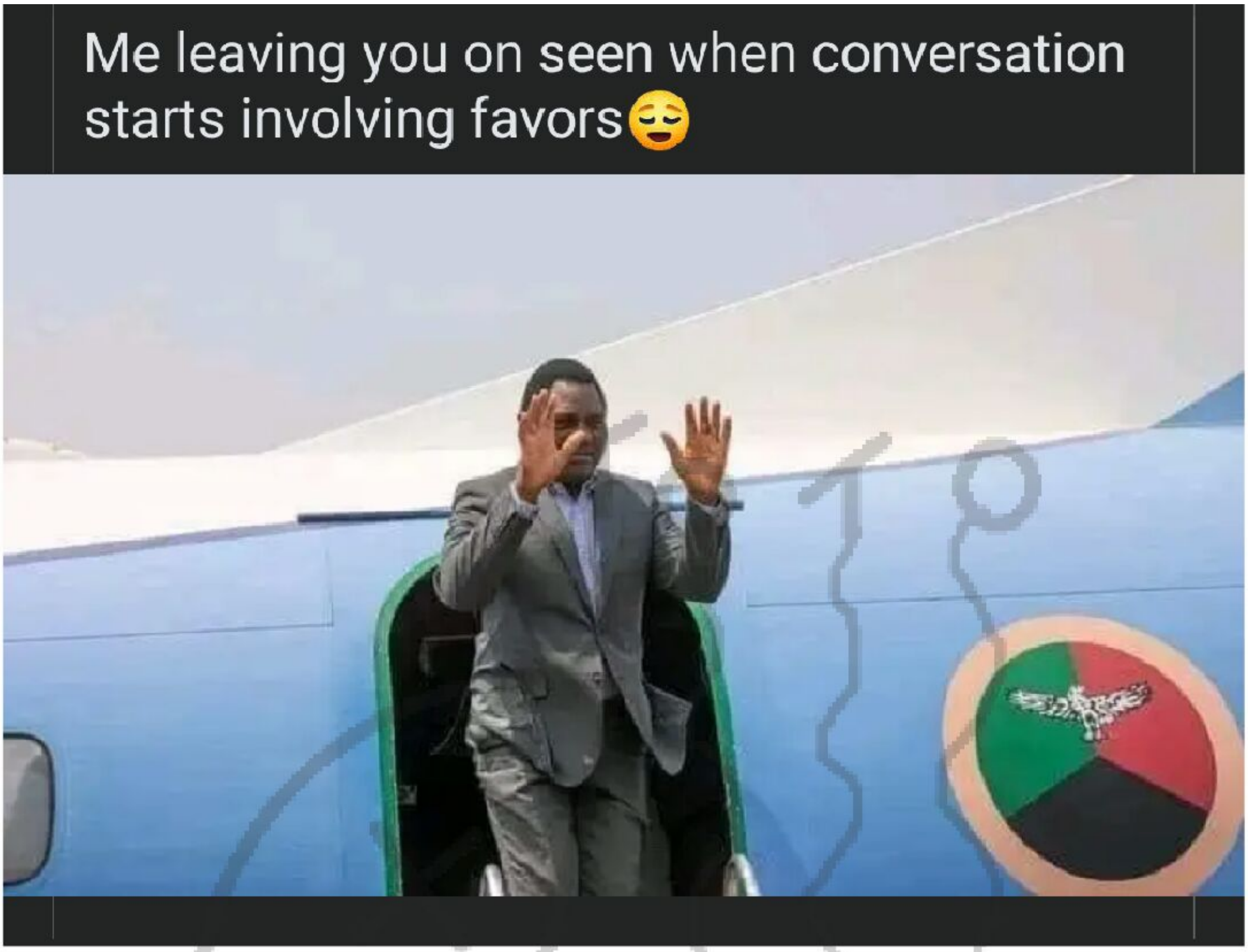


EXERCISES

1. A wheel accelerates uniformly from rest to an angular speed of 25 rad/s in 10 s. Consider a particle sticking at a point 10 cm from the wheel's center.
   (a) Find the angular acceleration of the particle. **[2.5 rad/s^2]**
   (b) Find the tangential and radial acceleration of the particle. **[0.25 m/s^2; 62.5m/s^2]**
   (c) How many revolutions has the wheel turned during this time interval? **[20 rev]**
   (d) Find the angular deceleration of the particle if the wheel comes to a full stop after 5 revolutions. **[−9.95 rad/s^2]**
2. As an approximation, assume the moon revolves about the Earth in a perfectly circular orbit with a radius $r = 3.85 \times 10^8$ m and takes 27.3 days to make a complete revolution (see figure 7.11). What is
   (a) the speed of the moon? **[1025 m/s]**
   (b) the radial acceleration of the moon toward the Earth's center? **[2.73×10^{-3} m/s^2]**

