000745611412 \text{ m}^3$$

$$= \underline{\underline{0.0007456 \text{ m}^3}}$$

Note: When converting units if the first digit of the converted figure or value is less than the first digit of the original value (metric value), we increase the number of significant figures (sf) in the converted unit (value) by one (more than the original value). So here we have

$$\begin{array}{ll} \underline{45.5} & \text{and } \underline{0.000745\dots} \\ (\text{original value}) & (\text{converted value}) \end{array}$$

$$\text{So } 4 > 0$$

hence the answer has 4 s.f
instead of 3 s.f

(4) Finding the thickness of the paint on a wall.

Data given

$$\text{Volume } (V) = 1 \text{ gallon}$$

$$1 \text{ gallon} \rightarrow 3.78 \times 10^{-3} \text{ m}^3$$

$$\text{Area} = 25.0 \text{ m}^2$$

$$\text{height} = h = ?$$

Diagrammatically



$$V = \pi r^2 h$$

$$A = \pi r^2$$

Solution

Let the volume of the paint

(V) = length × breadth × height

or specifically, let

$$\text{Volume} = \pi \times (\text{radius})^2 \times \text{height}$$

$$V = \pi r^2 \times h \quad \textcircled{1}$$

$$\text{where Area } (A) = \pi r^2 \quad \textcircled{2}$$

$$\textcircled{2} \text{ into } \textcircled{1}$$

$$V = A h \quad \textcircled{3}$$

Making h the subject gives

$$h = \frac{V}{A}$$

Substituting the given values now gives

$$h = \frac{3.78 \times 10^{-3} \text{ m}^3}{25.0 \text{ m}^2}$$

$$= \frac{3.78 \times 10^{-3} \text{ m}}{25.0}$$

$$= 0.0001512 \text{ m}$$

$$= \underline{0.000151 \text{ m}}$$

[Final should have 3 s.f according the division rules]

⑤ Calculating the Area of the plate with its uncertainty

Data given

$$\text{length} = (21.3 \pm 0.2) \text{ cm}$$

$$\text{width} = (9.8 \pm 0.1) \text{ cm}$$

$$A = m^2 = ?$$

$$\text{Area + Uncertainty} = m^2 = ?$$

Solution

Here we first find the Area of the plate without the uncertainty, we then find the uncertainty in measurements and add them. Finally we multiply the added (sum) of the uncertainties with the earlier found Area and write them down as follows.

$$\text{Area (A)} = \text{length (l)} \times \text{width (w)}$$

$$= (21.3 \times 9.8) \text{ cm}^2$$

$$= 208.74 \text{ cm}^2$$

$$= 210 \text{ cm}^2$$

[rounded off to the nearest 10 so that we should have 2 s.f according to multiplication rules]

Uncertainty

$$\frac{0.2}{21.3} \times 100\% + \frac{0.1}{9.8} \times 100\%$$

$$= 0.938967136\% + 1.020408163\%$$

$$= 1.959375299\%$$

$$= 2\% \quad [\text{Correct to 1 s.f}]$$

Finally

$$210 \text{ cm}^2 \pm \frac{2\%}{100\%} \times 210 \text{ cm}^2$$

$$= 210 \text{ cm}^2 \pm 4.2 \text{ cm}^2$$

$$= 210 \text{ cm}^2 \pm 4 \text{ cm}^2$$

- ⑥ Determining the number of significant figures (s.f)
 ⑦ 3.788×10^4 here 10^4 does not count it just
 Ans: 4 s.f in indicates the value of the number.

- ⑧ 2.26×10^{-6} same as (6)
 Ans: 3 s.f

⑨ Sorry ⑨ was skipped

- 78.9 ± 0.2 here ± 0.2 does not play a role in
 counting # of s.f & simply shows the uncertainty.
 Ans: 3 s.f So we only consider 78.9

⑩ 0.0053

Ans: 2 s.f

- (i) Zero before the decimal point (d.p) is not a s.f provided that there are no other numbers before the d.p other than zero.
 (ii) Provided that we only a zero before the (d.p) then zero next to or just after the (d.p) is not a (s.f)

(7) Determining the density of the sphere with its uncertainty
Data given

$$\text{Sphere's radius } (r) = (6.50 \pm 0.20) \text{ cm}$$

$$\text{Sphere's mass } (m) = (1.85 \pm 0.02) \text{ kg}$$

$$\text{Volume } (V) = m^3 = ?$$

$$\text{Density} = \text{kg/m}^3 = ?$$

Density and uncertainty = ?

