THE COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

DEPARTMENT OF PHYSICS

TEST ONE, 2015

INSTRUCTIONS: There are FOUR (4) questions in this test Answer **ALL** Questions. Write your **names**, **computer number** and lecture **group** on the front page of your answer booklet.

DURATION: TWO (2) HOURS

Question 1

(a) A unit of area, often used in measuring land areas is the *hectare*, defined as $10^4\ m^2$. An open-pit coal mine excavates 75 hectares, down to a depth of 26 m each year. What volume of Earth, in cubic kilometers is removed during this time? [3]

(b) A 12-hour-dial clock happens to gain 0.5 minutes each day. After setting the clock to the correct time at 12:00 noon, how many days must one wait until it again indicates the correct time? [3]

(c) Density is defined as mass per unit volume. The density of iron is 7.87 kg/m^3, and the mass of an iron atom is 9.27×10^{-26}kg. If atoms are cubical and tightly packed;

(i) What is the volume of an iron atom? [3]

(ii) What is the distance between the centers of two adjacent atoms? [2]

(d) Suppose the displacement s of an object moving in a straight line under uniform acceleration a as a function of time is given by the relation $s = k a^m t^n$, where k is a dimensionless constant. Use dimensional analysis to find the values of the power m and n. [5]

(e) Using dimensional analysis, determine which of the following equations are dimensionally correct?

(i) $s = s_0 \cos kt$, where k is a constant that has the dimension of the inverse of time. [2]

(ii) $v^2 = v_0 t + 2as$. The symbols are in their usual meaning. [2]

0.4223249345

Question 2

a) Vector **A** has x and y components of -8.7 cm and 15 cm, respectively; vector **B** has x and y components of 13.2 cm and -6.6 cm, respectively. If **A**-**B**+3**C**=0, what are the rectangular components of **C**? [5]

b) A airliner moving initially at 300 mph due East moves into a region where the wind is blowing at 100 mph in a direction 30^0 North of East.

(i) Sketch the new direction of the airliner. [5]

(ii) Find the new velocity of the plane. [6]

(iii) In what direction is it travelling? [4]

Question 3

(a) A velocity function describing the motion of a car is given by

$$v = at + bt^2$$

Where a = 8 $m.s^{-2}$ and b = 3 $m.s^{-3}$

i) Find the change in velocity of the car for the interval between $t_1 = 1s$ and $t_2 = 3s$ [3]

ii) Derive an expression for the instantaneous acceleration at any time [3]

(b) A stone is thrown from the top of a building upward at an angle 20^0 to the horizontal with an initial speed of 25 $m.s^{-1}$. If the height of the building is 50 m.

i) How long does it take the stone to reach the ground [3]

ii) What is the speed of the stone just before it strikes the ground? [assume g = 9.81 $m.s^{-2}$] [3]

(c) At t = 0, a particle moving in the x-y plane with constant acceleration, has velocity of $v_i = 4\hat{i} - 3\hat{j}$ $m.s^{-1}$ and is at the origin.
At t = 4s, the particles velocity is $v_f = 8\hat{i} + 5\hat{j}$ $m.s^{-1}$. Find

(i) The acceleration of the particle [2]

(ii) Its coordinates at any time t. [3]

(d) A car travelling at a constant speed of 60 $m.s^{-1}$ passes a trooper hidden behind a billboard. Two seconds after the speeding car passes the billboard, the trooper sets out from the billboard to catch it, accelerating at a constant rate of 4 $m.s^{-2}$. How long does it take her to overtake the car [3]

Question 4

(a) A body starts from the origin with a velocity of 3 m/s and an acceleration given by $a = 6t - 4$. Find

(i) Its velocity and [3]

(ii) Displacement at time t. [3]

(b) A ball is thrown straight upwards and returns to the throwers hand after 3 s in the air. A second ball is thrown at an angle of 30° with the horizontal.

i) At what velocity must the second ball be thrown so that it reaches the same height as the one thrown vertically? [4]

ii) What height does the ball reach? [3]

