$k(k+2)$
$(k+1)(k+3)$
36
2×18
3×12
$2(k+1)+7$
b^2-4ac
$2k+2+7$
$2k+9$ $(k+2)(2k+9)$
$k(2k+9)+2(2k+9)$
$P=36$
$S=13$
$F=4, 9$

MA110 - MATHEMATICAL METHODS TEST 2

Time allowed: Two hours thirty minutes (2:30)

Instructions:

1. You must write your **Name**, your **Computer Number** and **programme of study** on your answer sheet.
2. Calculators are **not** allowed in this paper.
3. There are four (4) questions in this paper, Attempt **All** questions and show **detailed** working for full credit

---

QUESTION ONE

a) Express $\frac{2x+1}{x^3-1}$ in partial fractions (5marks)

b) Find the centre and length of a radius of the given circle and graph it $x^2+y^2-10x=0$. (5marks)

c) Prove the result by induction: $1 \times 3 + 2 \times 4 + \cdots + n(n+2) = \frac{1}{6}n(n+1)(2n+7)$ (5marks)

d) Find the 4^{th} term in the binomial expansion $\left(2-\frac{x}{2}\right)^9$ (5marks)

e) If $xy = 64$ and $log_x y + log_y x = \frac{5}{2}$, Find x and y. (5marks)

QUESTION TWO

a) A is the point $(-1, 2)$, B is the point $(2, 3)$ and C is the point $(3, 5)$. P is a point which divides BC in the ratio 3 : 4 and Q lies on AB such that $AQ = \frac{2}{5}AB$.

(i) Find the coordinates of P (2.5 marks)

(ii) Find the coordinates of Q. (2.5 marks)

b) Find λ for which the matrix $\lambda I - A$ is a singular matrix if where I is an identity Matrix given that $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & -2 \\ 2 & -2 & 0 \end{pmatrix}$ (5 marks)

c) Show that the sum of the integers from 1 to n is $\frac{1}{2}n(n+1)$ (5marks)

$S_n = \frac{1}{2}n(n-1)d$

d) Solve the logarithmic equation : $\log(x-4)+\log(x-1)=1$ (5marks)

e) In the expansion of $(1+ax)^n$, the first three terms in ascending power of x are $1-\frac{5}{2}x+\frac{75}{8}x^2$, Find the values of n and a, and state the range of values of x for which the expansion is valid. (5marks)

QUESTION THREE

a) Find the radius of the circle with center at $C(-2,5)$ if the line $x+3y=9$ is a tangent line. (5marks)

b) Using geometrical progression, change $0.2\overline{14}$ to $\frac{a}{b}$ form , where a and b are integers and $b \neq 0$. (5marks)

c) Use mathematical induction to prove that the statement is true for all positive integers n given that 4^n-1 is divisible by 3 (5marks)

d) Graph $f(x)=log_{\frac{1}{2}}x$ by reflecting the graph of $g(x)=\left(\frac{1}{2}\right)^x$ a cross the line $y=x$ (5marks)

e) (i) Find the inverse of the matrix $A=\begin{pmatrix}3 & -1 & 2\\ 1 & 1 & 1\\ 2 & 2 & -1\end{pmatrix}$ (5marks)

(ii) Use your inverse to solve the system of linear equations

$$3x-y+2z=4$$
$$x+y+z=2$$
$$2x+2y-z=3$$

(3marks)

QUESTION FOUR

a) Write the following in sigma notation

(i) $1-\frac{2}{3}+\frac{4}{9}-\frac{8}{27}+\cdots$ (3marks)

(ii) $1^4+2^4+3^4+\cdots+n^4+(n+1)^4$ (2marks)

b) The number of grams of a certain radioactive substance present after t hours is given by the equation $Q=Q_0e^{-0.45t}$, where Q_0 represents the initial number of grams. How long will it take 2500 grams to be reduced to 1250 given $ln\left(\frac{1}{2}\right)=-0.693$ (5marks)

c) (i) Expand $(1+2x)^4$and $(1-2x)^4$ in ascending powers of x. (5marks)

(ii) Hence reduce $(1+2x)^4-(1-2x)^4$ to its simplest form. (3marks)

(iii) Using the results in (ii) evaluate $(1.002)^4-(0.998)^4$ (5marks)

$1+2x=1.002$
$2x=0.002$
$\boxed{x=0.001}$
$1-2x=0.998$
$-2x=-0.002$
$x=0.001$

$y=3\left(-\frac{12}{5}\right)+11$
$=-\frac{36}{5}+\frac{11}{1}$
$\frac{-36+55}{5}$
$\boxed{\frac{19}{5}}$

MA110 TEST 2 2023

Solutions

QUESTION 1

(a) $\frac{2x+1}{x^3-1}$ can be expressed as $\frac{A}{\text{Denominator}} + \frac{Bx+C}{\text{Denominator}}$

First we factorise Denominator (root factor theorem)

$$\frac{2x+1}{x^3-1} = \frac{2x+1}{(x^2+x+1)(x-1)}$$

$$\frac{2x+1}{x^3-1} = \frac{2x+1}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$$

multiply equation by $(x-1)(x^2+x+1)$ we get;