Solution

We proceed just as we did in question (5) except that here we multiply the uncertainty of the radius measurement by three to get the uncertainty of volume according to the rules. So we proceed as follows.

$$\text{Volume of the sphere } (V) = \frac{4}{3}\pi r^3 \quad \text{---(1)}$$

$$\text{Density } (\rho) = \frac{\text{mass}}{\text{volume}} = \frac{m}{V} \quad \text{---(2)}$$

① into ②

$$\rho = \frac{m}{\frac{4}{3}\pi r^3}, \quad r = \frac{6.50 \text{ cm} \times 10^{-2} \text{ m}}{1 \text{ cm}}$$

$$\begin{aligned} \rho &= \frac{1.85 \text{ kg}}{1.15034651 \times 10^{-3} \text{ m}^3} \\ &= 1608.211077 \text{ kg/m}^3 \end{aligned}$$

$$= 1610 \text{ kg/m}^3$$

$$\begin{aligned} V &= \frac{4}{3}\pi \times \left(\frac{6.50 \text{ cm}}{1 \text{ cm}} \times 10^{-2} \text{ m} \right)^3 \\ V &= 1.15034651 \times 10^{-3} \text{ m}^3 \end{aligned}$$

rounded to the nearest 10
in order to have 3 s.f
according to division rules

We now find the uncertainty in measurements

$$= 10.31185031\%$$

Finally density uncertainty and its uncertainty will be

$$\begin{aligned}
 \text{Density (d)} &= 1610 \text{ kg/m}^3 \pm \frac{10.31185031\%}{100\%} \times 1610 \text{ kg/m}^3 \\
 &= 1610 \text{ kg/m}^3 \pm 166.02079 \text{ kg/m}^3 \\
 &= 1610 \text{ kg/m}^3 \pm 166 \\
 &= \underline{\underline{(1.61 \pm 0.17) \times 10^3 \text{ kg/m}^3}}
 \end{aligned}$$

⑤ Computing the value of $\sum_{i=1}^4 x_i$ if $x_i = (2i + 1)$

Data given

$$i = 1 \rightarrow 4$$

$$\kappa_i = (z_i + 1)$$

$$\sum_{i=1}^4 x_i = ?$$

Solution

Since in any calculation the symbol \sum stands for summation, we add the values found as follows.

for $i = 1$; $x_i = 2^i + 1 = (2 \times 1) + 1 = 3$

$$i=2; x_i = 2^i + 1 = (2 \times 2) + 1 = 5$$

$$i=3; x_i = x_{i+1} = (2 \times 3) + 1 = 7$$

$$i=4; \quad n_i = 2^{i+1} = (2 \times 1) + 1 = 3$$

$$\text{Finally } \sum_{i=1}^4 x_i - (2i+1) = 3 + 5 + 7 + 9 - (2 \times 4) + 1 = 9$$

(9) Dimensional analysis; showing that $v^2 = ka^m s^n$ is dimensionally correct.

Data given

v = speed

a = acceleration

s = displacement.

k = constant (dimensionless)

Solution units of

We first start by identifying the dimensions of each given information.

$$v = m/s = L T^{-1}$$

$$a = m/s^2 = L T^{-2}$$

$$s = m = L$$

$k = 0 = 0$ = dimensionless so it can be neglected.

now

$$v^2 = k a^m s^n$$

$$v^2 = a^m s^n$$

Putting dimensions gives:

$$(L T^{-1})^2 = (L T^{-2})^n$$

$$[L T^{-1}]^2 = [L T^{-2}]^m [L]^n$$

$$[L]^2 [T^{-1}]^2 = [L]^{m+n} [T^{-2}]^m$$

$$\frac{L \cdot H.S}{L} = L^2$$

$$T = T^{-2}$$

$$R.H.S$$

$$L = L^{m+n}$$

$$T = T^{-2}$$

lets now find the values of m and n

$$L^2 = L^{m+n}$$

$$2 = m+n \quad \textcircled{1}$$

$$\frac{L^2}{T^{-2}} = \frac{L}{T^{-2m}}$$

$$-2 = -2m$$

$$m = 1 \quad \textcircled{2}$$

$$\textcircled{2} \text{ into } \textcircled{1}$$

$$2 = m+n$$

$$2 = 1+n$$

$$n = 2-1$$

$$n = 1$$

$$\text{now } V^2 \propto a^m s^n$$

$$V^2 = K a^m s^n \quad \text{let } K=1$$

$$[L T^{-2}] = [L T^{-2}]^m [L]^n$$

$$[L T^{-2}] = L^{m+1} T^{-2}$$

$$\underline{[L T^{-2}] = L T^{-2}}$$

Dimensionally Correct.

$$\text{LHS} = \text{RHS}$$

So since $m=n=1$ in this case we have shown that the given equation is dimensionally correct.

(10) Showing that $T = 2\pi\sqrt{\frac{L}{g}}$ is dimensionally correct.