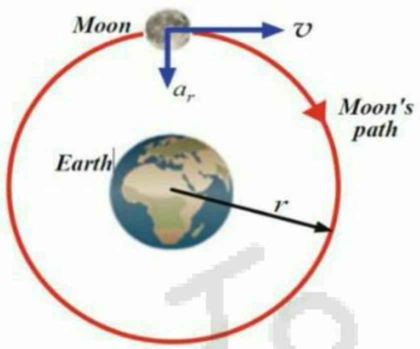


Figure 7.11 *See exercise 2*

3. A stone of mass 0.25 kg tied to the end of a string is whirled round a circle of radius 1.5 m with a speed of 40 rev/min in a horizontal plane. What is the tension in the string? What is the maximum speed with which the stone can be whirled around if the string can withstand a maximum tension of 200 N? **[6.57 N; 34.64 m/s]**
4. A 3.5 kg mass is allowed to spin. The rope it is attached to is 2 m long and makes an angle of 25 degrees with the vertical.
   (a) What is the tension in the rope? **[37.8 N]**
   (b) What is the mass's speed? **[1.97 m/s]**
5. A car has to move on a level turn of radius 45 m. If the coefficient of static friction between the tyre and the road is $\mu_s = 2.0$, find the maximum speed the car can take without skidding. Take g = 10 m/s^2. **[30 m/s]**
6. A park has a radius of 10 m. If a vehicle goes round it at an average speed of 18 km/hr, what should be the proper angle of banking? **[14^0]**
7. A small body is tied to the end of string of length 1 m and whirled in a vertical circle. What is
   (a) the minimum speed that the body must have at the highest point so that the string does not slacken? **[3.13 m/s]**
   (b) its speed at the lowest point if it has the above minimum speed at the highest point? **[7 m/s]**
8. In a rotor, a hollow vertical cylindrical structure rotates about its axis and a person rests against the inner wall. At a particular speed of the rotor, the floor below the person is removed and the person hangs resting against the wall without any floor. If the radius of the rotor is 2 m and the coefficient of static friction between the wall and the person is 0. 2, find the minimum speed at which the floor may be removed. Take g = 10 m/s^2. **[10 m/s]**
9. A particle has a non-uniform motion on a circular path of radius $r = 2$ m. At a given instant of time, the magnitude of its total acceleration a is 10 m/s^2 (see figure 7.12). At this instant, find:
   (a) the magnitude of both the centripetal and tangential accelerations. **[8.66 m/s^2, 5 m/s]**
   (b) the speed v of the particle. **[4.16 m/s]**

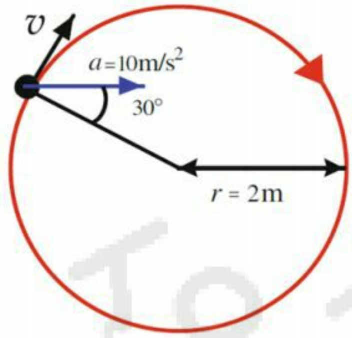


Figure 7.12 *See exercise 9*

10. A particle moves in a circle of radius 1 m. Its linear speed is given by $v = 4\,t$, where t is in second and v in metre/second. Find the radial and tangential acceleration at t = 0.5 s. Hence, find the magnitude and direction of the resultant acceleration. **[4 m/s^2; 4 m/s^2; 5.66 m/s^2; 45^0]**
11. The mass of Jupiter is $1.9 \times 10^{27}\,kg$ and that of the sun is $1.99 \times 10^{30}\,kg$. The mean distance of Jupiter from the sun is 7.8×10^{11}m. Calculate the gravitational force which the sun exerts on Jupiter, and the speed of Jupiter. **[4.1×10^{23} N; 1.3×10^4 m/s]**
12. Find the distance of a point from the earth's center where the resultant gravitational field due to the earth and the moon is zero. The mass of the earth is 6×10^{24} kg and that of the moon is 7.4×10^{22} kg. The distance between the earth and the moon is 4×10^5 km? Neglect effects of other planets. **[3.6×10^5 km]**
13. How far away from the surface of earth does the acceleration due to gravity become 4% of its value on the surface of earth? **[25600 km]**
14. An earth satellite makes a complete circuit around the earth in 90 minutes. If the orbit is circular, calculate the height of the satellite above the earth. **[279 km]**
15. A satellite of mass 1000 kg is supposed to orbit the earth at a height of 2000 km above the earth's surface. Compute its

