(114) marks

THE COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS

2016/17 academic year

second term test

MA110-Mathematical Methods

INSTRUCTIONS: (1) Attempt all questions

(2) Show detailed working for full credit

(3) Calculators are not allowed in this paper

TIME ALLOWED: Three (3) hours

1.a) Use G.P repeating decimal $0.\overline{42}$ to a/b form, where a and b are integers and $b \neq 0$. Express a/b reduced form. (+5)

b) Use mathematical induction to prove each of the sum formulas for the indicated sequence for all positive integers n

$$S_n = \frac{(5^{n+1}-5)}{4} \text{ for } a_n = 5^n$$ (+5) (+28)

c) Expand the function $\frac{x+5}{-3+5x-2x^2}$ in ascending powers of x, giving the first three terms and the gener term, and state the necessary restrictions on the values of x (+8)

d) Find the ratio in which the line-segment joining the points $(5, -4)$ and $(2, 3)$ is divided by the x-axis. (+5)

e) The line l has equation $2x - y - 1 = 0$. The line m passes through the point $A(0,4)$ and is perpendicular to the line l. Find an equation of m and show that the lines l and m intersect at the point $P(2,3)$. (+5)

2. a) Prove that $\log_{\frac{1}{x}} a = -\log_x a$ (+5)

b) Find the equation of the tangent at the point (2,1) of the circle $x^2 + y^2 - 4y - 1 = 0$ (+5)

c) Expand $(1+x)^{\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^3 and hence find an approximation for $\sqrt{1.08}$ (+5)

d) Find all possible values of λ given that the matrix $(\lambda I - A)$ is singular where $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$ and I is the id matrix. (+5) (+28)

e) Express $\log_9 xy$ in terms of $\log_3 x$ and $\log_3 y$. Hence solve for x and y the simultaneous equations

$$\log_9 xy = \frac{5}{2}$$
$$\log_3 x \log_3 y = -6$$

expressing your answers as simply as possible (+8)

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a) Find the common ratio and the sum of the first 10 terms of the series
$$\log x + \log x^2 + \log x^4 + \log x^8 + \cdots$$ +5

(b) Show that the sum of an infinite geometric sequence is given by $S_\infty = \frac{a_1}{1-r}, |r| < 1$ where a_1 is first term and r the common ratio. +5

c) Solve for real x the equation
$$6^{x+2} = 2(3^{2x})$$ +5

+28

d) Find the inverse of the matrix $A = \begin{pmatrix} 3 & -1 & 2 \\ 1 & 1 & 1 \\ 2 & 2 & -1 \end{pmatrix}$. Hence use your inverse to solve the system of linear equa +8

$$A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix}$$

e) Given that the equation of the circle in the form $x^2 + y^2 + Dx + Ey + F = 0$. Show that the centre is give $\left(-\frac{D}{2}, -\frac{E}{2}\right)$ and the radius $r = \sqrt{\frac{D^2 + E^2 - 4F}{4}}$. +5

4. a) Use mathematical induction to prove that the statement is true for all positive integers n
$7^n - 3^n, n \geq 1$ is divisible by 4 +5

b). Sketch each of the exponential functions and determine its domain and range.
$f(x) = -4^{x-1} + 1$ +5 and $g(x) = |2 - \log_3(x-2)|$ +5

c) Show that $\binom{n+1}{3} - \binom{n-1}{3} = (n-1)^2$. +5

+30

d) Write the following series in sigma notation and find their sums.
$$1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \cdots$$ +8

e) At what rate of interest compounded continuously will an investment of \$500 grow to \$1000 in 10 years? +5

(1) $0.\overline{42} = 0.424242\ldots$

$0.42 = 0.42 + 0.0042 + 0.000042 + \ldots$

$= \frac{42}{100} + \frac{42}{10000} + \frac{42}{1{,}000{,}000} + \ldots$

$= 42\underbrace{\left(\frac{1}{100} + \frac{1}{10{,}000} + \frac{1}{1000{,}000} + \ldots\right)}_{\text{Infinite geometric Series}}$

$S_\infty = \frac{a}{1-r}, \quad a = \frac{1}{100}$

$r = \frac{1/10000}{1/100} = \frac{1}{10{,}0\cancel{00}} \cdot \frac{1\cancel{00}}{1} = \frac{1}{100}$