Data given

T = period of simple pendulum

L = length of the Pendulum

g = acceleration due to gravity

Solution

We proceed similarly as in the previous question except that here the power for the RHS units is

$$T = s = T^1$$

$$L = m = L$$

$$g = m/s^2 = L T^{-2}$$

$$[T] = [L]^{\frac{1}{2}} [L^2 T^{-2}]^{\frac{1}{2}}$$

$$[T] = [L]^{\frac{1}{2} + \frac{1}{2}} [T]^{-2 + (-\frac{1}{2})}$$

$$T = L^{\frac{1}{2}} T'$$

$$\underline{T = T'} \quad \therefore LHS = RHS$$

Note: $\sqrt{\frac{1}{g}} = \sqrt(g^{-1}) = (g^{-1})^{\frac{1}{2}}$
 $= [L]^{\frac{1}{2}} [L T^{-2}]^{-\frac{1}{2}}$

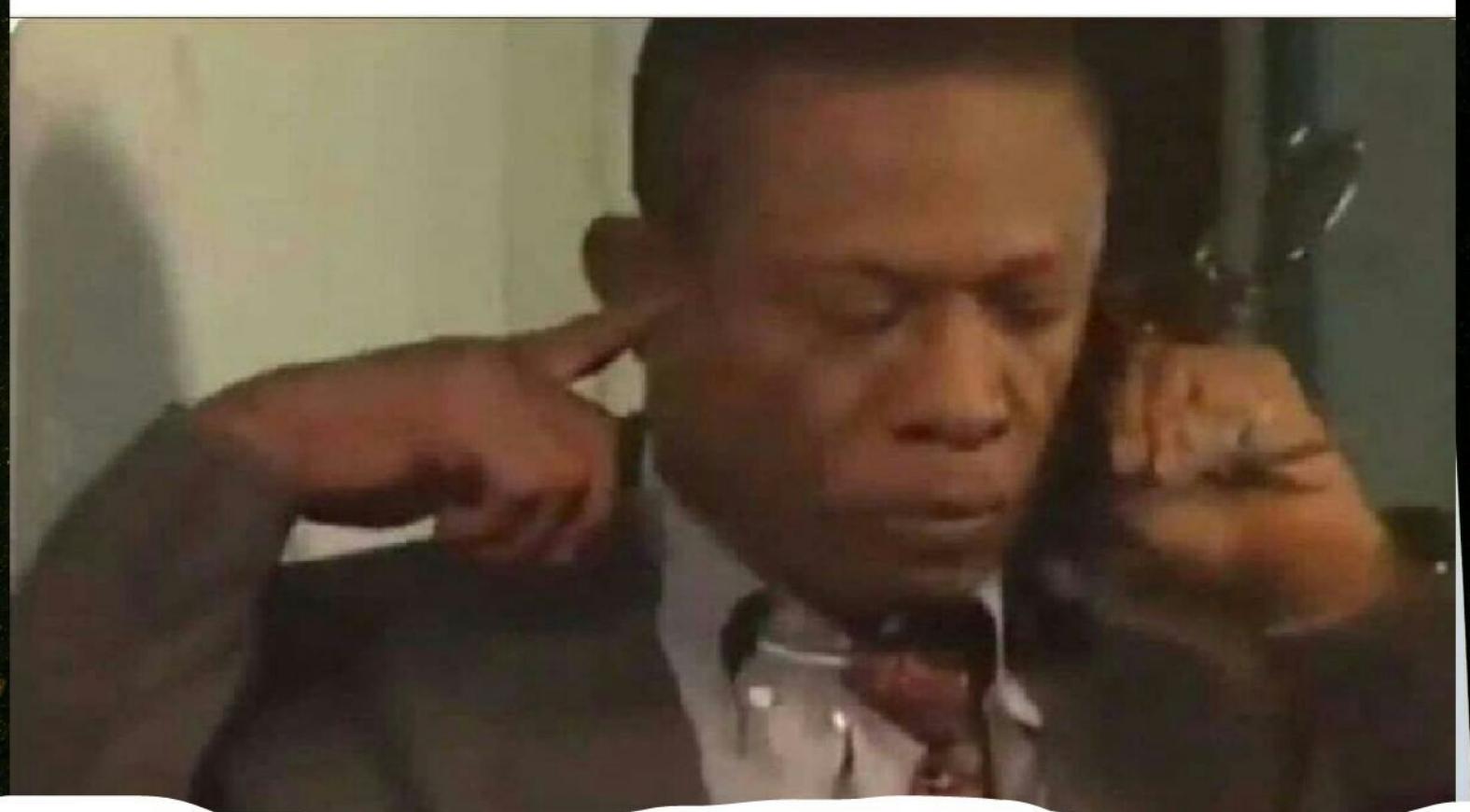
$(g^{-1})^{\frac{1}{2}}$
 $g^{-\frac{1}{2}}$

hence its dimensionally correct

When a short person views your status 😂

⌚ 0.5

Nobody HELB students when they hear people insulting HH



- (11) Expressing the values given in terms of prefixes

(a) Given value = 0.00085 litres
ANSWER = $0.85 \text{ ml} = 0.85 \text{ millilitres}$

(b) $5.44 \times 10^{-9} \text{ g} = 5.44 \times 10^{-2} \text{ nanograms}$

(c) 73 000 000 m = 73 Mega meters

(d) 9.450 s = 9450 milli seconds

Me collecting another **TOP FAN** badge from a page I had no idea I was following:



(12) Conversion of units.

