COMPOSITION OF FORCES

AIM: *To verify the law of parallelogram of forces, triangle law of forces, and Lami's theorem*

APPARATUS: Gravesand's apparatus (a wooden vertical board fitted with two frictionless pulleys), slotted weights with hangers, mirror strip, thread, spring balance, and set squares.

THEORY

Law of parallelogram of forces: If two forces acting simultaneously on a point object are represented in magnitude and direction by the two adjacent sides of a parallelogram, then the resultant of these two forces is represented in magnitude and direction by the diagonal of the parallelogram at the same point.

Law of triangle of forces: If two forces acting simultaneously on a point object are represented in magnitude and direction by the two sides of a triangle taken in order, then the third side taken in the opposite order completely represents in magnitude and direction the resultant of the two forces.

If three forces P, Q, and R act upon a point object such that the object is at rest or in equilibrium, then the three forces are completely represented by the three sides of a closed triangle.

Lami's theorem: $\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$

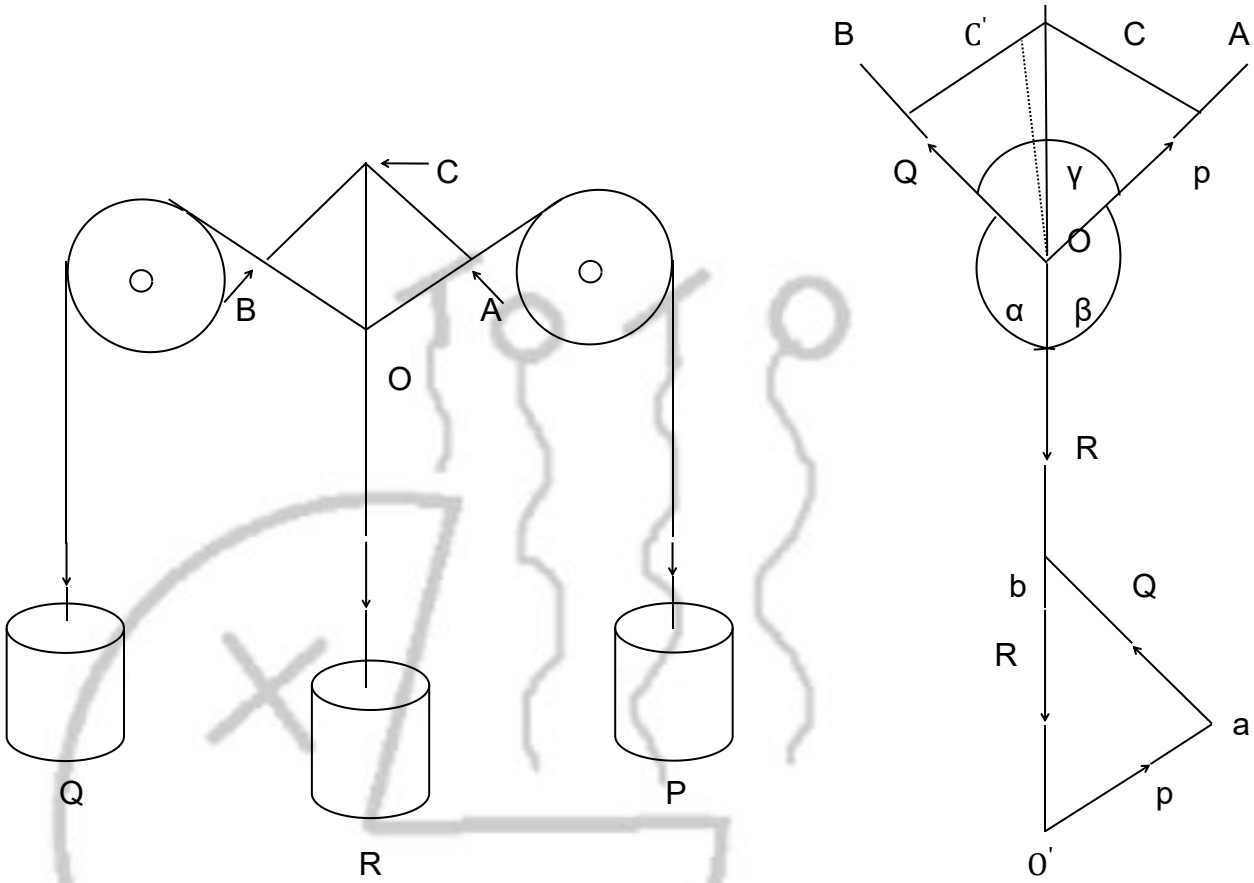

Figure 5.1: Gravesand's apparatus

PROCEDURE:

1. Gravesand's apparatus was placed on the table making sure that its board was vertical.
2. The pulleys were as free from friction as possible and were able to move freely. A sheet of white paper was fixed on the board.
3. A long thread was taken and two hangers were tied to its two ends. The third hanger was tied at the end of another thread. The other end of the second thread was connected to the center of the first thread.
4. The threads were placed as shown in figure 5.1
5. Slotted weights were added to the hangers. Weights were adjusted such that the knotted point O lied somewhere in the middle of the sheet of paper.
6. The point O was marked on the paper just close to the meeting point of the threads.
7. At the point O the three forces were in equilibrium. The strip of mirror was placed underneath the thread. The image of the thread was viewed in the mirror with the eye in such a position that the thread overlapped its image in the mirror. Two points were marked on the paper at the end of the strip of mirror just underneath the thread. These two points were joined with a line. Similarly, points underneath the other thread were marked. The three lines met at O.
8. The thread and hangers were removed.
9. The weights of P, Q and R were found using a spring balance and the values were tabulated.
10. The angles α, β and γ were measured.
11. The experiment was repeated twice with different values of P

DATA ANALYSIS

Converting masses

For trial 1

P=70 Q=70 R=60

$=\frac{70}{1000} \times 9.8$ $=\frac{70}{1000} \times 9.8$ $=\frac{60}{1000} \times 9.8$

=0.686N **=0.686N** **=0.588N**

For trial 2

P=40 Q=40 R=50

$=\frac{40}{1000} \times 9.8$ $=\frac{40}{1000} \times 9.8$ $=\frac{50}{1000} \times 9.8$

=0.392N **=0.392N** **=0.49N**

For trial 3

P=30 Q=30 R=40

$=\frac{30}{1000} \times 9.8$ $=\frac{30}{1000} \times 9.8$ $=\frac{40}{1000} \times 9.8$

=0.294N **=0.294N** **=0.392N**

Converting masses to lengths

For trial 1

OA=70 OB=70 OC=60

$=\frac{70}{10}\mathrm{g} \times 1cm$ $=\frac{70}{10}\mathrm{g} \times 1cm$ $=\frac{60}{10}\mathrm{g} \times 1cm$

=7cm **=7cm** **=6cm**

For trial 2

OA=40 OB=40 OC=50

$=\frac{40}{10}\mathrm{g} \times 1cm$ $=\frac{40}{10}\mathrm{g} \times 1cm$ $=\frac{50}{10}\mathrm{g} \times 1cm$

=4cm **=4cm** **=5cm**

For trial 3

OA=30 OB=30 OC=40

$=\frac{30}{10}\mathrm{g} \times 1cm$ $=\frac{30}{10}\mathrm{g} \times 1cm$ $=\frac{40}{10}\mathrm{g} \times 1cm$

=3cm **=3cm** **=4cm**

PARALLELOGRAM LAW OF FORCES

TABLE 1: LAW OF PARALLELOGRAM OF FORCES

No.	Forces (N)			Lengths (cm)			Resultant R'=OC'.x	Difference R-R'
	P	Q	R	P/x=OA	Q/x=OB	OC		
1	0.686	0.686	0.588	7	7	6	0.588	0.00
2	0.392	0.392	0.49	4	4	5	0.49	0.00
3	0.294	0.294	0.392	3	3	4	0.392	0.00

Trial 1

1cm => 10g

6cm => x

x cm = 10g X 6cm

x = 60.0g = 0.06kg

W = mg

W = 0.06kg X 9.8m/s^2

W = 0.588N

Difference in R - R'

R = 0.588N, R' = 0.588N

R – R' = 0.588 – 0.588

R – R' = 0.00N

Trial 2

1cm => 10g

5 cm => x

x cm = 10g X 5cm

x = 50g = 0.05kg

W = mg

W = 0.05kg X 9.8m/s^2

W = 0.49N

Difference in R - R'

R = 0.49N, R' = 0.49N

R – R' = 0.49N – 0.49N

R – R' = 0.00N

Trial 3

1cm => 10g

4cm => x

x cm = 10g X 4cm

x = 40g = 0.04kg

W = mg

W = 0.04kg X 9.8m/s^2

W = 0.392N

Difference in R - R'

R = 0.392, R' = 0.392N

R – R' = 0.3920 – 0.392

R – R' = 0.00N.

