THE COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS

TUTORIAL SHEET 2 : **MA110 - Mathematical Methods** 2023/2024

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1. Evaluate each of the following using the definition of Absolute value.

   a) $|x-2|=6$ b) $|2n+1|=11$ c) $\left|\frac{3}{k-1}\right|=4$ d) $\left|x-\frac{2}{3}\right|=\frac{3}{4}$ e) $|-4|$ f) $|4|$

   g) $|2x-3|\le 5$ h) $|5x-4|\le 8$

2. State the property that justifies each of the statements

   a) $x(2)=2(x)$ b) (7+4)+6=7+(4+6) c) 1(x)=x d) $\left(\frac{2}{3}\right)\left(\frac{3}{2}\right)=1$

3. Evaluate each of the following if x is a nonzero real number.

   a) $\frac{|x|}{x}$ b) $\frac{x}{|x|}$ c) $\frac{|-x|}{-x}$ d) $|x|-|-x|$

4. Evaluate each of the algebraic expressions for the given values of the variables

   a) $|x-y|-|x+y|; if\ x=-4\ and\ y=-7$

   b) $|3x+y|+|2x-4y|;\ if\ x=5\ and\ y=-3$

   c) $\left|\frac{x-y}{y-x}\right|\ if\ x=-6\ and\ y=13$

   d) $\left|\frac{2a-3b}{3b-2a}\right|\ if\ a=-4\ and\ b=-8$

5. Evaluate each of the following numerical expressions

   a) $(3^{-4}+4^{-1})^{-1}$ b) $2^{-3}+3^{-1}$ c) $\left(\frac{2^{-1}}{3^{-3}}\right)^{-2}$ d) $\left(\frac{3}{4}\right)^{-1}-\left(\frac{2}{3}\right)^{-1}$

6. Simplify each of the following ; express final results without using zero or negative integers as exponents

   a) $(a^2b^{-1}c^{-2})^{-4}$ b) $\left(\frac{x^{-2}}{y^{-3}}\right)^{-2}$ c) $\left(\frac{3x^2y}{4a^{-1}b^{-3}}\right)^{-1}$ d) $\left(\frac{24x^5y^{-5}}{-8x^6y^{-1}}\right)^{-3}$

7. Evaluate each of the following in simplest radical form. All variables represent positive real number

   a) $\sqrt{64x^4y^7}$ b) $\sqrt[3]{81x^5y^6}$ c) $\frac{\sqrt[3]{12xy}}{\sqrt[3]{3x^2y^5}}$ d) $\sqrt[4]{162x^6y^7}$ e) $\frac{\sqrt[3]{2y}}{\sqrt[3]{3x}}$ f) $\sqrt[3]{\frac{5}{2x}}$

8. Simplify the following

a) $2\sqrt{28}-3\sqrt{63}+8\sqrt{7}$ b) $4\sqrt[3]{2}+2\sqrt[3]{16}-\sqrt[3]{54}$ c) $\frac{2\sqrt{8}}{3}-\frac{3\sqrt{18}}{5}-\frac{\sqrt{50}}{2}$ d) $\frac{3\sqrt[3]{54}}{2}+\frac{5\sqrt[3]{16}}{3}$

e) $4\sqrt{50}-9\sqrt{32}$ f) $5\sqrt{12}+2\sqrt{3}$

9 . Multiply and express the results in simplest radical form. All variables represent non-Negative real numbers

a) $2\sqrt{3}(5\sqrt{2}+4\sqrt{10})$ b) $(2\sqrt{x}-3\sqrt{y})^2$ c) $(3\sqrt{x}+5\sqrt{y})(3\sqrt{x}-5\sqrt{y})$

d) $\sqrt{6y}(\sqrt{8x}+\sqrt{10y^2})$ e) $(\sqrt{x}+\sqrt{y})^2$

10. For each of the following, rationalize the denominator and simplify. All variables Represent positive real numbers.

a) $\frac{3}{\sqrt{5}+2}$ b) $\frac{\sqrt{x}}{\sqrt{x}-1}$ c) $\frac{5}{5\sqrt{2}-3\sqrt{5}}$ d) $\frac{3\sqrt{x}-2\sqrt{y}}{2\sqrt{x}+5\sqrt{y}}$ e) $\frac{5}{3-2\sqrt{3}}$

f) $\frac{7}{\sqrt{10}-3}$ g) $\frac{\sqrt{x}}{\sqrt{x}+2}$ h) $\frac{2\sqrt{x}}{\sqrt{x}-\sqrt{y}}$

11. Evaluate each of the following

a) $-8^{2/3}$ b) $-16^{5/4}$ c) $(0.01)^{3/2}$ d) $\left(\frac{1}{27}\right)^{-2/3}$

12. Perform the indicated operations and express the answers in simplest radical form.

a) $\frac{\sqrt[3]{16}}{\sqrt[6]{4}}$ b) $\frac{\sqrt[4]{x^9}}{\sqrt[3]{x^2}}$ c) $\sqrt{ab}\sqrt[3]{a^4b^5}$ d) $\sqrt[3]{x}\sqrt[5]{x^3}$

13. Rationalize the denominators and express the final answers in simplest radical form.

a) $\frac{5}{\sqrt[3]{x}}$ b) $\frac{2\sqrt{x}}{3\sqrt[3]{y}}$ c) $\frac{5\sqrt[3]{y^2}}{4\sqrt[4]{x}}$ d) $\frac{\sqrt{xy}}{\sqrt[3]{a^2b}}$

e) $\frac{\sqrt[3]{x}}{\sqrt{y}}$ f) $\frac{\sqrt[4]{x}}{\sqrt{y}}$ g) $\frac{3}{\sqrt[3]{x^2}}$

14. Simplify each of the following, expressing the final result as one radical.

a) $\sqrt{\sqrt[3]{2}}$ b) $\sqrt[3]{\sqrt[4]{3}}$ c) $\sqrt[3]{\sqrt{x^3}}$ d) $\sqrt{\sqrt[3]{x^4}}$

15. Add or subtract as indicated

a) $(5+3i)+(7-2i)(-8-i)$ b) $(4+i\sqrt{3})+(-6-2i\sqrt{3})$

c) $(5-7i)-(6-2i)-(1-2i)$ d) $\left(\frac{5}{8}+\frac{1}{2}i\right)-\left(\frac{7}{8}+\frac{1}{5}i\right)$

16. Write each of the following in terms of i, perform the indicated operations , and Simplify if possible.

a) $\sqrt{-4}\sqrt{-16}$ b) $\sqrt{-25}\sqrt{-9}$ c) $\frac{\sqrt{-36}}{\sqrt{-4}}$ d) $\frac{\sqrt{-64}}{\sqrt{-16}}$ f) $\frac{\sqrt{-18}}{\sqrt{-3}}$

17. Find each of the following products and express the answers in standard form

a) $(-2+5i)^2$ b) $(5+3i)(5-3i)$ c) $(1+i)(2-i)$ d) $(5i)(2+6i)$ e) $(-5i)(8i)$

f) $(5i)(2+6i)$

18. Find each of the following quotients and express the answers in standard form

a) $\frac{2+3i}{3i}$ b) $\frac{3-5i}{4i}$ c) $\frac{4+7i}{2-3i}$ d) $\frac{3-7i}{4i+2}$

e) $\frac{1+\sqrt{2}i}{\sqrt{3}-2i}$ f) $\frac{1+2i}{1-i}+\frac{1-2i}{1+3i}$

19. Plot each complex number and find its absolute value

a) $3+4i$ b) -4 c) $\frac{3}{5}-\frac{4}{5}i$ d) $-5i$ e) $1-2i$ f) $3-2i$

e) $\frac{1}{(2+i)(\sqrt{3}-2i)}$ f) $5-4i+\frac{5}{3-4i}$

20. Let $z_1=2+i$, $z_2=1-i\sqrt{3}$ and $z_3=3+4i$. Verify the following identities

(i) $\overline{z_1z_2}=\overline{z_1}\cdot\overline{z_2}$ (ii) $z_3\cdot\overline{z_3}=\overline{z_3}\cdot z_3=|z_3|^2$ (iii) $\overline{\left(\frac{z_1}{z_2}\right)}=\frac{\overline{z_1}}{\overline{z_2}}$