@ 100 L/hour to cubic meters /second

ANS: $\frac{100 \text{ L}}{\text{h}} \times \frac{\text{h}}{\text{L}} \times 2.778 \times 10^{-3} \text{ m}^3/\text{s}$
 $= 2.778 \times 10^{-3} \text{ m}^3/\text{s}$

④ 1 atm to Pascal

ANS: This is just a conversion factor we have here

$$1 \text{ atm} = 101325 \text{ Pa}$$

(C) 100 dyne to Newtons

ANS: $100 \text{ dyne} \times \frac{1 \text{ N}}{100000 \text{ dyne}} = 0.001 \text{ N}$

(13) Estimating the number of people to reach the moon from the earth.

Solution

Given that the distance from the earth to the moon is about 384.4 million meters. And given also that the average height of a human being is about 1.72m, then we estimate as follows

$$\begin{aligned}\text{People required} &= \frac{384.4 \text{ million meters}}{1.72 \text{ meters}} \\ &= \frac{384.4 \text{ million}}{1.72} \\ &= \frac{384.4 \times 1000000}{1.72} \\ &= 223488372.1 \\ &\approx 223.5 \text{ million people}\end{aligned}$$

(14) Finding the unit of mass in the given system.

Data given

$$\text{Force (F)} = 100 \text{ N}$$

$$\text{Distance} = 10 \text{ m}$$

$$\text{Time} = 100 \text{ seconds}$$

$$\text{Acceleration} = ?$$

$$\text{Mass} = ?$$

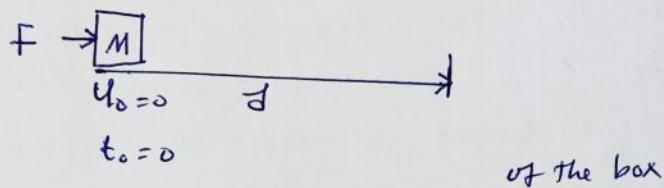
giv. Solution

Given that Force (F) = mass \times acceleration

$$F = ma \quad \text{--- (1)}$$

~~QED~~

lets have a clear picture of this given system. we assume we need to push a box of mass m with force F horizontally with a distance d .



of the box

let the initial velocity u_0 at time t_0 be equal to zero. given the distance d the box is to cover, let it undergo a certain acceleration a during the given period of time t .

$$d = u_0 t_0 + \frac{1}{2} a t^2 \quad \text{--- (2)}$$

$$d = \frac{1}{2} a t^2 \quad \text{--- (3) for } u_0 = t_0 = 0$$

making a the subject gives

$$a = \frac{2d}{t^2} \quad \text{--- (4)}$$

(4) into (1)

$$F = ma$$

$$F = m \frac{2d}{t^2} \quad \text{--- (5)}$$

making m the subject gives

$$m = \frac{F t^2}{2d} \quad \text{--- (6)}$$

lets now use (6) to find m .

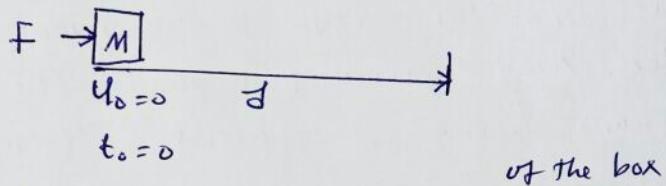
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$$d = \frac{1}{2} a t^2 \quad \text{--- (3) for } u_0 = t_0 = 0$$

making a the subject gives

$$a = \frac{2d}{t^2} \quad \text{--- (4)}$$

(4) into (1)

$$F = ma$$

$$F = m \frac{2d}{t^2} \quad \text{--- (5)}$$

making m the subject gives

$$m = \frac{F t^2}{2d} \quad \text{--- (6)}$$

lets now use (6) to find m .

$$\begin{aligned}
 m &= F t^2 / 2s \\
 &= \frac{100N \times (100)^2 s^{-2}}{2 \times 10m}, \quad N = \text{kg m/s}^2 \\
 &= \frac{100 \text{ kg m/s}^2 \times (100)^2 s^{-2}}{2 \times 10m} \\
 &= \frac{100 \text{ kg m/s}^2 \times (100)^2 s^{-2}}{2 \times 10m} \\
 &= \underline{\underline{50000 \text{ kg}}}
 \end{aligned}$$

