The Copperbelt University

School of Mathematics And Natural Sciences

Department of Mathematics

MA 110 : (Mathematical Methods I) : Deferred Test

August 9, 2023

Instructions

(1). You must write your **Name, Student Identification Number (SIN) and Programme** of study on your answer sheet. Calculators are not allowed. Time allowed is **1hr:30 minutes**

(2). There are **Four (4) questions** in this paper, for **deferred test 1, attempt questions 1 and 2** and **deferred test 2, attempt questions 3 and 4.**

QUESTION ONE

(a). Express $2.07\overline{2}$ as a fraction $\dfrac{a}{b}$ in its simplest form where a and b are integers and $b \neq 0$

(5 marks)

(b). Evaluate and Simplify $\dfrac{2^{n-1} - 8^n}{\frac{1}{2} - 4^n}$.

(5 marks)

(c). Rationalize the denominator of $\dfrac{1}{(\sqrt{2}+1)(\sqrt{3}-1)}$.

(5 marks)

(d). Determine the domain of the given function $f(x) = \sqrt{\dfrac{x+1}{x-1}}$.

(5 marks)

(e). Solve for x and y given that $\dfrac{1}{x+iy} + \dfrac{1}{1+3i} = 1$.

(5 marks)

QUESTION TWO

(a). Let $\mathcal{R}$, the set of all real numbers be the universal set.If $A = [-7, 8) \cup [11, \infty)$ and $B = [0, 20]$, find A' and $A \cap B$.

(5 marks)

(b). Determine whether the given function is odd, even or neither

$$f(x) = \frac{x^5 + x^3 + x}{2}.$$

(5 marks)

(c). Prove that $\sqrt{2}+\sqrt{3}$ is an irrational number using the fact that $\sqrt{6}$ is an irrational number.

(5 marks)

(d). Solve the inequality $\sqrt{2x+5} < \sqrt{9+x}$.

(5 marks)

(e). Sketch the graph of

$$f(x) = \begin{cases} 2x+3 & x < 0 \\ x^2 & 0 \leqslant x < 2 \end{cases}$$

$3 \times 3 \times 3 \times 3 \times 3B$

(5 marks)

(Total Marks: 25)

QUESTION THREE

(a). Change the repeating decimal $5.\overline{7}$ to its reduced form $\frac{a}{b}$, where a and b are integers and $b \neq 0$ using sum to infinity of a geometric series.

(5 marks)

(b). Express $\dfrac{3x^2+2x-9}{(x^2-1)^2}$ into a partial fraction.

$\dfrac{Ax+B}{(x^2-1)} + \dfrac{Cx+D}{(x^2-1)^2}$

(5 marks)

(c). Use Mathematical Induction to prove that $3^{2n} - 1$ is divisible by 5.

let $n=1$ $\quad 3^{(2)(2)} - 1$

$3^{2(1)} - 1 \quad 12 - 1 = 11$

$9 - 1 = 8$

(5 marks)

(d). Find the first term and the general expansion of $\dfrac{1}{(2-6x)^5}$ in ascending power of x. State the range of value of x for which this expansion is valid.

(e). Graph the function of $f(x) = 2^{(x-3)} + 2$ and obtain its inverse on the same axis.

(5 marks)

(Total Marks: 25)

QUESTION FOUR

(a) Find λ for which the matrix $\lambda I - A$ is a singular matrix if where I is an identity Matrix given that $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & -2 \\ 2 & -2 & 0 \end{pmatrix}$.

(5 marks)

(b) Find the center and radius of the circle whose equation is

$$x^2 + y^2 + 8x - 2y + 13 = 0.$$

(5 marks)

(c) Solve the equation $\log_3 x = -4\log_x 3 + 3$.

(5 marks)

(d) Write down the constant term in the expansion of $\left(x - \frac{1}{2x^2}\right)^{10}$.

(5 marks)

(e) Use crammer's method to solve the linear system of equation

$$\begin{aligned} x + 2z &= 9 \\ 2y + z &= 8 \\ 4x - 3y &= -2 \end{aligned}$$

(5 marks)

(Total Marks: 25)

THE END OF TEST

MA110 - MATHEMATICAL METHODS

Time allowed: Two hours (2:00 hours)

Instructions:

1. You must write your **Name**, your **Computer Number and programme of study** on your answer sheet.
2. Calculators are **not** allowed in this paper.
3. There are three (3) questions in this paper, Attempt **All** questions and show **detailed** working for full credit

---

QUESTION ONE

a) (i) If $C \subset D$, then simplify if possible

$$C' \cup D'$$

(2.5 marks)

2.5 (ii) Express $1.\overline{17}1717\ldots\ldots$ as a fraction $\frac{a}{b}$ in its simplest form where a and b are integers and $b \neq 0$. (2.5 marks)

b) Consider the binary operation $a * b = a + b - 2ab$, where $a\, and\, b$ are real numbers.

(i) Is $*$ a binary operation on the set of real numbers? Give reason for your answer. **(1) Mark**

(ii) Is the operation $*$ commutative? If not give a counter example. **(1)Mark**

(iii) Find the value of 1*(2*3) and (1*2)*3 and state whether * is associative **(3) Marks**

c) Given the rational function $f(x) = \frac{x+2}{x-2}$. Sketch its graph indicating its domain and range, all the asymptotes and intercepts. **(5 Marks)**

d) Prove that $\sqrt{2}$ is an irrational number **(5 Marks)**

e) Let $f(x) = \frac{x+1}{x-1}$ and $g(x) = \sqrt{x}$ .Find $(g\, o\, f)(x)$ and determine the domain (2, 2) (5 Marks)

(-1, 1)(-3, 2)

-3+2-2(

2-3-2(-3)(2) ; 2-3-2 2+2 -2(2)(2)

4-8

CamScanner

QUESTION TWO

4 a) Using the associative and distributive properties of unio and intersection of sets .Show that

$$A \cup B = (A \cap B) \cup (A \cap B') \cup (A' \cap B) \quad \textbf{(5 Marks)}$$

5 b) Let α and β be the roots of the quadratic equation $3x^2 + 2x + 5 = 0$. Find a quadratic equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ without calculating α and β **(5 Marks)**

3 c) Solve the given radical function inequality $\sqrt{2} - \sqrt{x+6} \leq -\sqrt{x}$ **(5 Marks)**

5 d) Solve for x and y given that:

$$\frac{x}{1+i} - \frac{y}{2-i} = \frac{1-5i}{3-2i} \quad \textbf{(5 Marks)}$$

5 e) Show that the function f defined by $f(x) = \frac{2x}{x-1}\ x \in R$, is a bijection on R on to $\{y \in R : y \neq 2\}$ **(5 Marks)**

QUESTION THREE

0 a) Use the Rational root theorem to solve $x^3 - 4x^2 + 8 = 0$ **(5 Marks)**

5 b) Rationalize the denominator $\frac{1}{(\sqrt{2}+1)(\sqrt{3}-1)}$ **(5 Marks)**

2.5 c) (i) Determine whether the function $f(x) = x^4 + x^2 + 1$ even, odd or neither. **(2.5 marks)**

2.5 (ii) Let $A = \{x \in \mathbb{R}: -4 \leq x < 2\}$ and $B = \{x \in \mathbb{R}: x \geq -1\}$.
Find a) $A \cap B$ b) A' **(2.5 marks)**

5 d) What are the dimensions of the largest rectangular field which can be enclosed by 1200 m of fencing? **(5 Marks)**

3 e) Sketch the graph of $f(x) = |2x+1|$. On the same diagram sketch also the graph of $g(x) = \sqrt{1-2x}$ and , hence, find the values such that $\sqrt{1-2x} > |2x+1|$ **(5 Marks)**

$\frac{x}{1+i} \cdot \frac{1-i}{1-i}$

$A = L \times B$

$= m \times m$

$A = [m][m]$

$A = [m]^{1+1}$

CamScanner

$k(k+2)$
$(k+1)(k+3)$
$2(k+1)+7$
$2k+2+7$
$2k+9 \quad (k+2)(2k+9)$
$k(2k+9)+2(2k+9)$

36
2×18
3×12
$b^2 - 4ac$
$P = 36$
$S = 13$
$F = 4, 9$

MA110 - MATHEMATICAL METHODS TEST 2

Time allowed: Two hours thirty minutes (2:30)

Instructions:

1. You must write your **Name,** your **Computer Number and programme of study** on your answer sheet.
2. Calculators are **not** allowed in this paper.
3. There are four (4) questions in this paper, Attempt **All** questions and show **detailed** working for full credit

---

QUESTION ONE

a) Express $\frac{2x+1}{x^3-1}$ in partial fractions (5marks)

b) Find the centre and length of a radius of the given circle and graph it $x^2 + y^2 - 10x = 0.$ (5marks)

c) Prove the result by induction: $1 \times 3 + 2 \times 4 + \cdots + n(n+2) = \frac{1}{6}n(n+1)(2n+7)$

(5marks)

d) Find the 4^{th} term in the binomial expansion $\left(2 - \frac{x}{2}\right)^9$ (5marks)

e) If $xy = 64$ and $log_x y + log_y x = \frac{5}{2}$, Find x and y. (5marks)

QUESTION TWO

a) A is the point $(-1, 2)$, B is the point $(2, 3)$ and C is the point $(3, 5)$. P is a point which divides BC in the ratio 3 : 4 and Q lies on AB such that $AQ = \frac{2}{5}AB$.

(i) Find the coordinates of P (2.5 marks)

(ii) Find the coordinates of Q. (2.5 marks)

b) Find λ for which the matrix $\lambda I - A$ is a singular matrix if where I is an identity Matrix given that $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & -2 \\ 2 & -2 & 0 \end{pmatrix}$ (5 marks)

c) Show that the sum of the integers from 1 to n is $\frac{1}{2}n(n+1)$ (5marks)

$S_n = \frac{1}{2}n(n-1)d$

d) Solve the logarithmic equation : $\log(x-4)+\log(x-1)=1$ (5marks)

e) In the expansion of $(1+ax)^n$, the first three terms in ascending power of x are $1-\frac{5}{2}x+\frac{75}{8}x^2$, Find the values of n and a, and state the range of values of x for which the expansion is valid. (5marks)

QUESTION THREE

a) Find the radius of the circle with center at $C(-2,5)$ if the line $x+3y=9$ is a tangent line. (5marks)

b) Using geometrical progression, change $0.2\overline{14}$ to $\frac{a}{b}$ form , where a and b are integers and $b\neq 0$. (5marks)

c) Use mathematical induction to prove that the statement is true for all positive integers n given that 4^n-1 is divisible by 3 (5marks)

d) Graph $f(x)=log_{\frac{1}{2}}x$ by reflecting the graph of $g(x)=\left(\frac{1}{2}\right)^x$ a cross the line $y=x$ (5marks)

e) (i) Find the inverse of the matrix $A=\begin{pmatrix}3 & -1 & 2\\ 1 & 1 & 1\\ 2 & 2 & -1\end{pmatrix}$ (5marks)

(ii) Use your inverse to solve the system of linear equations

$$3x-y+2z=4$$
$$x+y+z=2$$
$$2x+2y-z=3$$

(3marks)

QUESTION FOUR

a) Write the following in sigma notation $\sum_{n=1} n^4$

(i) $1-\frac{2}{3}+\frac{4}{9}-\frac{8}{27}+\cdots$ (3marks)

(ii) $1^4+2^4+3^4+\cdots+n^4+(n+1)^4$ (2marks)

b) The number of grams of a certain radioactive substance present after t hours is given by the equation $Q=Q_0e^{-0.45t}$, where Q_0 represents the initial number of grams. How long will it take 2500 grams to be reduced to 1250 given $ln\left(\frac{1}{2}\right)=-0.693$ (5marks)

c) (i) Expand $(1+2x)^4$and $(1-2x)^4$ in ascending powers of x. (5marks)

(ii) Hence reduce $(1+2x)^4-(1-2x)^4$ to its simplest form. (3marks)

(iii) Using the results in (ii) evaluate $(1.002)^4-(0.998)^4$ (5marks)

$1+2x=1.002$
$2x=0.002$
$\boxed{x=0.001}$
$1-2x=0.998$
$-2x=-0.002$
$x=0.001$

Learn Today, Lead Tomorrow 🍺 🍻

The Copperbelt University

School of Mathematics And Natural Sciences

Department of Mathematics

MA 110 : (Mathematical Methods I) : Test One

February 23, 2022

Instructions

(1). You must write your **Name, Computer number and Programme** of study on your answer sheet. Time allowed is **2 hours.**

(2). **Calculators** and use of **Cell phones** are **Not** allowed in this paper.

(3). There are **three (3) questions** in this paper, **attempt all the questions** and show detailed working for full credit.

QUESTION ONE

(a) Prove the De Morgan's law: $(A \cap B)' = A' \cup B'$.

(5 marks)

(b) Determine whether the function given is even,odd or neither

$$f(x) = x^6 + x^4 - 10x^2.$$

(5 marks)

(c) Given that $\sqrt{10}$ is irrational number, prove that $\sqrt{2} + \sqrt{5}$ is also an irrational number.

(5 marks)

(d) Graph the rational function : $f(x) = \dfrac{x^4 + 4}{(x-3)(x+2)}$.

(5 marks)

(e) Let $z = x + iy$ be a non-zero complex number. Given that $z + \dfrac{1}{z} = k$, where k [illegible] is a real number, prove that either z is a real number or $|z|$ [illegible]

(5 marks)

QUESTION TWO

(a) Express $3.3\overline{12}$ as a fraction $\frac{a}{b}$ in the simplest form where a and b are integers and $b \neq 0$.

(5 marks)

(b) Show that the two functions given below are inverses of each other:

$$f(x) = 4x - 3$$

and

$$g(x) = \frac{x+3}{4}.$$

(5 marks)

(c) If α and β are the roots of the quadratic equation $3x^2 + 6x - 15 = 0$, find the quadratic equation that has the roots α^2 and β^2.

(5 marks)

(d) Rationalize the denominator, expressing your answer in the form $a + b\sqrt{xy}$ where a and b are rational numbers

$$\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}.$$

(5 marks)

(e) Solve the polynomial equation given below using the **rational root theorem**

$$x^6 - x^5 - 4x^4 + 2x^3 + 5x^2 - x - 2 = 0.$$

(5 marks)

QUESTION THREE

(a) Define an operation $*$ on the set of real numbers by $x * y = \sqrt[x]{y}$ where x is the index of the radical and y is the radicand.

(i) Is $*$ is a binary operation on the set of real numbers? justify your answer.

(1 mark)

(ii) Evaluate $3 * (-64)$.

(1 mark)

(iii) Solve the equation : $3 * (2x - 3) = 3$.

(3 marks)

(b) Given the quadratic function $f(x) = -3x^2 + 6x - 5$. Express the function $f(x) = -3x^2 + 6x - 5$ in the vertex form $f(x) = a(x+q)^2 + r$ and **sketch its graph.**

(5 marks)

(c) Determine the domain of the function, expressing the answer in interval notation

$$f(x) = 3x + \sqrt{x^2 + 4x - 12}.$$

(5 marks)

(d) Solve the radical equation : $\sqrt{\sqrt{2y} - \sqrt{y-1}} = 1$.

(5 marks)

(e) Sketch the graph of $f(x) = |2x+1| - |x-2|$.

(5 marks)

THE END OF TEST

(d) Solve the radical equation : $\sqrt{\sqrt{2y} - \sqrt{}}$
(e) Sketch the graph of $f(x) = |2x+1| - |x$
THE END OF
Page 2 of 2

The Copperbelt University

School of Mathematics And Natural Sciences

Department of Mathematics

MA 110 : (Mathematical Methods I) : Test Two

Friday - July 22, 2022

Instructions

(1). You must write your **Name, Computer number and Programme** of study on your answer sheet. Time allowed is **2 hours.**

(2). **Calculators** and use of **Cell phones** are **Not** allowed in this paper.