$2x+1 = A(x^2+x+1) + (Bx+C)(x-1)$

let $x = 1$

$2(1)+1 = A(1^2+1+1) + (B(1)+C)(1-1)$

$3 = 3A + 0$

$A = \underline{\underline{1}}$

Expanding our first expression

$2x+1 = Ax^2 + Ax + A + Bx^2 - Bx + Cx - C$

$A - C = 1$

$-C = 1-1$

$C = \underline{\underline{0}}$

$Ax - Bx + Cx = 2x$

$A - B + C = 2$

$A - B = 2$

$-B = 2-1$ $B = \underline{\underline{-1}}$

$\therefore$ Our final answer is,

$$\frac{A}{(x-1)} + \frac{Bx+C}{x^2+x+1} = \frac{1}{(x-1)} + \frac{-x}{x^2+x+1}$$

(b) $x^2+y^2-10x=0$

the center can be obtained by converting the given equation to the standard equation i.e

$$x^2+y^2-10x=0$$
$$x^2-10x+y^2=0$$
$$x^2-10x+(-5)^2-(-5)^2+y^2=0$$
$$(x-5)^2-25+y^2=0$$
$$(x-5)^2+y^2=25$$

$x-5=0$ $\quad y=0$ $\quad r^2=25$

$x=5$ $\quad y=0$ $\quad r=\sqrt{25}=5$

$\therefore$ Center $= (5,0)$ radius $= 5$

Question 1(C)

Prove by induction $1\times 3+2\times 4+\dots+n(n+2)=\frac{1}{6}n(n+1)(2n+7)$.

Proof by Induction

let $n=1$

$a_n = n(n+2)$ $\qquad$ $S_n = \frac{1}{6}n(n+1)(2n+7)$

$a_1 = 1(1+2)$ $\qquad$ $S_1 = \frac{1}{6}(1)(1+1)(2(1)+7)$

$= 3$ $\qquad$ $= \frac{18}{6}$

$= 3$

$a_1 = S_1 = 3$ Statement is true for $n=1$.

let $n=k$

$a_k = k(k+2)$

$S_k = \frac{1}{6}(k)(k+1)(2k+7)$

Since $n=1$ is true, then we will assume that k is true for values of n.

let $n=k+1$

$a_{k+1} = (k+1)(k+1+2)$

$= k+1(k+3)$

and $S_{k+1} = \frac{1}{6}(k+1)(k+1+1)(2k+2+7)$

$S_{k+1} = \frac{1}{6}(k+1)(k+2)(2k+7)$

We will also assume that k+1 is true for value of n.

Using the identity $S_{k+1} = S_k + a_{k+1}$

$$S_{k+1} = \frac{(k)(k+1)(2k+7)}{6} + \frac{(k+1)(k+3)}{1}$$

$$S_{k+1} = \frac{(k)(k+1)(2k+7) + 2(k+1)(k+3)}{6}$$

$$= \frac{(k+1)\left(k(2k+7) + 2(k+3)\right.}{6}$$

$$= \frac{(k+1)\left((2k^2+7k) + 2k+6\right)}{6}$$

$$= \frac{(k+1)\left(2k^2+9k+6\right)}{6}$$

$$= \frac{(k+1)\left(k(2k+9) + 2(2k+9)\right)}{6}$$

$$= \frac{(k+1)(2k+9)(k+2)}{6}$$

Hence Proved.

(d) Find the 4^{th} term in the binomial $\left(2 - \frac{x}{2}\right)^9$

$$T_{r+1} = {}^nC_r \, a^{n-r} b^r$$

where $r + 1 = 4$

$r = 4 - 1 = 3$

$${}^nC_r = \frac{n!}{(n-r)!\,r!}$$

$$T_{3+1} = {}^9C_3 (2)^{9-3} \left(-\frac{x}{2}\right)^3$$

$$= {}^9C_3 \cdot (2)^6 \left(\frac{-x^3}{8}\right)$$

$$= {}^9C_3 \; \frac{-64x^3}{8}$$

$$= \frac{9!}{(9-3)!\,3!} \left(\frac{-64x^3}{8}\right)$$

$$= \frac{\cancel{9}^{3} \times \cancel{8}^{4} \times 7 \times \cancel{6!}}{\cancel{6!}\;\cancel{3} \times \cancel{2} \times 1} \left(\frac{-64x^3}{8}\right)$$

$$= 3 \times 4 \times 7 \times -8x^3$$

$$= \underline{\underline{-672x^3}}$$

The SPIRAL

Your INTUITION calls you to make gradual changes in your life that lead towards ongoing evolution, development, and transcendence. Pure consciousness can see past the repeating cycles of the mind and urges you to move beyond what you think you know.

1(e) $xy = 64 \quad \log_x y + \log_y x = \frac{5}{2}$,
Find ~~xy~~ x and y.

Solution

$\log_x y + \log_y x = \frac{5}{2}$ $\qquad \log_x x = 1$

$2(\log_x y + \log_y x) = 5$

$2\left(\log_x y + \frac{\log_x x}{\log_x y}\right) = 5$ $\qquad$ let $\log_x y = N$