For Table 2

LAW OF TRIANGLE OF FORCES

TABLE 2: LAW OF TRIANGLE OF FORCES

No.	Forces (N)			Lengths (cm)			$\frac{P}{O'a}$	$\frac{Q}{ab}$	$\frac{R}{bO'}$
	P	Q	R	O'a	Ab	bO'			
1	0.686	0.686	0.588	7.00	7.00	6	0.098	0.098	0.098
2	0.392	0.392	0.49	4.00	4.00	5	0.098	0.098	0.098
3	0.294	0.294	0.392	3.00	3.00	4	0.098	0.098	0.098

Trial 1

$$\frac{P}{O`a} = \frac{0.686}{7} = \mathbf{0.098}$$

$$\frac{Q}{ab} = \frac{0.686}{7} = \mathbf{0.098}$$

$$\frac{R}{bO`} = \frac{0.588}{6} = \mathbf{0.098}$$

Trial 2

$$\frac{P}{O`a} = \frac{0.392}{4} = \mathbf{0.098}$$

$$\frac{Q}{ab} = \frac{0.392}{4} = \mathbf{0.098}$$

$$\frac{R}{bO`} = \frac{0.49}{5} = \mathbf{0.098}$$

Trial 3

$$\frac{P}{O`a} = \frac{0.294}{3} = \mathbf{0.098}$$

$$\frac{Q}{ab} = \frac{0.294}{6.00} = \mathbf{0.098}$$

$$\frac{R}{bO`} = \frac{0.392}{4} = \mathbf{0.098}$$

For Table 3

LAMI'S THEOREM

TABLE 3: LAMI'S THEOREM

No.	Forces (N)			Angles(°)			$\frac{P}{\sin\alpha}$	$\frac{Q}{\sin\beta}$	$\frac{R}{\sin\gamma}$
	P	Q	R	A	β	γ			
1	0.686	0.686	0.588	116°	114°	130°	0.763	0.751	0.768
2	0.392	0.392	0.49	125°	130°	105°	0.479	0.512	0.507
3	0.294	0.294	0.392	130°	130°	100°	0.384	0.384	0.398

Trial 1

$$\frac{P}{\sin\alpha} = \frac{0.686}{\sin 116°} = \mathbf{0.763}$$

$$\frac{Q}{\sin\beta} = \frac{0.686}{\sin 114°} = \mathbf{0.751}$$

$$\frac{R}{\sin\gamma} = \frac{0.588}{\sin 130°} = \mathbf{0.768}$$

1. **Trial 2**

$$\frac{P}{\sin\alpha} = \frac{0.392}{\sin 125°} = \mathbf{0.479}$$

$$\frac{Q}{\sin\beta} = \frac{0.392}{\sin 130°} = \mathbf{0.512}$$

$$\frac{R}{\sin\gamma} = \frac{0.49}{\sin 105°} = \mathbf{0.507}$$

2. **Trial 3**

$$\frac{P}{\sin\alpha} = \frac{0.294}{\sin 130°} = \mathbf{0.384}$$

$$\frac{Q}{\sin\beta} = \frac{0.294}{\sin 130°} = \mathbf{0.384}$$

$$\frac{R}{\sin\gamma} = \frac{0.392}{\sin 100°} = \mathbf{0.398}$$

DISCUSSION

The magnitudes of R for trials 1, 2 and 3 were 0.588N, 0.49N and 0.392N respectively .The experiment was perfect in that the line RO passed through C. Thus R' =R. According to the triangle law of forces, $\frac{P}{O'a} = \frac{Q}{ab} = \frac{R}{bO'}$, which was found to be 0.098.

In trial 1, $\frac{P}{\sin\alpha}$ was found to be 0.763, $\frac{Q}{\sin\beta}$ was found to be 0.751 and $\frac{R}{\sin\gamma}$ was 0.768 while in trial 2 the results were 0.479, 0.512, and 0.507 respectively. The magnitudes of $\frac{P}{\sin\alpha}$, $\frac{Q}{\sin\beta}$, and $\frac{R}{\sin\gamma}$ in trial 3 were 0.384, 0.384 and 0.398. The values differed due to experimental errors such as when measuring the angles. To come up with more accurate results, the experiment has to be done repeatedly

CONCLUSION

The experiment was carried out using the Gravesand apparatus, in which slotted weight were added to the hangers of different masses, the weight were adjusted until the forces were in equilibrium. The experiment was a success, its aim was achieved the law of parallelogram of forces, triangle law of forces, and Lami's theorem were all verified.

REFERENCES

P.C. Simpemba, J. Simfukwe and M. Chengo, PH 110 Laboratory Manual, (2016), School of Mathematics and Natural Sciences, Department of Physical Sciences, Copperbelt University, Kitwe, Zambia.