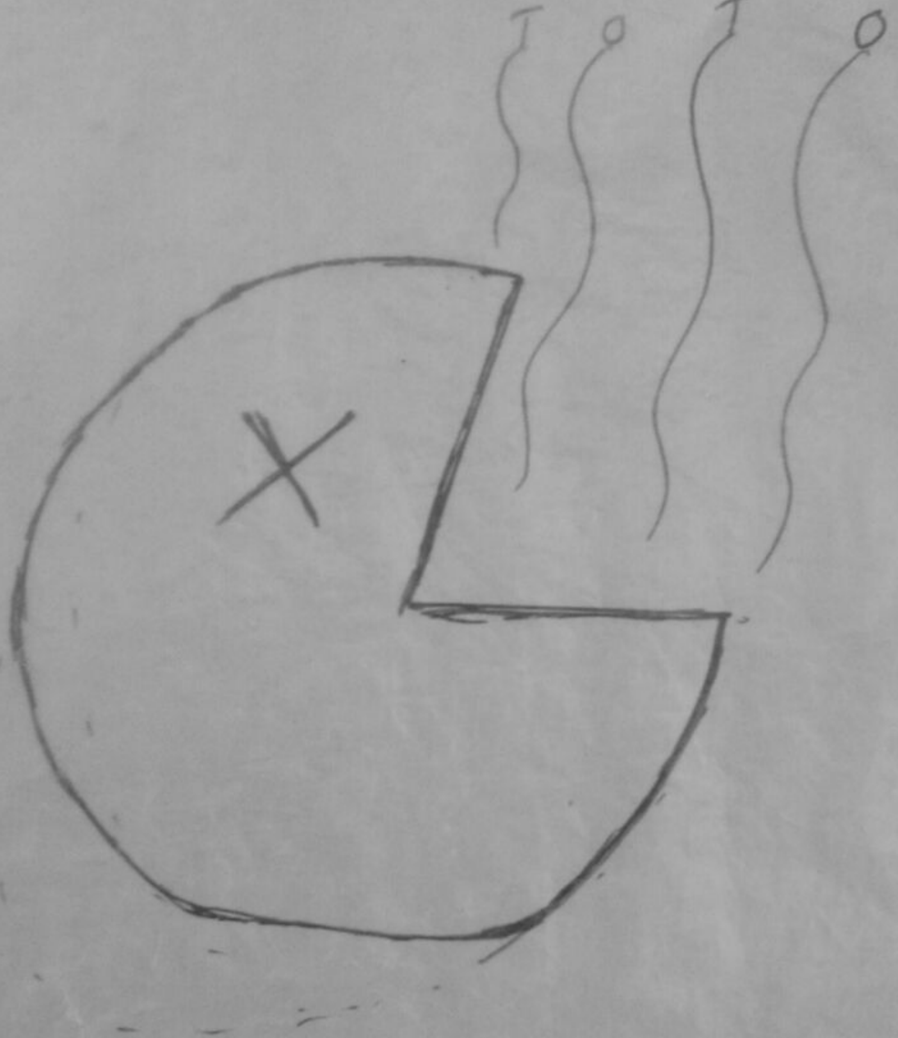
    (a) speed in the orbit. **[6. 90 km]**

    (b) time period. **[2.12 hours]**

    (c) kinetic energy. **[2.38×10^{10} J]**

    (d) potential energy of the earth–satellite system. **[-4.76×10^{10} J]**

16. A satellite of mass 200 kg orbits the earth at a height of 400 km above the surface. How much energy must be expended to rocket the satellite out of earth's gravitational influence? Radius of the earth = 6400 km; mass of the earth = 6×10^{24} Kg. **[5.88×10^9 J]**
17. A rocket is fired vertically upward with a speed of 9.8 km/s from the earth's surface. Find the maximum height attained by the rocket. Consider only earth's gravitation. **[20900 km]**
18. The distance of Neptune and Saturn from the sun is nearly 10^{13} m and 10^{12} m respectively. Assuming that they move in circular in circular orbits, then what will be the ratio of their periods? **[T_n:T_s = 31.6:1]**



TSI Tutorial Editron

2023 - Exercise !!