$2\left(N + \frac{1}{N}\right) = 5$

$2N + \frac{2}{N} = 5$ multiply by N

$2N^2 + 2 = 5N$ $\qquad$ Sum $= -5$ product $= 4$

$2N^2 - 5N + 2 = 0$ $\qquad$ Factors $= -4$ & -1

$(2N^2 - 4N)(-N + 2) = 0$

$2N(N-2) - 1(N-2) = 0$

$2N - 1 = 0 \quad N - 2 = 0$

$N = \frac{1}{2}$ or $N = 2$

$\log_x y = \frac{1}{2}$ or $\log_x y = 2$

$x^{\frac{1}{2}} = y$ or $x^2 = y$

Since $xy = 64$, then $y = \frac{64}{x}$

$x^{\frac{1}{2}} = \frac{64}{x}$ or $x^2 = \frac{64}{x}$

$x^{\frac{3}{2}} = 64$ or $x^3 = 64$

$x = \sqrt[3]{(64)^2} = 16$ $\qquad x = \sqrt[3]{64} = 4$

$y = \frac{64}{x}$; $y = \frac{64}{64^{\frac{2}{3}}}$

$y = \frac{64}{4}$

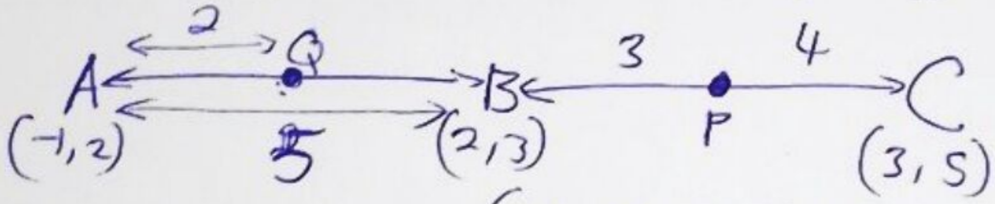
$y = 16$

$y = \frac{64}{64^{\frac{2}{3}}} \times \frac{64^{\frac{3}{2}}}{64^{\frac{3}{2}}}$?

$y = 64\sqrt[2]{64^3}$ Not taken seriously

QUESTION 2

(a) $A(-1,2)$ $B(2,3)$ $C(3,5)$



(i) $m.p = P = \left(\frac{mx_2 + x_1 n}{m+n}, \frac{my_2 + y_1 n}{m+n}\right)$

$P = \left(\frac{3(3)+4(2)}{3+4}, \frac{3(5)+4(3)}{3+4}\right)$

$P = \left(\frac{9+8}{7}, \frac{15+12}{7}\right)$

$P = \left(\frac{17}{7}, \frac{27}{7}\right)$

(ii) Coordinates of Q.

$$\text{Point } Q = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

$$= \left(\frac{2(2) + 3(-1)}{2+3}, \frac{2(3) + 3(2)}{2+3} \right)$$

$$= \left(\frac{4-3}{5}, \frac{6+6}{5} \right)$$

$$= \left(\frac{1}{5}, \frac{12}{5} \right)$$

(b) Find λ

$\lambda I - A$

$$\lambda\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} - \begin{pmatrix}1&0&2\\0&-1&-2\\2&-2&0\end{pmatrix}$$

note: $I = \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$ 3X3 matrix.

$$\begin{pmatrix}\lambda&0&0\\0&\lambda&0\\0&0&\lambda\end{pmatrix} - \begin{pmatrix}1&0&2\\0&-1&-2\\2&-2&0\end{pmatrix}$$

$$= \begin{pmatrix}\lambda-1&0&-2\\0&\lambda+1&2\\-2&2&\lambda\end{pmatrix}$$

Det = 0 therefore

$$\begin{vmatrix}\lambda-1&0&-2\\0&\lambda+1&2\\-2&2&\lambda\end{vmatrix} = 0$$

Top

$$\begin{vmatrix} \lambda-1 & 0 & -2 \\ 0 & \lambda+1 & 2 \\ -2 & 2 & \lambda \end{vmatrix} \begin{matrix} \lambda-1 & 0 \\ 0 & \lambda+1 \\ -2 & 2 \end{matrix}$$

Bottom

$$\Sigma \text{ top} = (-2\times -2)(\lambda+1) + (2\times 2)(\lambda-1)$$
$$= 4\lambda+4 + 4\lambda-4$$
$$= 8\lambda$$

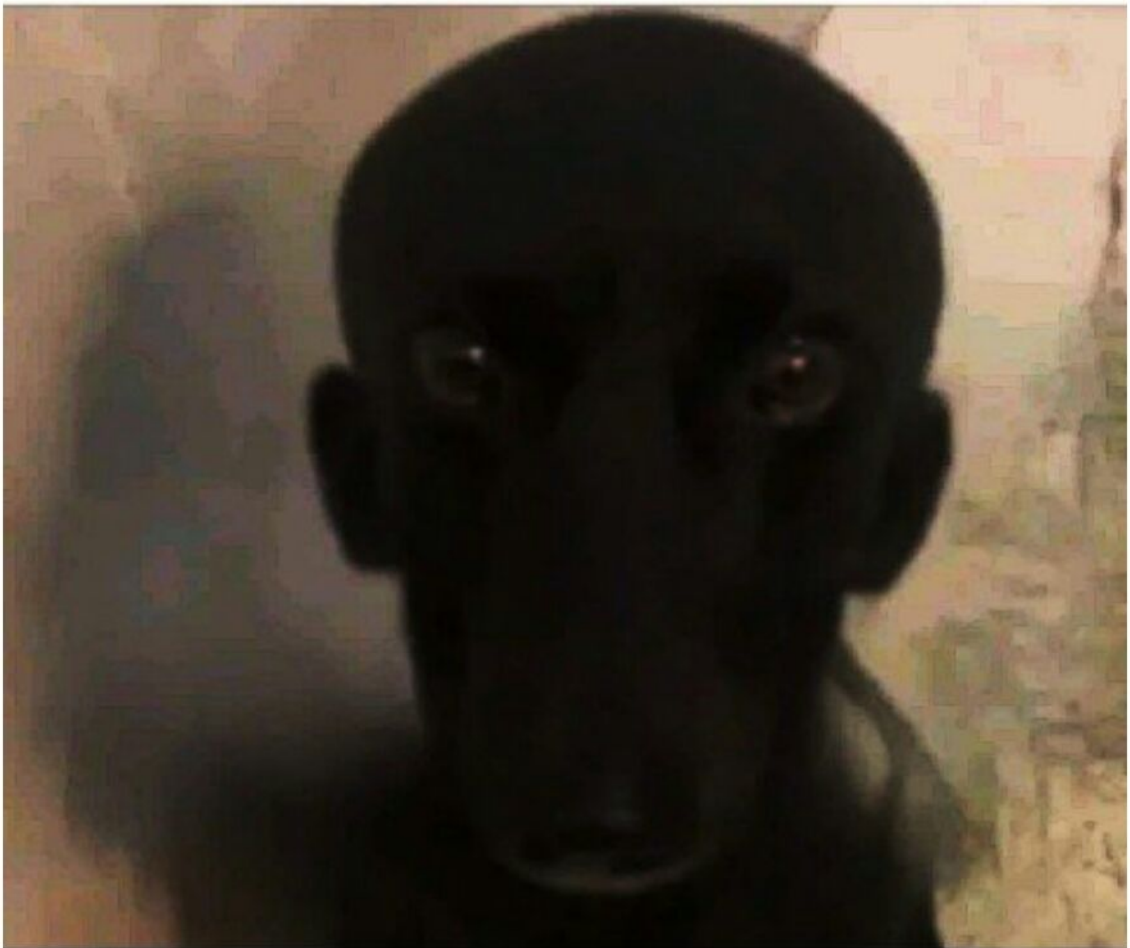
$$\Sigma \text{ bottom} = (\lambda-1)(\lambda+1)(\lambda) + 0 + 0$$
$$= (\lambda^2+\lambda-\lambda-1)(\lambda)$$
$$= \lambda^3-\lambda$$