$= 0.01$

$S_\infty = \frac{1/100}{1 - 1/100} = \frac{1/100}{\frac{99}{100}} = \frac{1}{100} \cdot \frac{100}{99}$

$= \frac{1}{99}$

So

$0.\overline{42} = 42\left(\frac{1}{99}\right) = \frac{42}{99} = 14/33$

$S_n = \frac{5^{n+1} - 5}{4}$ for $a_n = 5^n$

Mathematical induction

(i) for $n=1$,

$$S_1 = \frac{5^{1+1} - 5}{4} = \frac{5^2 - 5}{4} = \frac{20}{4} = 5$$

$$a_1 = 5^1 = 5 \quad - \text{ true.}$$

or $$S_n = 5^1 + 5^2 + \dots + 5^n = \frac{(5^{n+1} - 5)}{4}.$$

$$S_1 = 5^1 = 5 \quad - \text{ true.}$$

(ii) Assume S_n is true for $n=k$ i.e

$$5^1 + 5^2 + \dots + 5^k = \frac{5^{k+1} - 5}{4}$$

we prove for $n = k+1$,

$$S_{k+1} = \frac{5^{(k+1)+1} - 5}{4} = \frac{5^{k+2} - 5}{4}$$

we prove that $S_{k+1} = S_k + a_{k+1}$,

$a_{k+1} = 5^{k+1}$, so

$$S_{k+1} = \frac{5^{k+1} - 5}{4} + 5^{k+1}$$

$$= \frac{5^{k+1} - 5}{4} + \frac{4 \cdot 5^{k+1}}{4} = \frac{5^{k+1} - 5 + 4 \cdot 5^{k+1}}{4}$$

$$= \frac{5^{k+1}(1+4) - 5}{4} = \frac{5^{k+1} \cdot 5 - 5}{4} = \frac{5^{k+2} - 5}{4}$$

the statement is true for $n=k+1$,

$$S_n = \frac{5^{k+1}-5}{4} \text{ for } a_n = 5^n \text{ for}$$

all possible integers.

c) $$\frac{x+5}{-3+5x-2x^2} = \frac{x+5}{(1-x)(-3+2x)}$$

$$\frac{x+5}{(1-x)(-3+2x)} = \frac{A}{(1-x)} + \frac{B}{(-3+2x)}$$

$$\frac{x+5}{(1-x)(-3+2x)} = \frac{A(-3+2x)+B(1-x)}{(1-x)(-3+2x)}$$

$$x+5 = A(-3+2x) + B(1-x)$$

Critical values:

$x = 3/2$

$$3/2 + 5 = B(1-3/2)$$

$$3/2 + 5 = -B/2$$

$$B = -(3+10)$$

$$B = \underline{\underline{-13}}$$

$x = 1$

$$1+5 = A(-3+2)$$

$$6 = -A$$

$$A = \underline{\underline{-6}}$$

$\frac{3}{(-3+2x)}$

$$= \frac{-6}{(1-x)} - \frac{13}{(-3+2x)}$$

$$= -6(1-x)^{-1} - 13(-3+2x)^{-1}$$

$$= -6(1-x)^{-1} - 13\left(-3\left(1-\tfrac{2}{3}x\right)\right)^{-1}$$

$$= -6(1-x)^{-1} + \frac{13}{3}\left(1-\tfrac{2}{3}x\right)^{-1}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots$$

$$= -6(1+x+x^2+\cdots) + \frac{13}{3}\left(1+\tfrac{2x}{3}+\tfrac{4}{9}x^2+\cdots\right)$$

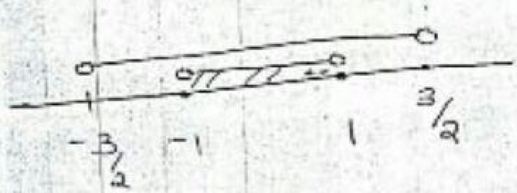
$$= (-6-6x-6x^2+\cdots) + \left(\tfrac{13}{3}+\tfrac{26}{9}x+\tfrac{52}{27}x^2 \cdots\right)$$

$$= -6\sum_{k=0}^{n} x^k + \frac{13}{3}\sum_{k=0}^{n}\left(\frac{2}{3}\right)^k x^k$$

