**THE COPPERBELT UNIVERSITY**
SCHOOL OF MATHEMATICS AND NATURAL SCIENCES
**DEPARTMENT OF PHYSICS**

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**PH 110 INTRODUCTORY PHYSICS**
**TUTORIAL SHEET 2 2023: SCALARS AND VECTORS**

1. Use the (i) graphical method and (ii) analytical (component method) to fully describe the resultant of the coplanar vector combination acting at point *O* in the figure below:

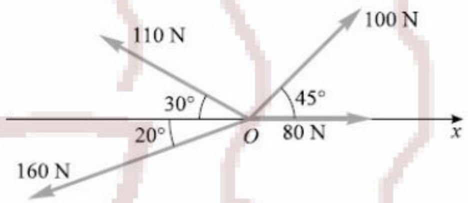
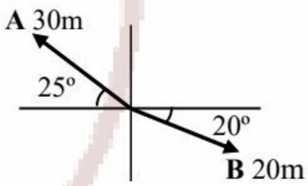



Figure 1.1

2. The resultant of three vectors is 60.0 N directed at 45° below the negative x-axis. One vector is along the positive x-axis and is of magnitude 20.0 N. The second vector has components -12.0 N in the x and 6.0 N in the y. Fully describe the third vector.

3. In Fig. 2.1 Find the direction and magnitude of:
   a) the vector sum **A** + **B**
   b) the vector difference **A**-**B**
   c) the vector difference **B**-**A**

Fig. 1.2




4. A rectangular parallelepiped has dimensions $a$, $b$ and $c$ as shown in Fig. 1.3
   (i) Obtain a vector expression for the face diagonal vector $\mathbf{R}_1$. What is the magnitude of this vector?

   (ii) Obtain a vector expression of the body diagonal vector $\mathbf{R}_2$ and prove that the magnitude of $\mathbf{R}_2$ is $\sqrt{a^2+b^2+c^2}$

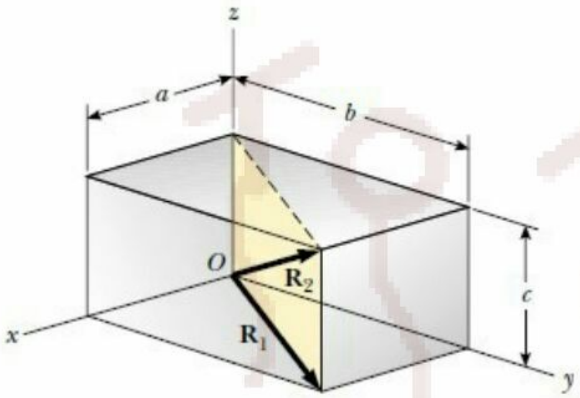



Fig. 1.3

5. The three finalists in a contest are brought to the centre of a large, flat field. Each is given a metre stick, a compass, a calculator, a shovel and the following three displacements:

   72.4 m, $32.0^0$ east of north;
   57.3 m, $36.0^0$ south of west;
   17.8 m straight south.

   The three displacements lead to a point where the keys to a new building are buried. Two contestants start measuring immediately; the winner first calculates where to go. What does the winner calculate in terms of magnitude and direction?

6. A ship is steaming due east at a speed of 12 $ms^{-1}$. A passenger runs across the deck at a speed of 5 $ms^{-1}$ toward north. What is the resultant velocity of the passenger relative to the sea?

7. The polar coordinates of a point are $r = 5.5$ m and $\theta = 240°$. What are the Cartesian coordinates of this point?

8. You find yourself pacing, in a deep thought about a physics problem. First you walk 12 meters due east. Then, you walk 6 meters due north. Then you doze off and find yourself 50 meters from your starting place, 30° north of east. How far did you walk while you were not paying attention?

9. Jelita walks 20 feet, 20° north of east. He then walks 32 feet, 40° north of west. Then 10 feet, 5° south of west. Then 65 feet, 69° south of east. What is the magnitude and direction of her resultant displacement?

10. Find the magnitude and angle of the resultant of the following displacement vectors:

    **A** = 5.0 m at E $37^0$ N
    **B** = 6.0 m at W $45^0$ N
    **C** = 4.0 m at W $30^0$ S

**D** = 3.0 m at E $60^0$ S

11. Find the cross product of $\mathbf{A}\times\mathbf{B}$ where $\mathbf{A}=2\hat{\imath}+3\hat{\jmath}$, $\mathbf{B}=-\hat{\imath}+2\hat{\jmath}$

12. Given a vector $\mathbf{A}=3\mathbf{i}+6\mathbf{j}-2\mathbf{k}$. Find another vector $\mathbf{B}$ which is parallel to vector $\mathbf{A}$ and has a magnitude of 17 units.