21. Solve for x and y given that:

a) $(x+iy)(4i)=8$ b) $\frac{1}{x+iy}+\frac{1}{1+3i}=1$

c) $\frac{x}{1+i}-\frac{y}{2-i}=\frac{1-5i}{3-2i}$

22. Let $z=x+iy$ be a non zero complex number

a) Express $\frac{1}{z}$ in the form $a+ib$

b) Given that $z+\frac{1}{z}=k$, where k is a real number, prove that either z is real or $|z|=1$

23 Express each of the following in the form $a+ib$ where a and b are real numbers:

a) $\frac{1}{i^3}$ b) i^{15} c) i^{1002}

24 . a) Express $\frac{\sqrt{3}+1}{\sqrt{3}-1}+\sqrt{3}-1$ in the form $a+b\sqrt{3}$ where a and b are rational numbers.

b) Rationalize the denominator of each of the following:

(i) $\frac{2\sqrt{3}-\sqrt{2}}{4\sqrt{3}}$ (ii) $\frac{x}{x+\sqrt{y}}$ (iii) $\frac{2\sqrt{7}+\sqrt{3}}{3\sqrt{7}-\sqrt{3}}$

(iv) $\frac{x-\sqrt{x^2-9}}{x+\sqrt{x^2-9}}$ (v) $\frac{1}{(\sqrt{2}+1)(\sqrt{3}-1)}$

(c) Rationalize the numerator in each of the following:

(i) $\frac{\sqrt{5+h}-3}{h}$ (ii) $\frac{\sqrt{3}+\sqrt{5}}{7}$ (iii) $\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}+\sqrt{x+h}}$

25a) Give a reason why $Z-a\ set\ of\ integers$, $N-a\ set\ ofnatural\ numbers$ are not Fields while $Q-a\ set\ of\ rational\ number, R-a\ set\ of\ real\ numbers\ and\ C-a\ set\ of\ complex\ numbers\ are\ Fields.$

b) Prove that if $a+c=b+c\ then\ a=b$ when $a,b,c\in R$

c) Prove that if $ac=bc\ then\ a=b$ when $a,b,c\in R$ and $c\neq 0$

Math Tutorial Sheet 2

Q.1

(a.) $|x-2| = 6$

$x-2 = 6$ or $x-2 = -6$

$x = 6+2$ $\quad$ $x = -6+2$

$\underline{x = 8}$ $\quad$ $\underline{x = -4}$

b.) $|2n+1| = 11$

$2n+1 = 11$ or $2n+1 = -11$

$2n = 11-1$ $\quad$ $2n = -11-1$

$2n = 10$ $\quad$ $2n = -12$

$\underline{n = 5}$ $\quad$ $\underline{n = -6}$

c.) $\left|\frac{3}{k-1}\right| = 4$

$\frac{3}{k-1} = 4$ or $\frac{3}{k-1} = -4$

$3 = 4k-4$ $\quad$ $3 = -4k+4$

$3+4 = 4k$ $\quad$ $3-4 = -4k$

$7 = 4k$ $\quad$ $-1 = -4k$

$\underline{k = \frac{7}{4}}$ $\quad$ $\underline{k = \frac{1}{4}}$

d) $|x - \frac{2}{3}| = \frac{3}{4}$

$x - \frac{2}{3} = \frac{3}{4}$ or $x - \frac{2}{3} = -\frac{3}{4}$

$x = \frac{3}{4} + \frac{2}{3}$ $\quad$ $x = -\frac{3}{4} + \frac{2}{3}$

$\underline{x = \frac{17}{12}}$ $\quad$ $\underline{x = \frac{-1}{12}}$

(e.) $|-4| = 4$

(f.) $|4| = 4$

(g) $|2x-3| \leq 5$

$-5 \leq 2x-3 \leq 5$

$-5+3 \leq 2x-3+3 \leq 5+3$

$-2 \leq 2x \leq 8$

$\frac{-2}{2} \quad \frac{2x}{2} \quad \frac{8}{2}$

$\underline{-1 \leq x \leq 4}$

h) $|5x-4| \leq 8$

$-8 \leq 5x-4 \leq 8$

$-8+4 \leq 5x-4+4 \leq 8+4$

$$\frac{-4}{5} \le \frac{5x}{5} \le \frac{12}{5}$$

$$-\frac{4}{5} \le x \le \frac{12}{5}$$

Q.2

(a) $x(2) = 2(x)$
-> Commutative property

(b) $(7+4)+6 = 7+(4+6)$
-> Associative

(c) $1(x) = x$
-> Identity property

(d) $\left(\frac{2}{3}\right)\left(\frac{3}{2}\right) = 1$
-> Inverse property

Q.3

(a) $\frac{|x|}{x} = \frac{x}{x} = \underline{\underline{1}}$

(b) $\frac{x}{|x|} = \frac{x}{x} = \underline{\underline{1}}$

c) $\frac{|-x|}{-x} = \frac{x}{-x} = \underline{\underline{-1}}$

d) $|x| - |-x| = x - x$
$= \underline{\underline{0}}$

Q 4

(a.) $|x-y| - |x+y|$

$x = -4,\ y = -7$

$|(-4)-(-7)| - |(-4)+(-7)|$
$|-4+7| - |-4-7|$
$|3| - |-11|$
$3 - 11$
$\underline{\underline{-8}}$

(b) $|3x+y| + |2x-4y|$

$x = 5,\ y = -3$

$|3(5)+(-3)| + |2(5)-4(-3)|$
$|15-3| + |10+12|$
$|12| + |22|$
$12 + 22$
$\underline{\underline{34}}$

c $\left|\dfrac{x-y}{y-x}\right|$

$x=-6 \quad , y=13$

$\left|\dfrac{(-6)-(13)}{13-(-6)}\right|$

$\left|\dfrac{-6-13}{13+6}\right|$

$\left|\dfrac{-19}{19}\right|$

$|-1|$

$\underline{\underline{1}}$

(d.) $\left|\dfrac{2a-3b}{3b-2a}\right|$

$a=-4 \quad , b=-8$

$\left|\dfrac{2(-4)-3(-8)}{3(-8)-2(-4)}\right|$

$\left|\dfrac{-8+24}{-24+8}\right|$

$\left|\dfrac{16}{-16}\right|$

$|-1| = \underline{\underline{1}}$

Q.5

(a.) $\left(3^{-4}+4^{-1}\right)^{-1}$

$= \left(\dfrac{1}{3^4}+\dfrac{1}{4}\right)^{-1}$

$= \left(\dfrac{4+3^4}{3^4 \cdot 4}\right)^{-1}$

$= \left(\dfrac{3^4 \cdot 4}{4+3^4}\right)$

• If you are in a test and you've run out of time you can leave it there since MA110 is a non-calculator course they'll understand.