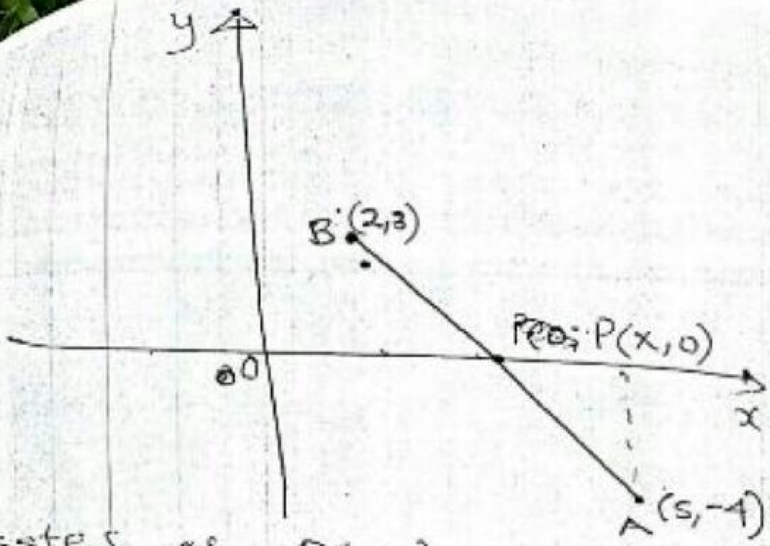
$$= -\frac{5}{3} - \frac{28}{9}x - \frac{110}{27}x^2 + \cdots$$

Validity: $(1-x)^{-1}$, $|x|<1$

$\left(1-\frac{2}{3}x\right)^{-1}$, $\left|-\frac{2}{3}x\right|<1 \Rightarrow |x|<\frac{3}{2}$



Validity of expansion: $(-1, 1)$



Coordinates of $P(x,y)$ are

$$\left(\frac{m x_2+n x_1}{m+n}, \frac{m y_2+n y_1}{m+n}\right) = (x,y) = (x,0)$$

$$\overset{(x_1,\,y_1)}{A(5,-4)} \text{ and } \overset{x_2,\,y_2}{B(2,3)}$$

$$\begin{cases} x = \dfrac{m x_2+n x_1}{m+n} \\[2ex] 0 = \dfrac{m y_2+n y_1}{m+n} \end{cases} \quad \begin{cases} x = \dfrac{2m+5n}{m+n} \\[2ex] 0 = \dfrac{3m+(-4)n}{m+n} \end{cases}$$

$$\begin{cases} x = \dfrac{2m+5n}{m+n} \\[2ex] 3m = 4n \Rightarrow \dfrac{m}{n} = \dfrac{4}{3} \end{cases}$$

$$n : m = 3 : 4$$

Let $(2,3) = (x_1, y_1)$

$(5,-4) = (x_2, y_2)$.

Coordinate of $P(x, 0)$

$$(x, 0) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

$$= \left(\frac{5m + 2n}{m+n}, \frac{-4m + 3n}{m+n}\right)$$

$$\frac{-4m + 3n}{m+n} = 0$$

$$-4m + 3n = 0$$

$$\frac{4m}{3m} = \frac{3n}{3m}$$

$$\frac{4}{3} = \frac{n}{m}$$

$$m : n = \underline{3 : 4}$$

$L: 2x - y - 1 = 0$

$m: ax + by + c = 0$

$L: 2x - y - 1 = 0$

$y = 2x - 1 \quad , m_1 = 2$

Gradient of line m is $m_2 = \frac{-1}{2}$

Equation of m,

$y - y_1 = m(x - x_1)$

$y - 4 = -\frac{1}{2}x$

$2y - 8 = -x$

$2y + x - 8 = 0$ or $y = \frac{8}{2} - \frac{x}{2}$

$y = 4 - x/2$

$\begin{cases} 2x - y = 1 \\ x + 2y = 8 \end{cases}$ $\begin{cases} 2x - y = 1 \\ -2x - 4y = -16 \end{cases}$