(1,a) <u>data</u>

$\omega_i = 0 rad/s$

$\omega_f = 25 rad/s$

$t = 10s$

$r = 10cm = 0.1m$

$$\alpha = \frac{\Delta\omega}{t}$$

$$= \frac{25 - 0}{10}$$

$$\underline{\underline{\alpha = 2.5 rad/s^2}}$$

b. $a_t = r\alpha$

$a_t = 0.1m \times 2.5 rad/s^2$

$= \cancel{0.25 rad/s}$

$\underline{\underline{a_t = 0.25 m/s^2}}$

$a_c = \frac{v^2}{r}$, where $v = r\omega$

$= \frac{(r\omega)^2}{r}$

$a_c = r\omega^2$

$= 0.1 \times (25)^2$

$\underline{\underline{a_c = 62.5 m/s^2}}$

c $s = ut + \frac{1}{2}at^2$

$\downarrow$

$\theta = \omega_i t + \frac{1}{2}\alpha t^2$

$= (0)(10) + \frac{1}{2}(2.5)(10)^2$

$\theta = 125 rad$

$\therefore \theta = 125\text{rad} \times \dfrac{1\text{rev}}{2\pi\text{rad}}$

$\theta = 19.8943789 \text{ rev}$

$\underline{\theta = 20 \text{ rev}}$

d $v^2 = u^2 + 2as \longrightarrow \omega_f^2 = \omega_i^2 + 2\alpha\theta$

$0^2 = 25^2 + 2\alpha\left(5\cancel{\text{revs}} \times \dfrac{2\pi\text{rad}}{1\text{rev}}\right)$

$-625 = 2\alpha 10\pi$

$\alpha = \dfrac{-625}{2(10\pi)}$

$\underline{\underline{\alpha = -9.947 \text{rad/s}^2}}$

2. $T = \dfrac{2\pi r}{v}$

$27.3 \text{days} \times \dfrac{24 \text{ hour}}{1 \text{ day}} \times \dfrac{3600\text{s}}{1 \text{ hour}}$

2358720

$v = \dfrac{2\pi r}{T}$

$= \dfrac{2\pi(3.85 \times 10^8)}{2358720}$

$\underline{\underline{v = 1025.57 \text{m/s}}}$

(b) $a_c = \dfrac{v^2}{r} = \dfrac{(1025.57)^2}{3.85 \times 10^8}$

$= 0.002731918$

$\underline{\underline{a_c = 2.73 \times 10^{-3} \text{ m/s}^2}}$

3, a) While moving in a kama circle the Tension on the rope is the centripetal force.

$T = F_c$

$T = \dfrac{mv^2}{r}$

$\dfrac{40\cancel{\text{rev}}}{\cancel{\text{min}}} \times \dfrac{1\cancel{\text{min}}}{60\text{s}} \times \dfrac{2\pi r}{1\text{rev}}$