but let me just finish it

$\underline{\underline{\dfrac{324}{85}}}$

(b.) $2^{-3}+3^{-1}$

$= \dfrac{1}{2^3}+\dfrac{1}{3}$

$= \dfrac{1}{8}+\dfrac{1}{3}$

$= \underline{\underline{\dfrac{11}{24}}}$

(c

(d

(c) $\left(\frac{2^{-1}}{3^{-3}}\right)^{-2}$

$= \left(\frac{2^{-1\times-2}}{3^{-2\times-2}}\right)$

$= \left(\frac{2^{2}}{3^{6}}\right)$

$= \frac{4}{729}$

(d) $\left(\frac{3}{4}\right)^{-1} - \left(\frac{2}{3}\right)^{-1}$

$= \left(\frac{4}{3}\right) - \left(\frac{3}{2}\right)$

$= \frac{4}{3} - \frac{3}{2}$

$= \frac{8-9}{6}$

$= \frac{-1}{6}$

Q.6

(a) $\left(a^{2} b^{-1} c^{-2}\right)^{-4}$

$a^{2\times-4}\ b^{-1\times-4}\ c^{-2\times-4}$

$a^{-8}\ b^{4}\ c^{8}$

$= \frac{b^{4}c^{8}}{a^{8}}$

b) $\left(\frac{x^{-2}}{y^{-3}}\right)^{-2}$

$= \left(\frac{x^{-2\times-2}}{y^{-3\times-2}}\right)$

$= \frac{x^{4}}{y^{6}}$

(c) $\left(\frac{3x^{2}y}{4a^{-1}b^{-3}}\right)^{-1}$

$= \frac{4a^{-1}b^{-3}}{3x^{2}y}$

$= \frac{4}{3x^{2}y\,a\,b^{3}}$

(d) $\left(\frac{24x^5y^{-5}}{-8x^6y^{-1}}\right)^{-3}$

Changing -3 to +3

$\left(\frac{-8x^6y^{-1}}{24x^5y^{-5}}\right)^3$

$\left(\frac{-8x^6y^5}{24x^5y}\right)^3$

$\left(\frac{-(1)(x)(y^4)}{(3)(1)(1)}\right)^3$

$\left(\frac{-xy^4}{3}\right)^3$

$= \frac{-x^3y^{12}}{27}$

Q. 7

(a) $\sqrt{64x^4y^7}$

$\sqrt{64}\sqrt{x^4}\sqrt{y^7}$

$(8)\sqrt{x^2 \times x^2}\sqrt{y^2 \times y^2 \times y^2 \times y}$

$8(x)(x)(y)(y)+(y)\sqrt{y}$

$8x^2y^3\sqrt{y}$

b) $\sqrt[3]{81x^5y^6}$

$\sqrt[3]{3 \times 27 \times x^3 \times x^2 \times y^3 \times y^3}$

$3(\sqrt{3x^2})\, x \times y \times y$

$3xy^2\sqrt[3]{3x^2}$

(c) $\frac{\sqrt[3]{12xy}}{\sqrt[3]{3x^2y^5}}$

$= \sqrt[3]{\frac{12xy}{3x^2y^5}}$

$= \sqrt[3]{\frac{4}{xy^4}}$

$= y\sqrt[3]{\frac{4}{xy}}$

$= \frac{\sqrt[3]{4}}{y\sqrt[3]{xy}}$

But in math its bad manners to leave a radicle as the denominator therefore conjugate the denominator

$\frac{\sqrt[3]{4}}{y\sqrt[3]{xy}} \times \frac{\sqrt[3]{(xy)^2}}{\sqrt[3]{(xy)^2}}$

$\frac{\sqrt[3]{4} \times \sqrt[3]{(xy)^2}}{y\sqrt[3]{(xy)^3}}$

$$\frac{y\sqrt[3]{4x^2y^2}}{xy}$$

$$\frac{\sqrt[3]{4x^2y^2}}{xy^2}$$

(d) $\sqrt[4]{162x^6y^7}$

Break down everything

$\sqrt[4]{\cancel{2\times 3\times 3\times 3\times 3\times x^2\times x^2\times x^2}}$

$162 = 2\times 3\times 3\times 3\times 3$
$= 2\times 3^4$

$\sqrt[4]{3^4\times 2\times x^4\times x^2\times y^4 + y^3}$

$3xy\sqrt[4]{2x^2y^3}$

(e) $\frac{\sqrt[3]{2y}}{\sqrt[3]{3x}}$

Conjugate the denominator

$\frac{\sqrt[3]{2y}}{\sqrt[3]{3x}}\left(\frac{\sqrt[3]{(3x)^2}}{\sqrt[3]{(3x)^2}}\right)$

$\frac{\sqrt[3]{2y}\;\sqrt[3]{3^2x^2}}{\sqrt[3]{(3x)^3}}$

$$\frac{\sqrt[3]{18x^2y}}{3x}$$

(F) $\sqrt[3]{\frac{5}{2x}}$

$\frac{\sqrt[3]{5}}{\sqrt[3]{2x}}$

$\frac{\sqrt[3]{5}}{\sqrt[3]{2x}}\times\left(\frac{\sqrt[3]{(2x)^2}}{\sqrt[3]{(2x)^2}}\right)$

$\frac{\sqrt[3]{5}\;\sqrt[3]{4x^2}}{\sqrt[3]{(2x)^3}}$

$\frac{\sqrt[3]{20x^2}}{2x}$

Q. 8

(a) $2\sqrt{28} - 3\sqrt{63} + 8\sqrt{7}$

$\rightarrow 2\sqrt{4 \times 7} - 3\sqrt{9 \times 7} + 8\sqrt{7}$

$\rightarrow 2(2)\sqrt{7} - 3(3)\sqrt{7} + 8\sqrt{7}$

$4\sqrt{7} - 9\sqrt{7} + 8\sqrt{7}$

$\underline{\underline{3\sqrt{7}}}$

(b) $4\sqrt[3]{2} + 2\sqrt[3]{16} - \sqrt[3]{54}$

$\Rightarrow 4\sqrt[3]{2} + 2\sqrt[3]{8 \times 2} - \sqrt[3]{27 \times 2}$

$4\sqrt[3]{2} + 2(2)\sqrt[3]{2} - (3)\sqrt[3]{2}$

$4\sqrt[3]{2} + 4\sqrt[3]{2} - 3\sqrt[3]{2}$

$\underline{\underline{5\sqrt[3]{2}}}$

(c) $\frac{2\sqrt{8}}{3} - \frac{3\sqrt{18}}{5} - \frac{\sqrt{50}}{2}$

$\frac{2\sqrt{4 \times 2}}{3} - \frac{3\sqrt{9 \times 2}}{5} - \frac{\sqrt{2 \times 25}}{2}$

$\frac{2(2)\sqrt{2}}{3} - \frac{3(3)\sqrt{2}}{5} - \frac{5\sqrt{2}}{2}$

$\frac{4\sqrt{2}}{3} - \frac{9\sqrt{2}}{5} - \frac{5\sqrt{2}}{2}$

$\underline{\underline{-\frac{89\sqrt{2}}{30}}}$

(d) $\frac{3\sqrt[3]{54}}{2} + \frac{5\sqrt[3]{16}}{3}$

$\frac{3\sqrt[3]{2 \times 3^3}}{2} + \frac{5\sqrt[3]{2 \times 2^3}}{3}$

$\frac{3(3)\sqrt[3]{2}}{2} + \frac{5(2)\sqrt[3]{2}}{3}$

$\frac{9\sqrt[3]{2}}{2} + \frac{10\sqrt[3]{2}}{3}$

$\underline{\underline{\frac{47\sqrt{2}}{6}}}$

(e) $4\sqrt{50} - 9\sqrt{32}$

$4\sqrt{2 \times 25} - 9\sqrt{2 \times 16}$

$4(5)\sqrt{2} - 9(4)\sqrt{2}$

$20\sqrt{2} - 36\sqrt{2}$

$\underline{\underline{-16\sqrt{2}}}$

(f) $5\sqrt{12} + 2\sqrt{3}$

$5\sqrt{4\times3} + 2\sqrt{3}$

$5(2)\sqrt{3} + 2\sqrt{3}$

$10\sqrt{3} + 2\sqrt{3}$

$\underline{12\sqrt{3}}$

Q. 9

(a) $2\sqrt{3}\,(5\sqrt{2} + 4\sqrt{10})$

$2\times5\sqrt{3}\sqrt{2} + 2\times4\sqrt{10}\sqrt{3}$

$10\sqrt{6} + 8\sqrt{30}$

$\underline{10\sqrt{6} + 8\sqrt{30}}$

(b) $(2\sqrt{x} - 3\sqrt{y})^2$

$(2\sqrt{x})^2 + 2(2\sqrt{x})(-3\sqrt{y}) + (-3\sqrt{y})^2$

$\underline{4x - 12\sqrt{xy} + 9y}$

(c) $(3\sqrt{x} + 5\sqrt{y})(3\sqrt{x} - 5\sqrt{x})$

* difference of the two squares

$(3\sqrt{x})^2 - (5\sqrt{y})^2$

$\underline{9x - 25y}$

(d) $\sqrt{6y}\,(\sqrt{8x} + \sqrt{10y^2})$

~~$\sqrt{6y}\sqrt{8x} + \sqrt{6y}\sqrt{10y^2}$~~

$\sqrt{6y}\,(\sqrt{8x} + \sqrt{10y^2})$

$\sqrt{6y}\,(2\sqrt{2x} + y\sqrt{10})$

$2\sqrt{6y\times2x} + y\sqrt{10\times6y}$

$2\sqrt{12xy} + y\sqrt{60y}$

$2\sqrt{4\times3xy} + y\sqrt{4\times15y}$

$2(2)\sqrt{3xy} + y(2)\sqrt{15y}$

$4\sqrt{3xy} + 2y\sqrt{15y}$

$\underline{4\sqrt{3xy} + 2y\sqrt{15y}}$

(c) $(\sqrt{x} + \sqrt{y})^2$

$(\sqrt{x})^2 + 2(\sqrt{x})(\sqrt{y}) + (\sqrt{y})^2$

$\underline{x + 2\sqrt{xy} + y}$

Q10

(a) $\dfrac{3}{\sqrt{5}+2}$

$$* \ \frac{3}{\sqrt{5}+2}\left(\frac{\sqrt{5}-2}{\sqrt{5}-2}\right)$$

$$\frac{3\sqrt{5}-6}{(\sqrt{5})^2-(2)^2}$$

$$\frac{3\sqrt{5}-6}{5-4} = \frac{3\sqrt{5}-6}{1}$$

$$= \underline{\underline{3\sqrt{5}-6}}$$

(b) $\dfrac{\sqrt{x}}{\sqrt{x}-1}$

$$* \ \frac{\sqrt{x}}{\sqrt{x}-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}+1}\right)$$