$-5y = -15$

$y = \underline{\underline{3}}$

$2x - y - 1 = 0$

$2x - 3 - 1 = 0 \quad P(2,3)$

$2x - 4 = 0$

$2x = 4$

$x = 2$

$\log_{1/x} a = -\log_x a$

RHS: Changing the base to x

$$\log_{1/x} a = \frac{\log_x a}{\log_x 1/x}$$

$$= \frac{\log_x a}{\log_x x^{-1}} = \frac{\log_x a}{-\log_x x} = -\log_x a$$

b) $x^2+y^2-4y-1=0$

$x^2+y^2-4y=+1$

$x^2+y^2-4y+4-4=1$

$x^2+(y-2)^2=5$

Centre $(0,2)$, $m = \frac{2-1}{1-2} = -\frac{1}{2}$

~~A(2,2)~~ $A(2,1)$

gradient of tangent 2

Equation: $y-y_1 = m(x-x_1)$

$y-1 = 2(x-2) \Rightarrow y-1 = 2x-4$

$\Rightarrow y-2x+3=0$

$y-2x=-3$

$(1+x)^{-1/2}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3$$

$$(1+x)^{1/2} = 1 + x/2 + \frac{\frac{1}{2}(-\frac{1}{2})}{2!}x^2 + \frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{3!}x^3$$

$$= 1 + x/2 + \left(\frac{-1}{4}\right)\frac{1}{2}x^2 + \left(\frac{3}{8}\right)\left(\frac{1}{6}\right)x^3 + \cdots$$

$$= 1 + x/2 - \frac{1}{8}x^2 + x^3/16 + \cdots$$

$$\sqrt{1.08} = \sqrt{1+0.08} = (1+0.08)^{1/2}$$

$$= 1 + \frac{0.08}{2} - \frac{(0.08)^2}{8} + \frac{(0.08)^3}{16}$$

$$= 1 + 0.04 - 0.0008 + 0.0000$$

$$= 1.032032$$

d) $$(\lambda I - A) = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} - \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} \lambda-2 & -1 & 0 \\ 0 & \lambda-2 & -1 \\ 0 & 0 & \lambda-2 \end{pmatrix}$$

$$\det(\lambda I - A) = \begin{vmatrix} \lambda-2 & -1 & 0 \\ 0 & \lambda-2 & -1 \\ 0 & 0 & \lambda-2 \end{vmatrix} = (\lambda-2)(\lambda-2)(\lambda-2) =$$

$$(\lambda-2)^3 = 0$$

$$\Rightarrow \lambda = 2$$

$$\log_9 xy = \log_9 x + \log_9 y$$

$$= \frac{\log_3 x}{\log_3 9} + \frac{\log_3 y}{\log_3 9}$$

$$= \frac{\log_3 x}{2} + \frac{\log_3 y}{2}$$

$$\begin{cases} \log_9 xy = 5/2 \\ \log_3 x \log_3 y = -6 \end{cases} \quad \begin{cases} \frac{\log_3 x}{2} + \frac{\log_3 y}{2} = 5/2 \\ \log_3 x \log_3 y = -6 \end{cases}$$

Let $v = \log_3 x$, $w = \log_3 y$.

$$\begin{cases} \frac{v}{2} + \frac{w}{2} = 5/2 \\ vw = -6 \end{cases} \quad \begin{cases} v + w = 5 \\ v(w) = -6 \end{cases}$$

$$\begin{cases} v = 5 - w \\ w(5 - w) = -6 \end{cases}, \quad \begin{cases} v = 5 - w \\ -w^2 + 5w + 6 = 0 \end{cases}$$

$$D = b^2 - 4ac$$
$$= 25 - 4(-1)(6)$$
$$= 25 + 24$$
$$= 49$$

$$w_{1,2} = \frac{-5 \pm 7}{2(-1)}$$

$$w_1 = \frac{-5+7}{-2} = \frac{2}{-2} = -1$$

$$w_2 = \frac{-5-7}{-2} = \frac{-12}{-2} = 6$$

$w_1 = -1$, $v_1 = 5 - (-1) = 6$

$w_2 = 6$, $v_2 = 5 - 6 = -1$

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$\log_3 x, \quad w = \log_3 y$

$\log_3 x = 6 \qquad \log_3 y = -1$

$x = 3^6 \qquad y = 3^{-1} \qquad \log_3 y = 6$

$y = 3^6$

$\begin{cases} x = 3^6 \\ y = \frac{1}{3} \end{cases} \qquad \begin{cases} x = -1 \\ y = 3^6 \end{cases} \qquad \log_3 x = -1$

$x = \frac{1}{3}$

Proof: $x = 3^6, \; y = \frac{1}{3} = 3^{-1}$

$$\frac{\log_3 x}{2} + \frac{\log_3 y}{2} = \frac{5}{2}$$

$$\frac{\log_3 3^6}{2} + \frac{\log_3 3^{-1}}{2} = 5$$

$$6\,\frac{\log_3 3}{2} + (-1)\frac{\log_3 3}{2} = \frac{5}{2}$$

$$\frac{6}{2} - \frac{1}{2} = \frac{5}{2}$$

$$\frac{5}{2} = \frac{5}{2}$$

$7^n - 3^n$, $n \geq 1$ is divisible by 4.