(3). There are **Four (4) questions** in this paper, **attempt all the questions** and show detailed working for full credit.

QUESTION ONE

(a) Find the center and radius of the circle whose equation is

$$x^2 + y^2 + 8x - 2y + 13 = 0.$$

(5 marks)

(b) Write down the constant term in the expansion of $\left(x - \dfrac{1}{2x^2}\right)^9$.

(5 marks)

(c) Prove that $\log_a\left(\dfrac{A}{B}\right) = \log_a(A) - \log_a(B)$.

(5 marks)

(d) Use **Crammer's method** to solve the linear system of the equation

$$\begin{aligned} 3x - 4y &= -11 \\ -5x + y &= 7. \end{aligned}$$

(5 marks)

QUESTION TWO

(a) Change the repeating decimal $3.\overline{7}$ to its reduced form $\frac{a}{b}$, where a and b are integers and $b \neq 0$ using sum to infinity of a geometric series.

(5 marks)

(b) Use Mathematical induction to prove that $2^n \geqslant n+1$ for all possible integer n.

(5 marks)

(c) Find the equation of the tangent at the point $(3, 1)$ on the circle

$$x^2 + y^2 - 4x + 10y - 8 = 0.$$

(5 marks)

(d) Graph the function $f(x) = 2^{(x-3)} + 2$ and obtain its inverse on the same axes.

(5 marks)

QUESTION THREE

(a) Solve $25^x - 5^x = 12$.

(5 marks)

(b) Express $\frac{2x^2 - 5x + 7}{(x-2)(x-1)^2}$ into a partial fraction.

(5 marks)

(c) What is the common ratio of the G.P. $(\sqrt{2} - 1) + (3 - 2\sqrt{2}) + \cdots$? Find the third term of progression.

(5 marks)

(d) How long will it take K2000 to double itself at 13% interest compounded continously?

(5 marks)

QUESTION FOUR

(a) Find the first term and the general expansion of $\frac{1}{(2-3x)^3}$ in ascending power of x. State the range of value of x for which this expansion is valid.

(5 marks)

(b) Show that the general term of an arithmetic sequence is given by

$$a_n = a_1 + (n-1)d.$$

(5 marks)

(c) Solve the equation $\log_2 x = \log_4(x+6)$.

(5 marks)

(d) Find the inverse of the matrix

$$A = \begin{pmatrix} 3 & -1 & 2 \\ 1 & 1 & 1 \\ 2 & 2 & -1 \end{pmatrix}.$$

(5 marks)

THE END OF TEST

The Copperbelt University

School of Mathematics and Natural Sciences

Department of Mathematics

MA 110 : Mathematical Methods I

Sessional Exam - 2021/22

August 29,2022

66
× 12
142
66
302

TIME ALLOWED : **3 HOURS** TOTAL MARKS : **100**

Instructions

(1). You must write your **Group, Student Identification Number (SIN) and Programme** of study on your answer booklet.

(2). Check that you have the **correct examination paper** in front of you.

(3). There are **Seven (7)** questions in this paper. Answer any **Five (5)** questions.

(4). All questions must be answered in the answer booklet provided **only**.

(5). Calculators are **not allowed** in this paper.

(6). Begin each question on a **new page**. And show all **your working** to obtain full marks.

(7). Write down the **number of questions** that you have answered on the cover of the examination answer booklet provided.

66
112
132
66
92

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

QUESTION ONE

(a). Prove De Morgans law:

$$(A \cup B)' = A' \cap B'.$$

(5 marks)

(b). If α and β are the roots of the quadratic equation $3x^2 + 6x - 15 = 0$, find the quadratic equation that has the roots α^2 and β^2.

(5 marks)

(c). Find the sum of

$$\sum_{k=0}^{\infty} 4\left(-\frac{2}{3}\right)^k.$$

(5 marks)

(d). Find the middle term of $\left(\frac{1}{x} - x^2\right)^{12}$, making sure that you simplifying your answer in simplest form.

(5 marks)

(Total Marks: 20)

QUESTION TWO

(a). Express $3.3\overline{12}$ as a fraction $\frac{a}{b}$ in simplest form where a and b are integers and $b \neq 0$.

(5 marks)

(b). Graph the rational function:

$$f(x) = \frac{x+3}{(x-3)(x+2)}.$$

(5 marks)

(c). Solve the radical equation:

$$\sqrt{\sqrt{2y} - \sqrt{y-1}} = 1.$$

(5 marks)

(d). Let $z = x + iy$ be a non-zero complex number. Given that $z + \frac{1}{z} = k$, where k is a real number, prove that either z is a real number or $|z| = 1$.

(5 marks)

(Total Marks: 20)

QUESTION THREE

(a). Solve the logarithmic equation:

$$\log_5 \sqrt{x} = \sqrt{\log_5 x}.$$

(5 marks)

(b). Use Mathematical induction to prove that $2^n \geqslant n + 1$ for all possible integer n.
(5 marks)

(c). Solve the equation involving absolute values:

$$| 3x - 1 | = | 2x + 3 |.$$

(5 marks)

(d). Find the period, amplitude and phase shift of $f(x) = 2\sin(2x - \pi\ )$ and sketch its graph for the interval $\frac{\pi}{2} \leqslant x \leqslant \frac{3\pi}{2}$.

(5 marks)

(Total Marks: 20)

QUESTION FOUR

(a). Prove that $\sqrt{2}$ is an irrational number.

(5 marks)

(b). Verify the trigonometric identity:

$$\frac{1 + \sin\theta}{\cos\theta} + \frac{\cos\theta}{1 + \sin\theta} = 2\sec\theta.$$

(5 marks)

(c). Solve the polynomial equation given below using the rational root theorem:

$$x^6 - x^5 - 4x^4 + 2x^3 + 5x^2 - x - 2 = 0.$$

(5 marks)

(d). Graph the function $f(x) = 2^{(x-3)} + 2$ and obtain its inverse on the same axis.
(5 marks)

(Total Marks: 20)

$4 \quad 12^2 - 4$ $\quad 3(-\frac{2}{5}) \quad 2(-\frac{2}{5})+3$

$144 \quad -\frac{6}{5} \quad -4+3$

QUESTION FIVE

(a). Show that the two functions given below are inverses of each other:

$$f(x) = \sqrt[3]{\frac{x-1}{3}}$$

and

$$g(x) = 3x^3 + 1.$$

(5 marks)

(b). Find the first four terms in the expansion of $\sqrt{4+2x}$. State the range of the values of x for which this expansion is valid.

(5 marks)

(c). The line AB is a diameter of a circle, where A and B are $(-4, 9)$ and $(10, -3)$ respectively. Find the equation of the circle.

(5 marks)

(d). Use the crammer's rule, to solve the system of linear equations

$$\begin{aligned} -x + 2y &= -5 \\ 3x + 2y - z &= -4 \\ 4x + 3z &= 13. \end{aligned}$$

4 + 9

(5 marks)

- 1

(Total Marks: 20)

QUESTION SIX

(a). Prove by Mathematical induction $3^{2n} - 1$ is divisible by 4.

(5 marks)

(b). If $a * b = 2a - b$ where a and b are real numbers. Solve the equation :

$$2x * (x * 3) = 5.$$

(5 marks)

(c). How long will it take K4,000 to grow to K12,000 at 13% interest compounded continously?

(5 marks)

(d). Find the equations of the tangents from the origin to the circle:

$$x^2 + y^2 - 10x - 6y + 25 = 0.$$

39
165
364
.3
192

49
+36
85

13
x5
65

(5 marks)

(Total Marks: 20)

QUESTION SEVEN

(a). The second term of an A.P is 15, and the firth term is 21. Find the common difference, the first term and the sum of the first ten terms.

(5 marks)

(b). Express $\dfrac{2x^2+x+2}{(x^2+1)^2}$ into a partial fraction.

(5 marks)

(c). Find the domain of the logarithmic function:

$$\log_2(x^2+2x-15),$$

expressing your final answer in interval notation.

(5 marks)

(d). Find the square roots of the complex number

$$15+8i.$$

(5 marks)

(Total Marks: 20)

END OF EXAM

$(-\infty,-5)\cup(3,\infty)$

$\dfrac{2x^2+x+2}{(x^2+1)^2}$

$2x^2+x+2 = \dfrac{Ax+B}{x^2+1} + \dfrac{Bx+B}{(x^2+1)^2}$

MA110 – Mathematical Method

Time allowed: Two hours (2:00 hours)

Instructions:

1. You must write your **Name,** your **Computer Number and programme of study** on your answer sheet.
2. Calculators are **not** allowed in this paper.
3. There are three (3) questions in this paper, Attempt **All** questions and show **detailed** working for full credit

---

QUESTION ONE

a) Express $2.07\overline{2}$ as a fraction $\frac{a}{b}$ in its simplest form where a and b are integers and $b \neq 0$. (5 Marks)

b) Given the rational function $f(x)=\dfrac{x^2+2}{x-1}$. Sketch its graph indicating its domain and range, all the asymptotes and intercepts. (5 Marks)

c) Given that $\sqrt{7}$ is an irrational number, Show that $2+\sqrt{7}$ is also an irrational number (5 Marks)

d) Verify that the two given functions are inverses of each other

$$f(x) = x^3 + 1 \text{ and } g(x) = \sqrt[3]{x-1}$$ (5 Marks)

e) Define an operation $*$ on the set of real numbers by $a * b = a^b$

i). Is $*$ a binary operation on the set of real numbers ? Give reason for your answer. (1 Mark)

ii). Is the operation commutative? (2 Marks)

iii). Evaluate $(3 * 2) * -2$ (2 Marks)

QUESTION TWO

a) Given that A and B are sets, simplify the following if possible

$$[(A \cap B)' \cap (A' \cup B)]'$$ (5Marks)

b) Determine the domain of the following function:

$$f(x) = \sqrt{\frac{x+1}{x-1}}$$ (5 Marks)

c) Let α and β be the roots of the quadratic equation $4x^2 + 3x - 2 = 0$

Find a quadratic equation whose roots are α^2 and β^2 (5 Marks)

d) Solve the given inequality $10 - \sqrt{2x+7} \leq 3$ (5 Marks)

e) Solve for x and y given that $\frac{1}{x+iy} + \frac{1}{1+3i} = 1$ (5 Marks)

QUESTION THREE

a) Sketch the graph of the function $k(x) = |2x-1| - |x+2|$ (5 Marks)

b) Using synthetic division find the quotient and the remainder when $f(x) = x^3 + 2x^2 + x - 2$ is divided by $x - (1+i)$. (5 Marks)

c) Let **R**, the set of real numbers be the universal set. If $A = [-7, 8) \cup [11, \infty)$ and $B = [0, 20]$, find the following sets and display them on the number line:

(i) A'. (2.5 Marks) (ii) $A \cap B$. (2.5 Marks)

d) Express $\frac{\sqrt{3}+1}{\sqrt{3}-1} + \sqrt{3} - 1$ in the form $a + b\sqrt{3}$ where a and b are rational numbers. (5 Marks)

e) Determine whether the function $f(x) = x^5 + x^3 + x$ is even, odd or neither. (5 Marks)

THE COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS AND NATURAL SCIENCE

2021 ACADEMIC YEAR **MA110 – Mathematical Method**

Test II

Time allowed: Two hours (2:00 h)

Instructions:

1. You must write your **Name**, your **Computer Number and Program of Study** on your answer sheet. (If **Part Time and Proceed and Repeat** then indicate)
2. Calculators are not allowed in this paper.
3. There are three (3) questions in this paper, attempt **All** questions and show **detailed** working for full credit.
4. Each equation carries **five (5) Marks** .

QUESTION ONE

a) Prove that $log_{\sqrt{2}}(4x^3) = 3log_{\sqrt{2}}(x) + 4$

b) Determine the sum $\sum_{n=3}^{12} 3(n - 2)$

c) Express $\frac{x^3-x^2-4x+1}{x^2-4}$ in partial fractions.

d) Prove the following result by induction:

$$1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1).$$

e) Let $B = \begin{pmatrix} -5 & 10 & 8 \\ 4 & -7 & -6 \\ -3 & 6 & 5 \end{pmatrix}$ and $A = \begin{pmatrix} -1 & 2 & 4 \\ 2 & 1 & -2 \\ -3 & 0 & 5 \end{pmatrix}$ and $BA = AB = I$,

where I is the 3×3 unit matrix. Solve the equation

$$\begin{pmatrix} -5 & 10 & 8 \\ 4 & -7 & -6 \\ -3 & 6 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 2 \end{pmatrix}$$

f) Use the binomial theorem to approximate the value of $\sqrt{0.98}$ up to the third term and determine where it is valid.

QUESTION TWO

a) Sketch and determine the domain and range of the following functions

(i) $f(x) = 2 - log_3(x-1)$

(ii) $f(x) = e^{x+1} + 2$

b) Write down the constant term in the expansion of $\left(2x^2 + \frac{1}{2x}\right)^6$

c) Solve for real values of x given the equation: $2e^x - 3e^{-x} + 5 = 0$

d) The three numbers, $n-2, n, n+3$, are consecutive terms of a geometrical progression. find n, and the term after $n+3$.

e) Find the equation of the perpendicular from the point $A(5,3)$ to the line $2x - y + 4 = 0$

QUESTION THREE

a) Find the centre and length of a radius of the given circle

$$x^2 + y^2 - 10x = 0.$$

b) At what rate of interest compounded continuously will an investment of \$500 grow to \$1000 in 10 years? (given that $ln2 = 0.6931$)

c) Let M be the matrix $\begin{pmatrix} 3 & 1 & -3 \\ 1 & 2z & 1 \\ 0 & 2 & z \end{pmatrix}$. Find two values of z for which M is non singular.

d) Show that $\binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r}$

e) Show that the sum of a geometric sequence is given by

$s_n = \frac{a_1 r^n - a_1}{r-1}$ $\quad$ *where* $r \neq 1$.

IN

THE COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS

2020/2021 ACADEMIC YEAR

MA 110 – Mathematical Methods
SESSIONAL EXAMINATION

TIME ALLOWED: Three (3) hours

INSTRUCTIONS:

1. You must write your **COMPUTER NUMBER** and **PROGRAM** on each answer booklet you have used.
2. There are **seven (7)** questions in this paper, Attempt any **five (5)** questions, each question consists of **a, b, c, d, e**. Each part question carries four **(4)** marks.
3. Calculators are **NOT** allowed in this paper.
4. Should you have any problem or if you need more answer booklet, put up your hand an invigilator will come to attend to you.

QUESTION ONE

a) Prove the De Morgan's law $(A \cap B)' = A' \cup B'$.

b) Sketch the polar equation $r^2 = cos2\theta$.

c) Prove the following result by induction: $S_n = \frac{n(n+1)(2n+1)}{6}$ for $a_n = n^2$.

d) Solve the logarithmic equation $\log_2 x = 8 + 9\log_x 2$.

e) Differentiate the given function using the first principle $f(x) = cosx$.

$\frac{1}{0}$

QUESTION TWO

a) Let **R**, the set of real numbers be the universal set. If $A = [-7, 8) \cup [11, \infty)$ and $B = [0, 20]$, find the following sets and display them on the number line:

(i) A'.

(ii) $A \cap B$.

$(A \cap B)'$

let $\cancel{x} \in (A \cap B)'$

Then $\cancel{x} \notin A$ ... $x \notin$

$x \notin A'$ and

b) Write down the constant terms in the expansion of $\left(2x^2 + \frac{1}{2x}\right)^6$.

c) Solve for real values of x, given that $sinhx + 4 = 4coshx$.

d) Evaluate the integral $\int x\, e^{2x} dx$.

e) Find the period, amplitude, and phase shift of the given function and sketch

$2\left(x^2 + \frac{1}{4x}\right)^6$

$$f(x) = -2 + 2\sin\left(2x - \frac{\pi}{2}\right).$$

$360 = 2\pi$

270

QUESTION THREE

a) Use Cramer's method to solve the following systems of linear equation

$$x - y + 2z = 1$$
$$2x + y + z = 2$$
$$x - 3y + z = 1$$

$8 \times 8 \times 8$

64

512

b) Let '$*$' be a binary operation on the set of real numbers defined by $a * b = a^b$, where a and b are real numbers.