13. Given the $\mathbf{A}=\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ and $\mathbf{B}=3\mathbf{i}+4\mathbf{j}+5\mathbf{k}$
    a) determine a unit vector perpendicular to both $\mathbf{A}$ and $\mathbf{B}$
    b) find the angle between $\mathbf{A}$ and $\mathbf{B}$

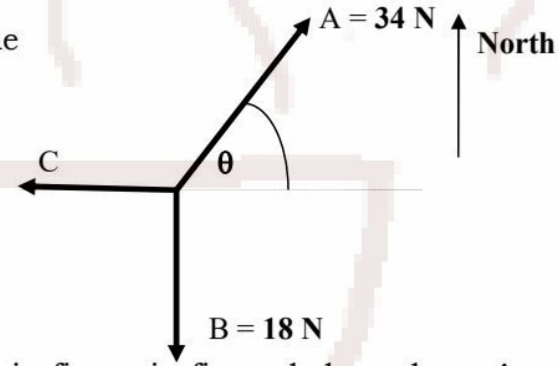
14. Find a vector whose length is 7 and which is perpendicular to each of the vectors $\mathbf{B}=2\mathbf{i}-3\mathbf{j}+5\mathbf{k}$ and $\mathbf{C}=\mathbf{i}+\mathbf{j}-\mathbf{k}$

15. If vector $\mathbf{A}=3\mathbf{j}$, $\mathbf{A}\times\mathbf{B}=3\mathbf{i}$ and $\mathbf{A}\bullet\mathbf{B}=12$. Find
    a) $\mathbf{B}$
    b) $\hat{\mathbf{B}}$

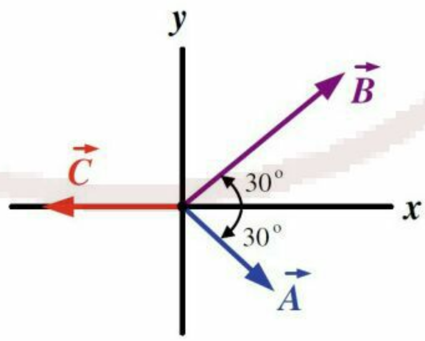
16. The vectors A, B, and C shown in the diagram alongside add together so that their resultant is zero.
Use the method of components to find the

(i) the bearing of vector A, and

(ii) the magnitude of vector C.




17. Three forces are acting on a body as shown in figure in figure below where $A$ = 10 N, $B$ = 20 N, and $C$ = 15 N. Find the magnitude and the direction of the resultant force acting on the body.



1

(i) # Graphical method you'll need a graph paper and a protractor to accurately measure your angles. Nomba me i have none of the two... kkk.

(ii) # Component Method

| | x-comp | y-comp |
|---|---|---|
| A | $80 \cos 0^\circ$ | $80 \sin 0^\circ$ |
| B | $100 \cos 45^\circ$ | $100 \sin 45^\circ$ |
| C | $110 \cos 150^\circ$ | $110 \sin 150^\circ$ |
| D | $160 \cos 200$ | $160 \sin 200$ |
| R | $-94.9029$ | $70.987$ |

$$|R| = \sqrt{x^2 + y^2}$$

$$= \sqrt{(94.9029)^2 + (70.987)^2}$$

$$= \sqrt{14045.714}$$

$$|R| = 118.5N$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\theta = \tan^{-1}\left(\frac{70.987}{94.9029}\right)$$

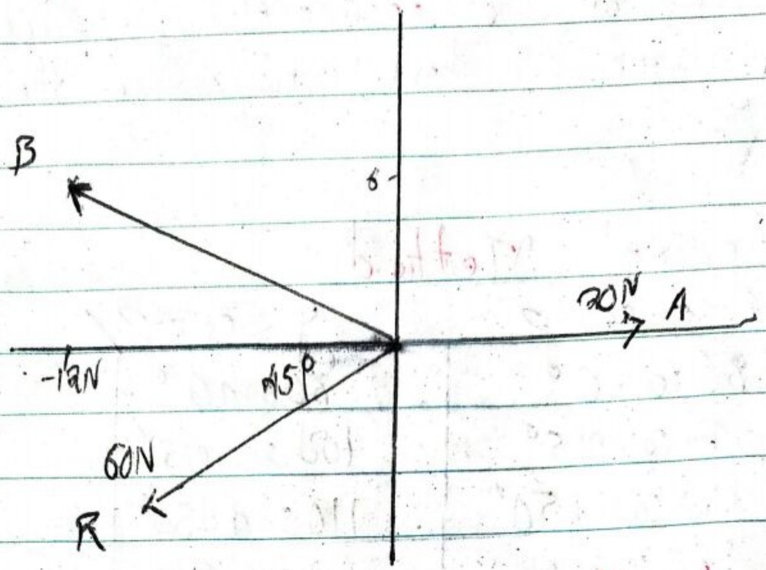
$$\theta = 36^\circ$$

$$\theta = 180^\circ - 36^\circ$$

$$\theta = 144^\circ$$

$\therefore$ The resultant vector of magnitude 118.5N lies in the direction $144^\circ$ from the (+) x-axis.

