

EXPERIMENT 3: HOOKE'S LAW AND VIBRATION

AIM: to measure the extension produced in a spring for various loads and calculate the spring constant.

APPARATUS: spiral spring, pointer, stand & clamp, a meter rule, masses, scale-pan & stop watch.

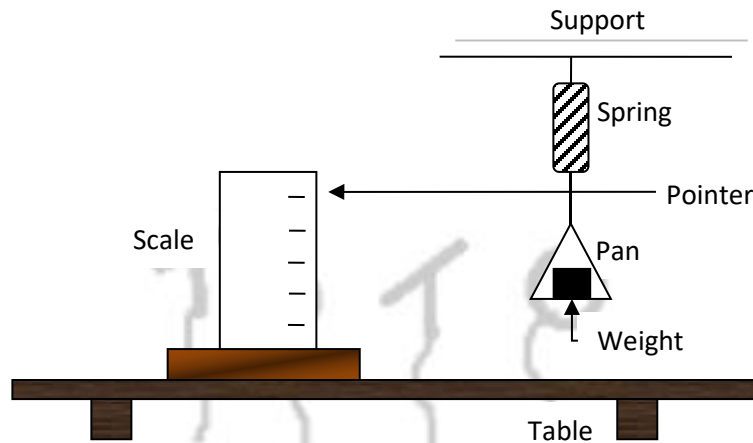


Figure 3.1 Hooke's law apparatus set up

THEORY

According to Hooke's law, a load attached to a spiral spring produces an extension proportional to the weight of the load.

$$E = Mg / k \quad (3.1)$$

Where E stands for the **extension** (or increase in length) of the spring, M stands for the mass of the load, and g stands for the acceleration of gravity ($g = 9.8 \text{ ms}$). Thus mg is the weight of the load. The spring constant k depends only on properties of the spring, not on its extension or load.

A method using equation (3.1) to determine k is called a **static method** because the measurements are performed in a static (not changing) situation.

When a loaded spring is stretched beyond its equilibrium position and then released, it will start vertical vibrations. The period t of the vibration is the time required to move from the upper end of the vibration to the lower end and back again.

Let M = mass of the applied load, m = mass of the scale pan, and s = mass of the spring.

Then we have the relation,

$$T = 2\pi\sqrt{\frac{(m0)}{K}}$$

Or $T^2 = 4\pi^2(m_0)/K$
(3.2)

Here k is the same spring constant as in equation (3.1). A method using equation (3.2) to determine k is called a **dynamical method** because the motion of the mass is essential to the measurement.

PROCEDURE

The apparatus was set up as shown in figure 1 and the pointer was fixed at the lower end of the spring such that it moved slightly over the vertical meter scale.

PART A

1. The “dead load” M_0 , was recorded from the pointer when no mass was added to the mass hanger.
2. Loads were added to the mass hanger and the readings of the pointer were recorded for each load (20g, 25g, 30g, 35g and 40g).
3. The extension was obtained by subtracting the reading of the pointer of one load with the “dead load”.

PART B

1. The pointer was removed from the mass hanger
2. A load M (25g) was added to the mass hanger and was set in vertical vibration by giving it a small additional downward displacement
3. The times for 20 vibrations were taken twice.
4. Step 2 and 3 were repeated with different loads added to M respectively (10g, 15g, 20g, 25g and 30g).
5. The masses of the spring and mass hanger were taken.

DATA ANALYSIS

PART A

Table 3.1 Extension of suspended masses

Mass on the mass hanger (g)		Pointer readings (mm)		Mass suspended on spring (g)	Extension E (mm)	Load M=mg (N)
m ₀	0	X ₀	20	0	00	0.000
m ₁	20	X ₁	44	20	24	0.196
m ₂	25	X ₂	51	25	31	0.245
m ₃	30	X ₃	57	30	37	0.294
m ₄	35	X ₄	64	35	44	0.343
m ₅	40	X ₅	71	40	51	0.392

Converting the masses to newton

Load M=mg

$$m_0 = \left(0g \times \frac{1kg}{1000g}\right) * 9.8m/s^2$$

$$= 0.000N$$

$$m_2 = \left(25g \times \frac{1kg}{1000g}\right) * 9.8m/s^2$$

$$= 0.245N$$

$$m_4 = \left(35g \times \frac{1kg}{1000g}\right) * 9.8m/s^2$$

$$= 0.343N$$

$$m_1 = \left(20g \times \frac{1kg}{1000g}\right) * 9.8m/s^2$$

$$= 0.196N$$

$$m_3 = \left(30g \times \frac{1kg}{1000g}\right) * 9.8m/s^2$$

$$= 0.294N$$

$$m_5 = \left(40g \times \frac{1kg}{1000g}\right) * 9.8m/s^2$$

$$= 0.392N$$

Extension

$$X_1 = (20-20)mm$$

$$= 000mm$$

$$X_1 = (51-20)mm$$

$$= 31mm$$

$$X_2 = (44-20)mm$$

$$= 24mm$$

$$X_2 = (57-20)mm$$

$$= 37mm$$

$$X_1 = (64-20)\text{mm}$$

$$=44\text{mm}$$

$$X_2 = (71-20)\text{mm}$$

$$=51\text{mm}$$

$$\text{Slope} = \frac{a}{b} = \frac{E}{Mg} \rightarrow k = \frac{1}{\text{slope}} \text{ N/m}$$

