

THE COPPERBELT UNIVERSITY
PHYSICS DEPARTMENT
SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

DEFERRED TEST 1

COURSE: PH 110

APRIL 2021

ANSWER ALL QUESTIONS

TIME: TWO (2) HOURS

QUESTION ONE

(a) Convert the following:

- i. 100 mg into kg [2]
- ii. 0.005 N into cgs system [2]
- iii. 0.1 cm^2 into m^2 [2]
- iv. 0.1 liters into cm^3 [2]
- v. $2.5 \times 10^{-10} \text{ m}$ into μm [2]

(b) State two (2) advantages of using the method of Estimations and Order of Magnitude in calculations. [2]

(c) State two (2) disadvantages of using the method of Estimations and Order of Magnitude in calculations. [2]

(d) Given the vectors $\mathbf{A} = 5\hat{i} + 3\hat{j} + 2\hat{k}$ and $\mathbf{B} = -4\hat{i} + 4\hat{j} + 7\hat{k}$. Find the angle between vectors \mathbf{A} and \mathbf{B} [5]

(e) A car initially at point X moves at constant velocity for 0.5 km eastward for 10 minutes to reach point Y and after point Y it suddenly changes to a constant velocity to move another 2 km eastward for 5 minutes to reach point Z.

(i) Calculate the average velocity in SI units for the scenario [3]

(ii) Calculate acceleration of the car from X to Z. [3]

QUESTION TWO

(a) Given that the three vectors $\mathbf{A} = 2\hat{i} + 3\hat{j} - 2\hat{k}$, $\mathbf{B} = -4\hat{i} + 3\hat{j} + 7\hat{k}$ and $\mathbf{C} = -\hat{i} + 4\hat{j} + 9\hat{k}$ act at one point in space. Find:

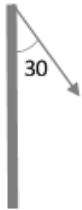
- (i) The resultant vector [6]
- (ii) The magnitude of the resultant vector [4]
- (iii) The unit vector of the resultant [4]
- (iv) The direction of the resultant with respect to the x-axis [4]

(b) Given the vectors $\mathbf{A} = 2\hat{i} + 3\hat{j} - 2\hat{k}$, $\mathbf{B} = -4\hat{i} + 3\hat{j} + 7\hat{k}$ and $\mathbf{C} = -\hat{i} + 4\hat{j} + 9\hat{k}$.

Find the scalar triple product $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$ [7]

QUESTION THREE

(a) A projectile is fired downwards with an initial velocity of 30 m/s at 30° angle to the vertical (as shown) and hits the ground after 10 s. Use $g = 9.82 \text{ m/s}^2$



Calculate:

- (i) The final velocity on impact [4]
- (ii) The height from which the projectile was fired [4]
- (iii) The range of the projectile [3]
- (iv) The x-component of the acceleration [2]

(b) A projectile is fired upward (vertically). It takes 30 seconds to come back to its starting point. Calculate:

- (i) The maximum possible height reached [4]
- (ii) The initial velocity [3]
- (iii) The velocity halfway upwards [3]
- (iv) Give one reason why in practice the time of ascent may be different from time of descent in projectile motions [2]

SOLUTION-QUESTION ONE

(a)

i. $1\text{g} = 1\text{kg}/1000 = 10^{-3}\text{ kg}$; $1\text{mg} = 1\text{kg}/1,000,000 = 10^{-6}\text{ kg}$; $100\text{mg} \Rightarrow 10^{-4}\text{ kg}$ [2]

ii. $0.005\text{ N} = 0.005\text{ kg.m/s}^2 = 0.005(1000)\text{g}(100)\text{cm/s}^2 = 100\text{ g.cm/s}^2$ [2]

iii. $1\text{cm} = 10^{-2}\text{m}$; $1\text{cm}^2 = 10^{-4}\text{m}^2$; $0.1\text{cm}^2 = 10^{-4}\text{m}^2 \times 10^{-1}\text{cm}^2 = 10^{-5}\text{ m}^2$ [2]

iv. $1\text{ L} = 1000\text{cm}^3$; $0.1\text{ L} = 100\text{ cm}^3$ [2]

v. $2.5 \times 10^{-10}\text{ m} = 2.5 \times 10^{-6} \times 10^{-4}\text{ m} = 2.5 \times 10^{-4}\text{ }\mu\text{m}$ [2]

(b) - Estimates serve as a partial check if the exact calculations are correct.

- Calculations can be carried out where limited information is available

- Can be used where it is difficult or impossible to get an exact answer in a calculation [2]

(c) - It does not give precise answers

- Values close to each other cannot easily be estimated apart [2]

(d) Given the vectors $\mathbf{A} = 5\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ and $\mathbf{B} = -4\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}$. Find the angle between vectors \mathbf{A} and \mathbf{B} [5]

$$\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}| |\mathbf{B}| \cos\theta; \cos\theta = \mathbf{A} \cdot \mathbf{B} / (|\mathbf{A}| |\mathbf{B}|); \theta = \cos^{-1} (\mathbf{A} \cdot \mathbf{B} / (|\mathbf{A}| |\mathbf{B}|))$$

$$\text{But, } \mathbf{A} \cdot \mathbf{B} = 5(-4) + 3(4) + 2(7) = -20 + 12 + 14 = 6;$$

$$|\mathbf{A}| = \sqrt{5^2 + 3^2 + 2^2} = \sqrt{38}$$

$$|\mathbf{B}| = \sqrt{(-4)^2 + 4^2 + 7^2} = \sqrt{81}$$

$$\text{Also } \sqrt{38} \times \sqrt{81} = 55.48$$

$$\text{Hence } \theta = \cos^{-1} (6/55.48); \theta = \cos^{-1} (0.108147); \theta = \cos^{-1} (0.108147) = 83.79^\circ$$

(e) A car initially at point X moves at constant velocity for 0.5 km eastward for 10 minutes to reach point Y and after point Y it suddenly changes to a constant velocity to move another 2 km eastward for 5 minutes to reach point Z.