2 # Given the resultant and the two vectors we can plot them as follows




# note that for B they have already given us the components.

| | x-comp | y-comp |
|---|---|---|
| A | $20 \cos 0°$ | $20 \sin 0°$ |
| B | $-12$ | $6$ |
| C | $C_x$ | $C_y$ |
| R | $60 \cos 225$ | $60 \sin 225$ |

# x-comp

$$20 \cos 0° + (-12) + C_x = 60 \cos 225°$$
$$8 + C_x = -42.426$$
$$C_x = -50.426$$

# y - comp

$$20 \sin 0 + 6 + C_y = 60 \sin 225$$
$$6 + C_y = -42.426$$
$$C_y = -48.426$$

$$|C| = \sqrt{C_x^2 + C_y^2}$$

$$= \sqrt{(-50.426)^2 + (-48.426)^2}$$

$$|C| = 69.9N$$

$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right)$$

$$\theta = \tan^{-1}\left(\frac{48.426}{50.426}\right)$$

$$\theta = 43.8^\circ$$

$$\theta = 180 + 48.8^\circ$$

$$\theta = 223.8^\circ$$

∴ The third vector has magnitude of 69.9N and is in the direction 223.8° from the (+) x-axis

3 # Pick your angles starting from the (+) - x-axis,

(a) A + B

| | x - comp | | y - comp | |
|---|---|---|---|---|
| A | 30 cos 155° | -27.18 | 30 sin 155° | 12.678 |
| B | 20 cos 340° | 18.798 | 20 sin 340° | -6.840 |
| R | -8.39538119 | | 5.83814983 | |

$$|R| = \sqrt{x^2 + y^2}$$
$$|R| = \sqrt{(8.395)^2 + (5.838 14)^2}$$
$$|R| = 10.23 m$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$
$$\theta = \tan^{-1}\left(\frac{5.838}{8.395}\right)$$

$$\theta = 34.8°$$

But according to the coordinates the correct $\theta$ must be in the 2nd quard since $x$ is (-) and $y$ is (+)

$$\theta = 180° - 34.8°$$
$$\theta = 145.2°$$

(b.) A - B

| | x - comp | y - comp |
|---|---|---|
| A | 30 cos 155 | 30 sin 155 |
| B | - 20 cos 340 | - 20 sin 340 |
| R | - 45.9830 | 19.5189 |

$$|R| = \sqrt{x^2 + y^2}$$
$$|R| = \sqrt{(-45.983)^2 + (19.5189)^2}$$
$$|R| = 49.95\,m$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$
$$\theta = \tan^{-1}\left(\frac{19.5189}{45.98}\right)$$
$$\theta = 23^\circ$$

$$\theta = 180^\circ - 23^\circ$$
$$\theta = 157^\circ$$

(c.) B - A

| | x-comp | y-comp |
|---|---|---|
| B | $20\cos 340^\circ$ | $20\sin 340^\circ$ |
| -A | $-30\cos 155^\circ$ | $-30\sin 155^\circ$ |
| R | 45.98308 | 19.518950 |

$$|R| = \sqrt{x^2 + y^2}$$
$$|R| = \sqrt{(45.983)^2 + (19.5189)^2}$$
$$|R| = 49.95\,m$$ # same as (b)

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$
$$\theta = \tan^{-1}\left(\frac{19.5189}{45.983}\right)$$

$$\theta = 23^\circ$$
$$\theta = 360^\circ - 23^\circ$$
$$\theta = 337^\circ$$

∴ Here $\theta$ is in the $4^{th}$ Quard.

Q.4

(i) # Refering from the kati ~~dig~~ diagram, using your spiritual eyes !!