$\omega = 4.18879 \text{rad/s}$

~~19.25 ...~~

$v = r\omega$

$= 1.5 \times 4.18879$

$v = 6.283\,m/s$

$T = \dfrac{mv^2}{r}$,

$= \dfrac{0.25 \times (6.283)^2}{1.5}$

$= 6.58\,N$

(ii) $T = \dfrac{mv^2}{r}$

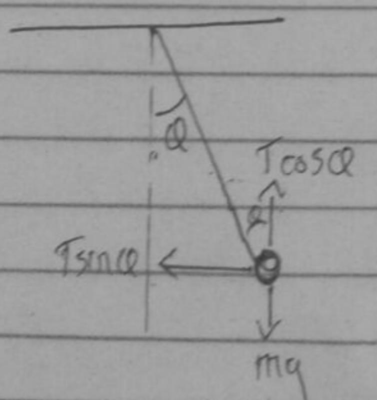
$Tr = mv^2$

$v = \sqrt{\dfrac{Tr}{m}}$

$v = \sqrt{\dfrac{(200)(1.5)}{0.25}}$

$v = 34.641\,m/s$

4



(a.) $T\cos\theta = mg$

$T = \dfrac{mg}{\cos\theta}$

$= \dfrac{3.5 \times 9.8}{\cos 25}$

$\underline{T = 37.85\,N}$

(b.) $T\sin\theta = \dfrac{mv^2}{r}$

$v = \sqrt{\dfrac{rT\sin\theta}{m}}$, wherer $r = l\sin\theta$

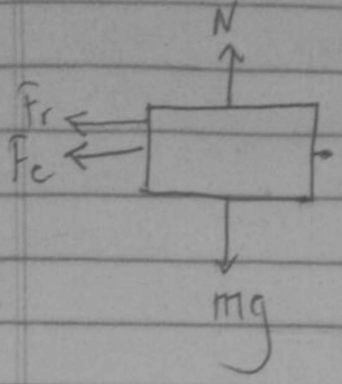
$= 2\sin 25$

$= 0.85\,m$

$v = \sqrt{\dfrac{0.85 \times 37.85}{3.5}}$

$v = 1.965\ m/s$

5



$$F_r = F_c$$

$$\mu N = \frac{mv^2}{r}$$

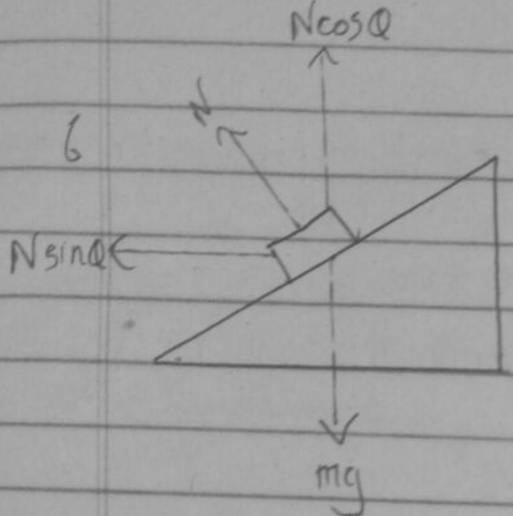
$$\mu \cancel{m} g = \frac{\cancel{m} v^2}{r}$$

$$v = \sqrt{\mu g r}$$

$$= \sqrt{2 \times 10 \times 4.5}$$

$$\underline{\underline{v = 30\ m/s}}$$

6



ΣF_y ; $\frac{N \cancel{\cos\theta}}{\cancel{\cos\theta}} = \frac{mg}{\cos\theta}$

$$N = \frac{mg}{\cos\theta}$$

ΣF_x ; $N\sin\theta = F_c$

$$N\sin\theta = \frac{mv^2}{r}$$

but $N = \frac{mg}{\cos\theta}$

$$\frac{18\ \cancel{km}}{\cancel{hr}} \times \frac{100\cancel{0}\ m}{\cancel{km}} \times \frac{1\ \text{hour}}{36\cancel{0}\cancel{0}\ s}$$

$5\ m/s$

$$\left(\frac{mg}{\cos\theta}\right)\sin\theta = \frac{mv^2}{r}$$

$$\cancel{m} g \tan\theta = \frac{\cancel{m} v^2}{r}$$

$$\tan\theta = \frac{v^2}{gr}$$

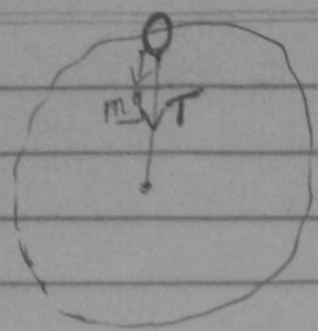
$$\tan\theta = \frac{(5)^2}{9.8 \times 10}$$

$$\tan\theta = 0.2551$$

$$\theta = \tan^{-1}(0.2551)$$

$$\underline{\underline{\theta = 14^\circ}}$$

7, a



$$T + mg = \frac{mv^2}{r}$$

at thy min speed $T = 0$

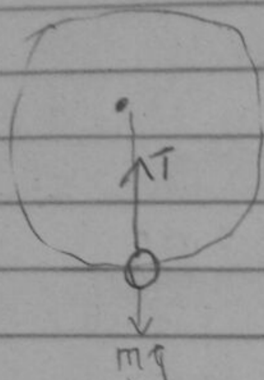
$$\cancel{m}g = \frac{\cancel{m}v^2}{r}$$

$$v = \sqrt{gr}$$

$$v = \sqrt{9.8 \times 1}$$

$$\underline{\underline{v = 3.13\ m/s}}$$

(b.)