(15) Conversion of units. (8.5 meters per second) to
 @ kilometer per meter

Ans:

$$\frac{8.5 \text{ m}}{\text{s}} \times \frac{60 \text{ s}}{1 \text{ minute}} \times \frac{1 \text{ Km}}{1000 \text{ m}} = \underline{\underline{0.51 \text{ Km/minute}}}$$

b) mile per hour

here we use the solution in @ above

Ans:

$$\begin{aligned}
 &\frac{0.51 \text{ km}}{\text{minute}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} \times \frac{1 \text{ mile}}{1.609 \text{ km}} \\
 &= 19.01802362 \text{ miles/hour} \\
 &= \underline{\underline{19 \text{ miles/hour}}}
 \end{aligned}$$

c) furlong per fortnight

Ans: We first convert miles per hour to yards/hour

so

$$\frac{19.018023 \text{ miles}}{\text{hour}} \times \frac{1760 \text{ yards}}{1 \text{ mile}}$$

$$= 33471.72157 \text{ yards/hour}$$

then we convert to furlong/hour as follows

$$\frac{33471.72157 \text{ yards}}{\text{hour}} \times \frac{\text{furlong}}{220 \text{ yards}}$$

$$= 152.1441889 \text{ furlong/hour}$$

Finally we convert furlong/hour to furlong/fortnight

$$\frac{152.1441889 \text{ furlong}}{\text{hour}} \times \frac{14 \text{ days}}{1 \text{ fortnight}} \times \frac{\text{hour}}{0.041666666 \text{ day}}$$

$$= 51120.44747 \text{ furlong/fortnight} \quad \text{Note: } 24 \text{ hours} \rightarrow 1 \text{ day}$$

$$= \underline{\underline{51000 \text{ furlong/fortnight}}} \quad \text{so } 1 \text{ hour} \rightarrow \frac{1}{24} \text{ day}$$

[Rounded off to the nearest hundred to make it become
Correct to 2. sf]

(16) Determining the volume of land to be removed for a year.

Data given

$$\text{height} = 26 \text{ m}$$

$$\text{Area} = ?$$

$$1 \text{ hectare} = 10^4 \text{ m}^2$$

$$\text{Volume} = \cancel{m^3} = ?$$

$$\text{Volume} = \cancel{Km^3} = ?$$

Solution

Since we don't know specifically the shape of the land to be cut out removed but just a few information, lets solve it in this form.

Let Volume (V) = length \times breadth \times height , length \times breadth = Area

$$V = Ah$$

$$V = Ah \quad \text{---(1)}$$

given that 1 hectare = 10^4 m^2

$V = 75 \times 10^4 \text{ m}^2 \times 26 \text{ m}$ the we are told 75 hectares will be removed, so

$$V = 260000 \text{ m}^3$$

$$A = \frac{75 \text{ hectares} \times 10^4 \text{ m}^2}{\text{hectare}}$$

We convert now

$$A = 75 \times 10^4 \text{ m}^2 \quad \text{---(2)}$$

$$h = 26 \text{ m} \quad \text{---(3)}$$

(2) and (3)
into (1)

$$V = 260000 \text{ m}^3 \times \left(\frac{1 \text{ km}}{1000 \text{ m}} \right)^3$$

$$= 2.6 \times 10^{-4} \text{ km}^3$$

(17) Dimensional analysis : finding the dimensions of a, b and c in the equation $V = at^2 + bt + c$

Data given

V = velocity

t = time

$a = b = c = \text{constant}$

Solution

According to the principle of homogeneity for dimensional analysis, units (similar) are added up divided, subtracted and divide multiplied.

So a, b, c should have similar units as those of t ,
And c should have units as those of V
This is because LHS should equalize RHS.
In terms of dimensions.

$$V = at^2 + bt + c$$

lets solve individually as follows

$$\textcircled{1} V = at^2 \quad , \quad V = LT^{-1}$$

$$t = T$$

$$\frac{LT^{-1}}{T^2} = a \frac{T^2}{T^2}$$

$$\therefore a = LT^{-3}$$

$$a = LT^{-1-2}$$

$$b = LT^{-2}$$

$$a = LT^{-3}$$

$$c = LT^{-1}$$

* lets try to prove.

$$\textcircled{2} V = bt$$

$$\frac{LT^{-1}}{T} = \frac{bT}{T}$$

$$b = LT^{-1-1}$$

$$b = LT^{-2}$$

$$V = at^2 + bt + c$$

$$[LT^{-1}] = [LT^{-3}][T]^2 + [LT^{-2}][T] + [LT^{-1}]$$

$$[L][T^{-1}] = [L][T^{-3+2}] + [L][T^{-2+1}] + [LT^{-1}]$$

$$[L][T^{-1}] = [L][T^{-1}] + [LT^{-1}] + [LT^{-1}]$$

$$\textcircled{3} V = c$$

$$LT^{-1} = c$$

$$\therefore c = LT^{-1}$$

$$LT^{-1} = LT^{-1} + LT^{-1} + LT^{-1}$$

$$LT^{-1} = 3LT^{-1}$$

, 3 is a

$$\therefore LHS = RHS$$

$$L = L$$

$$T^{-1} = T^{-1}$$

dimensionally
constant and
so it doesn't
count here

hence the dimensions for
a, b and c are correct.

(18) Dimensional analysis: Deriving dimensions of force F.