$$R_1 = ai + bj + ck \quad , \text{but } c=0$$

$$R_1 = ai + bj + 0k$$

$$\underline{\underline{R_1 = ai + bj}}$$

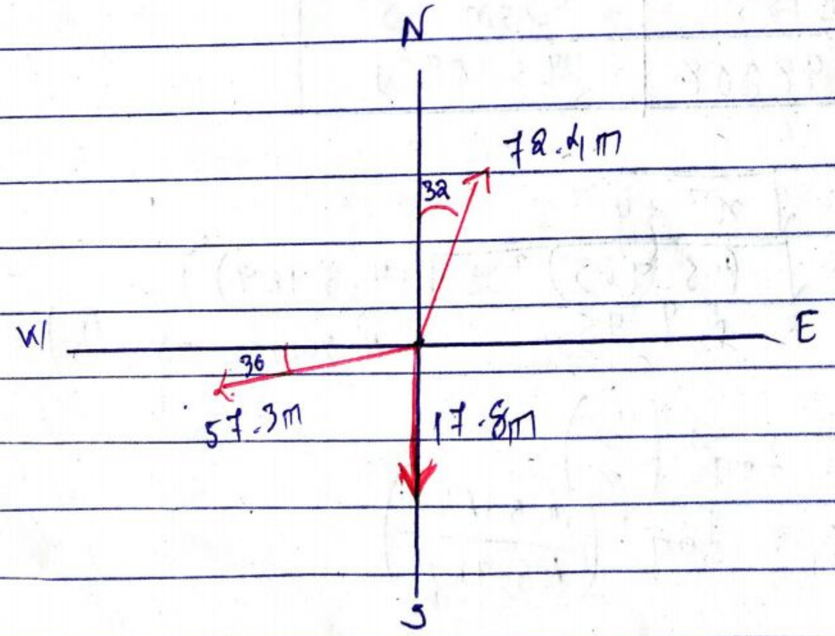
$$|R_1| = \sqrt{(a)^2 + (b)^2}$$

$$\underline{\underline{|R_1| = \sqrt{a^2+b^2}}}$$

(ii) $R_2 = ai + bj + ck$

$$|R_2| = \sqrt{a^2+b^2+c^2} \quad \text{hence shown}$$

5.



| | x- comp | y - comp |
|---|---|---|
| A | 72.4 cos 58° | 72.4 sin 58° |
| B | 57.3 cos 216° | 57.3 cos 216° |
| C | 17.8 cos 270° | 17.8 sin 270° |
| R | -7.9705 1905 | 9.9185872 |

$$|R| = \sqrt{x^2 + y^2}$$

$$|R| = \sqrt{(-7.99)^2 + (9.91)^2}$$

$$|R| = 12.74m$$

$$Q = \tan^{-1}\left(\frac{y}{x}\right)$$

$$Q = \tan^{-1}\left(\frac{9.918}{7.990}\right)$$

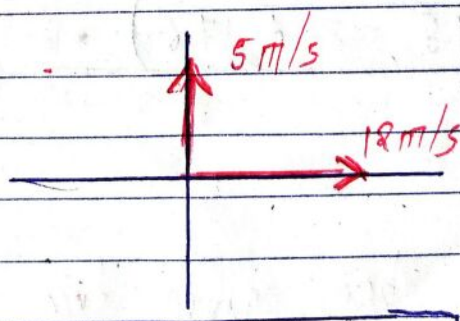
$$Q = 51.1°$$

$$Q = 180 - 51.1°$$

$$Q = 128.9°$$

∴ The winner needs to cover 12.74m 128.9° from the (+) x-axis.

6




| | x- comp | y - comp |
|---|---|---|
| A | 12 cos 0° | 12 sin 0° |
| B | 5 cos 90° | 5 sin 90° |
| R | 12 | 5 |

$$|R| = \sqrt{x^2 + y^2}$$
$$= \sqrt{(12)^2 + (5)^2}$$
$$|R| = 13 \, m/s$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$
$$\theta = \tan^{-1}\left(\frac{5}{12}\right)$$

$$\theta = 22.6^\circ$$

# The passenger has a velocity of 13 m/s at an angle 22.6° from the (+) x-axis

7 $r = 5.5 \, m$ $\quad \theta = 240^\circ$

# Polar coordinates are in the form $(r, \theta)$ catesian coordinates are expressed as $(x, y)$

$x = R \cos \theta$ $\quad\quad y = R \sin \theta$
$= 5.5 \cos 240^\circ$ $\quad\quad = 5.5 \sin 240^\circ$
$x = -2.75$ $\quad\quad y = -4.76$

$$(-2.75, -4.76)$$

8 # You have been given two vectors and the resultant vector we need to find the third vector, the one while you were not paying attention

7 data given

$\vec{D} = 50m, 30° NE$

$\vec{A} = 12m$ East $(0°)$

$\vec{B} = 6m$ North $(90°)$

Solution

Here vector $\vec{D}$ is the resultant vector showing the total movement. And the summation of x and y components make a vector that describes the full movent. So the vector required is $\vec{C}$ which we dont know

i.e $\vec{D} = \vec{A} + \vec{B} + \vec{C}$

$D = \vec{A}_x + \vec{A}_y + \vec{B}_x + \vec{B}_y + \vec{C}_x + \vec{C}_y$

$= 12\cos 0° \hat{i} + 12\sin 0° \hat{j} + 6\cos 90° \hat{i} + 6\sin 90° \hat{j}$