(i) Is $*$ a binary operation on the set of real number?

(ii) Evaluate $-2 * (3 * 2)$.

$\frac{\pi}{2} = 90°$

c) Given that $\sqrt{7}$ is an irrational number, show that $2 + \sqrt{7}$ is also an irrational number.

d) Use De Moivre's theorem to find $(1 + i)^{20}$ and express the results in $a + bi$ form.

e) Find the derivative of the function $3x + y^3 - 4y = 10x^2$.

$f(x) = \cos x$

$\frac{d}{dx} = -\sin x$

$(1+i)^n$

ud

$= xe^{2x}$

$z^n =$

QUESTION FOUR

a) At what rate of interest compounded continuously will an investment of \$500 grow to \$1000 in 10 years (given that $\ln 2 = 0.6931$).

b) Solve $\sin x \tan^2 x = \sin x$ for x, where $0 \le x \le 2\pi$.

c) Verify that the two given functions are inverses of each other

$$f(x) = x^3 + 1 \text{ and } g(x) = \sqrt[3]{x-1}.$$

d) Find λ for which $\lambda I - A$ is a singular matrix where I is an identity Matrix and

$$A = \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}.$$

e) Determine the interval on which $f(x) = \sqrt{4 - x^2}$ is continuous.

$\frac{\sin x \tan^2 x}{\sin x} = \frac{\sin x}{\sin x}$

$\tan^2 x = 1$

$\sqrt{\tan^2 x} = \sqrt{1}$

tan

QUESTION FIVE

a) Express $2.5\overline{17}$ as a rational number.

b) Find the equation of the perpendicular from the point $A(5,3)$ to the line

$$2x - y + 4 = 0.$$

c) Graph $f(x) = \log_{\frac{1}{2}} x$ by reflecting the graph of $g(x) = \left(\frac{1}{2}\right)^x$ across the line $y = x$.

$\lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} -$ $A = per$

d) Find the second roots of $8i$.

e) Evaluate the given limit $\lim_{x \to 3} \frac{x^3 - 27}{x - 3}$.

$A = pe^{rt}$ $\frac{450}{3}$ 150

QUESTION SIX

$1000 = 500$ $A = Pe^{rt}$

a) Solve the equation $y^{\frac{2}{3}} + y^{\frac{1}{3}} - 6 = 0$.

b) Verify that $\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$.

c) What is the common ratio of the G.P $(\sqrt{2} - 1) + (3 - 2\sqrt{2}) + \cdots$? Find the third term of the progression.

d) Verify the identity $\tanh^2 x = \frac{\cosh 2x - 1}{\cosh 2x + 1}$.

e) Show that the rectangle that has maximum area for a given perimeter is a square.

$\tan^{-1}$

QUESTION SEVEN

a) Use binomial theorem to find the value of $(1.01)^{10}$ up to the third term.

b) Solve $\frac{x-1}{x+2} > 2$, expressing the set of solutions in interval notation.

c) Find the exact value of $\tan 67.5°$.

$y - y = m(x - x$

d) Show that $\cos\theta + \sin\theta = \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right)$.

$\frac{x^3 - 27}{x - 3}$

e) Compute $\int_0^4 f(x)\,dx$ where $f(x) = \begin{cases} 2x & \text{if } x < 1 \\ 4 & \text{if } x \ge 1 \end{cases}$

$(x-3)(x+3)(x-3)$

$(x^2+9)(x-3)$

$x^3 + 3x^2 + 9x - 27$

The End.

TIME ALLOWED: Three (3) hours

1. a) The line $L: 4x - 5y + 20 = 0$ cuts the x - axis at A and cuts the y - axis at B.

(i) Find the coordinates of the points A and B ✓ $A(-5,0)$ ✓ $B(0,4)$ ✓

(ii) Find the equation of a line perpendicular to L and passing through the origin. $y = -\frac{5}{4}x$

b) Prove the given Logarithm property ✓

$$\log_a(xy) \equiv \log_a x + \log_a y$$

c) Use mathematical induction to prove that the statement is true for all positive integers n. $6^n - 1$ is divisible by 5 ✓ $\log_a x = m$

$a^m = x$

d) Find the term independent of x in the binomial expansion of $\left(2x^2 + \frac{4}{x}\right)^{12}$ ninth term ✓

e) Solve the simultaneous equations

$2\log y = \log 2 + \log x$ and $2^y = 4^x$ ✓ $x = 0$ $y = 0$ $x = \frac{1}{2}$, $y = 1$

2. a) If the 3rd term of an arithmetic sequence is 20 and the 7th term is 32, find the 25th term. $a = 14$ $d = 3$ $T_5 = 26$

b) State the domain of $f(x) = \ln(|x+3| - 1)$

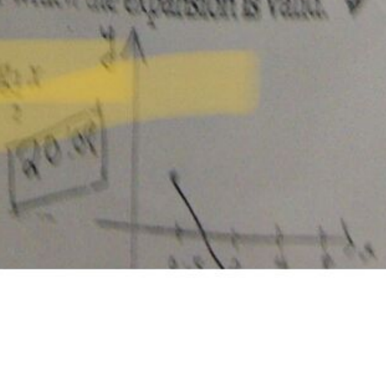
c) Let matrices $A = \begin{pmatrix} 3 & 0 \\ 1 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 4 & -2 & 1 \\ 0 & 2 & 3 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$. Evaluate $2B - C^T$. $\begin{bmatrix} 7 & -7 & -3 \\ -2 & 0 & 0 \end{bmatrix}$

d) (i) Express $\frac{8x+4}{(1-x)(2+x)}$ as partial fractions $\frac{4}{1-x} - \frac{4}{2+x}$

(ii) Hence or otherwise expand $\frac{8x+4}{(1-x)(2+x)}$ in ascending powers of x as far as the term in x^3

(iii) State the set of value of x for which the expansion is valid. ✓

e) Sketch the graph of $f(x) = \log_2 x$



3. a) Find the coordinates of a point P which divides the line segment joining A(2,5) a

B(-3,8) internally in the ratio 3:4. $\left(-\frac{3}{7}, 5\frac{4}{7}\right)$

b) Solve the logarithmic equation:

$$\log_3(2-3x) = \log_9(6x^2 - 19x + 2)$$ ✓

c) Using the formula for sum to infinity of geometric series, express $8.\bar{3}$ to reduced

where a and b are integers and $b \neq 0$.

d) (i) Find the inverse of the matrix $A = \begin{pmatrix} 3 & -1 & 2 \\ 1 & 1 & 1 \\ 2 & 2 & -1 \end{pmatrix}$ $A^{-1} = \frac{1}{-12}\begin{bmatrix} -3 & 4 \\ 3 & - \\ 0 & - \end{bmatrix}$

(ii) Use your inverse above to solve the system of linear equations

$$3x - y + 2z = 4$$
$$x + y + z = 2$$
$$2x + 2y - z = 3$$

$x = \frac{13}{12}$ $y = \frac{5}{12}$ $z = \frac{1}{3}$

e) Evaluate

$$\sum_{i=1}^{45}(5i+2)$$

$S_n = \frac{n}{2}(a + l)$

$S_{45} = \frac{45}{2}(7 + 227)$

$S_{45} = 5265$

4. a) Find λ for which the matrix $\lambda I - A$ is a singular matrix if where I is an identity Ma

$A = \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$.

b) Solve the exponential equation:

$$2^{2x} + 3(2^x) - 4 = 0$$

$2^x = -2^2$ invalid $2^x = 2^0$ $x = 0$

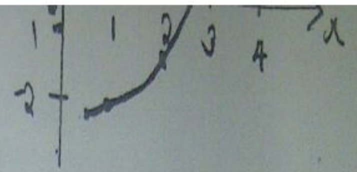
c) Use mathematical induction to prove that the statement is true for all positive integers n. $2^n \geq n + 1$ ✓

d) Given that $|x| > 2$, find the first four terms in the expansion of $\left(1 - \frac{2}{x}\right)^{\frac{1}{2}}$ in descending powers of x. By taking $x = 200$ use your expansion to find a va of $\sqrt{99}$. = 9.95

$1 - \frac{1}{x} + \frac{1}{x^2} - \frac{1}{x^3}$

e) Find the center and the radius of the circle given by the equation

$2x^2 + 2y^2 + 16x - 7y = 0$



a) Sketch the exponential function and determine its domain and range.

$f(x) = 2^{x-1} - 3$

b) Prove the logarithmic function

$$\log_{\sqrt{2}}(4x^3) \equiv 3\log_{\sqrt{2}}(x) + 4$$ prove ✓

c) Use the method of mathematical induction to the sum formula for the indicated sequence:

$S_n = \frac{3(3^n-1)}{2}$ for sequence $a_n = 3^n$. ✓

d) The points A (2, 3), B (3, 2), and C (-4, 3) lie on the circumference a circle.

i) Find the equation of the circle. $x^2 + y^2 + 2x + 2y - 23 = 0$

ii) Hence, find the centre and radius of the circle. $(-1, -1)$ $r = 5$

e) The number of grams of a certain radioactive substance present after t hours is given by the equation $Q = Q_0 e^{-0.45t}$, where Q_0 represents the initial number of grams. How long will it take 2500 grams to be reduced to 1250

$1250 = 2500 e^{-0.45t}$

$\frac{1250}{2500} = e^{-0.45t}$

$\ln \frac{1}{2} = \ln e^{-0.45t}$

THE COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

MA110 - MATHEMATICAL METHODS

SESSIONAL EXAMINATION

MONDAY 21ST SEPTEMBER, 2020

TIME: 09:00 - 12:00

Time allowed: 3 HOURS

Instructions to Candidates:

(1) Read the instructions very carefully.

(2) Check that you have the correct examination paper.

(3) Write clearly your Program of study, Group and Student identification number (SIN).

(4) There are SEVEN (07) questions in this paper. Answer any FIVE (05) questions.

(5) All questions carry equal marks. Students are NOT permitted to use a calculator.

(7) Begin each question on a new page. And show all your working to obtain full marks.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

1 4 6 4 1 $\quad \alpha^2 - \alpha\beta - \alpha\beta + \beta$

QUESTION ONE

(A) Prove theoretically the De-Morgan's law;

$$A' \cap B' = (A \cup B)'.$$

[5 marks]

(B) Given that the roots of the equation $2x^2 - 5x + 2 = 0$ are α and β. Find the positive value of the difference;

$$\alpha^4 - \beta^4.$$

[6 marks]

(C) Find the sum

$$\sum_{k=0}^{\infty} 4\left(-\frac{2}{3}\right)^k.$$

[4 marks]

(D) Find the middle term of $\left(\frac{1}{x} - x^2\right)^{12}$, making sure that you simplify your answer.

[5 marks]

[Total: 20 Marks]

QUESTION TWO

(A) Solve the following simultaneous equation

$$\log_3 x + \log_3 y = 2$$
$$\log_3(2y - 3) - 2\log_9 x = 1.$$

[5 marks]

(B) If $f(x) = x^3 - 3x^2 + x$. Find all the factors of $f(x+1)$.

[5 marks]

(C) Use mathematical induction to prove that

$$2^n > n + 4 \quad \text{for all} \quad n \geq 3$$

[5 marks]

(D) Sketch the following piece-wise function

$$f(x) = \begin{cases} \frac{1}{x}\,; & \text{if } -3 \leq x < 0 \\ \sqrt{x}\,; & \text{if } 0 \leq x \leq 4 \\ |x-4|\,; & \text{if } 4 < x \leq 7. \end{cases}$$

[5 marks]

[Total: 20 Marks]

QUESTION THREE

(A) Prove that $\sqrt[3]{2}+\sqrt{3}$ is irrational.

[5 marks]

(B) Solve the following equation completely;

$$f(x) = x^4 - (13+7i)x^3 - \left(11 - \frac{195}{2}i\right)x^2 + \left(143 - \frac{15}{2}i\right)x + \frac{195}{2}i = 0.$$

[5 marks]

(C) Decompose the following into partial fractions;

$$f(x) = \frac{x^5 - 3x^2 + 12x - 1}{x^3(x^2+x+1)(x^2+2)^3}.$$

[5 marks]

(D) Sketch the graphs of the following functions;

(i) $f(x) = 4 + \log_6\left(\frac{1}{3-x}\right)$

[2.5 marks]

(ii) $g(x) = \frac{1}{3} - \frac{27}{3^x}$.

[2.5 marks]

[Total: 20 Marks]

QUESTION FOUR

(A) Show that the first three terms in the expansion in ascending powers of x of $(1+8x)^{\frac{1}{4}}$ are the same as the first three terms of the expansion of $\frac{1+5x}{1+3x}$. Hence, use the fact that $(1+8x)^{\frac{1}{4}} \approx \frac{1+5x}{1+3x}$ to obtain $\sqrt[4]{1.16}$ as a fraction in its lowest terms.

[6 marks]

(B) Find the domain of $(f \circ g)(x)$ for the functions given by

$$f(x) = x^2 - 9 \text{ and } g(x) = \sqrt{9-x^2}.$$

[4 marks]

(C) Determine the range value(s) of m for which

$$mx^2 - 2(m+3)x + m - 1 = 0$$

have real roots.

[4 marks]

(D) Find the inverse of the matrix A given by

$$A = \begin{pmatrix} 3 & 4 & -1 \\ 2 & 0 & 7 \\ 1 & -3 & -2 \end{pmatrix}$$

[6 marks]

[Total: 20 Marks]

QUESTION FIVE

(A) Evaluate the complex number $\sqrt{17+10i}$.

[3 marks]

(B) (i) Suppose that $k2,000$ is invested in an account which offers $7\frac{1}{8}\%$ compounded monthly;

(a) Express the amount A in the account as a function of the term of the investment t in years.

[2 marks]

(b) How much is in the account after 5 years?

[1 marks]

(c) How long will it take for the initial investment to double?

[2 marks]

(ii) Suppose that the number of people N (in hundreds) at a local hospital who have been tested for COVID-19 can be modelled using the logistic equation

$$N(t) = \frac{84}{1 + 2799\, e^{-t}} \quad .$$

Where $t \geq 0$ is the number of days after 3rd March, 2020 when rumours of COVID-19 started spreading in Zambia.

(a) Find and interpret $N(0)$

[1.5 marks]

(b) Find and interpret the end behaviour of $N(t)$ of COVID-19 in Zambia.

[1.5 marks]

(c) How long until 4200 people have been tested for COVID-19?

[2 marks]

(C) Let $\star$ be a binary operation on $\mathcal{F}(A)$ defined by

$$f \star g = (f \circ g)(x), \quad \text{for all } f, g, h \in \mathcal{F}(A)$$

Where $f(x) = \sqrt{9 - x^2}$, $\quad g(x) = \dfrac{1}{x^2}$ and $h(x) = x^2 - 9$ for all $x \in \mathbb{R}$.

(i) Prove that the operation $\star$ is binary.

[1 marks]

(ii) Prove whether the operation $\star$ is associative and or commutative. Hence, find the value(s) of x if $h \star f = -9$.

[4 marks]

(D) Sketch the graph of $f(x) = (x-2)^2(x+1)$.

[2 marks]

[Total: 20 Marks]

QUESTION SIX

(A) Find the solution of a rational inequality

$$\frac{2x^2 - 7x - 4}{3x^2 - 14x + 11} > 2.$$

[5 marks]

(B) Express $\dfrac{(x^4 - 2x^3 - x^2 - 4x + 4)}{(x-3)(x^2+1)}$ into partial fractions.