$$\text{Det} = \Sigma \text{ bottom} - \Sigma \text{ top} = 0$$
$$= \lambda^3-\lambda-8\lambda = 0$$
$$= \lambda^3-9\lambda = 0$$
$$= \lambda(\lambda^2-9) = 0$$
$$\lambda = 0 \quad \text{or} \quad \sqrt{\lambda^2} = \sqrt{9}$$
$$\lambda = \pm 3$$

I Am Diode
22 minutes ago

@Jonathan Toliver jr.

after cutting a Pala
Dad: now you look smart
me:



Reply

(c) We can try to show that the sum of integers from 1 to n is $\frac{1}{2}(n)(n+1)$ by using mathematical induction.

$$1+2+3+\ldots n = \frac{1}{2}(n)(n+1)$$

let $n = 1$

$a_k = n$ and $S_k = \frac{1}{2}(n)(n+1)$

$a_k = 1$ and $S_k = \frac{1}{2}(1)(1+1)$

$S_k = \frac{1}{2}(2) = \underline{\underline{1}}$

Hence statement is true for positive values of n.

let $n = k$

$a_k = k$ and $S_k = \frac{1}{2}(k)(k+1)$

let $n = k+1$

$a_{k+1} = k+1$ and $S_{k+1} = \frac{1}{2}(k+1)(k+2)$

Since

$$S_{k+1} = a_{k+1} + S_k$$

then

$$S_{k+1} = (k+1) + \frac{1}{2}(k)(k+1)$$

$$= \frac{2(k+1) + (k)(k+1)}{2}$$

$$= \frac{(2+k)(k+1)}{2} = \frac{1}{2}(k+1)(k+2)$$

Hence Proved

(d) Solve $\log(x-4) + \log(x-1) = 1$

Solution

$\log(x-4) + \log(x-1) = 1$ is the same as

$\log(x-4)(x-1) = 1$ then apply BAM

Base to the power answer = middle part.

Base = 10 middle part = $(x-4)(x-1)$

answer = 1

$$10^1 = (x-4)(x-1)$$

$$10 = x^2 - x - 4x + 4$$

$$x^2 - 5x + 4 - 10 = 0$$

$$x^2 - 5x - 6 = 0$$

Sum = -5 product = -6

factors = -6 and 1

$$(x^2 - 6x) + (x - 6) = 0$$

$$x(x-6) + 1(x-6) = 0$$

$$x + 1 = 0 \qquad x - 6 = 0$$

$$x = -1 \qquad x = 6$$

(e) $(1+ax)^n$, 3 terms are $1 - \frac{5}{2}x + \frac{75}{8}x^2$

We can expand the expression Using binomial theorem.

$$a^n + na^{n-1}b + \frac{n(n-1)a^{n-2}b^2}{2!}$$

and apply comparisons!

$$1 + n(ax) + \frac{n(n-1)(ax)^2}{2!} = 1 - \frac{5}{2}x + \frac{75}{8}x^2$$

$$n(ax) = -\frac{5}{2}x \qquad \frac{n(n-1)(a^2x^2)}{2\times 1} = \frac{75}{8}x^2$$

$$\frac{1}{x}\left(nax = -\frac{5}{2}x\right)$$

$$\frac{na}{1} = -\frac{5}{2}$$

$$2na = -5$$

$$n = \frac{-5}{2a} \quad \text{(substitute in equation 2!)}$$

$$2\left(\frac{-\frac{5}{2a}\left(\frac{-5}{2a}-1\right)a^2x^2}{2} = \frac{75}{8}x^2\right)$$

$$\frac{-5}{2a}\left(\frac{-5}{2a}-1\right)a^2 = \frac{75}{4}$$

$$\frac{-5}{2a}\left(\frac{-5-2a}{2a}\right)a^2 = \frac{75}{4}$$

$$\frac{25+10a}{4a^2}(a^2) = \frac{75}{4}$$

$$\left[\frac{25+10a}{4} = \frac{75}{4}\right]\times 4$$

$$25 + 10a = 75$$

$$10a = 75 - 25$$

$$\frac{10a}{10} = \frac{50}{10}$$

$$a = 5$$

$$n = -\frac{5}{2(5)}$$

$$n = \frac{-5}{10}$$

$$n = \frac{-1}{2}$$

QUESTION 3

(a) Find the radius of the circle with center $C(-2,5)$ if the line $x+3y=9$ is a tangent line.