$$\frac{x+\sqrt{x}}{(\sqrt{x})^2-(1)^2}$$

$$= \underline{\underline{\frac{x+\sqrt{x}}{x-1}}}$$

(c) $\dfrac{5}{5\sqrt{2}-3\sqrt{5}}$

$$* \ \frac{5}{5\sqrt{2}-3\sqrt{5}}\left(\frac{5\sqrt{2}+3\sqrt{5}}{5\sqrt{2}+3\sqrt{5}}\right)$$

$$\frac{25\sqrt{2}+15\sqrt{5}}{(5\sqrt{2})^2-(3\sqrt{5})^2}$$

$$\frac{25\sqrt{2}+15\sqrt{5}}{25(2)-9(5)}$$

$$\frac{25\sqrt{2}+15\sqrt{5}}{50-45}$$

$$\frac{25\sqrt{2}+15\sqrt{5}}{5}$$

$$\frac{25\sqrt{2}}{5}+\frac{15\sqrt{5}}{5}$$

$$\underline{5\sqrt{2}+3\sqrt{5}}$$

(d) $\dfrac{3\sqrt{x}-2\sqrt{y}}{2\sqrt{x}+5\sqrt{y}}$

$$* \ \frac{3\sqrt{x}-2\sqrt{y}}{2\sqrt{x}+5\sqrt{y}}\left(\frac{2\sqrt{x}-5\sqrt{y}}{2\sqrt{x}-5\sqrt{y}}\right)$$

$$\frac{2\times3\sqrt{x}\sqrt{x}-3\times5\sqrt{x}\sqrt{y}-2\times2\sqrt{y}\sqrt{x}+2\times5}{(2\sqrt{x})^2-(5\sqrt{y})^2}$$

$$\frac{6x-15\sqrt{xy}-4\sqrt{xy}+10y}{4x-25y}$$

$$\underline{\underline{\frac{6x+10y-19\sqrt{xy}}{4x-25y}}}$$

(e) $\dfrac{5}{3-2\sqrt{3}}$

$$\frac{5}{3-2\sqrt{3}}\left(\frac{3+2\sqrt{3}}{3+2\sqrt{3}}\right)$$

$$\frac{15+10\sqrt{3}}{(3)^2-(2\sqrt{3})^2}$$

$$\frac{15+10\sqrt{3}}{9-4(3)}$$

$$\frac{15+10\sqrt{3}}{9-12}$$

$$\frac{15+10\sqrt{3}}{-3}$$

(f) $\frac{7}{\sqrt{10}-3}$

$$\frac{7}{\sqrt{10}-3}\left(\frac{\sqrt{10}+3}{\sqrt{10}+3}\right)$$

$$\frac{7\sqrt{10}+21}{(\sqrt{10})^2-(3)^2}$$

$$\frac{7\sqrt{10}+21}{10-9} = 7\sqrt{10}+21$$

(g) $\frac{\sqrt{x}}{\sqrt{x}+2}$

$$\frac{\sqrt{x}}{\sqrt{x}+2}\left(\frac{\sqrt{x}-2}{\sqrt{x}-2}\right)$$

$$\frac{x-2\sqrt{x}}{(\sqrt{x})^2-(2)^2}$$

$$\frac{x-2\sqrt{x}}{x-4}$$

(h) $\frac{2\sqrt{x}}{\sqrt{x}-\sqrt{y}}$

$$\frac{2\sqrt{x}}{\sqrt{x}-\sqrt{y}}\left(\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right)$$

$$\frac{2x+2\sqrt{xy}}{(\sqrt{x})^2-(\sqrt{y})^2}$$

$$\frac{2x+2\sqrt{xy}}{x-y}$$

Q 11

(a) $-8^{2/3}$

$$= -\left(\sqrt[3]{8}\right)^2$$
$$= -(2)^2$$
$$= -4$$

(b) $-16^{5/4}$

$$= -\left(\sqrt[4]{16}\right)^5$$
$$= -(2)^5$$
$$= -32$$

(c) $(0.01)^{3/2}$

remember no calculator

$$(0.01)^{3/2} = \left(\frac{1}{100}\right)^{3/2}$$

$$= \frac{1^{3/2}}{100^{3/2}}$$

$$\frac{(\sqrt{1})^3}{(\sqrt{100})^3} = \frac{1^3}{10^3}$$

$$= \frac{1}{1000}$$

(d) $\left(\frac{1}{27}\right)^{-2/3}$

$$\left(\frac{\sqrt[3]{1}}{\sqrt[3]{27}}\right)^{-2}$$

$$\left(\frac{1}{3}\right)^{-2}$$

$$\left(\frac{3}{1}\right)^{2}$$

$$9$$

Q.12

(a) $\frac{\sqrt[3]{16}}{\sqrt[6]{4}} = \frac{\sqrt[3]{16}}{\sqrt[3]{\sqrt{4}}}$

$$= \sqrt[3]{\frac{16}{\sqrt{4}}}$$

$$= \sqrt[3]{\frac{16}{2}}$$

$$= \sqrt[3]{8}$$

$$= 2$$

(b) $\frac{\sqrt[4]{x^9}}{\sqrt[3]{x^2}} = \cancel{\frac{\sqrt[4]{x^4\, x^4\, x}}{\sqrt[3]{x^2}}}$

$$= \frac{x^{9/4}}{x^{2/3}} = \cancel{x^2 \frac{\sqrt[4]{x}}{\sqrt[3]{x^2}}}$$

$$= x^{9/4 - 2/3} = \cancel{x^2 \frac{x^{1/4}}{x^{2/3}}}$$

$$= x^{19/12} =$$

$$= \sqrt[12]{x^{19}}$$

$$= x\sqrt[12]{x^7}$$

(c) $\sqrt{ab}\ \sqrt[3]{a^4b^5}$

$$(ab)^{1/2}\,(a^4b^5)^{1/3}$$

$$a^{1/2}\, b^{1/2}\, a^{4/3}\, b^{5/3}$$

$$a^{1/2}\, a^{4/3}\, b^{1/2}\, b^{5/3}$$

$$a^{1/2+4/3}\, b^{1/2+5/3}$$

$$a^{11/6}\, b^{13/6}$$

$$\sqrt[6]{a^{11}b^{13}}$$

$$\sqrt[6]{a^6a^5\, b^6\, b^6\, b^1}$$

$$ab^2\sqrt[6]{a^5b}$$

(d) $\sqrt[3]{x}\ \sqrt[5]{x^3}$

$x^{1/3}\ x^{3/5}$

$x^{1/3 + 3/5}$

$\underline{\underline{x^{14/15}}} = \underline{\underline{\sqrt[15]{14}}}$

Q.13

(a) $\dfrac{5}{\sqrt[3]{x}}$

$= \dfrac{5}{\sqrt[3]{x}} \left(\dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}}\right)$

$= \underline{\underline{\dfrac{5\sqrt[3]{x^2}}{x}}}$

(b) $\dfrac{2\sqrt{x}}{3\sqrt[3]{y}}$

$= \dfrac{2\sqrt{x}}{3\sqrt[3]{y}} \left(\dfrac{\sqrt[3]{y^2}}{\sqrt[3]{y^2}}\right)$