$a_1 + (n-1)d$

[5 marks]

(C) An arithmetic progression (AP) contains 20 terms. Given that the sum of the first 8 terms of an AP is 16 more than 3 times the sum of the first 5 terms.

(i) Find the relationship between the first term a and the common difference d.

[2 marks]

(ii) Given also that the sum of the last 6 terms is 183. Find the first term a and the common difference d.

[3 marks]

(D) Solve the system of linear equations using Cramer's rule;

$$\begin{aligned} p + 2r &= 9 \\ 2q + r &= 8 \\ 4p - 3q &= -2. \end{aligned}$$

$S_n = \frac{n}{2}(2a_1 + (n-1)d$

[5 marks]

[Total: 20 Marks]

QUESTION SEVEN

(A) Solve the equation;

$$\sqrt[3]{20-x}+x-6=10.$$

[5 marks]

(B) Given that $P(x)=2\ln(x)+1$ and $Q(x)=e^{(\frac{x^2}{2}-3x+12)}$. Find x such that $(P\circ Q)(x)=0$.

[5 marks]

(C) Use the principle of mathematical induction to prove that $3^{2n}-1$ is divisible by 4, for all positive integers n.

[5 marks]

(D) Graph the given rational function by use of intercepts and asymptotes;

$$f(x)=\frac{x^3-x^2-5x-3}{x^3+4x^2-3x-18}.$$

[5 marks]

[Total: 20 Marks]

END OF EXAMINATION PAPER

$3^{2k+2}-\frac{1}{3}=\frac{3x}{4}$

$4^{2k+2}-\frac{4}{3}=\frac{3x}{3}$

$4^{2k+2}-\frac{4}{9}=x$

$\frac{36^{2k+2}-4}{9}=x$

$36^{2k+2}-4=9x$

$9(4^{2k+2}-1)=9x$

$4^{2k+2}-1=x$

$x^3+4x^2-3x-18$

$x^2(x+4)-3(x+6)$

$1^{2k+2}-\frac{1}{3}=\frac{x}{12}$

$\frac{3^{2n}-1}{4}$

$\frac{3^{2k+1}-1}{4}$

1 2 1

1 3 3

$x^2-6x+25$

$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$6^3-3(6)^2x-18x^2-x^3$

$12^{2k+2}-\frac{12}{3}=x$

$12^{2k+2}-4=x$

$4(3^{2k+2}-1)=x$

1ab + 3ab + 3ab + 1ab

$1(6)^3$ $3(6)^2(-x)$ $3(6)(-x)^2$ $1(-x)^3$

$18x^2-x^3$

COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

DEPT OF PURE & APPLIED MATHEMATICS

MA 110 - MATHEMATICAL METHODS I | TEST 1

INSTRUCTIONS; 1. Attempt all Questions in this Paper without Using a Calculator.
2. Indicate clearly your Names,SIN and the Group you belong to.
3. Duration is **3 Hours Only.**

---

1. ***a.*** *i.)* Let $A = \{1, 2, 3\}$ and $B = \{2, 4\}$, Find $A \times B$. 1 Mk

   ii.) Prove that $(A^c)^c = A$ by Arbitrary Elementary Method. 4 Mks

   b. If the Operation $*$ is defined as, " *add the first number* **to** 8 *times the second number*"

   Find $(2 * 3) * 5$

   2 Mks

   c. Find the value of k given that when $2x^3 - 2kx^2 - 3x - 2$ is divided by $x - 2$, the Remainder is 40. 3 Mks

   d. If $g o f(x) = x$ and $g(x) = \dfrac{x+1}{x-1}$,

   i.) Find $f(x)$ ***ii.)*** Sketch the Graph of $f(x)$ and Find the Range of $f(x)$ 3 Mks, 4 Mks

2. ***a.*** Prove the De Morgan's Law: $A^c \cup B^c = (A \cap B)^c$ 5 Mks

   b. Solve the following Equations involving the Absolute value functions:

   $$|8x + 3| = |2x - 21|$$ 3 Mks

c. Solve the following inequation:

$$\frac{x-2}{x+1} \geq \frac{x-6}{x-2}$$ 5 Mks

d. Using Synthetic Division, show that both $x-2$ and $x+3$ are Factors of:

$$f(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18.$$

Hence, or otherwise Factorize $f(x)$ completely 4 Mks

3. *a.* Express the following in the form $\frac{a}{b}$, where $a, b \in \mathbb{Z}$ and $b \neq 0$.

   i.) **0.121212···** *ii.)* **1.3121212···** 1.5 Mks, 1.5 Mks

   b. Use the fact that $\sqrt{6}$ is Irrational to prove that $\sqrt{2} + \sqrt{3}$ is Irrational. 4 Mks

   c. Sketch the graphs of:

   i.) $f(x) = -|x+3| - 4$ *ii.)* $f(x) = 3 + \sqrt{3-x}$ 2.5 Mks, 2.5 Mks

   d. Solve the Polynormial Equation $x^4 - 6x^3 + 22x^2 - 30x + 13 = 0$ 5 Mks

4. *a.* Determine the vertex and Intercepts for the following Quadratic function:

$$f(x) = x^2 - 6x - 16$$ 2 Mks

   b. Sketch the graph of the Polynomial given by;

$$f(x) = (x-1)^2(x-3)^3(x+4)$$ 5 Mks

   c. Given that the roots of $x^2 + 3x + 17 = 0$ are α and β respectively. Find a Quadratic Function whose roots are $\alpha^3 + \beta^3$ and $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$. 5 Mks

   d. Given that set $A = \{1, 2, 3\}$ and set $B = \{2, 4, 6\}$, Determine whether the Operation;

$$A \circ B = P(A) - P(B).$$

   is Binary on the Universal Power set, $P(E)$. 5 Mks

5. *a.* If $x = \dfrac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}}$, find the value of $8x - x^2$. 4 Mks

b. Given the Functions, $f(x) = x^2 + 4$ and $g(x) = x - 9$.

Find the value of x for which $g[f(x)] = f[g(x)]$ 4 Mks

c. Write the Expression $f(x) = 2x^2 + 12x + 14$ in the form $f(x) = a(x+h)^2 + k$ where $a, h, k \in \mathbb{R}$.

Hence, state the turning point of $f(x)$. 4 Mks

d. Calculate the value(s) of x that are valid for the Equation below.

$$\left|\frac{x-2}{x+3}\right| = 4$$ 5 Mks

6. *a.* Simplify $-\dfrac{25}{2}\left[\dfrac{1+2i}{3+4i} - \dfrac{2-5i}{-i}\right]$ 3 Mks

b. Solve for x and y given that;

$$\frac{x}{1+i} - \frac{y}{2-i} = \frac{1-5i}{3-2i}$$ 5 Mks

c. Solve the Inequality below and present your answer in Interval Notation:

$$3x^2 + 2x + 2 < 2x^2 + x + 4$$ 4 Mks

d. Graph the Rational Functional by finding the Asymptotes and Intercept:

$$f(x) = \frac{5x^2 - 2}{1 - x}$$ 5 Mks

.... THE END......................

THE COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

MA 110 –MATHEMATICAL METHODS I

SESSIONAL EXAMINATION

OCTOBER, 2019

DURATION: 3 HOURS

TOTAL MARKS 100

INSTRUCTIONS TO CANDIDATES

1) Answer any FIVE (5) questions.
2) Marks are indicated on each question.
3) You must show **ALL** your workings clearly to get **FULL** marks.
4) Calculators are NOT ALLOWED in this examination.

$\frac{3}{2}\left(2 + 3^{k+1}\right)$

$3 + \frac{3(3^{k+1} - 1)}{2}$

$\frac{6 + 3(3^{k+1} - 1)}{2}$ 2

$\frac{3}{2}\left(3 + 3^{k+1} - 1\right)$

QUESTION ONE

a) Prove the De-Morgan's law;

$$(A \cap B)' = A' \cup B'$$

[5 marks]

b) Solve the inequality below and write the solution set in interval notation;

$$\frac{x+1}{x-1} \leq \frac{x+3}{x+2}$$

$3^y = x$

[5 marks]

c) Solve the following equation;

$9^y = x + 6$

$$log_3 x = log_9(x+6)$$

$= \frac{\log_3 9}{\log_3(x+6)} = \log_3 x$

[5 marks]

d) Use the principle of mathematical induction to prove that;

$$\sum_{i=1}^{n} 3^i = \frac{3(3^n - 1)}{2}$$

$\frac{2}{\log_3(x+6)} = \log_3 x$

$\log_3 x = \log_9(x+6)$

For all real numbers n **[5 marks]**

$\log_3 x = \frac{\log_3 9}{\log_3(x+6)}$

$3^y = x$

QUESTION TWO

a) Let $f(x) = \frac{3x^2+9x-20}{x^2+x-6}$

Resolve $f(x)$ into partial fractions **[5 marks]**

b) Find the term independent of x in the expansion of $\left(x^2 + \frac{1}{x^2}\right)^6$

[4 marks]

c) Let S be a set of integers and define $*$ as
$a * b = a^2 b$; where $a, b \in \mathbb{Z}$

$\log_3 x = \frac{2}{\log_3(x+6)}$

(i) Is $*$ a binary operation on S **[2 marks]**

(ii) Is $*$ commutative on S **[2 marks]**

(iii) Find $(2 * 3) * 4$ **[2 marks]**

d) Given that the quadratic equation $2x^2 - 6x - 10 = 0$ has roots α and β. Find a quadratic equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ **[5 marks]**

-4 -2.5 -1

~~x+3~~ -4 -3 -2 -1 0 1 2

x+3 - + +

x+2 - - +

+ - +

x+1 - + +

x-1 - - +

f(x) + − +

$x \in [1,-1] \cup [-3,-2]$

$0 \leq x \in [1,-1]$

$x \in [-3,-2]$.

Scanned by CamScanner

QUESTION THREE

a)

(i) Prove that $(A')' = A$ by arbitrary element **[3 marks]**

(ii) Let $A = \{1, 2, 3\}$ and $B = \{2, 4\}$

Find $(A \times B)$ and represent it on the $X\,0\,Y$ plane **[4 marks]**

b) Find the value of k if $x + 2$ is a factor of $x^3 - kx^2 + 3x + 7k$ **[3 marks]**

c) Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ find the determinant of $f(A)$ such that

$f(x) = 2x^2 - 6x - 10$ **[5 marks]**

d) There is a stack of logs in the backyard of a certain house. If there are 30 logs in the 1^{st} layer, 27 in the 2^{nd} layer, 24 in the 3^{rd} layer, 21 in the 4^{th} layer and so on with the last having three logs.

(i) How many layers are on the stack. **[2 marks]**

(ii) How many logs are in the stack **[3 marks]**

QUESTION 4

a) The numbers $3, x, (x + 6)$ form the first 3 terms of a **positive geometric sequence**. Find

i) the possible value(s) of x **[2 marks]**

ii) the tenth term of the sequence **[3 marks]**

b) Let $A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 3 & -1 \\ -3 & 1 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} -8 \\ 3 \\ 4 \end{pmatrix}$

i) Write down the inverse of A **[6 marks]**

ii) Find a matrix X such that $AX = B$ **[4 marks]**

c) Sketch the graph of the following polynomial and state the domain and the range.

$$f(x) = -(x + 3)(x + 2)(x - 1)^3$$ **[5 marks]**

QUESTION 5

a) Solve the following logarithmic simultaneous equations

$\log_x y = 2$ and $y \log 4 = x \log 8$ **[5 marks]**

b) The polynomial $2x^3 + ax^2 + bx + 1$ is exactly divisible by $2x - 1$ and $x + 1$. Find the values of a and b. Hence find the third factor of the expression.

[5 marks]

c) Solve for x and y where x and y are real numbers

$$\frac{x}{1+i} + \frac{y}{3+i} = 4 - 2i$$

[5 marks]

d) Graph the following rational function and state the domain and range of $f(x)$

$$f(x) = \frac{12}{(x-1)(x+2)}$$

[5 marks]

QUESTION SIX

a) Given that $f(x) = e^{\left(\frac{x^2}{2} - 3x + 12\right)}$ and $h(x) = 2\ln(x) + 1$

Find x such that

$$(h \circ f)(x) = 0$$

[5 marks]

b) Solve the following equation

$$\sqrt{(10 + 3\sqrt{x})} = \sqrt{x}\ .$$

[4 marks]

c) $log_2 X$, $log_2(X + 9)$, and $log_2(X + 45)$ are three consecutive terms of an arithmetic progression

i) Find X

ii) Find the 5th term as a single logarithm.

[5 marks]

d) Solve the system of equation by crammer's rule;

$$x + 2y + 3z = -5$$
$$3x + y - 3z = 4$$
$$-3x + 4y + 7z = -7$$

[6 marks]

$(1-x)(2-x)$

$\log_3 2 - \log_3 9$ $\quad 2 - x - 2x + x^2$

$3^x = 2 - 2$ $\quad (2 - 3x + x^2)^{-1}$ $\quad \log_3$

QUESTION 7

a) Find the first three terms of the binomial expansion of $\frac{12-7x}{(1-x)(2-x)}$

[5 marks]

b) Use the principle of mathematical induction to prove that

$$2^n > n$$

For all positive integers n

[5 marks]

c) Solve the equation involving absolute values;

$$\left|\frac{3x-1}{2}\right| = |2x+3|$$

$\log_3 2/9$

[5 marks]

d) Given the logarithmic function $f(x) = log_3\left(\frac{x+1}{9}\right)$

Simplify and sketch the graph of $f(x) = log_3\left(\frac{x+1}{9}\right)$ $(1-x)$

$\log_3 2/9$ $\quad \log_3 \frac{2}{3^2}$

[5 marks]

$\begin{cases} \frac{3x-1}{2} = 2x+3 \\ \frac{3x-1}{2} = -(2x+3) \end{cases}$

$3^x = 2/9$

$7k + (-4k) - 14 = 0$

$-2(2k+7)$ $\quad 32\frac{2}{3}$

$3k = 14$

$-4k - 14$

18
14
32

$\log_3(x+1) - \log_3 9 = y$ $\quad k = \frac{14}{3}$

$x^3 - kx^2 + 3x + 7k = 0$ $\quad 7k + 4k - 14 = 0$

$3^{x} (x+1) - 2 = y$ $\quad y = 0$ $\quad 18\frac{2}{3}$

$\frac{3k}{3} = \frac{14}{3}$

$-8-6$

$-14 - 18\frac{2}{3}$

$-32\frac{2}{3} + 32\frac{2}{3}$

$= 0$

1	$-k$	3	$7k$
	-2	$2k+4$	$-4k-14$
1	$-k-2$	$2k+7$	0

$3^2 = x+1$

$9 = x+1$

$x = 8$

$k = \frac{14}{3}$

$\frac{14}{4}$ $\quad \frac{56}{3}$ $\quad \frac{14}{7}$ $\quad \frac{98}{3}$ $\quad 14 \times 4 = 56$

$x^3 - \frac{14}{3}x^2 + 3x + 7 \times \frac{14}{3}$

$-24 - 56 - 36 + 98$ $\quad$ 24, 56, 36

$\left(x^3 - \frac{14}{3}x^2 + 3x + \frac{98}{3}\right) \times 3$ $\quad -8 - \frac{14}{3} \times 4 - 6 + \frac{98}{3}$ —GLASSMAN

$\boxed{3x^3 - 14x^2 + 9x + 98}$ $\quad -8 - \frac{56}{3} - 6 + \frac{98}{3}$

Scanned by CamScanner

BANDA PAUL
GROUP B

THE COPPERBELT UNIVERSITY
SCHOOL OF MATHEMATICS AND NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS

MA 110 TEST TWO | DURATION: **2 HRS** | SEPTEMBER 2019

Attempt all Questions in this Paper.