Using the conservatron of energy

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}\cancel{m}v^2 + \cancel{m}gh = \frac{1}{2}\cancel{m}v^2 + mgh \quad h=0$$

$$\frac{1}{2}v_0^2 + gh \pm \frac{1}{2}v_f^2$$

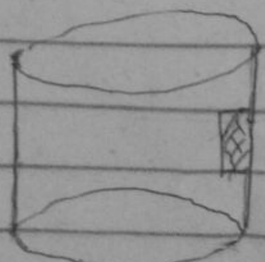
$$\frac{1}{2}(3.13)^2 + (9.8)(2) = \frac{1}{2}v_f^2$$

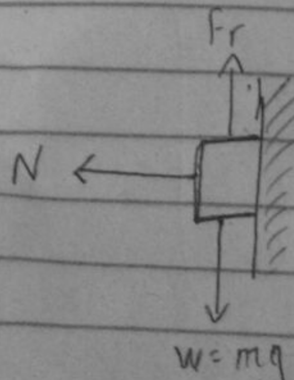
$$24.5 = \frac{1}{2}v_f^2$$

$$v_f = \sqrt{49}$$

$$\underline{\underline{v_f = 7\ m/s}}$$

8.





ΣF_y ; $F_r = mg$

$$\frac{\mu N}{\cancel{\mu}} = \frac{mg}{\mu}$$

$$N = \frac{mg}{\mu}$$

$\Sigma f_x ; \quad N = f_c$

$$N = \frac{mv^2}{r}$$

$$\frac{mg}{\mu} = \frac{mv^2}{r}$$

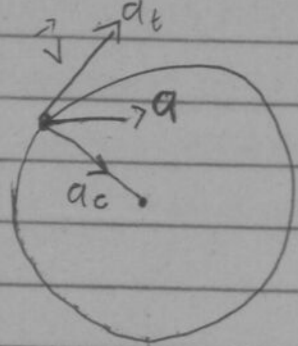
$$\frac{g}{\mu} = \frac{v^2}{r}$$

$$v^2 = \frac{rg}{\mu}$$

$$v = \sqrt{\frac{2 \times 10}{0.2}}$$

$$\underline{\underline{v = 10\,m/s}}$$

9. a



From that figure you can deduce that to find a_c and a_t simply resolve a into its components where a_c will be the x-axis.

$$a_c = a\cos\theta \qquad\qquad a_t = a\sin\theta$$

$$= 10\cos 30^\circ \qquad\qquad = 10\sin 30$$

$$\underline{\underline{a_c = 8.66\,m/s^2}} \qquad\qquad \underline{\underline{a_t = 5\,m/s^2}}$$

(b.) $a_c = \frac{v^2}{r}$

$$v = \sqrt{a_c r}$$

$$v = \sqrt{8.66 \times 2}$$

$$\underline{\underline{v = 4.16\,m/s}}$$

10. $v = 4t$

$$a_t = \frac{dv}{dt}(4t) = 4$$

$$\therefore \underline{\underline{a_t = 4\,m/s^2}}$$

(b.) $a_c = \frac{v^2}{r}$, where $v = 4(0.5) = 2$

$$= (2)^2$$

$$\underline{\underline{a_c = 4\,m/s^2}}$$

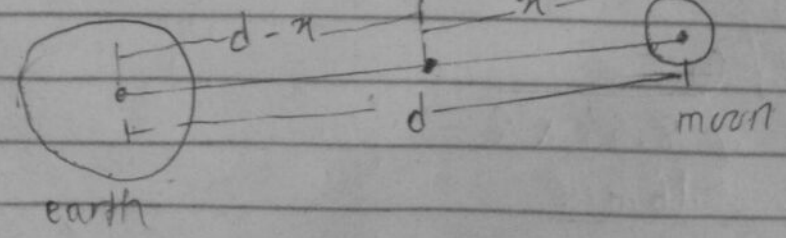
$$a = \sqrt{a_t^2 + a_c^2}$$
$$= \sqrt{4^2 + 4^2}$$
$$a = 5.66 \, m/s^2$$

$$\alpha = \tan^{-1}\left(\frac{a_t}{a_c}\right)$$
$$= \tan^{-1}\left(\frac{1}{1}\right)$$
$$\alpha = 45^\circ$$