(i) average velocity:

$$u = 0.5\text{km}/10\text{ min} = \frac{500\text{m}}{600\text{s}} = 0.833\text{ m/s}$$

$$v = 2\text{km}/5\text{ min} = \frac{2000\text{m}}{300\text{s}} = 6.667\text{ m/s}$$

$$\text{Average velocity} = (v+u)/2 = (6.667 + 0.833)/2 = 7.5\text{ m/s} \quad [3]$$

(ii) acceleration from X to Z:

$$\text{Acceleration} = (v-u)/t; \text{Acceleration} = (6.667-0.833)/900 = 6.48 \times 10^{-3}\text{ m/s}^2; [3]$$

SOLUTION - QUESTION TWO

(a) vectors $\mathbf{A} = 2\hat{i} + 3\hat{j} - 2\hat{k}$, $\mathbf{B} = -4\hat{i} + 3\hat{j} + 7\hat{k}$ and $\mathbf{C} = -\hat{i} + 4\hat{j} + 9\hat{k}$

(i) The resultant vector \mathbf{R} :

$$\mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C} = (2-4-1)\hat{i} + (3+3+4)\hat{j} + (-2+7+9)\hat{k} = -3\hat{i} + 10\hat{j} + 14\hat{k} \quad [6]$$

(ii) magnitude of \mathbf{R} : $R = \sqrt{(-3)^2 + 10^2 + 14^2} = \sqrt{305} = 17.46 \quad [4]$

(iii) Unit vector of \mathbf{R} : $\hat{R} = \frac{\mathbf{R}}{R} = \frac{-3\hat{i} + 10\hat{j} + 14\hat{k}}{17.46} \quad [4]$

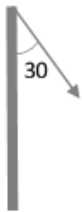
(iv) direction of \mathbf{R} : $\tan(\theta) = 10/3 = 3.333$; $\theta = \tan^{-1}(3.333) = 73.3^\circ \quad [4]$

(b) Find the scalar triple product $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$:

$$\begin{aligned} (\mathbf{B} \times \mathbf{C}) &= \begin{vmatrix} 2 & 3 & -2 \\ -4 & 3 & 7 \\ -1 & 4 & 9 \end{vmatrix} = 2(3 \times 9 - 7 \times 4) - 3(-4 \times 9 + 1 \times 7) - 2(-4 \times 4 + 1 \times 3) \\ &= -2 + 87 + 26 = 111 \end{aligned} \quad [7]$$

SOLUTION - QUESTION THREE

(a) A projectile is fired downwards with an initial velocity of 30 m/s at 30° angle to the vertical (as shown) and hits the ground after 10 s. Use $g = 9.82 \text{ m/s}^2$



(i) The final velocity on impact

$$u_y = 30 \cos(30) = 25.981 \text{ m/s}; \quad u_x = 30 \sin(30) = 15 \text{ m/s};$$

$$v_y = u_y + a_y t = 25.981 + 9.82(10) = 124.181 \text{ m/s}$$

$$v_x = u_x + a_x t = 15 + 0(10) = 15 \text{ m/s}$$

$$v = \sqrt{v_y^2 + v_x^2} = \sqrt{124.181^2 + 15^2} = 125.08 \text{ m/s} \quad [4]$$

(ii) The height from which the projectile was fired

$$v_y^2 = u_y^2 + 2a_y S_y \quad ; \quad S_y = \frac{v_y^2 - u_y^2}{2a_y} = (124.181^2 - 25.981^2)/2(9.82)$$

$$= (15420.928 - 675.012)/19.64 = 14745.916/19.64 = 750.81 \text{ m} \quad [4]$$

(iii) The range of the projectile

$$S_x = \frac{(v_x + u_x)t}{2} = \frac{2v_x t}{2} = v_x t = 15(10) = \mathbf{150\ m} \quad [3]$$

(iv) x-component of the acceleration = $\mathbf{0\ m/s^2}$ due to constant velocity [2]

(b) A projectile is fired upward (vertically). It takes 30 seconds to come back to its starting point. Calculate:

(i) The maximum possible height reached:

$$\text{Time taken to go up is } 30/2 = 15\ \text{s}; v_y = u_y + a_y t \Rightarrow 0 = u_y - 9.82(15)$$

$$u_y = 9.82(15) =$$

$$S_y = \frac{(v+u)t}{2} = \frac{(0+u)15}{2} = \frac{15u}{2} = \frac{9.82(15)(15)}{2} = \quad [4]$$

(ii) The initial velocity

$$u_y = 9.82(15) = \quad [3]$$

(iii) The velocity halfway upwards

$$v = \frac{u_y}{2} = \frac{9.82(15)}{2} = \quad [3]$$

(iv) Give one reason why in practice the time of ascent may be different from time of descent in projectile motions: $\mathbf{Air\ resistance}$ [2]