1. *a.* *i.* Show that

$$\frac{3}{x-2}+\frac{4}{x-1}=\frac{7x-11}{(x-2)(x-1)}$$ 2Mks

ii. Hence or Otherwise Solve the Equation below

$$\frac{3}{x-2}+\frac{4}{x-1}=\frac{7}{x}$$ 3Mks

b. Given that the 3^{rd} term of an Arithmetic Sequence is 20 and the 7^{th} term is 12. Find

i. The common difference and the First term of the Sequence 4Mks

ii. The 20^{th} term of the Sequence 3Mks

2. *a.* Using Mathematical Induction, Prove that

$2^n > n$ For all Positive Integer Values of n. 6Mks

b. *i.* Show that the First three terms in the Expansion of $(1+8x)^{\frac{1}{4}}$ in ascending powers of x are the same as the First three terms in the Expansion of $\frac{(1+5x)}{(1+3x)}$. 4Mks

ii. Hence or Otherwise, Evaluate $(16.16)^{\frac{1}{4}}$. 3Mks

3. a. Find the sum of the Infinity Geometric Sequence $\sum_{k=1}^{\infty} 4(0.6)^{k-1}$ 6Mks

/ b. Let $f(x) = \dfrac{2x^4 - 17x - 1}{(x-2)(x^2+5)}$, Resolve $f(x)$ into Partial Fractions 7Mks

4. a. Prove by Mathematical Induction that for all Positive integers n, the number $3^{2n} - 1$ is divisible by 8. 6Mks

/ b. Find the value of r if the coefficient of x^r and x^{r+1} are equal in the Binomial Expansion of $(3x+2)^{19}$. 6Mks

$$\boxed{T_n = T_{r+1} = {}^nC_r\, a^{n-r} b^r}$$

dalitso

THE COPPERBELT UNIVERSITY
SCHOOL OF MATHEMATICS AND NATURAL SCIENCES
DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MA 110 - MATHEMATICAL METHODS I
TEST 1

DATE: 8th December 2017
DURATION: 3 Hours
MARKS: 100

READ THE FOLLOWING INSTRUCTIONS

1. Write your **NAME, PROGRAM, COMPUTER NUMBER AND GROUP on the cover of your answer sheet.**
2. This is a **THREE Hours** test. Cell phones are **NOT** allowed.
3. Attempt **ALL** questions. Answers to questions should fully be explained. A correct but unclear answer will not get full marks.
4. No pencil work (except for graph sketching) or any work in red ink will be marked.
5. Use of correction fluid or "**Tip-Ex**" and **calculators** are **NOT** allowed.

Question 1

(a) Let $E = \{3,4,5,6\}$, $F = \{0,2,4,6,8\}$ and the universal set $X = \{0,1,2,\ldots,10\}$. Find $E \cap F'$. **[1 mark]**

(b) Let

$$f(x) = \frac{2}{x^2-2}, \text{ and } g(x) = \frac{1}{\sqrt{x+1}}$$

Find $(g \circ f)(-2)$. **[4 marks]**

(c) Use long division to divide $4x^3 - 7x^2 - 11x + 5$ by $4x + 5$. **[3 marks]**

(d) If A and B are subsets of the universal set U such that $A \subset B$. Simplify

(i) $A' \cup B'$ (ii) $A - B$. **[2+2 marks]**

(e) Find the possible values of λ and k if the expression $3x^4 + \lambda x^3 + kx + 4$ is exactly divisible by $x - 1$ and leaves a remainder of 18 when divided by $x + 2$. **[5 marks]**

Question 2

(a) Prove the De-Morgan's law; $(A \cap B)' = A' \cup B'$. **[5 marks]**

(b) Is the function $f(x) = |x| + x^2$ even, odd or neither? Justify your answer. **[3 marks]**

(c) Solve the inequality below and write the solution set in interval notation

$$\frac{x+4}{x+1} \leq \frac{x-2}{x-4}.$$ **[5 marks]**

(d) Use Factor Theorem and synthetic division to factorise $f(x) = 6x^4 - 19x^3 + 17x^2 - x - 3$ completely. **[4 marks]**

Question 3

(a) Sketch the following piecewise defined function

$$f(x) = \begin{cases} |x+2| & \text{if } -4 \leq x < 0 \\ x^2 & \text{if } 0 \leq x \leq 2 \\ 3 & \text{if } 2 < x \leq 3 \end{cases}$$ **[4 marks]**

(b) Prove that $\sqrt{2} + \sqrt{3}$ is irrational. **[4 marks]**

(c) Solve the polynomial equation $2x^5 - 5x^4 + x^3 + x^2 - x + 6 = 0$. **[5 marks]**

(d) Solve the following inequality and write the solution in interval notation $3|2x+1| + 1 \leq 7$. **[4 marks]**

Question 4

(a) Let the universal set be the set of real numbers, with $A = (3,8]$, $B = (2,7)$, $C = [1,5]$ and $D = [6,\infty)$. Find (i) $(A \cup C)$ (ii) $(B \cap D)$ (iii) $(A \cup C) - (B \cap D)$ **[1+1+2 marks]**

(b) Is the binary operation $*$ defined by $a * b = a + b - ab$ both commutative and associative. Justify your answer. $a \times b = b \times a$ **[3 marks]**

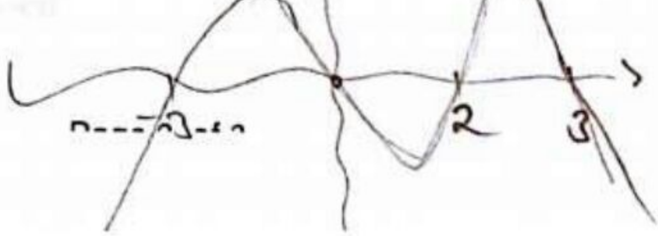
(c) Sketch the graph of the polynomial $f(x) = -(x-3)(x-2)^3(x+1)^2$. **[5 marks]**

(d) Solve the equation $|2x + 11| = 7$. **[2 marks]**

(e) The roots of the equation $x^2 - 9x + K = 0$ are α and $\alpha + 1$. Find the value of K. **[3 marks]**

3 cross
2 – torch
– out

polynomial terms



Scanned by CamScanner

Question 5

(a) Express

$$\frac{\sqrt{3}+1}{\sqrt{3}-1}+\sqrt{3}-1$$

in the form $a+b\sqrt{3}$ where a and b are rational numbers. **[4 marks]**

(b) Verify that

$$f(x)=4x-5 \text{ and } g(x)=\frac{x+5}{4}$$

are inverse functions of each other. **[4 marks]**

(c) Express $f(x)=1-6x-x^2$ in the form $f(x)=a(x+h)^2+k$ where a, h and k are rational numbers. Hence, write down the coordinates of the turning point of the graph $f(x)=1-6x-x^2$.

[5 marks]

(d) Solve the equation $2x-11\sqrt{x}+12=0$. **[4 marks]**

Question 6

(a) Solve for x and y where x and y are real numbers

$$(x+yi)-i=i(x+yi)+5.$$ **[4 Marks]**

(b) The equation $Kx^2-2Kx+2K=1$ where K is a constant has two real solutions.

(i) Show that K satisfies the inequality

$$K^2-K\leq 0.$$ **[2 Marks]**

(ii) Hence, find the set of all possible values of K. **[3 Marks]**

(c) For the following rational function

$$f(x)=\frac{2x^2-2}{x^2-4},$$

(i) Determine the x-intercepts and the y-intercept. **[3 Marks]**

(ii) Find the horizontal and the vertical asymptotes. **[2 Marks]**

(iii) Sketch the graph of f. **[3 Marks]**

dalitso tembo
group B

THE COPPERBELT UNIVERSITY
SCHOOL OF MATHEMATICS AND NATURAL SCIENCES
DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MA 110 - MATHEMATICAL METHODS I

TEST 2

DATE: 6th June 2018
DURATION: 3 Hours
MARKS: 100

READ THE FOLLOWING INSTRUCTIONS

1. Write your **NAME, PROGRAM, COMPUTER NUMBER AND GROUP on the cover of your answer sheet.**
2. This is a **THREE Hours** test. Cell phones are **NOT** allowed.
3. Attempt **ALL** questions. Answers to questions should fully be explained. A correct but unclear answer will not get full marks.
4. No pencil work (except for graph sketching) or any work in red ink will be marked.
5. Use of correction fluid or "**Tip-Ex**" and **calculators** are **NOT** allowed.

Question 1

(a) (i) Write down the first four terms of the sequence 1, 2, 3, 4

$$a_0 = -2, \quad a_n = \frac{a_{n-1}}{(n+1)(n+2)} \quad \text{for } n \geq 1 \text{ and } n \in \mathbb{N}$$ **[3 marks]**

−2, −3,

(ii) Find the coordinates of the point C on the line joining the points $A(-1,2)$ and $B(-9,14)$ which divides AB internally so that

−1, −4, −5, −6

$$\frac{AC}{CB} = \frac{1}{3}.$$ **[3 marks]**

(b) The second term of a geometric progression is 3 and the 5th term is $\frac{81}{8}$. Find the 8th term.

[5 marks]

(c) Three points have coordinates $A(2,9)$, $B(4,3)$ and $C(2,-5)$. The line through C with gradient $\frac{1}{2}$ meets the line AB produced at D.

(i) Find the coordinates of D.

[4 marks]

(ii) Find the equation of the line through D perpendicular to the line $5y - 4x = 17$. [2 marks]

Question 2

(a) Prove the logarithmic law: [2 marks]

$$\log_r x^z = z \log_r x.$$

(b) Solve the logarithmic equation [5 marks]

$$\sqrt{\log_5 x} = \log_5 \sqrt{x}.$$

(c) Using the Principle of Mathematical Induction, prove for all natural numbers n the following:

(i) $$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$$ [5 marks]

(ii) $3^{2n} - 1$ is divisible by 8. [5 marks]

Question 3

(a) Find the standard equation of the circle passing through the point $(-2, 1)$ and is tangent to the line $3x - 2y = 6$ at the point $(4, 3)$. [8 marks]

(b) Find the sum of the geometric series

$$\sum_{n=1}^{\infty} \left(-\frac{2}{3}\right)^{n-1}.$$ [4 marks]

(c) Write $0.3\overline{45}$ as a ratio of two integers a and b in its lowest terms using the geometric progression series. [5 marks]

Question 4

(a) (i) Using the Binomial Theorem, evaluate $(1.02)^3$. [3 marks]

(ii) Find the 6th term of the expansion $(1 + 2y)^8$. [3 marks]

(b) Expand $(1 + 3x)^{\frac{4}{3}}$ up to the third term. State the range of values of x for which the expansion is valid. [4 marks]

(c) Find the partial fraction decomposition of each of the following:

(i) $\frac{2x}{x^2 - 1}$. [3 marks]

307
− 45
352 99 ×3 207

99 ×3 7

8 + 1 − 9
8 +

12 ×6 72 9,9 8,9

−9 − 12
21
=

(ii) $\dfrac{3x^2-2x+5}{(x-1)(x^2+5)}$. [4 marks]

Question 5

(a) Find the values of x such that

$$\begin{vmatrix} 3 & 1 & -3 \\ 1 & 2x & 1 \\ 0 & 2 & x \end{vmatrix} = 0.$$ [5 marks]

5 − 12
−7

(b) Find the inverse of the matrix

1 − 12
−11

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ 6 & 1 & 7 \end{bmatrix}.$$ [7 marks]

−18
−11

(c) Use the Principle of Mathematical Induction to prove that

$$2^n > n$$

for all positive integers n. [5 marks]

4 − 12
−16

Question 6

(a) (i) Show that the functions

$$f(x) = 3 + \log_2(x-2) \quad \text{and} \quad g(x) = 2 + 2^{x-3}$$

are inverse functions of each other. [4 Marks]

(ii) Sketch the graph of $g(x)$. [3 Marks]

(b) Solve the following equation

$$2e^{2x} - 5e^{x} - 3 = 0.$$ [4 marks]

28 − 30
−21

(c) Use Cramer's Rule to solve the system of equations

$$\begin{aligned} x + 2y + 3z &= 5 \\ 2x - 3y - z &= 3 \\ -3x + 4y + 5z &= 3 \end{aligned}$$ [6 marks]

5 6 91   9 41 63 28

16 ×2   55 26   12 ×2   2315 18   345

Scanned by CamScanner

dali+80
5

THE COPPERBELT UNIVERSITY
SCHOOL OF MATHEMATICS AND NATURAL SCIENCES
DEPARTMENT OF PURE AND APPLIED MATHEMATICS

SESSIONAL EXAMINATION FOR 2017/ 2018 ACADEMIC YEAR

MA 110 - MATHEMATICAL METHODS I
EXAMINATION

dali+86

DATE: 7th August 2018
DURATION: 3 Hours
MARKS: 100

READ THE FOLLOWING INSTRUCTIONS

1. The paper consists of pages 1 to 6 (questions pages 2 to 6). Check whether your paper is complete.
2. This is a **3 (Three)** hours examination. Cell phones are **NOT** allowed.
3. There are **7 (Seven)** questions in this paper. Each question carries equal marks. Attempt any **5 (five)** of the questions.
4. Please indicate clearly in the answer booklet provided the questions answered.
5. Answers to questions should fully be explained. A correct but unclear answer will not get full marks.
6. No pencil work (except for graph sketching) or any work in red ink will be marked.
7. Use of correction fluid or **"Tip-Ex"** is **NOT** allowed.