$+ \vec{C}\cos\theta + C\sin\theta$

so since $C = C_x + C_y$ we now write

$C = \vec{D} - (\vec{A} + \vec{B})$ , $\vec{D} = D_x + D_y$

$C = (50\cos 30° \hat{i} + 50\sin 30° \hat{j}) - (\vec{A} + \vec{B})$

$C = [50\cos 30° - (12\cos 0° + 6\cos 90°)]\hat{i} + \cancel{[50\sin 30° -}$

$+ [50\sin 30° - (12\sin 0° + 6\sin 90°)\hat{j}]$

$= \cancel{50}$

$= \cancel{43.30127019\hat{i} - 12\hat{i}}$

$(43.30127019 - 12)\hat{i} + (25 - 6)\hat{j}$

$= 31.30127019\hat{i} + 19\hat{j}$

$= 31.3\hat{i} + 19\hat{j}$

the the magnitude of C will be

$C = \sqrt{31.30127019^2 + 19^2}$

$= 36.61651971 m$

$= 36.62m$

and the direction is

$$\tan\theta = \frac{c_y}{c_x}$$

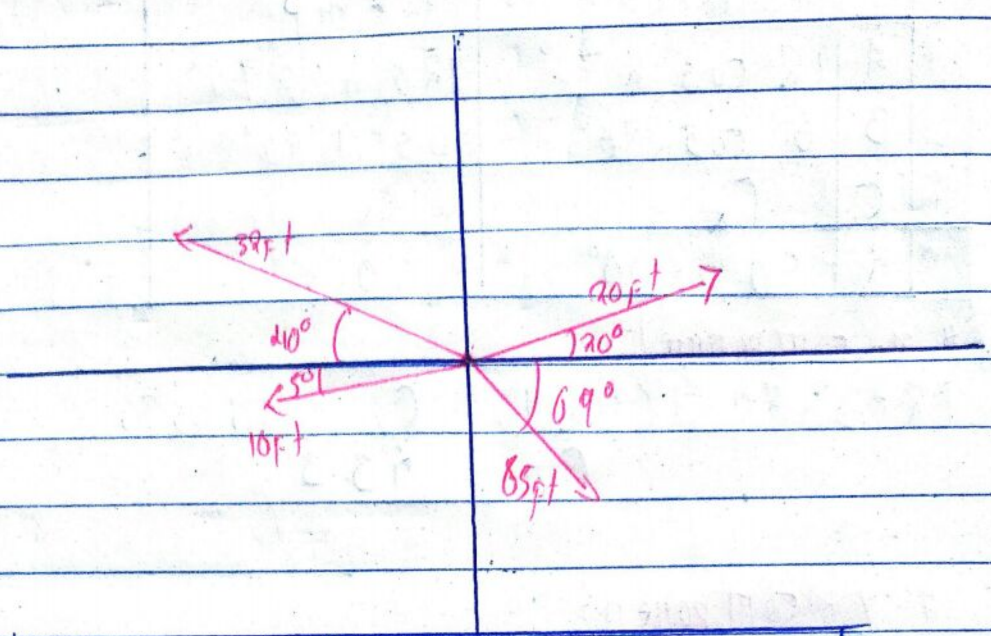
$$= \frac{19m}{31.30127019m}$$

$$\theta = 31.25792331°$$

$$= \underline{\underline{31.3°}}$$

∴ The distance moved while not paying attention was 36.62m at and angle of 31.3° in the first quadrant.

Q. 9




| | x - comp | y - comp |
|---|---|---|
| A | 20 cos 20° | 20 sin 20° |
| B | 32 cos 140° | 32 sin 140° |
| C | 10 cos 185° | 10 sin 185° |
| D | 85 cos 291° | 85 sin 291° |
| R | 7.6123 | -34.14467877 |