$\dfrac{2\sqrt{x}\ \sqrt[3]{y^2}}{3y}$

$\dfrac{2x^{\frac{1}{2}\times\frac{3}{3}}\ y^{\frac{2}{3}\times\frac{2}{2}}}{3y}$

$\dfrac{2x^{3/6}\ y^{4/6}}{3y}$

$= \dfrac{2\sqrt[6]{x^3y^4}}{3y}$

(c) $\dfrac{5\sqrt[3]{y^2}}{4\sqrt[4]{x}}$

$= \dfrac{5\sqrt[3]{y^2}}{4\sqrt[4]{x}} \left(\dfrac{\sqrt[4]{x^3}}{\sqrt[4]{x^3}}\right)$

$= \dfrac{5\sqrt[3]{y^2}\ \sqrt[4]{x^3}}{4x}$

$= \dfrac{5y^{\frac{2}{3}\times\frac{4}{4}}\ x^{\frac{3}{4}\times\frac{3}{3}}}{4x}$

$= \dfrac{5y^{8/12}\ x^{9/12}}{4x}$

$= \underline{\underline{\dfrac{5\sqrt[12]{y^8x^9}}{4x}}}$

(d) $\dfrac{\sqrt{xy}}{\sqrt[3]{a^2b}}$

$\dfrac{\sqrt{xy}}{\sqrt[3]{a^2b}} \left(\dfrac{\sqrt[3]{ab^2}}{\sqrt[3]{ab^2}}\right)$

$\dfrac{\sqrt{xy}\ \sqrt[3]{ab^2}}{ab}$

$\dfrac{(xy)^{\frac{1}{2}\times\frac{3}{3}}(ab^2)^{\frac{1}{3}\times\frac{2}{2}}}{ab}$

$\dfrac{(xy)^{3/6}\ (ab^2)^{2/6}}{ab} = \dfrac{\sqrt[6]{x^3y^3a^2b^4}}{ab}$

(e) $\dfrac{\sqrt[3]{x}}{\sqrt{y}}$

$$\frac{\sqrt[3]{x}}{\sqrt{y}}\left(\frac{\sqrt{y}}{\sqrt{y}}\right)$$

$$\frac{\sqrt[3]{x}\,\sqrt{y}}{y}$$

$$= \frac{x^{\frac{1}{3}\times\frac{2}{2}}\, y^{\frac{1}{2}\times\frac{3}{3}}}{y}$$

$$= \frac{x^{2/6}\, y^{3/6}}{y}$$

$$= \frac{\sqrt[6]{x^2y^3}}{y}$$

(f) $\dfrac{\sqrt[4]{x}}{\sqrt{y}}$

$$\frac{\sqrt[4]{x}}{\sqrt{y}}\left(\frac{\sqrt{y}}{\sqrt{y}}\right)$$

$$\frac{\sqrt[4]{x}\,\sqrt{y}}{y}$$

$$\frac{x^{\frac{1}{4}\times\frac{2}{2}}\, y^{\frac{1}{2}\times\frac{4}{4}}}{y}$$

$$\frac{x^{2/8}\, y^{4/8}}{y}$$

$$\frac{\sqrt[8]{x^2y^4}}{y}$$

(g) $\dfrac{3}{\sqrt[3]{x^2}}$

$$\frac{3}{\sqrt[3]{x^2}}\left(\frac{\sqrt[3]{x}}{\sqrt[3]{x}}\right)$$

$$\frac{3\sqrt[3]{x}}{x}$$

Q14

(a) $\sqrt{\sqrt[3]{2}} = \sqrt[6]{2}$

(b) $\sqrt[3]{\sqrt[4]{3}} = \sqrt[12]{3}$

(c) $\sqrt[3]{\sqrt{x^3}} = \sqrt[6]{x^3} = x^{3/6}$

$= x^{1/2}$

$= \sqrt{x}$

(d) $\sqrt{\sqrt[3]{x^4}} = \sqrt[6]{x^4} = x^{4/6}$

$= x^{2/3}$

$= \sqrt[3]{x^2}$

Q.15

(a) $(5+3i)+(7-2i)(-8-i)$

$(5+3i)+(-56-7i+16i-2)$

$(5+3i)+(-58+9i)$

$(5+3i)+(-58+9i)$

$5-58+3i+9i$

$\underline{\underline{-53+12i}}$

(b) $(4+i\sqrt{3})+(-6-2i\sqrt{3}$

$4-6+i\sqrt{3}-2i\sqrt{3}$

$\underline{\underline{-2-i\sqrt{3}}}$

(c) $(5-7i)-(6-2i)-(1-2i)$

$(5-7i)-6+2i-1+2i$

$5-6-1-7i+2i+2i$

$\underline{\underline{-2-3i}}$

(d) $\left(\frac{5}{8}+\frac{1}{2}i\right)-\left(\frac{7}{8}+\frac{1}{5}i\right)$

$\frac{5}{8}+\frac{1}{2}i-\frac{7}{8}-\frac{1}{5}i$

$\frac{5}{8}-\frac{7}{8}+\frac{1}{2}i-\frac{1}{5}i$

$\underline{\underline{-\frac{1}{4}+\frac{3}{10}i}}$

Q 16

(a) $\sqrt{-4}\sqrt{-16}$

$(2i)(4i)$

$8(-1)$

$\underline{\underline{-8}}$

(b) $\sqrt{-25}\sqrt{-9}$

$(5i)(3i)$

$15(-1)$

$\underline{\underline{-15}}$

(c) $\frac{\sqrt{-36}}{\sqrt{-4}}$

$\frac{6i}{2i}=\underline{\underline{3}}$

(d) $\frac{\sqrt{-64}}{\sqrt{-16}}$

$\frac{8i}{4i}=\underline{\underline{2}}$

(F) $\frac{\sqrt{-18}}{\sqrt{-3}}$

$\rightarrow \frac{\sqrt{-9\times 2}}{\sqrt{-3}}$

$$\frac{3i\sqrt{2}}{i\sqrt{3}}$$

$$\frac{3\sqrt{2}}{\sqrt{3}}$$

$$\frac{3\sqrt{2}}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$$

$$\frac{3\sqrt{2}\sqrt{3}}{3}$$

$$\underline{\underline{\sqrt{6}}}$$

Q.17

(a) $(-2+5i)^2$

$= (-2)^2 + 2(-2)(5i) + (5i)^2$

$4 - 20i - 25$

$\underline{\underline{-21 - 20i}}$

(b) $(5+3i)(5-3i)$

difference of two squares!!

$(5)^2 - (3i)^2$

$25 + 9$

$\underline{\underline{34}}$

(c) $(1+i)(2-i)$

$= 2 - i + 2i + 1$

$\underline{\underline{3 + i}}$

(d) $(5i)(2+6i)$

$\underline{\underline{10i - 30}}$

(e) $(-5i)(8i)$

$\underline{\underline{40}}$

(f) $(5i)(2+6i)$

Q.18

(a) $\frac{2+3i}{3i}$

$\rightarrow \frac{2+3i}{3i}\left(\frac{-3i}{-3i}\right)$

$$\frac{-6i+15}{9}$$

$$\frac{15-6i}{9}$$

$$\frac{15}{9} - \frac{6i}{9}$$

$$\underline{\underline{\frac{5}{3} - \frac{2}{3}i}}$$

(b) $\frac{3-5i}{4i}$

$$\frac{3-5i}{4i}\left(\frac{-4i}{-4i}\right)$$

$$\frac{-12i - 20}{16}$$

$$\frac{-20 - 12i}{16}$$

$$\frac{-20}{16} - \frac{12}{16}i$$

$$\# \quad \frac{-5}{4} - \frac{3}{4}i$$

(c) $\dfrac{4+7i}{2-3i}$

$$\frac{4+7i}{2-3i}\left(\frac{2+3i}{2+3i}\right)$$

$$\frac{8+12i+14i-21}{(2)^2-(3i)^2}$$

$$\frac{-13+26i}{4+9}$$

$$\frac{-13+26i}{13}$$

$$\frac{-13}{13} + \frac{26}{13}i$$

$$\# \quad -1+2i$$

(d) $\dfrac{3-7i}{4i+2}$

$$\frac{3-7i}{4i+2}\left(\frac{4i-2}{4i-2}\right)$$

$$\frac{12i-6+28+14i}{(4i)^2-(2)^2}$$

$$\frac{22+26i}{-16-4}$$

$$\frac{22+26i}{-20}$$

$$\frac{22}{-20} + \frac{26i}{-20}$$

$$\# \quad \frac{-11}{10} - \frac{13}{10}i$$

(e) $\dfrac{1+\sqrt{2}i}{\sqrt{3}-2i}$

$$\frac{1+\sqrt{2}i}{\sqrt{3}-2i}\left(\frac{\sqrt{3}+2i}{\sqrt{3}+2i}\right)$$

$$\frac{\sqrt{3}+2i+\sqrt{6}i-2\sqrt{2}}{(\sqrt{3})^2-(2i)^2}$$

$$\frac{\sqrt{3}-2\sqrt{2}+2i+\sqrt{6}i}{3+4}$$

$$\# \quad \frac{\sqrt{3}-2\sqrt{2}}{7} + \frac{2i+\sqrt{6}i}{7}$$

(f) $\dfrac{1+2i}{1-i} + \dfrac{1-2i}{1+3i}$

$$\frac{1+2i}{1-i}\left(\frac{1+i}{1+i}\right) + \frac{1-2i}{1+3i}\left(\frac{1-3i}{1-3i}\right)$$