Solution

the radius is simple the perpendicular distance to the circle.

$$r = \frac{|ax+by+c|}{\sqrt{a^2+b^2}}$$

$$C = (\underset{x}{-2}, \underset{y}{5}) \qquad x+3y-9=0$$

$$a=1 \quad b=3 \quad c=-9$$

$$r = \frac{|(1)(-2)+3(5)+(-9)|}{\sqrt{1^2+3^2}}$$

$$r = \frac{|-2+15-9|}{\sqrt{1+9}}$$

$$r = \frac{4}{\sqrt{10}} = \frac{4\sqrt{10}}{10} \text{ units}$$

(b) Using geometric progression, Change $0.2\overline{14}$ to $\frac{a}{b}$ form, where a and b are intergers and $b \neq 0$.

Solution

$0.2\overline{14}$ is written as

$0.2\overline{14} + 0.2\overline{14}\,\overline{14} + 0.2141 4\overline{14} + \ldots$

$\boxed{*0.2 + 0.0\overline{14} + 0.000\overline{14}}$ (Consider this)

$$r = \frac{0.000\overline{14}}{0.0\overline{14}} = \frac{14}{100000} \div \frac{14}{1000}$$

$$= \frac{\cancel{14}}{100\,000} \times \frac{1000}{\cancel{14}}$$

$$r = \frac{1000}{100\,000} = \frac{1}{100}$$

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{0.014}{1 - \frac{1}{100}} = \frac{14}{1000} \div \frac{99}{100}$$

$$= \frac{14}{1000} \times \frac{100}{99}$$

$$= \frac{14}{990} + \frac{2}{10}$$

$$= \frac{14 + 198}{990}$$

$$= \frac{106}{495}$$

(c) Proof By Induction ($4^n - 1$ is divisible by 3)

let $n = 1$ $\quad 4^n - 1$

$4^1 - 1 = 3$ $\quad$ 3 is divisible by 3

hence statement is true.

let $n = k$

$4^n - 1 = 4^k - 1 = 3x$ So we assume that this statement is true for values of n.

let $n = k+1$

$4^{k+1} - 1 = 3x$ Since the statement for k was true, then this statement is also true for +ve Values of n.