Data given

F = force

m = mass

v = velocity

r = radius of
the circle

solution

here we are told that

$$F \propto mvr$$

$$F = k m v r, \quad k = \text{dimensionally constant}$$

lets assign the powers of mvr
by x, y and z accordingly

so

$$F = km^x v^y r^z \quad \text{let } k=1$$

$$F = m^x v^y r^z \quad \textcircled{1}$$

we now right down the ~~units~~ units and dimensions

$$F = \cancel{\text{kg}} \text{m/s}^2 = \text{kg m s}^{-2} = M L T^{-2}$$

$$m = \cancel{\text{kg}} = M$$

$$v = \text{m/s} = L T^{-1}$$

$$r = m = L$$

now we go back to the equation

$$\begin{aligned} F &= [m]^x [v]^y [r]^z \\ &= [M]^x [L T^{-1}]^y [L]^z \\ &= [M]^x [L]^{y+z} [T^{-y}]^y \end{aligned}$$

$$M L T^{-2} = [M^x] [L^{y+z}] [T^{-y}]$$

$$\text{LHS} = \text{RHS}$$

$$M = M^x$$

$$\therefore x=1 \quad \textcircled{1}$$

$$L = L^{y+z}$$

$$y+z=1 \quad \text{---(2)}$$

$$\bar{r}^{-z} = \bar{r}^{-y}$$

$$-z = -y$$

$$\therefore y = z \quad \text{---(3)}$$

(3) into (2)

$$2+z=1$$

$$z=1-2$$

$$z=-1$$

So we now have

$$x = 1$$

$$y = z$$

$$z = -1$$

lets now recall the equation (1)

$$F = m^x v^y r^z$$

we now replace x, y, z with

$$1, 2, -1$$

$$F = m^1 v^2 r^{-1}$$

$$F = \frac{mv^2}{r}$$

$$\therefore F = \frac{mv^2}{r}$$

This is the formula for Centripetal force F derived dimensionally.

Never in my life have I wanted to steal a sign more than I do right now



(19)

$$x = \frac{2\pi r^3 u^2 p}{9P}$$

(i) To show that the equation is ^{not} correct, the dimensions of L.H.S \neq dimensions of R.H.S

R.H.S ;

$$\rightarrow \frac{2\pi r^3 u^2 p}{9P}$$

$$\rightarrow \frac{[\lambda]^3 [\lambda T^{-1}]^2 [M \lambda^{-3}]}{M \lambda^2 T^{-3}}$$

$$\rightarrow \frac{(\lambda^3)(\lambda^2 T^{-2})(M \lambda^{-3})}{M \lambda^2 T^{-3}}$$

$$\rightarrow \frac{\lambda^{3+2-3} T^{-2}}{\lambda^2 T^{-3}}$$

$$\rightarrow \frac{\cancel{\lambda^2} T^{-2}}{\cancel{\lambda^2} T^{-3}}$$

$$\rightarrow \frac{1}{T^{-1}}$$

$$\rightarrow T$$

But L.H.S;