(c) Show that the maximum range of a projectile is given by $R = \frac{v_0^2}{g}$ [4]

(d) Derive the equation of motion $v^2 = v_o^2 + 2a(x - x_0)$ [3]

QUESTION ①

Given 1 hectare $= 10^4 m^2$, 75 hectares , 26m

1 hectare $= 10^4 m^2$ we can find number of m^2 in 75 hectares

$$75 \text{ hectares} \times \frac{10^4 m^2}{1 \text{hectare}} = 7.5 \times 10^5 m^2$$

Volume = Area × depth

$V = 7.5 \times 10^5 m^2 \times 26 m$

$V = 1.95 \times 10^7 m^3$

∴ volume in cubic kilometers

$$= 1.95 \times 10^7 m^3 \times \left(\frac{10^{-3} km}{1m}\right)^3$$

$$= 1.95 \times 10^7 \cancel{m^3} \times \frac{10^{-9} km^3}{1\cancel{m^3}}$$

$$= \underline{\underline{1.95 \times 10^{-2} km^3}}$$

3

ⓑ 0.5 min — 1 day

12 hrs — x

$$12 hrs \times \frac{60 min}{1 hr} = 720 min$$

0.5 min — 1 day

720 min — x

$$x = \frac{720 \cancel{min} \times 1 day}{0.5 \cancel{min}}$$

$x = \underline{1440 \text{ days}}$

∴ one must wait for 1440 days

3

ⓒ Density $= 7.87 kg/m^3$, mass $= 9.27 \times 10^{-26} kg$

(i) $\rho = \frac{m}{V}$ $V = \frac{m}{\rho}$

$$V = \frac{9.27 \times 10^{-26} kg}{7.87 kg/m^3}$$

$\underline{\underline{V = 1.18 \times 10^{-26} m^3}}$

3

...nce the atoms are cubical, then

$V = L^3$

from (i) $V = 1.18 \times 10^{-26} m^3$

$1.18 \times 10^{-26} m^3 = L^3$

$L^3 = 1.18 \times 10^{-26} m^3$

$L = \sqrt[3]{1.18 \times 10^{-26} m^3}$

$L = 2.28 \times 10^{-9} m$

2

$\therefore$ the distance between two adjacent atoms is 2.28×10^{-9} m

(d) $S = K a^m t^n$ $\quad [S] = [K][a^m][t^n]$ —— (i)

$[S] = L$, $[a] = \frac{L}{T^2}$ $[t] = T$

by convention $[K] = 1$

Equation (i) can be written as

$L = 1\left(\frac{L}{T^2}\right)^m (T)^n$

$L = (LT^{-2})^m T^n$

$L = L^m T^{-2m} T^n$

$L = L^m T^{n-2m}$ —————— (ii)

We can introduce the dimension of time on the L.H.S of equation (ii) by inspection and L.H.S = R.H.S

$L^1 T^0 = L^m T^{n-2m}$

$m = 1$ $\quad n - 2(1) = 0 \Rightarrow n = 2$

$\therefore$ $m = 1$ and $n = 2$

5

$S = S_0 \cos kt$

$[S] = L$, $[S_0] = L$, $[K] = \frac{1}{T}$, $[t] = T$

$[S] = [S_0][\cos kt]$

$L = L\left[\cos\left(\frac{1}{T} \times T\right)\right]$

$L = L\ (\cos 1)$ when the dimensions of K is $\frac{1}{T}$, cos kt becomes dimensionless

$L = L \times 1$ 2

$L = L$

∴ the equation is dimensionally correct.

(ii) $V^2 = V_0 t + 2as$

$[V] = \frac{L}{T}$, $[V_0] = \frac{L}{T}$, $[t] = T$, $[a] = \frac{L}{T^2}$, $[S] = L$

$[V^2] = [V_0][t] + [a][s]$

$(LT^{-1})^2 = (LT^{-1})(T) + (LT^{-2})(L)$

$L^2T^{-2} = L + L^2T^{-2}$ 2

Since L.H.S $\neq$ R.H.S

∴ the equation is not dimensionally correct.