$$(4^k - 1 = 3x) \times 4$$

$$4^{k+1} - 4 = 4(3x)$$

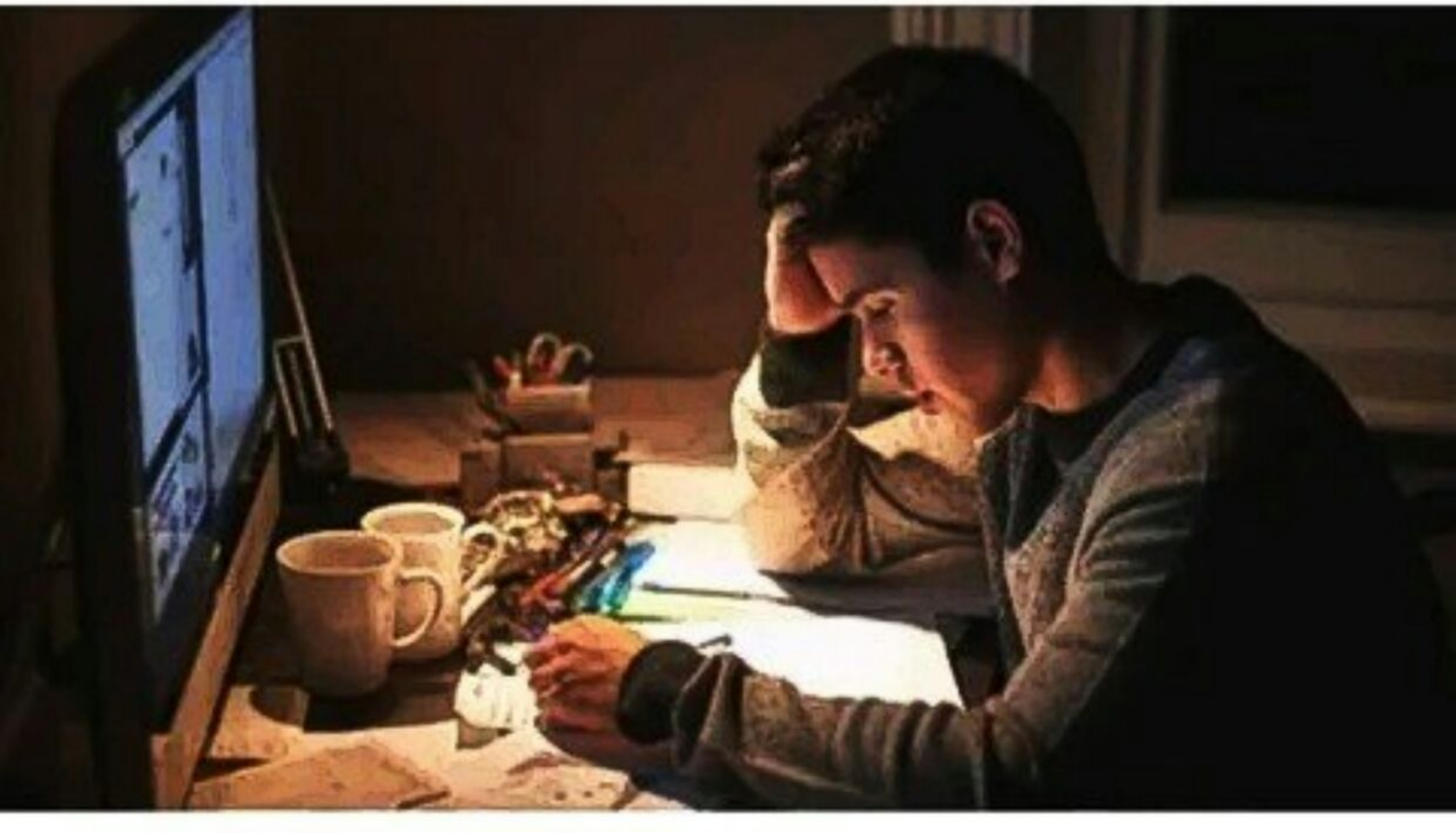
$$4^{k+1} - 1 - 3 = 4(3x)$$

$$4^{k+1} - 1 = 4(3x) + 3$$

$$4^{k+1} - 1 = 3(4x+1)$$

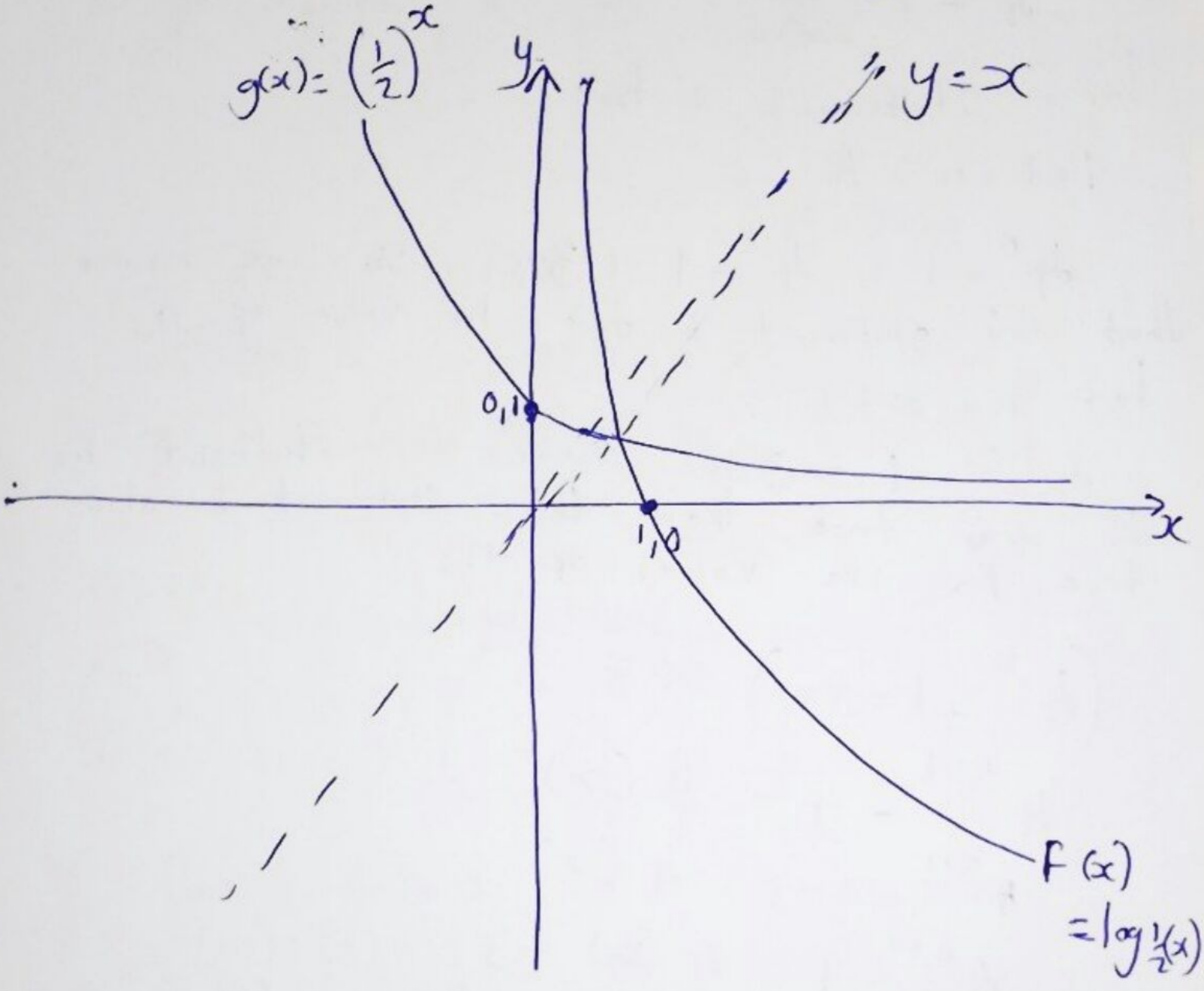
Hence Proved

I'm really bad at studying but I don't know why



I feel like I'll be rich in future

(d) Graph $f(x) = \log_{\frac{1}{2}} x$ by reflecting the graph of $g(x) = \left(\frac{1}{2}\right)^x$ across a line $y = x$



No Sweating

Mune!