NOTE: Answer any 5 (five) of the following 7 (seven) questions

$S_n = \frac{n}{2}(2a + (n-1)d)$

Question 1 **[20 marks]**

(a) Given that E is the universal set and A and B are subsets of E, simplify

$$A \cap (B - A).$$

[5 Marks]

(b) The remainder obtained when the polynomial $2x^3 + ax^2 - 6x + 1$ is divided by $(x + 2)$ is twice the remainder obtained when the same polynomial is divided by $(x - 1)$. Find the value of a. **[5 marks]**

(c) Two straight lines L_1 and L_2 have equations

$$\frac{x-1}{3} = \frac{y+1}{-2} \quad \text{and} \quad \frac{x-4}{4} = \frac{y-1}{6}$$

respectively. Express each equation in the form $y = mx + c$ where m and c are real numbers. Hence, show that L_1 and L_2 are perpendicular. **[5 marks]**

(d) How many terms of the arithmetic sequence

$$24, 22, 20, \ldots$$

are needed to give a sum of 150? **[5 Marks]**

$\frac{-5}{-5+4} = \frac{-5}{-1}$ $\frac{2(12)-15}{12}$

$\frac{-10-15}{-5}$ $\frac{+25}{+5}$ $= -5$ $= 5$ $\frac{24-15}{12}$

Question 2 **[20 marks]**

(a) Prove the De-Morgan's law;

$$(A \cup B)' = A' \cap B'.$$

[5 Marks]

(b) Solve the inequality below and write the solution set in interval notation

$$\frac{x-2}{x+1} \geq \frac{x-6}{x-2}.$$

[5 marks]

(c) Solve the logarithmic equation

$$\log_3 x - \frac{4}{\log_3 x} + 3 = 0.$$

[5 marks]

(d) Use the Principle of Mathematical Induction to prove that

$$3^n \geq 2n + 1$$

for all positive integers n. **[5 marks]**

Question 3 **[20 marks]**

(a) The numbers

$$(k+4),\ k,\ \text{and}\ (2k-15)$$

form the first three terms of a geometric sequence where k is a positive constant. Find the possible value(s) of k. **[5 Marks]**

(b) Prove that $\sqrt{10}$ is an irrational number. **[5 marks]**

(c) Find the center and radius of a circle whose equation is

$$x^2 + y^2 = 2x + 6y - 2.$$

[5 Marks]

(d) Evaluate the expression

$$\cos(\arcsin(2x))$$

as an algebraic expression in x where $0 \leq x \leq \frac{1}{2}$. **[5 Marks]**

$\sqrt{9}$ $\sqrt{2\times4}$ $2\sqrt{2}$

$\sqrt{2 \times 4}$ $\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$2\sqrt{2}$

upper limit

Question 4 [20 marks]

(a) Find the amplitude, period, and phase shift of the following trigonometric function

$$f(x) = -1 + \cos(2x + \pi).$$

Hence, sketch the graph of the function f. **[5 marks]**

(b) Prove the following trigonometric identity

$$\frac{1}{\cos\theta + 1} + \frac{1}{\cos\theta - 1} = -2\csc\theta\cot\theta.$$

[5 marks]

(c) Find the three cube roots of $8i$. **[5 marks]**

(d) If $0 \leq \theta \leq 2\pi$, find all the solutions of the trigonometric equation

$$\sec^2\theta + \tan^2\theta = 3.$$

[5 marks]

π 2π

$\frac{\pi}{2}$ π $\frac{3\pi}{2}$ 2π

Question 5 [20 marks]

$\tan = y/x$

$y/0$

(a) Find the partial fraction decomposition of the following expression

$$\frac{8x + 4}{(1 - x)(x + 2)}.$$

Hence, or otherwise, expand the expression in ascending powers of x as far as the term in x^2, stating the set of values of x for which the expansion is valid. **[5 marks]**

(b) Determine whether or not the binary operation $*$ defined on $\mathbb{Z}$ by $x * y = 1 - 2xy$ is both commutative and associative. **[5 marks]**

(c) Given that the roots of the equation $2x^2 + 5x - 3 = 0$ are α and β. Find an equation with integer coefficients whose roots are $\alpha\beta^2$ and $\alpha^2\beta$. **[5 marks]**

(d) Solve the given equation giving your answer in terms of natural logarithms

$$10\sinh x + \cosh x = 5.$$

upper limit

$-1 + 1 = 0$

lower limit

$-1 - 1$

$A\cos(bx+c)+D$ 4t

$\sin^2 x + \cos^2 x = 1$

$2\cos\theta + 2$

[5 marks]

Question 6 [20 marks]

(a) If

$$M = \begin{bmatrix} -5 & 10 & 8 \\ 4 & -7 & -6 \\ -3 & 6 & 5 \end{bmatrix}, \text{ and } N = \begin{bmatrix} -1 & 2 & 4 \\ 2 & 1 & -2 \\ -3 & 0 & 5 \end{bmatrix},$$

prove that $MN = NM$ [5 marks]

(b) Solve the equation

$$\sqrt{y+7} + 3 = \sqrt{y+4}.$$

[5 marks]

(c) Verify that

$$f(x) = \frac{3}{4x-4} \text{ and } g(x) = \frac{3+4x}{4x}$$

are inverse functions of each other. [5 marks]

(d) Express $f(x) = 1 - 6x - x^2$ in the form $f(x) = a(x+h)^2 + k$ where a, h and k are rational numbers. Hence, find the maximum or minimum value of the function $f(x) = 1 - 6x - x^2$. [5 marks]

Question 7 [20 marks]

(a) For the following rational function

$$f(x) = \frac{2-2x^2}{x^2-4},$$

(i) Determine the x- and the y- intercepts. **[3 Marks]**

(ii) Find the horizontal and the vertical asymptotes. **[3 Marks]**

(iii) Hence, sketch the graph of f. **[3 Marks]**

(b) If $\sin\theta = -\frac{3}{5}$, and $\cos\theta < 0$, find the value of:

(i) $1-2\cos\theta$. **[3 marks]**

(ii) $\sqrt{\cot\theta - 1}$. **[2 marks]**

(c) Use Cramer's Rule to solve the system of equations

$$\begin{aligned} x-2y+z &= -4 \\ y+2z &= 4 \\ 2x+3y-2z &= 2 \end{aligned}$$

[6 marks]

$x = 5/3$ $y = 10/9$

$f(x) = 0$

x-intercepts

-END OF EXAMINATION-

10/9 + 2

1

16 + 18 / 9

28

32 - 2

20

32 - 12 + 10

20 + 10

30

THE COPPERBELT UNIVERSITY
SCHOOL OF MATHEMATICS AND NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS
2016/17 academic year
second term test
MA110-Mathematical Methods

INSTRUCTIONS: (1) Attempt all questions
(2) Show detailed working for full credit
(3) Calculators are not allowed in this paper

TIME ALLOWED: Three (3) hours

1.a) Use G.P repeating decimal $0.\overline{42}$ to $^{a}/_{b}$ form, where a and b are integers and $b \neq 0$. Express $^{a}/_{b}$ in reduced form .

b) Use mathematical induction to prove each of the sum formulas for the indicated sequence for all positive integers n

$$S_n = \frac{(5^{n+1}-5)}{4} \text{ for } a_n = 5^n$$

c) Expand the function $\frac{x+5}{-3+5x-2x^2}$ in ascending powers of x, giving the first three terms ~~and the general term~~, and state the necessary restrictions on the values of x

d) Find the ratio in which the line-segment joining the points (5, - 4) and (2, 3) is divided by the x-axis.

e) The line l has equation $2x - y - 1 = 0$. The line m passes through the point $A(0,4)$ and is perpendicular to the line l. Find an equation of m and show that the lines l and m intersect at the point $P(2,3)$.

2. a) Prove that $\log_{\frac{1}{x}} a = -\log_x a$

b) Find the equation of the tangent at the point (2,1) of the circle $x^2 + y^2 - 4y - 1 = 0$

c) Expand $(1+x)^{\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^3 and hence find an approximation for $\sqrt{1.08}$

d) Find all possible values of λ given that the matrix $(\lambda I - A)$ is singular where $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$ and I is the identity matrix.

e) Express $\log_9 xy$ in terms of $\log_3 x$ and $\log_3 y$. Hence solve for x and y the simultaneous equations

$$\log_9 xy = \frac{5}{2}$$

$$\log_3 x \log_3 y = -6$$

expressing your answers as simply as possible

3. (a) Find the common ratio and the sum of the first 10 terms of the series

$$\log x + \log x^2 + \log x^4 + \log x^8 + \cdots$$

(b) Show that the sum of an infinite geometric sequence is given by $S_\infty = \frac{a_1}{1-r}, |r| < 1$ where a_1 is the first term and r the common ratio.

c) Solve for real x the equation

$$6^{x+2} = 2(3^{2x})$$

d)Find the inverse of the matrix $A = \begin{pmatrix} 3 & -1 & 2 \\ 1 & 1 & 1 \\ 2 & 2 & -1 \end{pmatrix}$ .Hence use your inverse to solve the system of linear equations

$$A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix}$$

e) Given that the equation of the circle in the form $x^2 + y^2 + Dx + Ey + F = 0$.Show that the centre is given by $\left(-\frac{D}{2}, -\frac{E}{2}\right)$ and the radius $r = \sqrt{\frac{D^2 + E^2 - 4F}{4}}$.

4. a) Use mathematical induction to prove that the statement is true for all positive integers n

$7^n - 3^n, n \geq 1$ is divisible by 4

b). Sketch each of the exponential functions and determine its domain and range.

$f(x) = -4^{x-1} + 1$ and $g(x) = |2 - \log_3(x-2)|$

c) Show that $\begin{pmatrix} n+1 \\ 3 \end{pmatrix} - \begin{pmatrix} n-1 \\ 3 \end{pmatrix} = (n-1)^2$.

d) Write the following series in sigma notation and find their sums.

$$1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \ldots$$

e) At what rate of interest compounded continuously will an investment of \$500 grow to \$1000 in 10 years?

THE COPPERBELT UNIVERSITY
SCHOOL OF MATHEMATICS AND NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS
2015/2016 Academic Year
MA110 - Mathematical Methods
Test I
5th September, 2015.

Time Allowed : Three hours (3:00 hrs)

Instructions:

1. You must write your **Name,** your **Computer Number and programme of study** on your answer sheet.
2. Calculates are **not** allowed in this paper.
3. There are five (5) questions in this paper, Attempt **All** questions and show **detailed** working for full credit.

Question 1

a) If C *and* D are disjoint, simplify if possible $(C \cup D)'$

b) Express $2.\overline{143}$ in the form of $\frac{a}{b}$ where a and b are integers, $b \neq 0$.

c) Rationalize the denominator and express the final answer in simplest radical form for $\frac{5\sqrt[3]{y^2}}{4\sqrt[4]{x}}$

d) Sketch and determine the domain and the range of the function $f(x) = \frac{2}{x^2+4}$

e) Prove that if $a + c = b + c$ *then* $a = b$ when $a, b, c \in R$.

Question 2

a) Let binary operation $*$ defined $a * b = a - b + ab$ where a and $b \in R$, solve $|x * 2| = 1$.

b) Rationalize the denominator of $\frac{1}{(\sqrt{2}+1)(\sqrt{3}-1)}$

c) Determine whether $f(x) = x^2 + 1$ is even, odd or neither.

d) Solve the equation $\sqrt{-2x-7} + \sqrt{x+9} = \sqrt{8-x}$.

e) Solve. $x^4 + 3x - 2 = 0$.

Question 3

a) Solve for x and y given that $\frac{x}{1+i} - \frac{y}{2-i} = \frac{1-5i}{3-2i}$ ✓

b) Sketch $f(x) = 2 + 3\sqrt{-x+1}$ and determine its range and domain.

c) The roots of the equation $2x^2 + 6x - 15 = 0$ are $\propto$ and β. Find the value of $(\alpha - \beta)$.

d) Prove that $\sqrt{3}$ is an irrational number.

e) Solve $\frac{3x+2}{x-1} > 0$ expressing the set of solution sets in interval notation.

Question 4

a) Verify that the two given functions are inverses of each other $f(x) = x^3 + 1$ and $g(x) = \sqrt[3]{x-1}$ ✓

b) Express $5 - x - 2x^2$ in the form $a - b(x+c)^2$ and hence or otherwise find its maximum value and the value of x where this occurs.

c) Using the associative and distributive properties of union and intersection of sets . Show that ✓

$$A \cup B = (A \cap B) \cup (A \cap B') \cup (A' \cap B)$$

d) Solve for x given $|3x+1| < |4-2x|$.

e) What type of roots does the equation $5x^2 - 3x + 1 = 0$ have ?

Question 5

a) Determine whether $f(x) = x^2 - 2$ is one-to-one . If it is, find the inverse and graph both the function and its inverse.

b) Given that $z + \frac{1}{z} = k$, where k is a real number, prove that either z is real or $|z| = 1$

c) Given the set $X = \{0, 1, 2, 3\}$. Determine whether the operations $+$, $-$, $\times$ are binary operations on X ✓

d) Determine whether or not $x + 3i$ is a factor of $f(x) = x^4 + 14x^2 + 45$.

e) Solve $\frac{4}{x-2} + \frac{x}{x+1} = \frac{x^2-2}{x^2-x-2}$.

f) Sketch $f(x) = |x^2 + 5x + 4| - 2$ and determine its domain and range.

END

THE COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS

2015/16 ACADEMIC YEAR

SESSIONAL EXAM

MA110 – MATHEMATICS

TIME ALLOWED: Three (3) Hours. 18th JUNE, 2016

INSTRUCTIONS:

(1) You must write your Computer number and program on each answer booklet you have used.

(2) There are Seven (7) questions in this paper. Attempt Any Five (5). Each question consists of a,b,c,d,e. All questions carry equal marks.

(3) Calculators are NOT allowed in this paper

(4) Should you have any problem or if you are in need of more answer booklets, put up your hand an invigilator will come and attend to you.

Scanned by CamScanner

i). Is $*$ commutative on real number? Justify your answer.

ii). Find $-1 * (4 * 9)$

c) Sketch the graph of the polar equation $r = 2 + 4\cos\theta$

d) Use De Moivre's theorem to find $\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^{15}$, and express the results in $a + bi$ form.

e) Prove the identity that $\cot(\alpha + \beta) = \frac{\cot\alpha\cot\beta - 1}{\cot\alpha + \cot\beta}$

QUESTION FOUR

a) Solve $\sin x \tan^2 x = \sin x$ for x, where $0 \leq x < 2\pi$.

b) Verify that the two given functions are inverses of each other

$f(x) = x^3 + 1$ and $g(x) = \sqrt[3]{x - 1}$

c) Solve for real values of x, given that $\sinh x + 4 = 4\cosh x$

d) Use De Moivre's theorem to find $(1 + i)^{10}$ and express the results in $a + bi$ form

e) Find λ for which $\lambda I - A$ is a singular matrix where I is an identity Matrix and

$$A = \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}.$$

QUESTION FIVE

a) Find the general solution of $\tan x + 1 = \sec x$

b) Graph $f(x) = \log_{1/2} x$ by reflecting the graph of $g(x) = \left(\frac{1}{2}\right)^x$ across the line $y = x$.

c) Find the second cube roots of $1 - i$. Express the roots in $a + bi$ form if they are exact. Otherwise, leave them in trigonometric form

d) Express $2.5\overline{17}$ as a rational number

e) Find the perpendicular distance of the given line $3y = 2x - 12$ from the point(0,4).

QUESTION SIX

a) Solve the equation $y^{2/3} + y^{1/3} - 6 = 0$

b) Verify that $cosA + cosB = 2cos\frac{A-B}{2}cos\frac{A-B}{2}$

c) What is the common ratio of the $G.P.$ $(\sqrt{2}-1) + (3-2\sqrt{2}) + \cdots$? Find the third term of the progression.

d) Find the second cube roots of 8. Express the roots in $a + bi$ form if they are exact. Otherwise, leave them in trigonometric form

e) Verify the identity $\tanh^2 x = \frac{\cosh 2x - 1}{\cosh 2x + 1}$.

QUESTION SEVEN

a) Evaluate the expression $sin(tan^{-1}\sqrt{3})$

b) Use the binomial theorem finds the values of $(1.01)^{10}$ up to the third term.

c) Solve $\frac{x-1}{x+2} > 2$, expressing the set of solution sets in interval notation.

d) Find an exact value for tan 67.5°

e) Show that $cos\theta + sin\theta = \sqrt{2}sin\left(\theta + \frac{\pi}{4}\right)$

1. a) i) Prove the De Morgan's laws

$$(A \cup B)' = A' \cap B'.$$

ii) Show that

$$X = (X \cap Y) \cup (X \cap Y').$$

b) i)Express the following in the form of $\frac{a}{b}$ where a and b are intergers, $b \neq 0$.

$$0.5333333\ldots$$

ii) Given that $\sqrt{5}$ is an irrational number, prove that $\sqrt{5} + 1$ is an irrational number .

c) Determine whether the function f is even, odd or neither.

$$f(x) = x^3 + x\,.$$

2. a) Express $\frac{\sqrt{x}}{2\sqrt{x}-1}$ in the form $a + b\sqrt{x}$, where a and b are real numbers.

b) i) Express $1 + 4i + \frac{5}{2-i}$ in form $a + bi$ and find its absolute value(modulus).

ii) Solve for x and y given that:

$$\frac{x+iy}{2+i} = 5 - i$$

c) Let $'*'$ be a binary operation on the set of real numbers defined by $a * b = 2^{b-a}$, where a and b are real numbers .

i). Is $*$ commutative on real numbers? Justify your answer.

ii). Find $-1 * (4 * 9)$.

d) Sketch the graph of the following radical function:

$$f(x) = 2 + \sqrt{-x+2}\,.$$

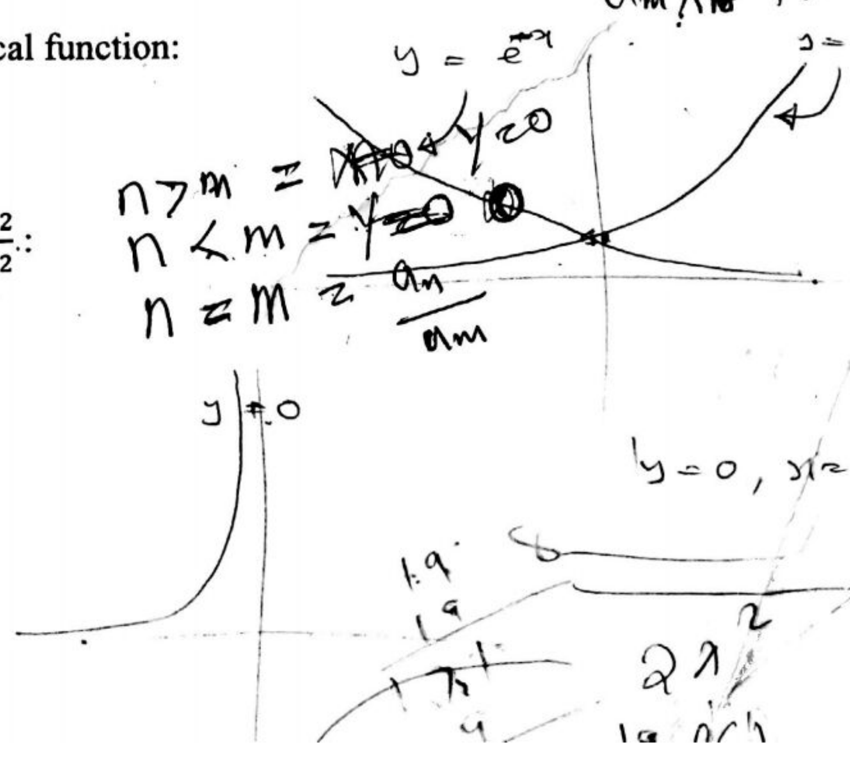
3. a) Given the rational function $f(x) = \frac{x+2}{x-2}$:

i). Find the domain of $f(x)$

ii). Find the vertical asymptotes

iii). Find the horizontal asymptotes

iv). Sketch the graph of $f(x)$.