QUESTION 2

a ~~A = (-8.~~

$\underset{\sim}{A} = \begin{pmatrix} -8.7\text{ cm} \\ 15\text{cm} \end{pmatrix}$ $\underset{\sim}{B} = \begin{pmatrix} 13.2\text{cm} \\ -6.6\text{cm} \end{pmatrix}$ let vector c components be x cm and y cm

∴ $\underset{\sim}{C} = \begin{pmatrix} x\text{cm} \\ y\text{cm} \end{pmatrix}$

we know that

$\underset{\sim}{A} - \underset{\sim}{B} + 3\underset{\sim}{C} = 0$

$\underline{a} + 3\underline{c} = 0$

$$\begin{pmatrix} -8.7\cancel{cm} \\ 15 \end{pmatrix} - \begin{pmatrix} 13.2 \\ -6.6 \end{pmatrix} + 3\begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$\begin{pmatrix} -8.7 \\ 15 \end{pmatrix} - \begin{pmatrix} 13.2 \\ -6.6 \end{pmatrix} + \begin{pmatrix} 3x \\ 3y \end{pmatrix} = 0$$

$-8.7 - 13.2 + 3x = 0$

$-21.9 + 3x = 0$

$\frac{3x}{3} = \frac{+21.9}{3}$

$x = 7.3$ cm

$15 - (-6.6) + 3y = 0$

$15 + 6.6 + 3y = 0$

$21.6 + 3y = 0$

$\frac{3y}{3} = \frac{-21.6}{3}$

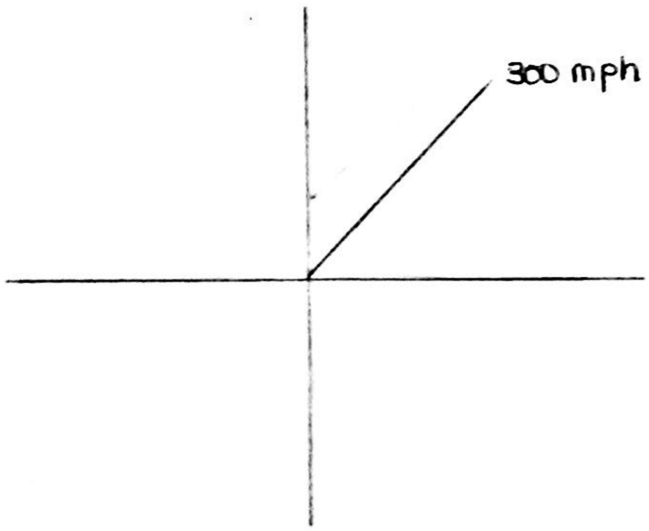
$y = -7.2$ cm

∴ the rectangular components of $\underline{c}$ are

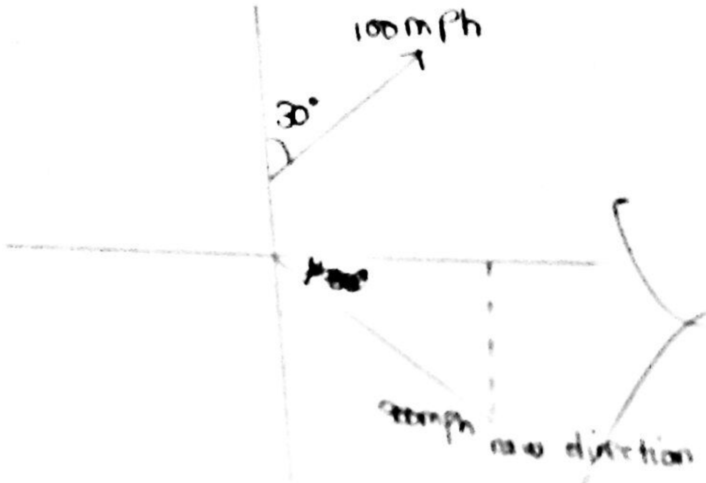
$x = 7.3$ cm and $y = -7.2$ cm

5

(b)