(c) (1) Inverse of $A = \frac{1}{\det}$ (Adjoint)

Top

$$\text{Det} = \left|\begin{matrix} 3 & -1 & 2 \\ 1 & 1 & 1 \\ 2 & 2 & -1 \end{matrix}\right| \begin{matrix} 3 & -1 \\ 1 & 1 \\ 2 & 2 \end{matrix}$$

Bottom

$\sum \text{Tot} = 4 + 6 + 1 = 11$

$\sum \text{Bot} = -3 + (-2) + 4 = -1$

$\text{Det} = \sum \text{bottom} - \sum \text{top}$

$= -1 - (+11) = \underline{\underline{-12}}$

<u>Matrix of Co-factors</u>

$$\begin{pmatrix} 3 & -1 & 2 \\ 1 & 1 & 1 \\ 2 & 2 & -1 \end{pmatrix} \begin{pmatrix} \overset{+}{a} & \overset{-}{b} & \overset{+}{c} \\ \overset{-}{d} & \overset{+}{e} & \overset{-}{f} \\ \overset{+}{g} & \overset{-}{h} & \overset{+}{I} \end{pmatrix}$$

$a = (1 \times -1) - (2) = -3$

$-b = (-1 \times 1) - (2) = 3$

$c = (2 - 2) = 0$

$-d = (1 - 4) = 3$

$e = (-3 - 4) = -7$

$-f = (6 - (-2)) = -8$

$g = (-1 - 2) = -3$

$-h = (3 - 2) = -1$

$I = 3 - (-1) = 4$

$$\text{Co factors} = \begin{pmatrix} -3 & 3 & 0 \\ 3 & -7 & -8 \\ -3 & -1 & 4 \end{pmatrix}$$

$$\text{Transpose of Co factors} = \begin{pmatrix} -3 & 3 & -3 \\ 3 & -7 & -1 \\ 0 & -8 & 4 \end{pmatrix}$$

$$A^{-1} = \frac{1}{-12} \begin{pmatrix} -3 & 3 & -3 \\ 3 & -7 & -1 \\ 0 & -8 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{3}{12} & \frac{-3}{12} & \frac{3}{12} \\ \frac{-3}{12} & \frac{7}{12} & \frac{1}{12} \\ 0 & \frac{8}{12} & \frac{-4}{12} \end{pmatrix}$$

(e)(ii) $\frac{1}{-12}\begin{pmatrix} -3 & 3 & -3 \\ 3 & -7 & -1 \\ 0 & -8 & 4 \end{pmatrix}\begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{-12}\begin{pmatrix} (-3\times 4)+(3\times 2)+(-3\times 3) \\ (3\times 4)+(-7\times 2)+(-1\times 3) \\ 0+(-8\times 2)+(4\times 3) \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{-12}\begin{pmatrix} -12+6-9 \\ 12-14-3 \\ -16+12 \end{pmatrix}$$

Multiply by $-\frac{1}{12}$ we get

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{-15}{-12} \\ \frac{-5}{-12} \\ \frac{-4}{12} \end{pmatrix}$$

$\therefore$ $x = \frac{5}{4}$ $\quad y = \frac{5}{12}$ $\quad z = \frac{1}{3}$

QUESTION 4

(i) $1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \dots$ In Sigma Form

$$r = \frac{-\frac{2}{3}}{1} = \frac{-\frac{2}{3}}{1}$$

$$a_n = a_1 r^{n-1}$$

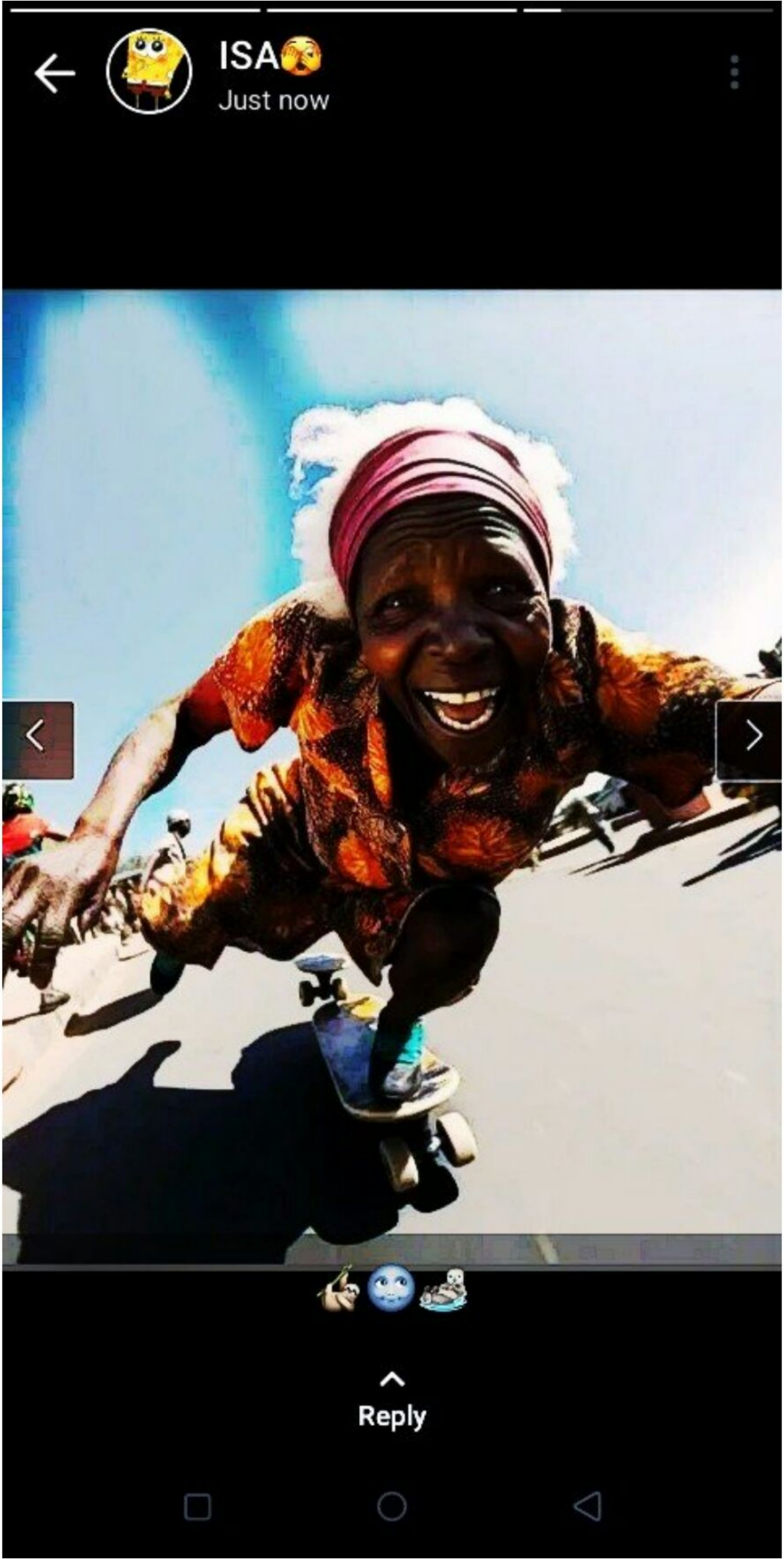
$$\sum_{n=1}^{n_f=\infty} 1\left(\frac{-2}{3}\right)^{n-1}$$