Scanned by CamScanner

b) Given the universal set $U = [1,12]$ where $A = \{1,2,3,4,5,6,7,8,9\}$ and $B = [1,5)$ .Find

i) $A \cap B$ ii) $A - B$.

c) i) Verify that the two given functions are inverses of each other

$$f(x) = x^3 + 1 \text{ and } g(x) = \sqrt[3]{x-1}.$$

ii) Determine the domain of functions:

$$f(x) = x + \sqrt{x^2 + 4x - 12} .$$

4. a).Use the Rational root theorem to solve the following equation:

$$3x^4 + 5x^3 - 5x^2 - 5x + 2 = 0 .$$

b). i) Solve each of the following equation

$$\left|\frac{x+1}{x-4}\right| \leq 3$$

ii). Redefine $k(x) = |2x - 1| - |x + 2|$ by removing the modulus, hence sketch the graph of function:

c) Let $f(x) = 3x^2 + 12x + 5$ be a quadratic function.

i) By completing the square, express $f(x)$ in the form $f(x) = a(x+p)^2 + q$ where *a, p* and *q* are constants.

ii)State the maximum or the minimum point of the function *f*.

iii)Sketch the graph of the function $f(x) = 3x^2 + 12x + 5$

d) i)Find the values of k if the equation $x^2 + (k-2)x + 10 - k = 0$ has equal roots.

ii) The roots of the equation $2x^2 + 6x - 15 = 0$ are α and β. Find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$.Hence find the quadratic equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$.

$$\frac{\beta^2 + \alpha^2}{\alpha^2\beta^2}$$

THE COPPERBELT UNIVERSITY
SCHOOL OF MEDICINE
DEPARTMENT OF MATHEMATICS
TEST ONE 2018/19 ACADEMIC YEAR

MA110 - Mathematical methods

INSTRUCTIONS **TIME ALLOWED: 2 hours 30 minutes**

(1)You must write your **NAME,POROGRAM, COMPUTER NUMBER AND GROUP on the cover of your answer sheet.**

(2)There are five questions in this paper.Attempt all Questions.

(3) Show all neccessary working and **number the pages** in your answer sheet

(4) **Calculators are NOT** allowed in this paper

QUESTION ONE

(a) Given that A, B and C are sets, simplify the following if possible
$[(A\cap B)'\cap(A'\cup B)]'$

(b) Rationalize the denominator of $\dfrac{1}{(\sqrt{2}+1)(\sqrt{3}-1)}$

(c) Sketch the graph of $f(x)=\begin{cases}2x+3 & if\ x<0\\ x^2 & if\ 0\le x<2.\\ 1 & if\ x\ge 2\end{cases}$

(d) Prove that $\sqrt{2}$ is an irrational number.

(e) Using synthetic division find the quotient and the remainder when $f(x)=x^3+2x^2+x-2$ is divided by $x-(1+i)$.

$2(x^2-\frac{3}{2}x)-4$
$2(x-\frac{3}{4})^2-\frac{9}{4}$
$(1+i)(3+i)$
$3+i+3i-1$
$2+4i$
$(1+i)(3+4i)$
$3+4i+3i-4$
$-1+7i$
20

QUESTION TWO

(a) Express $2.07\overline{2}$ as a fraction $\dfrac{a}{b}$ in its simplest form where a and b are integers and $b\neq 0$.

207.2
1865.0
900
180
373

(b) Express $f(x)=2x^2-3x-4$ in the form $f(x)=a(x+p)^2+q$ where a, p and q are constants indicating the axis of symmetry and the coordinates of its maximum or the minimum point.

(c) Define an operation *on the set of real numbers by $a*b=a+b-2\sqrt{ab}$

i). Is $*$ a binary operation on the set of real numbers? Give reason for your answer.

ii). Evaluate $(1*-1)*2$ and $1*(-1*2)$ and state whether $*$ is associative.

(d) Let $f(x) = \frac{7}{x-4}$ and $g(x) = \frac{2}{x}$, find $(f \circ g)\left(-\frac{1}{2}\right)$ and determine its domain of $(f \circ g)$

(e) Determine whether the function $f(x) = \frac{x}{\sqrt{x^2+4}}$ is one-to-one. If it is, find the inverse and graph both the function and its inverse

$\frac{-1}{\sqrt{5}}$ $\frac{0}{2}$

QUESTION THREE

(a) State and prove one of de-Morgan's laws.

(b) Let **R**, the set of real numbers be the universal set. If $A = [-7, 8) \cup [11, \infty)$ and $B = [0, 20]$, find the following sets and display them on the number line:

(i) A'. (ii) $A \cap B$.

(c) Let α and β be the roots of the quadratic equation $4x^2 + 3x - 2 = 0$ $\sqrt{1}$
Find the sum $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

(d) Sketch the graph of the function $k(x) = |2x - 1| - |x + 2|$ $-x-2$

QUESTION FOUR

$-2x+1+x+2$

$3-x$

(a) Solve the inequality $|x - 1| > 1 - x^2$

(b) Given the rational function $f(x) = \frac{x^2+2}{x-1}$. Sketch its graph indicating its domain and range, all the asymptotes and intercepts.

(c) Determine the domain of the given function: $f(x) = \sqrt{\frac{x+1}{x-1}}$

(d) Let $z = x + iy$ be a non zero complex number. Given that $z + \frac{1}{z} = k$, where k is a real number, prove that $|z| = 1$

(e) Solve for x and y given that $\frac{1}{x+iy} + \frac{1}{1+3i} = 1$

QUESTION FIVE

(a) Simplify $\frac{2^n - 6^n}{1 - 3^n}$

(b) Sketch $f(x) = 2 + 3\sqrt{-x+1}$ and determine its range and domain

(c) Rationalize the denominator $\frac{5\sqrt[3]{z^2}}{4\sqrt[4]{x}}$ and express the final answer in simplest radical form.

(d) If the equation $x^2 - (p-2)x + 1 = p(x-2)$ is satisfied by only one value of x, What are the possible values of p

(e) Use the definition of the absolute value, evaluate $\left|x - \frac{2}{3}\right| = \frac{3}{4}$

Bertha Muhinte Hambaya
(18130F65)

76

THE COPPERBELT UNIVERSITY, SCHOOL OF MEDICINE

2019 ACADEMIC YEAR **MA110 – Mathematical Method**

Test II

Time allowed: Two hours (2:00)

Instructions:

1. You must write your **Name**, your **Computer Number and Program of Study** on your answer sheet. (If **Part Time and Proceed and Repeat** then indicate)
2. Calculators are not allowed in this paper.
3. There are five (5) questions in this paper, attempt **All** questions and show **detailed** working for full credit.

QUESTION ONE

a) Determine the sum $\sum_{n=3}^{12} 3(n-2)$

b) Express $\frac{3x^2+2x-9}{(x^2-1)^2}$ in partial fractions 4

c) Prove the following result by induction: $1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1)$

d) The line $y = 2x - 8$ meets the co-ordinates axes at A and B. the line AB is a diameter of the circle. Find the equation of the circle.

e) Solve the logarithmic equation: $log_{10}a + log_a 100 = 3$

QUESTION TWO

a) Prove that $log_{\sqrt{2}}(4x^3) = 3log_{\sqrt{2}}(x) + 4$

b) Let M be the matrix $\begin{pmatrix} 3 & 1 & -3 \\ 1 & 2z & 1 \\ 0 & 2 & z \end{pmatrix}$. Find two values of z for which M is non singular.

c) Sketch $f(x) = 2 - log_3(x-1)$ and determine the domain and range

d) Write down the constant term in the expansion of $\left(2x^2 + \frac{1}{2x}\right)^6$

e) Solve for real values of x given the equation:$2e^x - 3e^{-x} + 5 = 0$

QUESTION THREE

a) The three numbers, $n-2, n, n+3$, are consecutive terms of a geometrical progression. find n, and the term after $n+3$.

b) The circle, centre (p, q) radius 25 meets the x −axis at $(-7,0)$ and $(7,0)$, where $q > 0$. Find the values of p and q.

c) Use mathematical induction to prove that the statement is true for all positive integers n given that $2^n \geq n$.

d) Verify that $f(x) = b^x$ and $y = log_b x$ where b and $x > 0$ are inverses of each other.

e) At what rate of interest compounded continuously will an investment of \$500 grow to \$1000 in 10 years? (given that $ln2 = 0.6931$)

QUESTION FOUR

a) Let $D = \begin{bmatrix} 0 & -3 \\ -2 & 1 \end{bmatrix}$, Evaluate $(I_2 - D)^2$ where I is a 2 by 2 matrix.

b) Show that $\binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r}$

c) Find the first four terms in the binomial expansion of $(-x+2)^6$ and determine where it is valid

d) Find the radius of a circle centre at $C(-2,5)$ if the line $x + 3y = 9$ is a tangent line.

e) Given that $log_a x^2 y = p$ and prove that $log_a\left(\frac{x}{y^2}\right) = q$, find $log_a x$ and $log_b y$ in terms of p and q. (4)

QUESTION FIVE

a) Find the centre and length of a radius of the given circle

$$4x^2 + 4y^2 - 6x + 10y - 1 = 0.$$

b) Show that the sum of a geometric sequence is given by

$$S_n = \frac{a_1 r^n - a_1}{r-1} \quad where\ r \neq 1$$

c) Let $B = \begin{pmatrix} -5 & 10 & 8 \\ 4 & -7 & -6 \\ -3 & 6 & 5 \end{pmatrix}$ and $A = \begin{pmatrix} -1 & 2 & 4 \\ 2 & 1 & -2 \\ -3 & 0 & 5 \end{pmatrix}$ and $BA = AB = I$, where I is the 3×3 unit matrix. Solve the equation (4)

$$\begin{pmatrix} -5 & 10 & 8 \\ 4 & -7 & -6 \\ -3 & 6 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 2 \end{pmatrix}$$

d) Find the equation of the perpendicular from the point $A(5,3)$ to the $2x - y + 4 = 0$

e) Use mathematical induction to prove that $9^n - 1$ is divisible by 4 for all positive integers n.

The Copperbelt University

School of Mathematics And Natural Sciences

Department of Mathematics

MA 110 : (Mathematical Methods I) : Deferred Test

August 15, 2022

Instructions

(1). You must write your **Name, Student Identification Number (SIN)** and **Programme** of study on your answer sheet. Calculators are not allowed. Time allowed is **1hr:30 minutes**

(2). There are **Four (4)** questions in this paper, for **deferred test 1, attempt questions 1 and 2** and **deferred test 2, attempt questions 3 and 4.**

QUESTION ONE

(a). Express $2.07\overline{2}$ as a fraction $\frac{a}{b}$ in its simplest form where a and b are integers and $b \neq 0$

(5 marks)

(b). Evaluate and Simplify $\dfrac{2^{n-1} - 8^n}{\frac{1}{2} - 4^n}$.

(5 marks)

(c). Rationalize the denominator of $\dfrac{1}{(\sqrt{2}+1)(\sqrt{3}-1)}$.

(5 marks)

(d). Determine the domain of the given function $f(x) = \sqrt{\dfrac{x+1}{x-1}}$.

(5 marks)

(e). Solve for x and y given that $\dfrac{1}{x+iy} + \dfrac{1}{1+3i} = 1$.

(5 marks)

QUESTION TWO

(a). Prove the De Morgans law $(A \cap B)' = A' \cup B'$. **(5 marks)**

(b). Determine whether the given function is odd, even or neither

$$f(x) = \frac{x^3 + 2x}{2}.$$

(5 marks)

(c). Prove that $\sqrt{2}+\sqrt{3}$ is an irrational number using the fact that $\sqrt{6}$ is an irrational number.

(5 marks)

(d). Find the square root of a complex number $15 + 8i$.

(5 marks)

(e). Sketch the graph of

$$f(x) = \begin{cases} 2x+3 & x < 0 \\ x^2 & 0 \leqslant x < 2 \end{cases}$$

(5 marks)

(Total Marks: 25)

QUESTION THREE

(a). Change the repeating decimal $5.\overline{7}$ to its reduced form $\frac{a}{b}$, where a and b are integers and $b \neq 0$ using sum to infinity of a geometric series.

(5 marks)

(b). Express $\dfrac{3x^2 + 2x - 9}{(x^2-1)^2}$ into a partial fraction.

$x^4 - 2x^2 + 2$

$(x^2-1)(x^2-1)$

$x^4 + x^2 - x^2 + 2$

(5 marks)

(c). Use Mathematical Induction to prove that $3^{2n} - 1$ is divisible by 8. 4

(5 marks)

(d). Find the first term and the general expansion of $\dfrac{1}{(2-6x)^5}$ in ascending power of x. State the range of value of x for which this expansion is valid.

(5 marks)

(e). Graph the function of $f(x) = 2^{(x-3)} + 2$ and obtain its inverse on the same axis.

(5 marks)

(Total Marks: 25)

QUESTION FOUR

(a) Prove that $\log_a(A^C) = C\log_a(A)$.

(5 marks)

(b) Find the center and radius of the circle whose equation is

$$x^2 + y^2 + 8x - 2y + 13 = 0.$$

(5 marks)

(c) Solve the equation $\log_3 x - 4\log_x 3 + 3 = 0$.

(5 marks)

(d) Write down the constant term in the expansion of $\left(x - \dfrac{1}{2x^2}\right)^9$.