QUESTION (2)



300 cos 330°

2

(ii) 100 cos 30°

= 86.6 mph

1

new velocity of plane

= 86.6 mph + 300 mph

= $\underline{\underline{386.6 \text{ mph}}}$

(iii) $\theta = \tan^{-1} \dfrac{386.6}{300}$

$\underline{\underline{\theta = 52.1° \text{ below the x-axis}}}$

QUESTION ③

$v = at + bt^2$ $a = 8\,m.s^{-2}$ $b = 3\,m.s^{-3}$

(i) $v = 8\,m/s^2\, t + 3\,m/s^3\, t^2$

v at $t_1 = 1s$

$v = 8\,m/s^2\,(1s) + 3\,m/s^3\,(1s)^2$

$v = 8\,m/s^2 \times 1s + 3\,m/s^3 \times 1s^2$

$v = 8\,m/s + 3\,m/s = \underline{11\,m/s}$

v at $t_2 = 3s$

$v = 8\,m/s^2\,(3s) + 3\,m/s^3\,(3s)^2$

$v = 8\,m/s^2\,(3s) + 3\,m/s^3\,(9s^2)$

$v = 24\,m/s + 27\,m/s$

$v = 51\,m/s$

3

Change in velocity $= v(t_2) - v(t_1)$

$= 51\,m/s - 11\,m/s$

$= \underline{40\,m/s}$

(ii) $v = at + bt^2$

at $t_1 = t$, $v = at + bt^2$

let $t_2 = t + \Delta t$

velocity at t_2 is

$v = a(t + \Delta t) + b(t + \Delta t)^2$

$v = at + a\Delta t + b(t^2 + 2t\Delta t + (\Delta t)^2)$

$v = at + a\Delta t + bt^2 + 2bt\Delta t + b\Delta t^2$

$$a_{av} = \frac{v_2 - v_1}{t_2 - t_1}$$

$$a_{av} = \frac{at + \cancel{at}\, a\Delta t + bt^2 + 2bt\Delta t + b\Delta t^2 - (at + bt^2)}{\cancel{t} + \Delta t - \cancel{t}}$$

$$a_{av} = \frac{a\cancel{t} + \cancel{bt^2} + a\Delta t + 2bt\Delta t + b\Delta t^2 - \cancel{at} - \cancel{bt^2}}{\Delta t}$$

$$a_{av} = \frac{a\Delta t + 2bt\Delta t + b\Delta t^2}{\Delta t} = \frac{\Delta t\,(a + 2bt + b\Delta t)}{\Delta t}$$

$$\frac{\cancel{\Delta t}(a + 2bt + b\Delta t)}{\cancel{\Delta t}} = a + 2bt$$

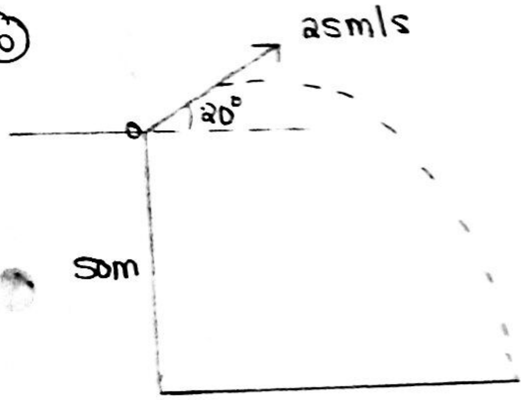
$$a = \lim_{\Delta t \to 0} \left(\frac{\Delta v}{\Delta t}\right) = a + 2bt$$

3

$\therefore$ the expression for instantaneous acceleration is

$\underline{\underline{a + 2bt}}$

(b)



taking the top of the building as our reference point (origin)