(i) for $n = 1$

$7^1 - 3^1 = 7 - 3 = 4$. — true for $n = 1$.

(ii) Suppose $7^n - 3^n$ is divisible by 4 for $n = k$ i.e $7^k - 3^k = 4x$.

Prove for $n = k+1$.

Since

$$7^k - 3^k = 4x$$

Multiplying throughout by 7,

$$7 \cdot 7^k - 7 3^k = 7 \cdot 4x$$

$$7^{k+1} = 7 \cdot 3^k + 7 \cdot 4x,$$

Since $7 = 3+4$, so

$$7^{k+1} = (3+4)3^k + 7 \cdot 4x$$

$$7^{k+1} = 3 \cdot 3^k + 4 \cdot 3^k + 7 \cdot 4x$$

$$7^{k+1} = 3^{k+1} + 4 \cdot 3^k + 7 \cdot 4x$$

$$7^{k+1} - 3^{k+1} = 4 \cdot 3^k + 7 \cdot 4x$$

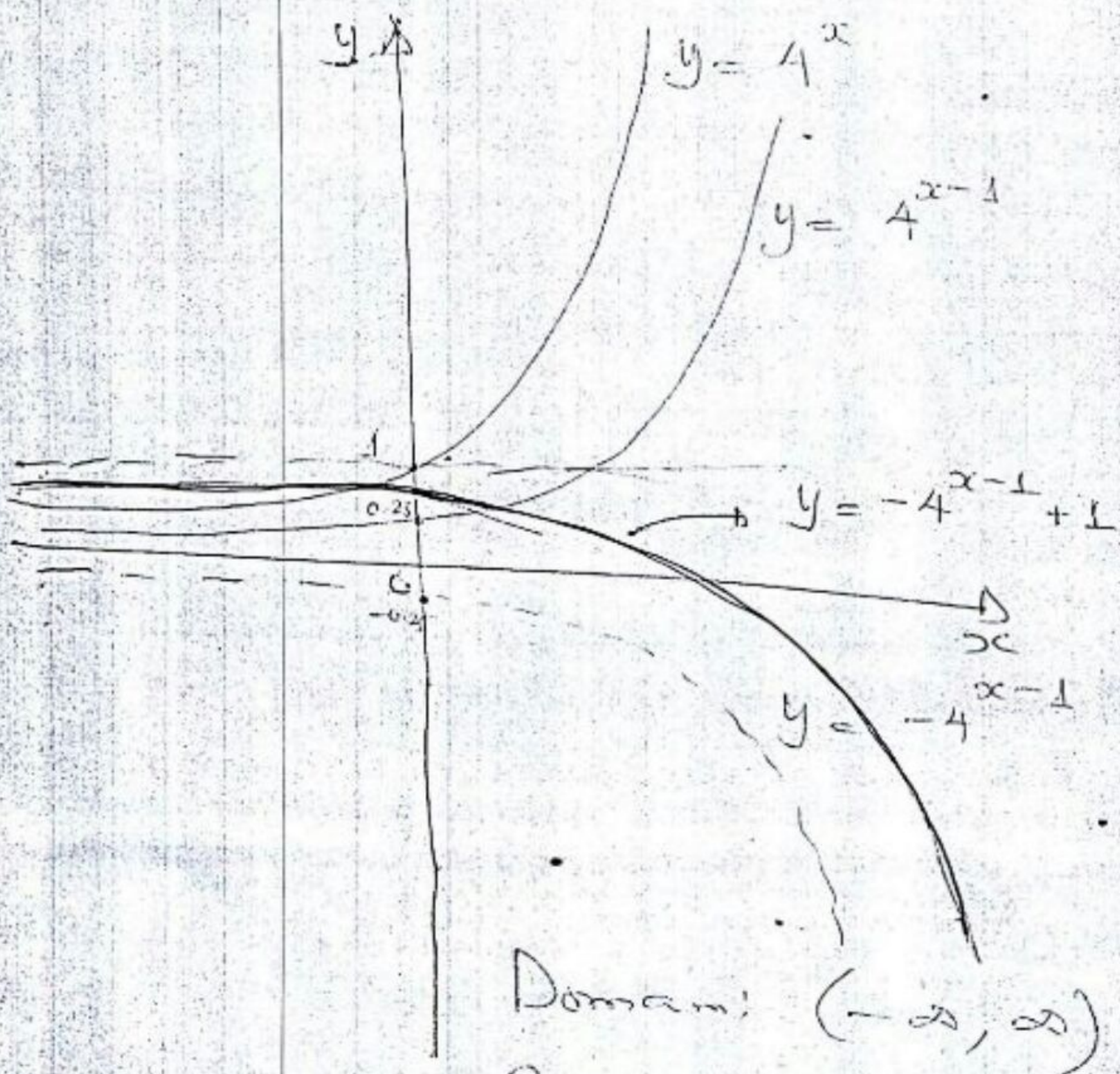
$= 3^{k+1} = 4(3^k + 7)$

Let $y = 3^k + 7$, where $y \in \mathbb{Z}^+$,

So $7^{k+1} - 3^{k+1} = 4y$.