(5 marks)

(e) Use crammer's method to solve the linear system of equation

$$\begin{aligned} x + 2z &= 9 \\ 2y + z &= 8 \\ 4x - 3y &= -2 \end{aligned}$$

(5 marks)

(Total Marks: 25)

THE END OF TEST

5.7
0.7
5.14

57
0.007
5.707

THE COPPERBELT UNIVERSITY
SCHOOL OF MATHEMATICS AND NATURAL SCIENCES
DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MA 110 - MATHEMATICAL METHODS I
TEST 1

DATE: 8th December 2017
DURATION: 3 Hours
MARKS: 100

READ THE FOLLOWING INSTRUCTIONS

1. Write your **NAME, PROGRAM, COMPUTER NUMBER AND GROUP on the cover of your answer sheet.**
2. This is a **THREE Hours** test. Cell phones are **NOT** allowed.
3. Attempt **ALL** questions. Answers to questions should fully be explained. A correct but unclear answer will not get full marks.
4. No pencil work (except for graph sketching) or any work in red ink will be marked.
5. Use of correction fluid or **"Tip-Ex"** and **calculators** are **NOT** allowed.

Question 1

(a) Let $E = \{3,4,5,6\}$, $F = \{0,2,4,6,8\}$ and the universal set $X = \{0,1,2,\ldots,10\}$. Find $E \cap F'$.

[1 mark]

(b) Let

$$f(x) = \frac{2}{x^2 - 2}, \text{ and } g(x) = \frac{1}{\sqrt{x+1}}$$

Find $(g \circ f)(-2)$.

[4 marks]

(c) Use long division to divide $4x^3 - 7x^2 - 11x + 5$ by $4x + 5$.

[3 marks]

(d) If A and B are subsets of the universal set U such that $A \subset B$. Simplify

(i) $A' \cup B'$. **(ii)** $A - B$.

[2+2 marks]

(e) Find the possible values of λ and k if the expression $3x^4 + \lambda x^3 + kx + 4$ is exactly divisible by $x - 1$ and leaves a remainder of 18 when divided by $x + 2$.

[5 marks]

$x = 1$

$x = -2$

Question 2

[5 marks]

(a) Prove the De-Morgan's law; $(A \cap B)' = A' \cup B'$.

(b) Is the function $f(x) = |x| + x^2$ even, odd or neither? Justify your answer. [3 marks]

(c) Solve the inequality below and write the solution set in interval notation [5 marks]

$$\frac{x+4}{x+1} \leq \frac{x-2}{x-4}.$$

(d) Use Factor Theorem and synthetic division to factorise $f(x) = 6x^4 - 19x^3 + 17x^2 - x - 3$ completely. [4 marks]

$\frac{b^2 - 4ac}{2a}$

Question 3

(a) Sketch the following piecewise defined function [4 marks]

$$f(x) = \begin{cases} |x+2| & \text{if } -4 \leq x < 0 \\ x^2 & \text{if } 0 \leq x \leq 2 \\ 3 & \text{if } 2 < x \leq 3 \end{cases}$$

$K = \frac{81 - 4(1)(c)}{2}$

$\frac{76}{2}$

$\frac{38}{76}$

$K = 38c$

(b) Prove that $\sqrt{2} + \sqrt{3}$ is irrational. [4 marks]

(c) Solve the polynomial equation $2x^5 - 5x^4 + x^3 + x^2 - x + 6 = 0$. [5 marks]

(d) Solve the following inequality and write the solution in interval notation [4 marks]

$$3|2x+1| + 1 \leq 7.$$

Question 4

(a) Let the universal set be the set of real numbers, with $A = (3,8]$, $B = (2,7)$, $C = [1,5]$ and $D = [6,\infty)$. Find **(i)** $(A \cup C)$ **(ii)** $(B \cap D)$ **(iii)** $(A \cup C) - (B \cap D)$ **[1+1+2 marks]**

(b) Is the binary operation $*$ defined by $a * b = a + b - ab$ both commutative and associative. Justify your answer. **[3 marks]**

(c) Sketch the graph of the polynomial $f(x) = -(x-3)(x-2)^3(x+1)^2$. **[5 marks]**

(d) Solve the equation $|2x + 11| = 7$. **[2 marks]**

(e) The roots of the equation $x^2 - 9x + K = 0$ are α and $\alpha + 1$. Find the value of K. **[3 marks]**

$x^2 - 6x - 3x + K$

$x(x-6) - 3(x-K)$

$-x + 3 = 0$ $-b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

$-x = -3$

$-81 - 4$

$+1 - 2$

-3

$-6 - 3$

$-3 + 1$

-2

$x + 2$

$x + 1$ | $2x^5 + 5x^4 + x^3 +$

$2x^5 + 5x^4 + x^3 + x^2 - x + 6$

5 8 3 3 -3 9

2 3

2 3 -2

2 5

$x + 3 = 0$

Question 5

(a) Express

$$\frac{\sqrt{3}+1}{\sqrt{3}-1}+\sqrt{3}-1$$

in the form $a+b\sqrt{3}$ where a and b are rational numbers.

[4 marks]

(b) Verify that

$$f(x) = 4x-5 \text{ and } g(x) = \frac{x+5}{4}$$

are inverse functions of each other.

[4 marks]

(c) Express $f(x) = 1-6x-x^2$ in the form $f(x) = a(x+h)^2+k$ where a, h and k are rational numbers. Hence, write down the coordinates of the turning point of the graph $f(x) = 1-6x-x^2$.

[5 marks]

(d) Solve the equation $2x-11\sqrt{x}+12 = 0$.

[4 marks]

Question 6

(a) Solve for x and y where x and y are real numbers

$$(x+yi)-i = i(x+yi)+5.$$

[4 Marks]

(b) The equation $Kx^2-2Kx+2K = 1$ where K is a constant has two real solutions.

(i) Show that K satisfies the inequality

$$K^2-K \leq 0.$$

[2 Marks]

(ii) Hence, find the set of all possible values of K.

[3 Marks]

(c) For the following rational function

$$f(x) = \frac{2x^2-2}{x^2-4},$$

(i) Determine the x-intercepts and the y-intercept.

[3 Marks]

(ii) Find the horizontal and the vertical asymptotes.

[2 Marks]

(iii) Sketch the graph of f.

[3 Marks]

COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

DEPT OF PURE & APPLIED MATHEMATICS

MA 110 - MATHEMATICAL METHODS I | TEST 1 2018

INSTRUCTIONS; 1. Attempt all Questions in this Paper without Using a Calculator.
2. Indicate clearly your Names,SIN and the Group you belong to.
3. Duration is **3 Hours Only.**

---

1. *a. i.)* Let $A = \{1, 2, 3\}$ and $B = \{2, 4\}$, Find $A \times B$. 1 Mk

ii.) Prove that $(A^c)^c = A$ by Arbitrary Elementary Method. 4 Mks

b. If the Operation $*$ is defined as, " *add the first number* **to** 8 *times the second number*"

Find $(2 * 3) * 5$ 2+8x5

2 Mks

c. Find the value of k given that when $2x^3 - 2kx^2 - 3x - 2$ is divided by $x - 2$, the Remainder is **40**. 3 Mks

d. If $gof(x) = x$ and $g(x) = \dfrac{x+1}{x-1}$,

i.) Find $f(x)$ *ii.)* Sketch the Graph of $f(x)$ and Find the Range of $f(x)$ 3 Mks, 4 Mks

2. *a.* Prove the De Morgan's Law: $A^c \cup B^c = (A \cap B)^c$ 5 Mks

b. Solve the following Equations involving the Absolute value functions:

$$|8x + 3| = |2x - 21|$$ 3 Mks

c. Solve the following inequation:

$$\frac{x-2}{x+1} \geq \frac{x-6}{x-2}$$ 5 Mks

d. Using Synthetic Division, show that both $x-2$ and $x+3$ are Factors of:

$$f(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18.$$

Hence, or otherwise Factorize $f(x)$ completely 4 Mks

3. *a.* Express the following in the form $\frac{a}{b}$, where $a, b \in \mathbb{Z}$ and $b \neq 0$.

   i.) **0.121212···** *ii.)* **1.3121212···** 1.5 Mks, 1.5 Mks

** *b.* Use the fact that $\sqrt{6}$ is Irrational to prove that $\sqrt{2} + \sqrt{3}$ is Irrational. 4 Mks

   c. Sketch the graphs of:

   i.) $f(x) = -|x+3| - 4$ *ii.)* $f(x) = 3 + \sqrt{3-x}$ 2.5 Mks, 2.5 Mks

   d. Solve the Polynormial Equation $x^4 - 6x^3 + 22x^2 - 30x + 13 = 0$ 5 Mks

4. *a.* Determine the vertex and Intercepts for the following Quadratic function: Vertex $\left[-\frac{b}{2a}\right.$

$$f(x) = x^2 - 6x - 16$$ 2 Mks

b. Sketch the graph of the Polynomial given by;

$$f(x) = (x-1)^2(x-3)^3(x+4)$$ 5 Mks

c. Given that the roots of $x^2 + 3x + 17 = 0$ are α and β respectively. Find a Quadratic Function whose roots are $\alpha^3 + \beta^3$ and $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$. 5 Mks

roots
Sum $\alpha + \beta = -$
product $\alpha\beta = \frac{c}{a}$

* *d.* Given that set $A = \{1, 2, 3\}$ and set $B = \{2, 4, 6\}$, Determine whether the Operation;

$$A \circ B = P(A) - P(B).$$

is Binary on the Universal Power set, $P(E)$. 5 Mks

Hence it is Binary on the $P(E)$ since $P(E) = P(A) \cup P(B)$

intersection the complement.

$-5(-2)$
$10+$

$-5(-1)+16$
$5+16 = 2$

$\cdot 5 + 17 - 13$
$5 + 4$

$1 - 6 + 22 - 30 + 13$

$2 + 11$
-1
$-2 + 11 - 18 + 9$
$9 - 9 = 0$

$11 - 2$
9
$18 - 9$

5. *a.* If $x = \dfrac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}}$, find the value of $8x - x^2$. 4 Mks

× *b.* Given the Functions, $f(x) = x^2 + 4$ and $g(x) = x - 9$.

Find the value of x for which $g[f(x)] = f[g(x)]$ 4 Mks

x *c.* Write the Expression $f(x) = 2x^2 + 12x + 14$ in the form $f(x) = a(x+h)^2 + k$

where $a, h, k \in \mathbb{R}$.

Hence, state the turning point of $f(x)$. 4 Mks

d. Calculate the value(s) of x that are valid for the Equation below.

$$\left|\frac{x-2}{x+3}\right| = 4$$ 5 Mks

6. *a.* Simplify $-\dfrac{25}{2}\left[\dfrac{1+2i}{3+4i} - \dfrac{2-5i}{-i}\right]$ 3 Mks

b. Solve for x and y given that;

$$\frac{x}{1+i} - \frac{y}{2-i} = \frac{1-5i}{3-2i}$$ 5 Mks

c. Solve the Inequality below and present your answer in Interval Notation:

$$3x^2 + 2x + 2 < 2x^2 + x + 4$$ 4 Mks

d. Graph the Rational Functional by finding the Asymptotes and Intercept:

$$f(x) = \frac{5x^2 - 2}{1 - x}$$ 5 Mks

1. **(a)** **(i)** Use laws of sets to simplify $[A \cup (A \cup B^c)^c]^c$

(ii) Rationalise the expression $\dfrac{\sqrt{5}+1}{\sqrt{5}-\sqrt{3}}$.

(b) **(i)** Express $0.30\overline{5}$ as a fraction in its lowest terms.

(ii) Solve $(x-2)(x+1)(x+4) > 0$

(c) Given the complex numbers

$$z_1 = p + 2i \text{ and } z_2 = 1 - 2i$$

where p is an integer,

(i) find $\dfrac{z_1}{z_2}$ in the form $a + bi$, where a and b are real numbers. Give your answer in its simplest form in terms of p.

(ii) Given that $\left|\dfrac{z_1}{z_2}\right| = 13$, find the possible values of p.

[4,8,8]

2. **(a)** **(i)** Express $\sqrt{5}+2+\dfrac{1}{\sqrt{5}-2}$ in the form $A+B\sqrt{C}$, where $A,\ B$ and C are rational.

(ii) Prove that $\sqrt{3}$ is irrational.

(b) Find two numbers whose sum is 24 and whose product is the maximum possible value.

(c) Given that $f(x) = 3x^2 + 5x - 2$,

(i) complete the square of $f(x) = 3x^2 + 5x - 2$

(i) solve the quadratic equation $3x^2 + 5x - 2 = 0$

(ii) find the minimum value of $f(x) = 3x^2 + 5x - 2$ and the value of x for which this occurs

(iv) sketch the curve $f(x) = 3x^2 + 5x - 2$, showing where the curve cuts the axes.

[8,5,7]

[**Turn Over...**]

3. **(a)** **(i)** Find the values of x for which $|4-x| > 1 + |x+1|$.

(ii) Find the values of x for which $\dfrac{x-2}{x+2} > 4$.

(b) **(i)** Sketch the graph of $f(x) = \dfrac{1}{x-2} + 1$ for $x \neq 2$.

(ii) Sketch the graph of $y = \sqrt{x+2} - 1$.

(c) **(i)** Factorise $x^3 + y^3$ completely.

(ii) Show that $(x+1)$ is a factor of $x^3 + 2x^2 - 5x - 6$ and hence factorise the expression fully.

[10,4,6]

4. **(a)** **(i)** If $\sin\theta = \dfrac{8}{17}$ and θ is obtuse, find, without using a calculator, the values of $\sec\theta$ and $\cot\theta$.

(ii) Find one point of intersection of $y = \sqrt{3}\sin x$ and $y = \cos x$, where x is in radians.

(b) Given that $y = 2\cos\left(x - \dfrac{\pi}{2}\right)$,

(i) find the amplitude, period and phase shift of y

(ii) sketch the graph of y

(c) If $x = a\sin\theta$, simplify $\dfrac{x}{\sqrt{a^2 - x^2}}$.

[9,6,5]

[Turn Over...]

5. **(a)** Evaluate the following limits:

   (i) $\lim_{x \to 2} \dfrac{x-2}{x^2-4}$

   (ii) $\lim_{x \to 0} \dfrac{\sqrt{2+x}-\sqrt{2}}{x}$

   (b) Let the function f be defined as follows:

$$f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{if } x \neq 2 \\ 7 & \text{if } x = 2 \end{cases}$$

   Is f continuous at $x = 2$?

   (c) Given that $f(x) = \dfrac{x^2+2x-8}{x^2-4}$,

   (i) find the vertical asymptotes of $f(x)$

   (ii) sketch the graph of $f(x)$

   (iii) find $\lim_{x \to -2^+} f(x)$ and $\lim_{x \to -2^-} f(x)$.

   (iv) is $f(x)$ continuous at $x = -2$?

[6,5,9]

6. **(a)** Use first principles to find the derivative of $f(x) = \dfrac{1}{\sqrt{x+1}}$

   (b) Find $\dfrac{dy}{dx}$ if

   (i) $y = e^{-x}\sin(x^2+1)$

   (ii) $y = \left(\dfrac{x-2}{x-\pi}\right)^{-2}$

   (c) A farmer has $50m$ of metal railing with which to form two adjacent sides of a rectangular enclosure, the other two sides being two existing walls, meeting at right angles. Let the length of one side of the enclosure be xm.

   (i) Draw a diagram and write down an expression for the length of the other side of the enclosure.

   (ii) Obtain the expression for the area, Am^2, of the enclosure in terms of x.

[**Turn Over...**]

(iii) Find $\frac{dA}{dx}$ and solve $\frac{dA}{dx} = 0$.

(iv) Hence find the dimensions of the enclosure which gives the maximum area and find the maximum area.

[5,8,7]

7. **(a)** Evaluate the following integrals

(i) $\int \frac{e^x}{2+e^x} dx$

(ii) $\int x^2 \cos x dx$

(b) Find y in terms of x given that

$$\frac{d^2y}{dx^2} = 6x - 1$$

and that when $x = 2$, $\frac{dy}{dx} = 4$ and $y = 0$.

(c) Find the area enclosed by the x-axis, the curve $f(x) = x^2 - 3x + 2$ and straight lines $x = 0$ and $x = 3$.

[8,6,6]

END!