From the graph

$$\text{Slope} = \frac{(49-30)(10^{-3})\text{m}}{(0.4-0.24)\text{N}} = 0.119\text{N/m}$$

$$k = \frac{1}{0.119}$$

+

$$=8.40\text{N/m}$$

PART B

Mass of the mass hanger, $m = 5\text{g}$

Mass of the spring, $s = 2.7\text{g}$

Effective mass of the spring, $s/3 = 0.9\text{g}$

Mass of pointer, $p = 11.8\text{g}$

Table 3.2 Time of vibrations for different loads.

Load M (g)	M + m + p + s/3 (g)	Time for 20 vibrations set 1(sec) set 2		Mean time for 20 vibrations (sec)	Time for 1 vibration (sec)	T^2 (sec) ²
25	42.7	8.95	9.14	9.05	0.453	0.205
35	52.7	9.64	9.34	9.49	0.475	0.227
40	57.7	10.08	10.10	10.09	0.505	0.255
45	62.7	10.20	10.56	10.38	0.519	0.270
50	67.7	11.15	11.10	11.13	0.558	0.311
55	72.7	11.43	11.49	11.46	0.573	0.328

$$M + m + p + \frac{s}{3} \text{ (g)}$$

- a) $25+5+11.8+0.9= \mathbf{42.7g}$
- b) $35+5+11.8+0.9= \mathbf{52.7g}$
- c) $40+5+11.8+0.9= \mathbf{57.7g}$
- d) $45+5+11.8+0.9= \mathbf{62.7g}$
- e) $50+5+11.8+0.9= \mathbf{67.7g}$
- f) $55+5+11.8+0.9= \mathbf{72.7g}$

$$\text{Time for 1 vibration (sec)} = \frac{\text{mean time}}{20}$$

$$\text{i)} \quad \frac{9.05}{20} = \mathbf{0.453}$$

$$\text{ii)} \quad \frac{9.49}{20} = \mathbf{0.475}$$

$$\text{iii)} \quad \frac{10.09}{20} = \mathbf{0.505}$$

$$\text{iv)} \quad \frac{10.38}{20} = \mathbf{0.519}$$

$$\text{v)} \quad \frac{11.15}{20} = \mathbf{0.558}$$

$$\text{vi)} \quad \frac{11.46}{20} = \mathbf{0.573}$$

$$\text{Slope} = \frac{b}{a} = \frac{(m+m+p+\frac{s}{3})}{T^2} \rightarrow k = 4\pi^2(\text{slope}) \text{ N/m}$$

(3.3)

From the graph

$$\text{Slope} = \frac{(77-50)(10^{-3})\text{kg}}{(0.35-0.225)\text{s}^2} = \mathbf{0.216\text{N/m}}$$

$$k = 4\pi^2(0.216)$$

$$= \mathbf{8.53\text{N/m}}$$

The values of k in both A and B are almost equal with a difference of 0.13N/m.

Discussion

In part A, a spring was hung vertically with a mass hanger attached to the lower end of the spring and masses from 20g-50g were added. The down location of the spring was measured once it came to rest. A graph of extension versus the magnitude of displacement resulted in the expected straight line in the range of forces examined and is consistent with Hooke's law, the slope of this line is 0.119N/m and from this the spring constant was calculated and found to be 8.40N/m.

In part B, k was determined dynamically using the period of an oscillating mass. The time for twenty oscillations was measured for five different masses; for each mass the period of oscillation was measured two times using different oscillation amplitudes. The period of the mass oscillating vertically on a spring depends on the spring constant and the mass of the oscillating object but not on the amplitude of oscillation. The measurements confirmed that the amplitude of oscillation, within experimental uncertainty, did not affect the period. A graph of $M + m + p + \frac{s}{3}$ (g) against T^2 is a straight line and consistent with the theory that the period is a function of the effective mass of the spring and the spring constant of the spring, the slope of the graph was found to be 0.216N/m and the spring constant 8.53N/m.

Conclusion

The amount the spring stretches plotted against the mass added to the hanger gives a straight line that goes through the origin. This means that the extension of the spring is directly proportional to the stretching force applied. Besides measuring the spring constant using two very different methods, we verified the linear relationship between period squared and load for a vertically oscillating spring and observed that the amplitude of the oscillations didn't affect the period. Thus the experiment was a success.

Reference

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