 x λ

$$\lambda \neq T$$

hence shown

(ii) To get the correct formula, we must rid of T and add L

mathematically speaking

$$\rightarrow T \times \frac{L}{T}$$

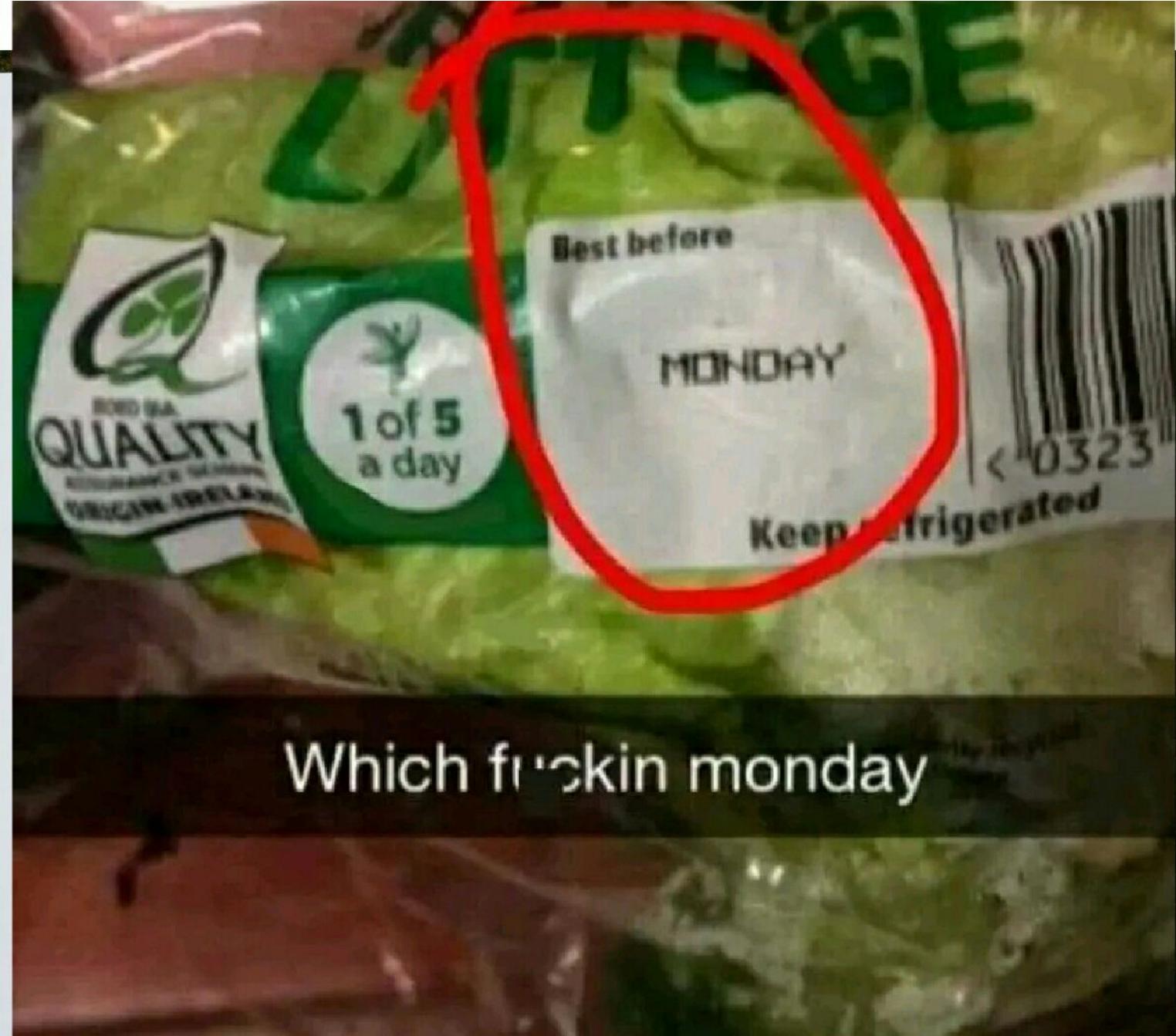
$$\rightarrow T \times LT^{-1}$$

a quantity that is close to what we have added (LT^{-1}) is velocity (m/s)

\therefore To get the correct formula simply multiply the R.H.S with velocity (v)

$$\left[\frac{2\pi r^3 u^2 \rho}{9\rho} \right] \times u$$

$$x = \underline{\underline{\frac{2\pi r^3 u^3 \rho}{9\rho}}}$$



Which frickin monday

(20) Determining the exponents of x, y and z in the given equation using dimensional analysis

Data given

v = speed

P = pressure

ρ = density

V = Volume

C = dimensionally constant.

We proceed as follows

We write the dimensions and units of the given data.

$$V = m/s = LT^{-1}$$

$$\begin{aligned} P &= kg \cdot m s^{-2} m^{-2} = [M] [LT^{-2}] [L^2] \\ P &= kg m^{-3} = [M] [L^{-3}] \\ V &= m^3 = [L^3] \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \textcircled{2}$$

now we have

$V = CP^x g^y \nu^z$ -① as the given equation, C is dimensionally constant.
so we replace ② into ①

$$[LT^{-1}] = [M] [LT^{-2}] [L^{-2}]^x [ML^{-3}]^y [L^3]^z$$

$$[L][T^{-1}] = [M^{x+y}] [L^{-x+3z-3y}] [T^{-2x}]$$

$$[L][T^{-1}] = [M^{x+y}] = [L^{-x+3z-3y}] [T^{-2x}]$$

we now proceed as follows
 $LHS = RHS$

$$T^{-1} = T^{-2x}$$

$$-1 = -2x$$

$$x = \frac{1}{2} \quad \textcircled{1}$$

$$L = L^{-x+3z-3y}$$

$$1 = -x + 3z - 3y \quad , x = \frac{1}{2}$$

$$1 = -\frac{1}{2} + 3z - 3y$$

$$3z - 3y = \frac{3}{2} \quad \textcircled{2}$$

$$M^o = M^{x+y}$$

$$o = x + y$$

$$y = -x$$

$$y = -\frac{1}{2}$$

now

$$1 = -x + 3z - 3y$$

$$1 = 3z - \frac{1}{2}y + \frac{3}{2}$$

$$1 = 3z + 1$$

$$z = 0$$

$$\therefore x = \frac{1}{2}$$

$$y = -\frac{1}{2}$$

$$z = 0$$

$$\text{hence } V = C P^x g^y V^z \quad \text{where } C=1$$

$$V = P^k g^{-l} V^0$$

$$V = \left(\frac{P}{g}\right)^{\frac{k}{l}}$$

$$\underline{V = C \sqrt[k]{\frac{P}{g}}^l}$$

(21) Dimensional analysis; showing that distance equals the given units.