$a = -g = -9.8\ m/s^2$

vertical distance will be negative because we have chosen building top as origin

$V_{ix} = V_0 \cos\theta$

$V_{ix} = (25 m/s) \cos 20°$

$V_{ix} = 23.5 m/s$

$V_{iy} = V_0 \sin\theta$

$V_{iy} = (25 m/s) \sin 20°$

$V_{iy} = 8.55\ m/s$

Vertical distance

$y = V_{iy}t - \frac{1}{2}gt^2$

$-50 = 8.55t - \frac{1}{2}(9.81)t^2$

$-50 = 8.55t - 4.905t^2$

$4.905t^2 - 8.55t - 50 = 0$

$a = 4.905,\ b = -8.55,\ c = -50$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(-8.55) \pm \sqrt{(-8.55)^2 - 4(4.905)(-50)}}{2(4.905)}$$

$$t = \frac{8.55 \pm \sqrt{73.1025 + 981}}{9.81}$$

$$t = \frac{8.55 \pm 32.47}{9.81}$$

$$t = \frac{8.55 + 32.47}{9.81} \quad \text{or} \quad t = \frac{8.55 - 32.47}{9.81}$$

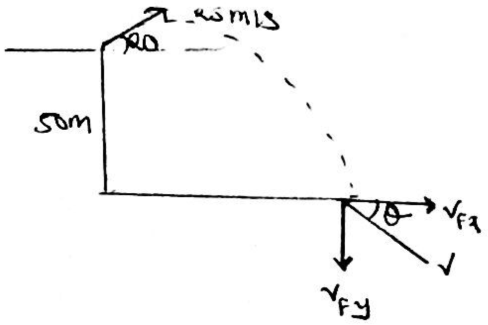
$\underline{t = 4.18_s}$ or $\underline{t = -2.44_s}$