So $7^n - 3^n$ is divisible by 4 for any $n \in \mathbb{Z}^+$

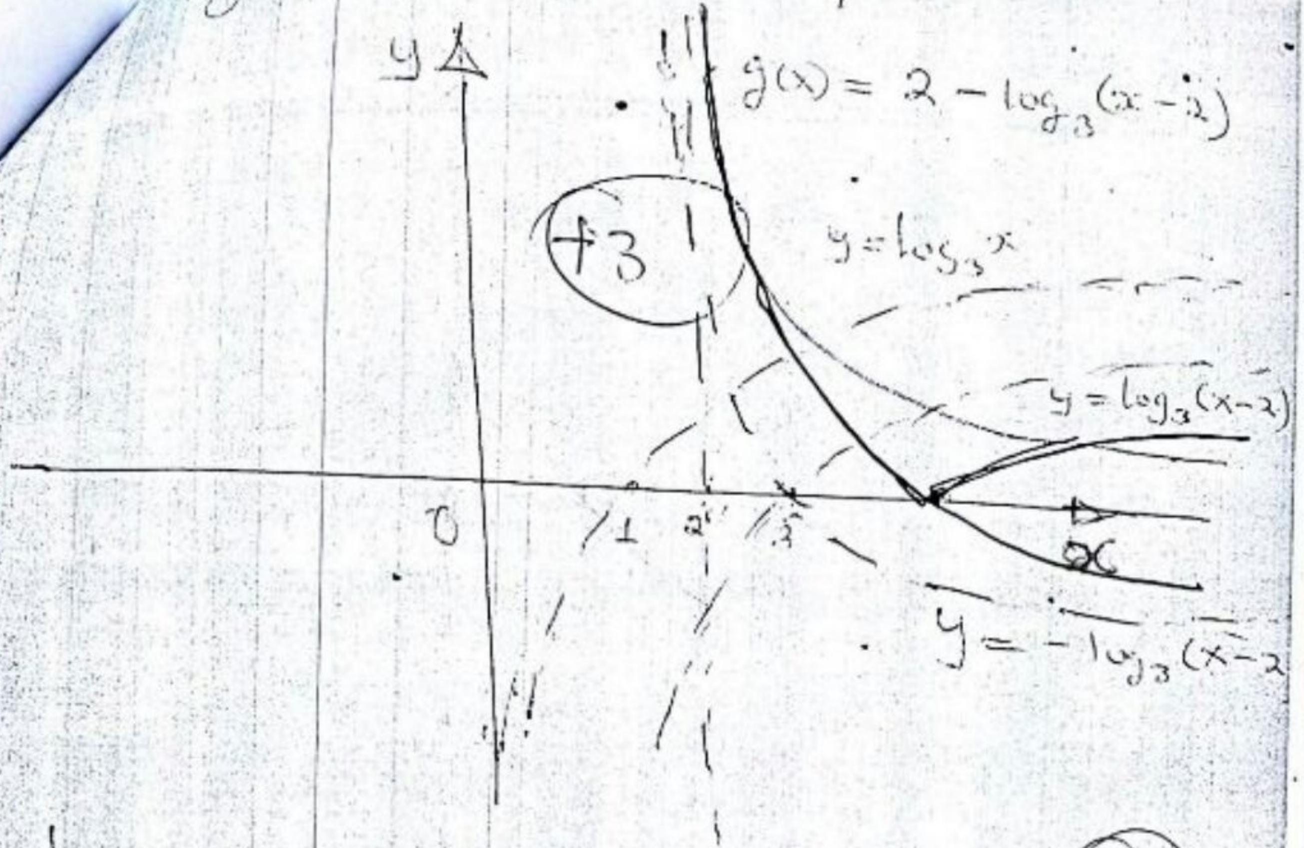
b) (i) $f(x) = -4^{x-1} + 1$



Domain: $(-\infty, \infty)$

Range: $y < 1$.

$g(x) = |2 - \log_3(x-2)|$



Domain: $(x-2) > 0 \Rightarrow x > 2$ +1

or

$(2, \infty)$

Range: $y \geq 0 \Rightarrow [0, \infty)$ +1

$$\binom{n+1}{3} + \binom{n-1}{3} = (n-1)^2$$

RHS:

(1) $$\frac{(n+1)!}{(n+1-3)!\,3!} + \frac{(n-1)!}{(n-1-3)!\,3!}$$

$$\Rightarrow \frac{(n+1)!}{(n-2)!\,3!} + \frac{(n-1)!}{(n-4)!\,3!}$$

$\underline{\text{N.B}}$ $(n-2)! = (n-2)(n-3)(n-4)!$

$$\Rightarrow \frac{(n+1)!}{(n-2)!\,3!} + \frac{(n-1)!\,(n-2)(n-3)}{(n-2)!\,3!}$$

$$\Rightarrow \frac{(n+1)! + (n-1)!\,(n-2)(n-3)}{(n-2)!\,3!}$$

$$\Rightarrow \frac{(n+1)n(n-1)! + (n-1)!\,(n-2)(n-3)}{(n-2)!\,3!}$$

$$\Rightarrow \frac{(n-1)!\,(n^2+n-(n^2-5n+6))}{(n-2)!\,3!}$$

$$\Rightarrow \frac{(n-1)!\,(n^2+n-n^2+5n-6)}{(n-2)!\,3!}$$

$$\Rightarrow \frac{(n-1)!\,(6n-6)}{(n-2)!\,3!}$$

$$\Rightarrow \frac{(n-1)!\,6(n-1)}{(n-2)!\,3!}$$

$$\Rightarrow \frac{(n-1)\cancel{(n-2)!}\,\cancel{6}(n-1)}{\cancel{(n-2)!}\,\cancel{3!}}$$

$$\Rightarrow (n-1)^2$$

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$1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \cdots$