$$|R| = \sqrt{x^2 + y^2}$$

$$= \sqrt{(7.6123)^2 + (-34.1444)^2}$$

$$|R| = 34.98 \text{ ft}$$

$$Q = \tan^{-1}\left(\frac{y}{x}\right)$$

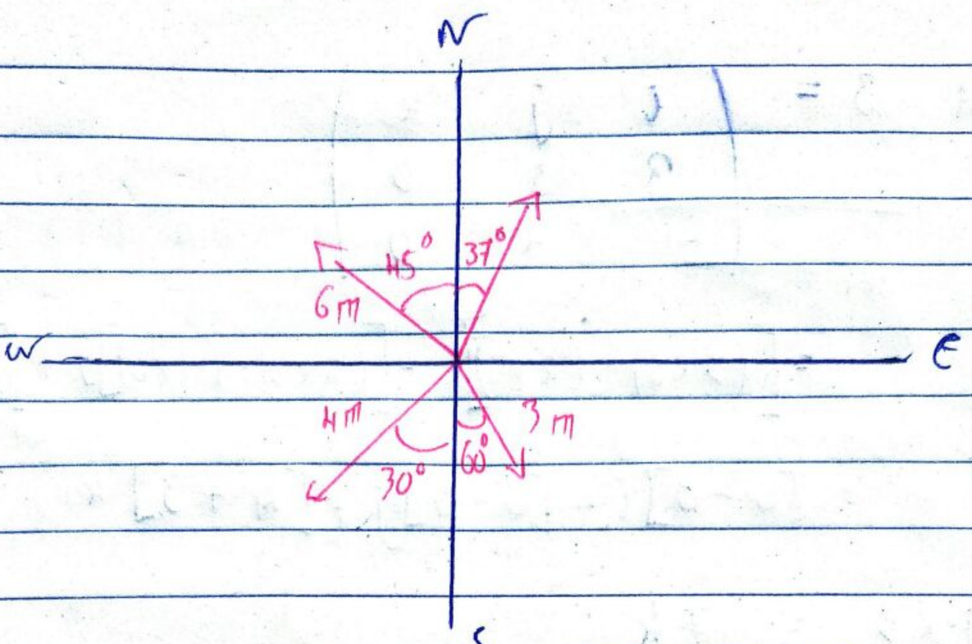
$$Q = \tan^{-1}\left(\frac{34.1444}{7.612}\right)$$

$$Q = 77.4°$$

$$Q = 360° - 77.4°$$

$$Q = 282.6°$$

Q.10




| | x-comp | y-comp |
|---|---|---|
| A | $5\cos 53^\circ$ | $5\sin 53^\circ$ |
| B | $6\cos 135^\circ$ | $6\sin 135^\circ$ |
| C | $4\cos 240^\circ$ | $4\sin 240^\circ$ |
| D | $3\cos 330^\circ$ | $3\sin 330^\circ$ |
| R | $-0.642$ | $3.27$ |

$$|R| = \sqrt{x^2 + y^2}$$

$$= \sqrt{(-0.642)^2 + (3.27)^2}$$

$$|R| = 3.33\,m$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\theta = \tan^{-1}\left(\frac{3.27}{0.642}\right)$$

$$\theta = 78.9^\circ$$

$$\theta = 180 - 78.9^\circ$$

$$\theta = 101.1^\circ$$

$A = 2i + 3j \qquad B = -i + 2j$

(11) $A \times B = \begin{pmatrix} i & j & k \\ 2 & 3 & 0 \\ -1 & 2 & 0 \end{pmatrix}$

$= [(3\times0)-(2\times0)]i - [(2\times0)-(-1\times0)]j + [(2\times2)-(-1\times3)]$

$= [0-0]i - [0-0]j + [4+3]k$

$A \times B = 7k$

12 $|A| = \sqrt{(3)^2 + (6)^2 + (-2)^2}$

$= \sqrt{49}$

$|A| = 7$

# To find the vector parallel to A we say

$\frac{|A|}{|B|} \times \bar{A}$

$\frac{7}{17}(3i + 6j - 2k)$

13 (a) The vector perpendicular to the two vectors is just finding cross product.

$A \times B = \begin{pmatrix} i & j & k \\ 1 & 2 & 3 \\ 3 & 4 & 5 \end{pmatrix}$

$= [(2\times5)-(4\times3)]i - [(1\times5)-(3\times3)]j + [(1\times4)-(3\times2)]k$

$A \times B = -2i + 4j - 2k$

$4 + 16 + 4$

now finding the unit vector of this vector.

$$\frac{A \times B}{|A \times B|}$$

$$|A \times B| = \sqrt{(-2)^2 + (4)^2 + (-2)^2}$$
$$= \sqrt{24}$$
$$= \sqrt{24}$$
$$|A \times B| = \sqrt{24}$$

$$\frac{-2i + 4j - 2k}{\sqrt{24}}$$

(b) # find $A \cdot B$

$$A \cdot B = (i + 2j + 3k) \cdot (3i + 4j + 5k)$$
$$= 3 + 8 + 15$$
$$A \cdot B = 26$$

# find also $|A|$ and $|B|$

$$|A| = \sqrt{(1)^2 + (2)^2 + (3)^2}$$
$$|A| = \sqrt{14}$$

$$|A| = \sqrt{(3)^2 + (4)^2 + (5)^2}$$
$$|A| = \sqrt{50}$$

$$A \cdot B = |A||B| \cos Q$$

$$\cos Q = \frac{A \cdot B}{|A||B|}$$

$$Q = \cos^{-1}\left(\frac{A \cdot B}{|A||B|}\right)$$

$$Q = \cos^{-1}\left(\frac{26}{\sqrt{14}\sqrt{50}}\right)$$

$$\theta = \cos^{-1}(0.982707629)$$

$$\theta = 10.7^\circ$$

(14.) Here we first find the cross product of B and C, then its magnitude. Then use the formula