(ii) $1^4 + 2^4 + 3^4 + \dots + n^4 + (n+1)^4$

$$n_f = (n+1)^4$$

$$\therefore \sum_{n=1}^{n_f=(n+1)^4} (n)^4$$

YES I GOT FULL MARKS FOR THIS!


ISA
Just now
Reply

(b) We have $Q = Q_0 e^{-0.45t}$, Q_0 is initial

$$1250 = 2500 e^{-0.45t}$$

$$= \frac{1250}{2500} = \frac{2500}{2500} e^{-0.45t}$$

$$= \frac{1}{2} = e^{-0.45t}$$

Introduce ln men!

$$= \ln \frac{1}{2} = \ln e^{-0.45t}$$

$$= \frac{\cancel{-0.45}t}{\cancel{-0.45}} = \frac{\ln \frac{1}{2}}{-0.45}$$

$$t = \frac{-0.693}{-0.45}$$

$$= \frac{693}{1000} \div \frac{45}{100}$$

$$= \frac{693}{450} \text{ hours}$$

and you, feel happy

(C) (i) $(1+2x)^4$ and $(1-2x)^4$

$$a^n + na^{n-1}b + \frac{n(n-1)\,a^{n-2}b^2}{2!}$$

awe let's use pascals 

$$\begin{array}{c} 1 \\ 1\ \ 1 \\ 1\ \ 2\ \ 1 \\ 1\ \ 3\ \ 3\ \ 1 \\ 1\ \ 4\ \ 6\ \ 4\ \ 1 \end{array}$$

$$(1+2x)^4 = 1^4 + 4(2x)^1 + 6(2x)^2 + 4(2x)^3 + 1(2x)^4$$

$$= 1^4 + 4(2x) + 6(2x)^2 + 4(2x)^3 + (2x)^4$$

$$= 1 + 8x + 24x^2 + 32x^3 + 16x^4$$

$$(1-2x)^4 = 1^4 + 4(-2x) + 6(-2x)^2 + 4(-2x)^3 + 1(-2x)^4$$

$$= 1 - 8x + 6(4x^2) + 4(-8x^3) + 1(16x^4)$$

$$= 1 - 8x + 24x^2 - 32x^3 + 16x^4$$

(ii) $(1+2x)^4 - (1-2x)^4$

We will use the expansions in (i) men!

So $(1+2x)^4 - (1-2x)^4$

$= 1+8x+24x^2+32x^3+16x^4 - 1+8x-24x^2+32x^3-16x^4$

$= \underline{\underline{16x+64x^3}}$

(iii) Same situation with (ii) men

$(1+0.002)^4 - (1-0.002)^4$

$x = 0.002$

We will write

$16x+64x^3 = 16(0.002) + 64(0.002)^3$

$= 16\left(\frac{2}{1000}\right) + 64\left(\frac{2}{1000}\right)^3$

$= \frac{32}{1000} + 64\left(\frac{8}{1000\,000}\right)$

$= \frac{32\,000 + 512}{1000\,000}$

$= \frac{32512}{1\,000\,000}$

$= \underline{\underline{0.032512}}$

GRADE	RANGE	CBU GRADE POINT EQUIVALENT	
		FULL COURSE	HALF COURSE
A+	≥ 86%	5	2.5
A	76% - 85%	4	2
B+	68% - 75%	3	1.5
B	62% - 67%	2	1
C+	56% - 61%	1	0.5
C	50% - 55%	0	0
D+	40% - 49%	0	0
D	≤39%	0	0

Where there are 12 courses

Distinction	:	42.5 and above
Merit	:	30.5 - 42
Credit	:	18.5 - 30
Pass	:	0 – 18