So last night I was reading in the book of Numbers, and then I realized, I don't have yours.

$$\frac{1+i+2i-2}{(1)^2-(i)^2}+\frac{1-3i-2i-6}{(1)^2-(3i)^2}$$

$$\frac{-1+3i}{1+1}+\frac{-5-5i}{1+9}$$

$$\frac{-1+3i}{2}+\frac{-5-5i}{10}$$

$$\frac{5(-1+3i)+(-5-5i)}{10}$$

$$\frac{-5+15i-5-5i}{10}$$

$$\frac{-10+10i}{10}$$

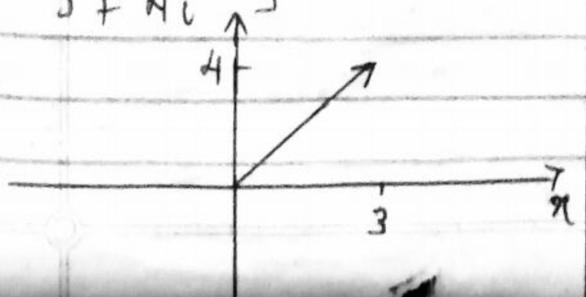
$$\frac{-10}{10}+\frac{10i}{10}$$

$$\underline{\underline{-1+i}}$$

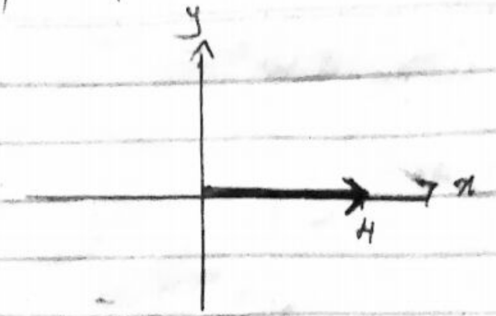
Q.19

Complex numbers can be represented on the Argand diagram, where the y-axis is the imaginary axis and x-axis is the real axis

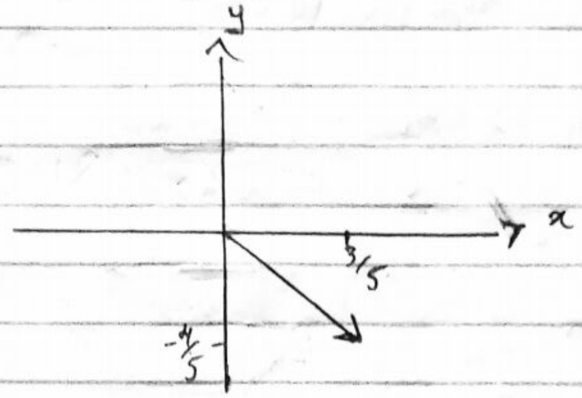
(a) $3+4i$



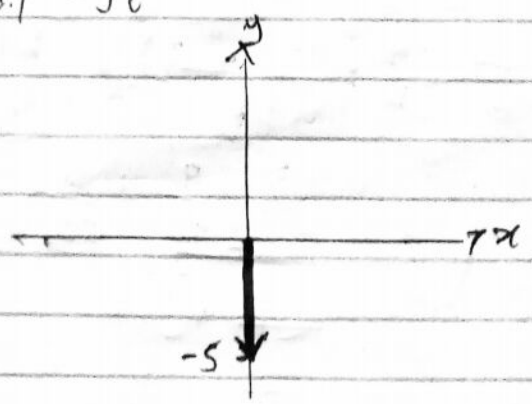
(b.) -4



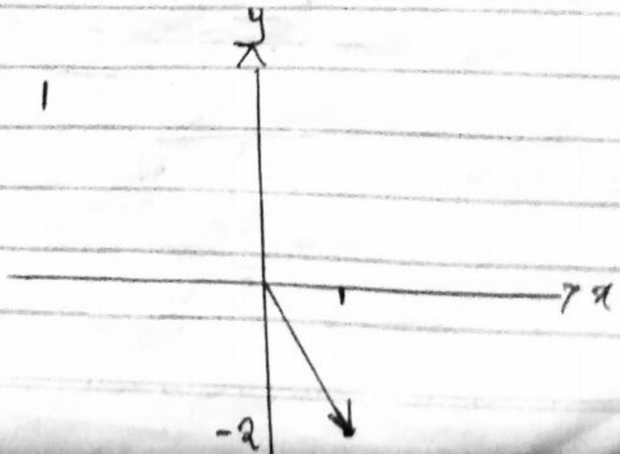
(c.) $\frac{3}{5}-\frac{4}{5}i$



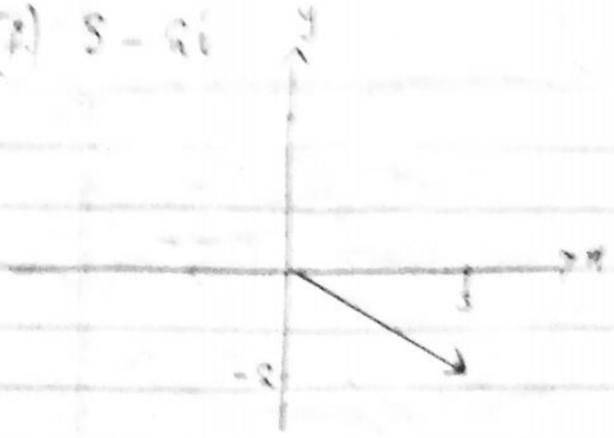
(d.) $-5i$



(e.) $1-2i$



(f) $3 - 2i$



Absolute values:

(a) $3 + 4i = \sqrt{3^2 + 4^2}$
$= \sqrt{25}$
$= \underline{5}$ units

(b) $-4 = \sqrt{(-4)^2}$
$= \sqrt{16}$
$= \underline{4}$ units

(c) $\frac{3}{5} - \frac{4}{5}i = \sqrt{\left(\frac{3}{5}\right)^2 + \left(-\frac{4}{5}\right)^2}$
$= \sqrt{\frac{9}{25} + \frac{16}{25}}$
$= \sqrt{1}$
$= \underline{1}$ units.