$$A = \frac{7 \cdot B \times C}{|B \times C|}$$

$$B \times C = \begin{pmatrix} i & j & k \\ 2 & -3 & 5 \\ 1 & 1 & -1 \end{pmatrix}$$

$$= [(-3 \times -1) - (1 \times 5)]\,i - [(2 \times -1) - (1 \times 5)]\,j + [(2 \times 1) - (1 \times -3)]\,k$$

$$= [-2]\,i - [-7]\,j + [5]\,k$$

$$B \times C = -2i + 7j + 5k$$

$$|B \times C| = \sqrt{(-2)^2 + (7)^2 + (5)^2}$$

$$= \sqrt{78}$$

$$A = \frac{7}{\sqrt{78}}(-2i + 7j + 5k)$$

(15) $A = 3j$ , $A \times B = 3i$ $A.B = 12$

(a) $A.B = 12$

# Vector A in full can be written as ;

$A = 0i + 3j + 0k$

# then B ;

$B = xi + yj + zk$

$A.B = (0i + 3j + 0k).(xi + yj + zk)$

$0 + 3y + 0 = 12$

$3y = 12$

$y = 4$

$A = 0i + 3j + 0k$

$B = xi + \cancel{y}4j + zk$

$$A \times B = \begin{vmatrix} i & j & k \\ 0 & 3 & 0 \\ x & 4 & z \end{vmatrix}$$

$\cancel{[(0 \times 4) - (x \times 3)]i}$

$\cancel{(0 - 3x)i}$

$\cancel{-3xi = 3i}$

$\cancel{-3x = 3}$

$\cancel{x = -1}$

$[(3 \times z) - (4 \times 0)]i$

$[3z - 0]i = 3i$

$3zi = 3i$

$3z = 3$

$z = 1$

$\therefore B = 0i + 4j + k$

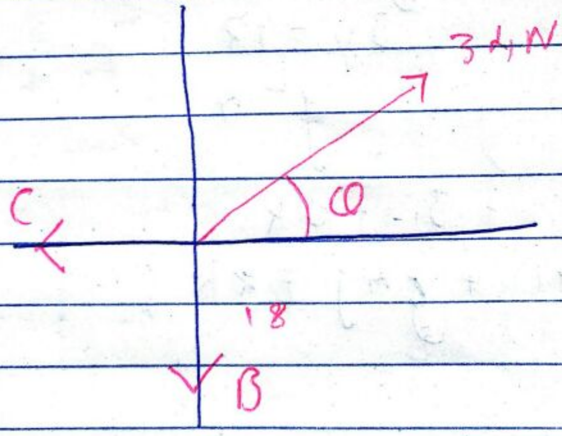
$\underline{B = 4j + k}$

(b) $\hat{B}$ [this is unit vector

$$|B| = \sqrt{(4)^2 + (1)^2}$$
$$|B| = \sqrt{17}$$

$$\therefore \hat{B} = \frac{B}{|B|}$$
$$\hat{B} = \frac{4j + k}{\sqrt{17}}$$

(16,i)




(i) since resultant a zero
# using the x-comp

$$A_x + B_x + C_x = 0$$
$$34\cos\theta + 18\cos 270° + C\cos 180° = 0$$
$$34\cos\theta + C(-1) = 0$$
$$34\cos\theta = C \quad \dots (i)$$

# This will be useful later,
# using the y-comp

$$A_y + B_y + C_y = 0$$

$$34\sin\theta + 18\sin 270 + C\sin 180 = 0$$

$$34\sin\theta - 18 = 0$$

$$\frac{34\sin\theta}{34} = \frac{18}{34}$$

$$\sin\theta = \frac{18}{34}$$

$$\theta = \sin^{-1}\left(\frac{18}{34}\right)$$

$$\theta = 32^\circ$$

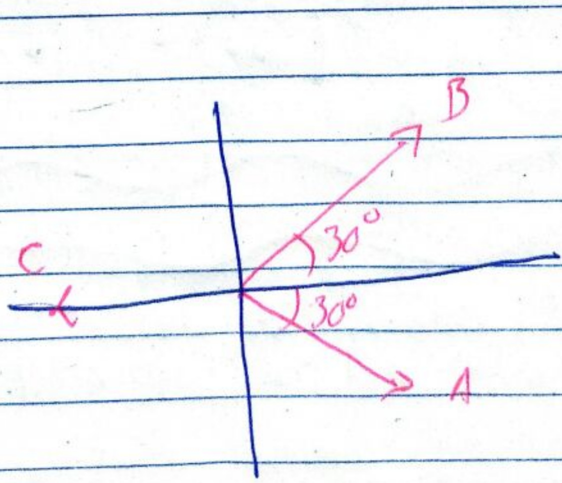
(ii) Finding the magnitude of C lets use the x-comp equation we made;