3

$\therefore$ time taken to reach the ground

$= \underline{\underline{4.18_s}}$

$v_{ix} = 23.5\, m/s$



$v_{ix} = v_{fx} = 23.5\, m/s$

vertical velocity at $t = 4.18_s$

$v_{fy} = v_{iy} - gt$

$v_{fy} = 8.55\, m/s - (9.8\, m/s^2 \times 4.18_s)$

$v_{fy} = -32.4\, m/s^2$

$\therefore$ Speed $= \sqrt{(23.5 m/s)^2 + (-32.4 m/s)}$

$= 40 m/s$

$\therefore$ speed $= \underline{\underline{40 m/s}}$

3

(c) $v_i = 4\hat{i} - 3\hat{j}\ m.s^{-1}$ at $t = 0_s$

$v_f = 8\hat{i} + 5\hat{j}\ m.s^{-1}$ at $t = 4_s$

(i) acceleration $= \dfrac{\Delta v}{\Delta t} = \dfrac{v_f - v_i}{t_f - t_i}$

$$a = \frac{[(8\hat{i} + 5\hat{j}) - (4\hat{i} - 3\hat{j})]\, m/s}{4_s - 0_s}$$

$$a = \frac{[(8\hat{i} - 4\hat{i}) + 5\hat{j} - (-3\hat{j})]\, m/s}{4_s}$$

$$a = \frac{(4\hat{i} + 5\hat{j} + 3\hat{j})\, m/s}{4_s}$$

$$a = \frac{4\hat{i} + 8\hat{j}}{4_s}\ m/s$$

~~$a = 2\hat{i}$~~

$\underline{\underline{a = \hat{i} + 2\hat{j}\ m/s^2}}$

2

$x_i + v_{ix}t + \frac{1}{2}a_x t^2$

$t=0$, $x_i = 0$ $v_x = 4\hat{i}$

$x_f = v_{ix}t + \frac{1}{2}at^2$

$x_f = 4\hat{i}t + \frac{1}{2}(\hat{i})(t^2)$

$$x_f = 4\hat{i}t + \frac{\hat{i}t^2}{2}$$

$y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2$

at $t=0$, $y_i = 0$, $v_y = -3\hat{j}$

$y_f = (-3\hat{j})t + \frac{1}{2}(\hat{j})(t)^2$

$$y_f = -3t\hat{j} + \hat{j}t^2$$

3

(d) $\cancel{x_{car} = 60m + 4t}$

trooper 4m/s² Cars car

$x_{car} = 120m + 60t$

$x_{trooper} = \frac{1}{2}at^2$ since it is starting from rest

$x_{trooper} = \frac{1}{2}(4)(t^2)$

$= 2t^2$

$x_{trooper} = x_{car}$

$2t^2 = 120 + 60t$

$\frac{2t^2}{2} - \frac{60t}{2} - \frac{120}{2} = \frac{0}{2}$

$t^2 - 30t - 40 = 0$

$$t = \frac{-(-30) \pm \sqrt{(-30)^2 - 4(1)(-40)}}{2}$$

$$t = \frac{30 \pm \sqrt{900 + 1600}}{2}$$

$t = \frac{30 \pm 50}{2}$ $\quad t = \frac{30-50}{2}$

$t = \frac{-20}{2} = -10s$

$t = \frac{30+50}{2}$

$t = \frac{80}{2} = 40s$

∴ time taken to overtake $= \underline{40s}$

QUESTION (4)

a) $a = 6t - 4$

Conditions

$t=0$, $v_i = 3\,m/s$, $s_i = 0$

(i) $v = \int 6t - \int 4 + c$

$v = \frac{6t^2}{2} - 4t + c$

$v = 3t^2 - 4t + c$

with the above conditions

c, at $t=0$, $v_i = 3\,m/s$

$3 = 3t^2 - 4t + c$

$3 = 3(0) - 4(0) + c$

$c = 3$

$v = 3t^2 - 4t + 3$

3

(ii) $s = \int 3t^2 - \int 4t + \int 3 + c$

$s = \frac{3t^3}{3} - \frac{4t^2}{2} + 3t + c$

$s = t^3 - 2t^2 + 3t + c$

at $t=0$, $s=0$

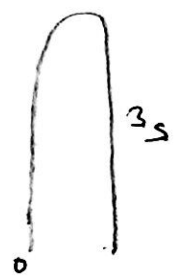
$0 = (0)^3 - 2(0)^2 + 3(0) + c$

$c = 0$

$\therefore$ $s = t^3 - 2t^2 + 3t$

3

(b)



First ball

$t = 3s$ $v_0 = 0\,m/s$

$v_F = v_i + at$

$v_F = 3(9.8)$

$v_F = 29.4\,m/s$

$v_{iy} = v_0 \sin\theta$

$h = \frac{v_0^2 \sin^2\theta}{2g}$ 1/2

$\frac{v_0^2 \sin^2\theta}{2g} = 44.1$

$\frac{v_0^2 \sin^2\theta}{\sin^2\theta} = \frac{2g \times 44.1}{\sin^2\theta}$

$h = \frac{1}{2}(9.8)(3)^2$

$h = \frac{1}{2}(9.8)(9)$

$h = 44.1\,m$

$$\frac{\sin^2\theta}{\sin^2\theta} = \frac{2g\ 44.1}{\sin^2\theta}$$

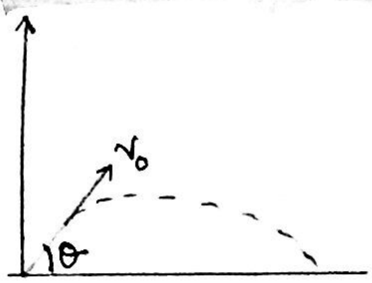
$$v_0^2 = \frac{2g \times 44.1}{\sin^2\theta}$$

$$v_0 = \sqrt{\frac{2 \times 9.8 \times 44.1}{(\sin 30)(\sin 30)}}$$

$$v_0 = \sqrt{\frac{864.36}{0.25}}$$

$$\underline{\underline{v_0 = 58.8\,m/s}}$$

(iv) Max height = $\underline{\underline{44.1\,m}}$ as in part b(i)



t_m

taking up as positive

time taken to reach max height

at max height $v_f = 0\,m/s$, $g = -9.8\,m/s^2$

$\cancel{v_f = v_i} - gt$ $\qquad v_0 = \quad v_{ix} = v_0 \cos\theta$

$\cancel{0 = v_0 - gt}$ $\qquad v_{iy} = v_0 \sin\theta$

$v_f = v_{iy} - gt$ $\qquad$ time for max range = 2 (time max height)