(d) $-5i = \sqrt{(-5)^2}$
$= \sqrt{25} = \underline{5}$ units

(e) $1 - 2i = \sqrt{(1)^2 + (-2)^2}$
$= \sqrt{1 + 4}$
$= \underline{\sqrt{5}}$ units

(f) $3 - 2i = \sqrt{(3)^2 + (-2)^2}$
$= \sqrt{9 + 4}$
$= \underline{\sqrt{13}}$ units

(g) $\frac{1}{(2+i)(\sqrt{3} - 2i)}$

$\left(\frac{1}{2+i}\right)\left(\frac{1}{\sqrt{3} - 2i}\right)$

$\frac{1}{2+i}\left(\frac{2-i}{2-i}\right) \times \frac{1}{\sqrt{3}-2i}\left(\frac{\sqrt{3}+2i}{\sqrt{3}+2i}\right)$

$\frac{2-i}{(2)^2 - (i)^2} \times \frac{\sqrt{3}+2i}{(\sqrt{3})^2 - (2i)^2}$

$\frac{2-i}{4+1} \times \frac{\sqrt{3}+2i}{3+4}$

$\frac{2-i}{5} \times \frac{\sqrt{3}+2i}{7}$

$\frac{2\sqrt{3} + 4i - \sqrt{3}i + 2}{35}$

$\underline{\frac{2\sqrt{3}+2}{35} + \frac{4i - \sqrt{3}i}{35}}$

(8)

$$5-4i+\frac{5}{3-4i}$$

$$\frac{5}{3-4i}\left(\frac{3+4i}{3+4i}\right)$$

$$\frac{15+20i}{(3)^2-(4i)^2}$$

$$\frac{15+20i}{9+16}$$

$$\frac{15+20i}{25}$$

$$\frac{15}{25}+\frac{20i}{25}$$

$$\frac{3}{5}+\frac{4}{5}i$$

$$5-4i+\frac{3}{5}+\frac{4}{5}i$$

$$\# \ \frac{28}{5}-\frac{16}{5}i$$

(5) $\frac{28}{5} - \frac{16}{5} i$

20) $Z_1 = 2+i, \quad Z_2 = 1 - i\sqrt{3}, \quad Z_3 = 3+4i$

(i) $\overline{Z_1 Z_2} = \bar{Z}_1 \cdot \bar{Z}_2$ $\bar{Z}_1$ jst means conjugate

R.H.S

$\bar{Z}_1 = 2 - i, \quad \bar{Z}_2 = 1 + i\sqrt{3}$

$\bar{Z}_1 . \bar{Z}_2 = (2-i)(1+i\sqrt{3})$

$= 2 + 2i\sqrt{3} - i + 1\sqrt{3}$

$\bar{Z}_1 . \bar{Z}_2 = 2 + \sqrt{3} + (2\sqrt{3} - 1)i$

L.H.S

$Z_1 Z_2 = (2+i)(1-i\sqrt{3})$

$= 2 - 2i\sqrt{x} + i + \sqrt{3}$

$= 2 + \sqrt{3} - 2i\sqrt{x} + i$

$Z_1 Z_2 = 2 + \sqrt{3} - (2\sqrt{x} - 1)i$

But $\overline{Z_1 Z_2} = 2 + \sqrt{3} + (2\sqrt{x} - 1)i$

L.H.S = R.H.S

Hencen shown

(ii) $Z_3 . \bar{Z}_3 = \bar{Z}_3 . Z_3 = |Z_3|^2$

$\bar{Z}_3 = 3 - 4i$

$Z_3 . \bar{Z}_3 = (3+4i)(3-4i)$

$= 9 + 16$

$Z_3 . \bar{Z}_3 = 25$

$\bar{Z}_3 . Z_3 = (3-4i)(3+4i)$

$= 9 + 16$

$\bar{Z}_3 . Z_3 = 25$

$|Z_3| = \sqrt{3^2 + 4^2}$

$|Z_3|^2 = \left(\sqrt{3^2+4^2}\right)^2$

$= 3^2 + 4^2$

$= 9 + 16$

$|Z_3|^3 = 25$

Hence langad!

21 a) $(x+iy)(4i) = 8$

$4ix - 4y = 8 + 0i$

$\frac{-4y}{-4} = \frac{8}{-4}$ $\quad$ $4ix = 0i$

$y = -2$ $\quad$ $4x = 0$

$x = 0$

b) $\frac{1}{x+iy} + \frac{1}{1+3i} = 1$

$\frac{1+3i + x + iy}{(x+iy)(1+3i)} = 1$

$\frac{1 + x + 3i + iy}{x + 3xi + iy - 3y} = 1$

$\frac{1 + x + 3i + iy}{x - 3y + 3xi + iy} = \frac{1}{1}$

$1 + \cancel{x} + 3i + \cancel{iy} = \cancel{x} - 3y + 3xi + \cancel{iy}$

$1 + 3i = -3y + 3xi$

$\frac{-3y}{-3} = \frac{1}{-3}$ $\quad$ or $\quad$ $3i = 3xi$

$y = -1/3$ $\quad$ $\frac{3x}{3} = \frac{3}{3}$

$x = 1$

Q.21

$(x+iy)(4i) = 8$

$4ix - 4y = 8$

equating: real to real
fake to fake

$-4y = 8 \qquad 4ix = 0$

$\underline{x = 0}$

$\underline{y = -2}$

(b) $\frac{1}{x+iy} + \frac{1}{1+3i} = 1$

$\frac{1+3i+x+iy}{(x+iy)(1+3i)} = 1$

$\frac{1+x+3i+iy}{x+3xi+iy-3y} = 1$

$\frac{1+x+3i+iy}{x-3y+3xi+iy} = 1$

since every thing on your left is equal to 1 meaning your numerator must be equal to the denominator.

$\therefore 1+x+3i+iy = x-3y+3xi+iy$

Again real to real
fake to fake

$1+x = x-3y$

$1 = -3y$

$\underline{y = -\frac{1}{3}}$

$3i+iy = 3xi+iy$

$3+y = 3x+y$

$3 = 3x$

$\underline{x = 1}$

(C.) $\frac{x}{1+i} - \frac{y}{2-i} = \frac{1-5i}{3-2i}$

Rationalise each one by one L.H.S

$\frac{x}{1+i}\left(\frac{1-i}{1-i}\right) - \frac{y}{2-i}\left(\frac{2+i}{2+i}\right) =$

$\frac{x-xi}{(1)^2-(i)^2} - \frac{2y+yi}{(2)^2-(i)^2} =$

$\frac{x-xi}{2} - \frac{2y+yi}{5} =$

$\frac{5(x-xi)-2(2y+yi)}{10}$

$$\frac{5x - 5xi - 4y - 2yi}{10}$$

$$\frac{5x - 4y - 5xi - 2yi.}{10}$$

Now taking the R.H.S

$$\frac{1-5i}{3-2i}\left(\frac{3+2i}{3+2i}\right)$$

$$\frac{3+2i-15i+10}{(3)^2-(2i)^2}$$

$$\frac{13-13i}{13}$$

$1-i$

L.H.S = R.HS

$$\frac{5x - 4y - 5xi - 2yi}{10} = 1-i$$

$$\frac{5xy-4x}{10} = 1 \quad or \quad \frac{-5xi-2yi}{10} = -i$$

$5x-4y = 10 \mid -5x - 2y = -1$

$$+\begin{array}{|l} 5x - 4y = 10 \\ -5x - 2y = -10 \end{array}$$

$$-6y = 0$$

$$\underline{y = 0}$$

$5x - 4y = 10$

$5x - 4(0) = 10$

$5x = 10$

$\underline{x = 2}$

22 $z = x + iy$

(a) $\frac{1}{z}$ in the form $a + ib$

$\frac{1}{x+iy}$ kati, then you conjugate the bottom

$$\frac{1}{x+iy}\left(\frac{x-iy}{x-iy}\right)$$

$$\frac{x-iy}{x^2-(iy)^2}$$

$$\frac{x-iy}{x^2+y^2}$$

writing this in the form $a+ib$

$$\frac{x-iy}{x^2+y^2} = \underline{\underline{\frac{x}{x^2+y^2} - \frac{iy}{x^2+y^2}}}$$

(b) $z + \frac{1}{z} = k$

So here we know that $z = x+iy$

and that $\frac{1}{z} = \frac{x-iy}{x^2+y^2}$, ryt

before we splitted it into the form $a+ib$

therefore

$$z + \frac{1}{z} = k$$

$$\frac{x+iy}{1} + \frac{x-iy}{x^2+y^2} = k$$

$$\frac{(x+iy)(x^2+y^2) + (x-iy)}{x^2+y^2} = k$$

$$\frac{x^3 + xy^2 + x^2yi + y^3i + x - iy}{x^2+y^2}$$

so on the numerator part i'll group them like this

$(x^3 + xy^2)$ & $(x^2yi + y^3i)$

What i mean is this

$$\frac{x^3+xy^2}{x^2+y^2} + \frac{x^2yi+y^3i}{x^2+y^2} + \frac{x-iy}{x^2+y^2}$$

$$\frac{x(x^2+y^2)}{x^2+y^2} + \frac{iy\cancel{(x^2+y^2)}}{x^2+y^2} + \frac{x-iy}{x^2+y^2}$$

$$x + iy + \frac{x-iy}{x^2+y^2}$$

$x + iy + \frac{x}{x^2+y^2} - \frac{iy}{x^2+y^2}$

Collect the real part and

the imaginery part.