$$34\cos\theta = C$$

$$C = 34\cos 32^\circ$$

$$C = 28.8\text{ N}$$

17



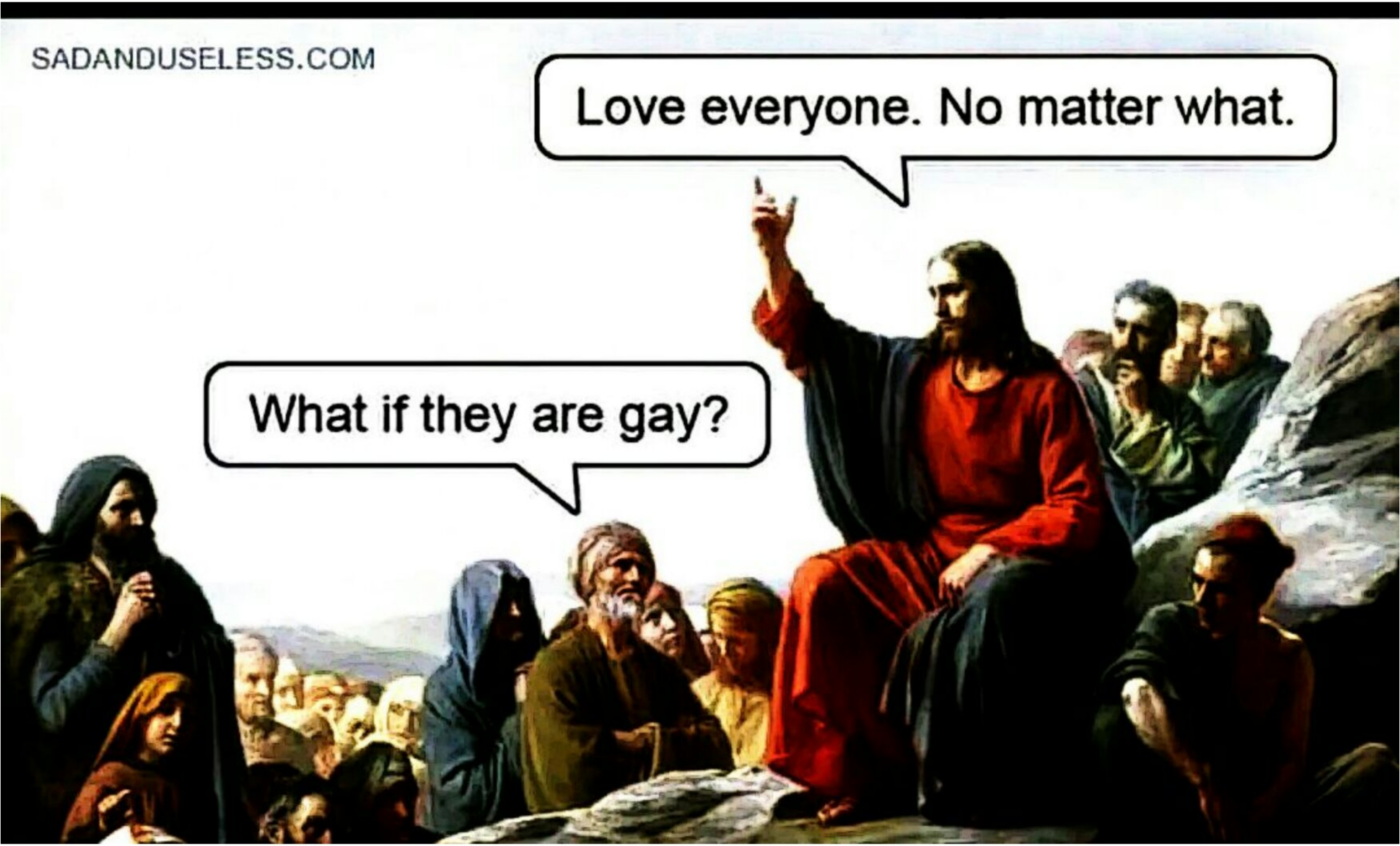
| | x - comp | y - comp |
|---|---|---|
| A | $10\cos 30^\circ$ | $10\sin 30^\circ$ |
| B | $20\cos 330^\circ$ | $20\sin 330^\circ$ |
| C | $15\cos 180^\circ$ | $15\cos 180^\circ$ |
| R | $10.98$ | $-5$ |

$$|R| = \sqrt{x^2 + y^2}$$
$$= \sqrt{(10.98)^2 + (-5)^2}$$
$$|R| = 12.07\ N$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$
$$\theta = \tan^{-1}\left(\frac{5}{10.98}\right)$$
$$\theta = 24.5^\circ$$

$$\theta = 360^\circ - 24.5^\circ$$

$$\underline{\theta = 335.5^\circ}$$


SADANDUSELESS.COM
Love everyone. No matter what.
What if they are gay?