$a_1 = 1$, $r = \frac{(-2/3)}{1} = -2/3$, $|r| = |-2/3| <$

$S_n = \frac{a(1-r^n)}{1-r}$ or $S_\infty = \frac{a_1}{1-r}$

$$\sum_{k=1}^{n} \left(-\frac{2}{3}\right)^{k-1}$$

2) $A = Pe^{rt}$, $P = \$500$

$A = \$1000$,

$t = 10$ years

$10\cancel{0}\cancel{0} = 5\cancel{0}\cancel{0} e^{10r}$

$2 = e^{10r}$

$\ln 2 = \ln e^{10r}$

$10r = \ln 2$

$r = \frac{\ln 2}{10} = \frac{\ln 2}{10}$

Question three

$\log x + \log x^2 + \log x^4 + \log x^8 + \cdots$

$$r = \frac{\log x^2}{\log x} = \frac{2\log x}{\log x} = 2$$

$a = \log x, \quad n = 10, \quad r = 2 > 1$

$$S_n = \frac{a(1-r^n)}{1-r} \quad \text{or} \quad S_n = \frac{a(r^n-1)}{r-1}$$

$$S_{10} = \frac{a(r^n-1)}{r-1} = \log x\left[\frac{2^{10}-1}{2-1}\right]$$

$$= [2^{10}-1]\log x$$

$$= (1024-1)\log x$$

$$= (1023)\log x$$

$$S_{10} = \log x^{1023}$$

The sum of n terms of a G.P

is $S_n = \frac{a_1(1-r^n)}{1-r}$, rewriting

$$S_n = \frac{a_1}{1-r} - \frac{a_1 r^n}{1-r}$$

If we consider very large values of n i.e $n \to \infty$ (n approaches infinity), r^n will tend towards zero because $|r| < 1$. So S_n also approaches $\frac{a_1}{1-r}$ for large values of n since $\frac{ar^n}{1-r} \to 0$.