11 $$F = \frac{GmM}{r^2}$$
$$= \frac{(6.67 \times 10^{-11}) \times (1.9 \times 10^{27}) \times (1.99 \times 10^{30})}{(7.8 \times 10^{11})^2}$$
$$F = 4.15 \times 10^{23} N$$

(b) $$V_o = \sqrt{\frac{GM}{r}}$$
$$= \sqrt{\frac{(6.67 \times 10^{-11}) \times (1.99 \times 10^{30})}{(7.8 \times 10^{11})}}$$
$$V_o = 13044.942 \, m/s$$
$$V_o = 1.3 \times 10^4 \, m/s$$

12



$$\frac{\cancel{G} M_e \cancel{m}}{(d-x)^2} = \frac{\cancel{G} M_m \cancel{m}}{x^2}$$
$$\frac{(d-x)^2 M_m}{x^2} = \frac{M_e x^2}{x^2}$$

$$\frac{(d-x)^2}{x^2} \cdot \frac{\cancel{M_m}}{\cancel{M_m}} = \frac{M_e}{M_m}$$

$$\sqrt{\left(\frac{d-x}{x}\right)^2} = \sqrt{\frac{M_e}{M_m}}$$

$$\frac{d-x}{x} = \sqrt{\frac{6\times10^{24}}{7.4\times10^{22}}}$$

$$\frac{4\times10^5 - x}{x} = 9.004503378$$

$$4\times10^5 = 10.00450338x$$

$$x = 39981.9946$$

this value of x is the distance between the moon and the point.

$\therefore$ distance from the earth $= d - x$

$= (4\times10^5) - 39981.9946$

$= 360018.0054$ km

$= 3.6\times10^5$ km

13 $$g' = g\left[\frac{R}{R+h}\right]^2$$, ~~and g~~

where g' = gravity after the reduction

g = original gravity

R = radius of the earth

h = height above the earth

notice the $g' = 4\%$ of $g = \frac{4}{100}g$

$$\frac{4}{100}\cancel{g} = \cancel{g}\left(\frac{R}{R+h}\right)^2$$

$$\sqrt{\frac{4}{100}} = \sqrt{\left(\frac{R}{R+h}\right)^2}$$

$$\frac{R}{R+h} = \frac{2}{10} = \frac{1}{5}$$

$$R + h = 5R$$

$h = 4R$, where $R = 6400 km$

$$h = 4(6400)$$

$$\underline{h = 25600 km}$$

14 $$(T)^2 = \left(2\pi\sqrt{\frac{r^3}{GM}}\right)^2$$

$$90 min \times \frac{60 sec}{1 min}$$

$$5400 s$$

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$\frac{4\pi^2 r^3}{4\pi^2} = \frac{T^2 GM}{4\pi^2}$$

$$\sqrt[3]{r^3} = \sqrt[3]{\frac{T^2 GM}{4\pi^2}}$$

$$r = \sqrt[3]{\frac{T^2 GM}{4\pi^2}}$$

$$r = \sqrt[3]{\frac{5400^2 \times (6.67 \times 10^{-11})(5.98 \times 10^{24})}{4\pi^2}}$$

$$r = 6654032.743$$

$$R + h = 6654032.743$$

$h = 6654032.743 - R$, where $R = 6.38 \times 10^6 m$

$$h = 274032.743 m$$

$$\underline{h = 274 km}$$

due to the fact that they did not specify for us in the question which mass & radius to use, thats why the answer is slightly diferent from Mr Mumaz !!

15 $V_0 = \sqrt{\frac{GM}{r}}$, where M is mass of the earth and

$r = R + h = 6400 km + 2000 km = 8400 km$

$$V_0 = \sqrt{\frac{(6.67 \times 10^{-11}) \times (6 \times 10^{24})}{8.4 \times 10^6}}$$

$V_c = 6902.380542\ m$

$V_c = 6.9\ km$

(b) $T = 2\pi\sqrt{\dfrac{r^3}{GM}}$

$= 2\pi\sqrt{\dfrac{(8.4\times10^6)^3}{(6.67\times10^{-11})\times(6\times10^{24})}}$