Data given.

d = distance

f = frequency

I = intensity

P = density

k = dimensionally constant

solution

It's quiet the same as what we did in question 21
so let

$$d \propto f^x I^y \rho^z$$

$$d = K f^x I^y \rho^z$$

let assign the powers of the given units as follows

$$d = K f^x I^y \rho^z, K \text{ is dimensionally constant}$$

$$d = f^x I^y \rho^z \quad \textcircled{1}$$

so now we need to show that

$$d = K \sqrt[xyz]{\left(\frac{I}{\rho}\right)} \text{ as follows.}$$

dimensions of the given units

$$\begin{aligned} f &= T^{-1} \\ I &= M L^2 T^{-3} \\ \rho &= M L^{-3} \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \textcircled{2}$$

@ into \textcircled{1}

$$d = f^x I^y \rho^z$$

$$[d] = [T^{-1}]^x [M L^2 T^{-3}]^y [M L^{-3}]^z$$

$$L = [T^{-1}]^x [M^{y+z}] [L^{0y-3z}] [T^{-3y}]$$

$$L = [T]^{-x-3y} [M]^{y+z} [L]^{0y-3z} \quad \textcircled{2}$$

we can also write \textcircled{2} as

$$[M^0 T^0] [L] = [T]^{-x-3y} [M]^{y+z} [L]^{0y-3z}$$

now lets find the values of x, y and z

$$LHS = RHS$$

$$M^0 = M^{y+z}$$

$$0 = y+z$$

$$-y = z \quad -\textcircled{1}$$

$$\overline{L}^0 = \overline{L}^{-x - 3y}$$

$$0 = -x - 3y$$

$$x = -3y \quad -\textcircled{2}$$

$$L' = L^{0y - 3z}$$

$$1 = -3z$$

$$z = -\frac{1}{3} \quad -\textcircled{3}$$

$\textcircled{3}$ into $\textcircled{1}$

$$-y = z$$

$$-y = -\frac{1}{3}$$

$$y = \frac{1}{3} \quad -\textcircled{4}$$

$\textcircled{4}$ into $\textcircled{2}$

$$x = -3y$$

$$x = -3 \times \frac{1}{3}$$

$$x = -1$$

$$\therefore x = -1$$

$$y = \frac{1}{3}$$

$$z = -\frac{1}{3}$$

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Plug in the values of x, y and z into equation ① gives

$$d = f^x I^y g^z$$

$$d = f^{-1} I^{\frac{1}{3}} g^{-\frac{1}{3}}$$

$$d = \frac{1}{f} \left(\frac{I}{g} \right)^{\frac{1}{3}}$$

$$d = \frac{1}{f} \sqrt[3]{\frac{I}{g}}$$

$$\therefore d = K \frac{1}{f} \left(\sqrt[3]{\frac{I}{g}} \right)$$

Hence the above equation is dimensionally constant.

(22) Dimensional analysis: finding the dimensions of a and b in the given equation

Data given

P = Pressure

V = Volume

a, b = Constants

T = Temperature

R = Universal gas constant

$$(P + \frac{a}{V^2})(V - b) = RT$$

Solution

We proceed as we did in question 19. So here the units of b should be that of V and units of a should be that of P and V .

$$\text{So } b = v, v = L^3$$

$$\therefore b = L^3$$

$$\frac{a}{v^2} + p \quad \text{here } a = PV^2$$

$$\text{So } a = \frac{F}{A} V^2$$

$$\boxed{\frac{M L T^{-2}}{L^2}}$$

$$= [M][L][T^{-2}][L^2][L^3]^2$$

$$= [M][L]^{1+2+6}[T^{-2}]$$

$$[M][L]^9[T]^{-2}$$

$$\therefore a = M L^9 T^{-2}$$

$$b = L^3$$

(23) Dimensional analysis; find the expression for viscosity dimensionally.

Given data

η = viscosity

m = mass

v = speed

d = distance (diameter)

Solution:

lets proceed as follows (same as question 21)

$$\eta \propto m d v$$

$$\eta = K m d v, K \text{ is dimensionless constant.}$$

$$\eta = m d v$$

$$\eta = m^x d^y v^z$$

$$\eta = [M]^x [d]^y [v]^z$$

$$[M]^{-1} [T]^{-1} = [M]^x [L]^y [L T^{-1}]^z$$

$$[M]^x [L]^{-1} [T]^{-1} = [M]^x [L]^{y+z} [T]^{-z}$$

We now find the values of x, y and z as follows

$$LHS = RHS$$

$$M' = M^x$$

$$1 = x \quad \text{---(1)}$$

$$L^{-1} = L^{y+z}$$

$$-1 = y+z \quad \text{---(2)}$$

$$T^{-1} = T^{-z}$$

$$-1 = -z$$

$$z = 1 \quad \text{---(3)}$$

We now substitute the values of x, y and z in the equation

$$\eta = m^x d^y v^z$$

$$\eta = m' d^{-2} v'$$

$$\eta = \frac{m v}{d^2}$$

As required

then

$$-1 = y+z$$

$$-1 = y+1$$

$$y = -1 - 1$$

$$y = -2.$$