Thus, the sum of an infinite G.P is $S_\infty = \frac{a_1}{1-r}$

$$6^{x+2} = 2(3^{2x})$$

$$\ln 6^{x+2} = \ln 2(3^{2x})$$

$$(x+2)\ln 6 = \ln 2 + \ln 3^{2x}$$

$$x\ln 6 + 2\ln 6 = \ln 2 + 2x\ln 3$$

$$x\ln 6 - 2x\ln 3 = \ln 2 - 2\ln 6$$

$$x(\ln 6 - 2\ln 3) = \ln 2 - 2\ln 6$$

$$x = \frac{\ln 2 - \ln 36}{\ln 6 - \ln 9} = \frac{\ln 2/36}{\ln 6/9}$$

$$x = \frac{\ln 1/18}{\ln 2/3}$$

$$A = \begin{pmatrix} 3 & -1 & 2 \\ 1 & 1 & 1 \\ 2 & 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & -1 & 2 \\ 1 & 1 & 1 \\ 2 & 2 & -1 \end{pmatrix}$$

Matrix of Cofactors

$$C = \begin{pmatrix} \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix} & -\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix} & \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} \\ -\begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix} & \begin{pmatrix} 3 & 2 \\ 2 & -1 \end{pmatrix} & -\begin{pmatrix} 3 & -1 \\ 2 & 2 \end{pmatrix} \\ \begin{pmatrix} -1 & 2 \\ 1 & 1 \end{pmatrix} & -\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix} & \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix} \end{pmatrix}$$

$$C = \begin{pmatrix} -3 & 3 & 0 \\ 3 & -7 & -8 \\ -3 & -1 & 4 \end{pmatrix} \quad +4$$

$$C^T = \begin{pmatrix} -3 & 3 & -3 \\ 3 & -7 & -1 \\ 0 & -8 & -4 \end{pmatrix} = \operatorname{adj} A$$

$\operatorname{Det} A = -12$

$$A^{-1} = \frac{1}{\det A} \operatorname{adj} A$$

+1

$$= \frac{-1}{12} \begin{pmatrix} -3 & 3 & -3 \\ 3 & -7 & -1 \\ 0 & -8 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} -3/-12 & 3/-12 & -3/-12 \\ 3/-12 & -7/-12 & -1/-12 \\ 0 & -8/-12 & -4/-12 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 1/4 & -1/4 & 1/4 \\ -1/4 & 7/12 & 1/12 \\ 0 & 2/3 & -1/3 \end{pmatrix}$$

+3

$$A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1} \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1/4 & -1/4 & 1/4 \\ -1/4 & 7/12 & 1/12 \\ 0 & 2/3 & -1/3 \end{pmatrix} \begin{pmatrix} \text{[illegible]} \\ 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5/4 \\ 5/12 \\ 1/3 \end{pmatrix}$$

<u>Proof:</u> $\begin{pmatrix} 3 & -1 & 2 \end{pmatrix} \begin{pmatrix} 5/4 \\ 5/12 \\ 1/3 \end{pmatrix} = \underline{\underline{4}}$

$$x^2+y^2+Dx+Ey+F=0$$

$$x^2+Dx+y^2+Ey+F=0$$

$$x^2+Dx+\left(\frac{D}{2}\right)^2-\left(\frac{D}{2}\right)^2+y^2+Ey+\left(\frac{E}{2}\right)^2-\left(\frac{E}{2}\right)^2+F$$

$$\left(x+\frac{D}{2}\right)^2+\left(y+\frac{E}{2}\right)^2=\frac{D^2}{4}+\frac{E^2}{4}-F$$

$$\left(x+\frac{D}{2}\right)^2+\left(y+\frac{E}{2}\right)^2=\frac{D^2+E^2-4F}{4}$$

Centre: $\left(-\frac{D}{2}, -\frac{E}{2}\right)$

Radius: $r^2=\frac{D^2+E^2-4F}{4}$

$$r=\sqrt{\frac{D^2+E^2-4F}{4}}$$