$$\underbrace{x + \frac{x}{x^2+y^2}}_{\text{Real part}} + \underbrace{iy - \frac{iy}{x^2+y^2}}_{\text{Imaginery part}}$$

now remember they told us that

K is a real number, and at your age you must know that real numbers don't have imaginery parts

$\therefore$ if ;

$$K = x + \frac{x}{x^2+y^2} + iy - \frac{iy}{x^2+y^2}$$

Getting the imaginery part

$$iy - \frac{iy}{x^2+y^2} = 0$$

$$iy\left(1 - \frac{1}{x^2+y^2}\right) = 0$$

$$iy = 0 \quad \text{or} \quad 1 - \frac{1}{x^2+y^2} = 0$$

$$y = 0 \qquad 1 = \frac{1}{x^2+y^2}$$

$$\frac{1}{1} = \frac{1}{x^2+y^2}$$

$$x^2+y^2 = 1$$

now remember that ~~z=a+ib~~ $z = x + iy$, and to find $|z|$ we say;

$$|z| = \sqrt{x^2+y^2}$$

but $\boxed{x^2+y^2 = 1}$

$$|z| = \sqrt{1}$$

$$\therefore |z| = 1$$

hence proved

2	162
3	81
3	27
3	9
3	3
	1

Q. 23

(a) $\frac{1}{i^3} = \frac{1}{-i}$

$= \frac{1}{-i}\left(\frac{i}{i}\right)$

$= \frac{i}{-(-1)}$

$= i$

(b) $i^{15} = (i^4)^3 \times i^3$

$= (1)^3 \times i^3$

$= -i$

(c) $i^{1002} = (i^4)^{250} \times i^2$

$= (1)^{250} \times (-1)$

$= -1$

Q. 24

(a.) $\frac{\sqrt{3}+1}{\sqrt{3}-1} + \sqrt{3} - 1$

$\frac{\sqrt{3}+1}{\sqrt{3}-1}\left(\frac{\sqrt{3}+1}{\sqrt{3}+1}\right)$

$\frac{(\sqrt{3})^2 + 2(\sqrt{3})(1) + (1)^2}{(\sqrt{3})^2 - (1)^2}$

$\frac{3 + 2\sqrt{3} + 1}{3 - 1}$

$\frac{4 + 2\sqrt{3}}{2}$

$2 + \sqrt{3}$

Finally add $\sqrt{3} - 1$

$2 + \sqrt{3} + \sqrt{3} - 1$

$1 + 2\sqrt{3}$

$\therefore \frac{\sqrt{3}+1}{\sqrt{3}-1} + \sqrt{3} - 1 = 1 + 2\sqrt{3}$

(b)

(i) $\frac{2\sqrt{3} - \sqrt{2}}{4\sqrt{3}}$

$\frac{2\sqrt{3} - \sqrt{2}}{4\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$

$\frac{2(3) - \sqrt{6}}{4(3)}$

$\frac{6 - \sqrt{6}}{12}$

(ii) $\frac{x}{x+\sqrt{y}}$

$$\frac{x}{x+\sqrt{y}} \times \left(\frac{x-\sqrt{y}}{x-\sqrt{y}}\right)$$

$$\frac{x^2 - x\sqrt{y}}{(x)^2 - (\sqrt{y})^2}$$

$$\frac{x^2 - x\sqrt{y}}{x^2 - y}$$

(iii) $\frac{2\sqrt{7}+\sqrt{3}}{3\sqrt{7}-\sqrt{3}}$

$$\frac{2\sqrt{7}+\sqrt{3}}{3\sqrt{7}-\sqrt{3}} \left(\frac{3\sqrt{7}+\sqrt{3}}{3\sqrt{7}-\sqrt{3}}\right)$$

$$\frac{6(7) + 2\sqrt{21} + 3\sqrt{21} + (3)}{(3\sqrt{7})^2 - (\sqrt{3})^2}$$

$$\frac{42 + 5\sqrt{21} + 3}{9(7) - 3}$$

$$\frac{45 + 5\sqrt{21}}{60}$$

$$\frac{45}{60} + \frac{5\sqrt{21}}{60}$$

$$\frac{3}{4} + \frac{\sqrt{21}}{12}$$

(iv) $\frac{x-\sqrt{x^2-9}}{x+\sqrt{x^2-9}}$

$$\frac{x-\sqrt{x^2-9}}{x+\sqrt{x^2-9}} \left(\frac{x-\sqrt{x^2-9}}{x-\sqrt{x^2-9}}\right)$$

$$\frac{(x-\sqrt{x^2-9})^2}{(x)^2 - (\sqrt{x^2-9})^2}$$

$$\frac{x^2 - 2x\sqrt{x^2-9} + x^2 - 9}{x^2 - (x^2-9)}$$

$$\frac{2x^2 - 2x\sqrt{x^2-9} - 9}{x^2 - x^2 + 9}$$

$$\# \ \frac{2x^2 - 2x\sqrt{x^2-9} - 9}{9}$$

(iv) $\frac{1}{(\sqrt{2}+1)(\sqrt{3}-1)}$

$\frac{1}{\sqrt{2}+1} \times \frac{1}{\sqrt{3}-1}$

$\frac{1}{\sqrt{2}+1}\left(\frac{\sqrt{2}-1}{\sqrt{2}-1}\right) \times \frac{1}{\sqrt{3}-1}\left(\frac{\sqrt{3}+1}{\sqrt{3}+1}\right)$

$\frac{\sqrt{2}-1}{(\sqrt{2})^2-(1)^2} \times \frac{\sqrt{3}+1}{(\sqrt{3})^2-(1)^2}$

$\frac{\sqrt{2}-1}{2-1} \times \frac{\sqrt{3}+1}{3-1}$

$\frac{(\sqrt{2}-1)(\sqrt{3}+1)}{(1)\ (2)}$

$\frac{\sqrt{6}+\sqrt{2}-\sqrt{3}-1}{2}$

(C) $\frac{\sqrt{5+h}-3}{h}$

$\frac{\sqrt{5+h}-3}{h}\left(\frac{\sqrt{5+h}+3}{\sqrt{5+h}+3}\right)$

$\frac{(\sqrt{5+h})^2-(3)^2}{h(\sqrt{5+h}+3)}$

$\frac{5+h-9}{h\sqrt{5+h}+h3}$

$\frac{h-4}{h\sqrt{5+h}+3h}$

(ii) $\frac{\sqrt{3}+\sqrt{5}}{7}$

$\frac{\sqrt{3}+\sqrt{5}}{7}\left(\frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}-\sqrt{5}}\right)$

$\frac{(\sqrt{3})^2-(\sqrt{5})^2}{7(\sqrt{3}-\sqrt{5})}$

$\frac{3-5}{7\sqrt{3}-7\sqrt{5}}$

$\frac{-2}{7\sqrt{3}-7\sqrt{5}}$

(iii) $\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}+\sqrt{x+h}}$

$\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}+\sqrt{x+h}}\left(\frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}\right)$

$\frac{(\sqrt{x})^2-(\sqrt{x+h})^2}{h(x)+h(\sqrt{x})(\sqrt{x+h})+(\sqrt{x})(\sqrt{x+h})+x+h}$

$\frac{x-(x+h)}{xh+h\sqrt{x^2+xh}+\sqrt{x^2+xh}+x+h}$

$\frac{x-x-h}{xh+h\sqrt{x^2+xh}+\sqrt{x^2+xh}+x+h}$

\# $-h$

$xh + h\sqrt{x^2+xh} + \sqrt{x^2+xh+xh}$

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(a.)

(b) $a+c = b+c$

\# adding the additive inverse of c on both sides of the equation we get;

$a+c-c = b+c-c$

$a+0 = b+0$

$a = b$

Hence proved

(c) $ac = bc$

\# multiplying through by the multiplicative inverse of c on both sides;

$\left[ac = bc\right] \times \frac{1}{c}$

$a\cancel{c} \times \frac{1}{\cancel{c}} = b\cancel{c} \times \frac{1}{\cancel{c}}$

$a = b$

hence ~~pr~~ proved

xth

h

h