$T = 7646.457083\ s$

$7646.457083\ \cancel{s} \times \dfrac{1\ hour}{3600\ \cancel{s}}$

$\underline{\underline{T = 2.124\ hours}}$

c $K.E = \frac{1}{2}mv^2$

$= \frac{1}{2}(1000)\times(6902.380542)^2$

$\underline{\underline{K.E = 2.382\times10^{10}\ J}}$

d $U = -\dfrac{GmM}{r}$

$= -\dfrac{(6.67\times10^{-11})\times(1000)\times(6\times10^{24})}{8.4\times10^6}$

$\underline{\underline{U = -4.76\times10^{10}\ J}}$

16 Required Energy = final energy − Initial energy

$= 0 - \left(-\dfrac{GmM}{2(R+h)}\right)$

$= \dfrac{GmM}{2(R+h)}$

$= \dfrac{(6.67\times10^{-11})\times(200)\times(6\times10^{24})}{2(6400\,000m + 400\,000m)}$

$= 5885294118$ J

Required Energy $= 5.89 \times 10^9$ J

17. $$E_i = E_f$$

$$\frac{1}{2}\cancel{m}v^2 - \frac{G\cancel{m}M}{R} = -\frac{G\cancel{m}M}{R+h}$$

$$\# \quad \frac{1}{2}v^2 - \frac{GM}{R} = -\frac{GM}{R+h}$$

Recall that $g = \frac{GM}{R^2}$, first imagine if we multiplied g by R it would give us $\frac{GM}{R}$ ryt !?

also writing $\frac{GM}{R+h}$ in terms of g would mean, what if we multiplied g by R^2 this time and ~~would~~ wouldn't it give us something GM

$\therefore$ that equation can be written as ;

$$\frac{1}{2}v^2 - gR = -\frac{gR^2}{R+h}$$

$$\left(\frac{1}{2}v^2 - gR\right)(R+h) = -gR^2$$

$$\frac{1}{2}v^2R + \frac{1}{2}v^2h - \cancel{gR^2} - gRh = -\cancel{gR^2}$$

AH wait no chill, this method of solving for h is going to complicated lets do it in another way

$$\frac{1}{2}v^2 - gR = -\frac{gR^2}{R+h}$$

$$\frac{v^2 - 2gR}{2} \times -\frac{gR^2}{R+h}$$

$$\frac{(\cancel{v^2 - 2gR})(R+h)}{\cancel{v^2 - 2gR}} = \frac{-2gR^2}{v^2 - 2gR}$$

$$R + h = \frac{-2gR^2}{v^2 - 2gR}$$

$$h = \frac{-2gR^2}{v^2 - 2gR} - R$$ much better !!

$$\cancel{h = \frac{-2 (9.8) \times (6400)^2}{(9.8)^2 - 2 (9.8)(6400)} - 64}$$

$$\cancel{= \frac{-802816000}{-125343.96}}$$

$$h = \frac{-2 \times (9.8)(6400 \times 10^3)^2}{(9.8 \times 10^3)^2 - (2 \times 9.8 \times 6400 \times 10^3)} - 6400000$$

$$= \frac{-8.02816 \times 10^{14}}{-2.94 \times 10^7} - 6400000$$

$$= 27306666.67 - 6400000$$

$$= 20906666.67 \text{ m}$$

$\underline{h = 20906 \text{ km}}$ ... close enough

18 $$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$$

$$\sqrt{\left(\frac{T_n}{T_s}\right)^{\cancel{2}}} = \sqrt{\left(\frac{r_n}{r_s}\right)^3}^{2}$$

$$\frac{T_n}{T_s} = \left(\frac{r_n}{r_s}\right)^{3/2}$$

$$\frac{T_n}{T_s} = \left(\frac{10^{13}}{10^{12}}\right)^{3/2}$$

$$\cancel{\frac{10^{13\times 3/2}}{10^{12\times 3/2}}}$$

$$\cancel{\frac{\left(\sqrt{10^{13}}\right)^3}{\left(\sqrt{10^{12}}\right)^3}} = \cancel{\frac{\left(10^6\sqrt{10}\right)^3}{\left(10^6\right)^3}}$$

$$\frac{T_n}{T_s} = \left(\frac{10}{1}\right)^{3/2}$$

$$\frac{T_n}{T_s} = \frac{10^{3/2}}{1^{3/2}}$$

$$\frac{T_n}{T_2} = \frac{31.62}{1}$$

$$\therefore T_n : T_2 = 31.62 : 1$$