$0 = v_0 \sin\theta - gt$ $\qquad t = \dfrac{2v_0 \sin\theta}{g}$

$\dfrac{\cancel{g}t}{\cancel{g}} = \dfrac{v_0 \sin\theta}{g}$

$t = \dfrac{v_0 \sin\theta}{g}$

Max range $R = v_{ix} t$ where $v_{ix} = v_0 \cos\theta$

$R = v_0 \cos\theta \times \dfrac{2v_0 \sin\theta}{g}$

$R = \dfrac{2v_0^2 \cos\theta \sin\theta}{g}$

$R = \dfrac{v_0^2 (2\cos\theta \sin\theta)}{g}$ $\qquad$ where $2\cos\theta\sin\theta = \sin 2\theta$

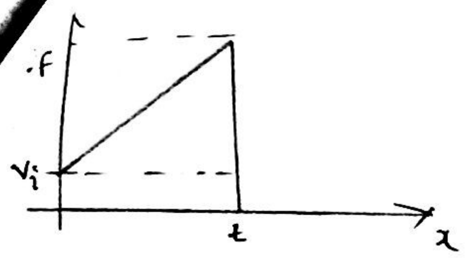
$2\sin\theta\cos\theta = \sin 2\theta$

$R = \dfrac{v_0^2 \sin 2\theta}{g}$ $\qquad$ at max range $\theta = 45°$

$R = \dfrac{v_0^2 \sin(90°)}{g}$ $\qquad \sin 90° = 1$

$R = \dfrac{v_0^2}{g}$ Hence shown

4



$$a = \frac{v_f - v_i}{t}$$

$$v_f = v_i + at \quad \text{———} \quad (i)$$

Average velocity

$$\frac{v_f + v_i}{2}$$

$$\text{Area} = v_i t + \tfrac{1}{2}(v_f - v_i)t$$

$$= v_i t + \left(\tfrac{1}{2}v_f - \tfrac{1}{2}v_i\right)t$$

$$= v_i t + \tfrac{1}{2}v_f t - \tfrac{1}{2}v_i t$$

$$= \tfrac{1}{2}v_i t + \tfrac{1}{2}v_f t$$

$$= \tfrac{1}{2}(v_i + v_f)t \quad \text{———} \quad (ii)$$

But $v_f = v_i + at$ $\qquad$ $\Delta x =$ Average velocity $\times$ time

$$\tfrac{1}{2}(v_i + v_i + at)t = \Delta x$$

$$\tfrac{1}{2}(2v_i + at)t = \Delta x$$

$$v_i t + \tfrac{1}{2}at^2 = \Delta x \quad \text{———} \quad (iii)$$

Making t subject from eq (i), Substitute in eq (iii)

$$t = \frac{v_f - v_i}{a}$$

$$\Delta x = v_i\left(\frac{v_f - v_i}{a}\right) + \tfrac{1}{2}a\left(\frac{v_f - v_i}{a}\right)^2$$

$$\Delta x = \frac{v_i v_f - v_i^2}{a} + \tfrac{1}{2}a\left(\frac{v_f^2 - 2v_i v_f + v_i^2}{a^2}\right)$$

$$\Delta x = \frac{v_i v_f - v_i^2}{a} + \frac{v_f^2 - 2v_i v_f + v_i^2}{2a}$$

$$\frac{2v_i v_f - 2v_i^2 + v_f^2 - 2v_i v_f + v_i^2}{2a}$$

$$\Delta x = \frac{v_f^2 - v_i^2}{2a}$$

$$2a\,\Delta x = v_f^2 - v_i^2$$

$$v_f^2 = v_i^2 + 2a\,\Delta x$$

$$v_f^2 = v_i^2 + 2a(x - x_0)$$

$$\therefore\ v^2 = v_0^2 + 2a(x - x_0)$$ Hence shown

3