TORQUE AND EQUILIBRIUM

AIM: To investigate the turning effect of forces (torque) using a pivoted meter rule and known unequal masses, and to use these to determine a value for the mass of the rule by applying the conditions for static equilibrium.

APPARATUS: Meter Rule, pivot, known masses and spring balance.

THEORY:

The torque of a force about a point O is the product of the magnitude of the force and the perpendicular distances (d) between the point O and line of action of the force.

$$T = f.d \qquad (4.1)$$

For a body to be in equilibrium under the action of two or more forces;

1. The vector sum of the forces must be zero, and
2. The sum of the torques about any point in the clockwise direction is equal to the sum of the torque about the same point in the anti-clockwise direction.

The weight of a mass is the force exerted by the earth on the mass and this weight acts vertically downwards. The magnitude of the forces is

$$W = Mg \qquad (3.2)$$

Where M is the mass in kg, g is in ms^{-2} ($9.8ms^{-2}$), and W is in Newton. Note that in many experiments, it is not always necessary to calculate the Mg since g will often cancel out from both sides of the equation.

PROCEDURE

For all parts of the experiment, the meter rule was placed on the pivot with the scale side up. The known masses M_1 and M_2 (which were not equal), made of small slotted brass weights, were hung under the meter rule with pieces of string.

1. The meter rule was placed on the pivot and the center on gravity was determined.
2. The meter rule was pivoted at its center of gravity. M_1 was hung at a distance x_1, and M_2 at a distance x_2 from the center of gravity G until it was balanced. The values M_1, x_1, M_2 and x_2 were noted and the torques were calculated.
3. With the experiment as in part 2 (Figure A), X_1 was doubled and the new position of M_2 which was giving equilibrium was found.

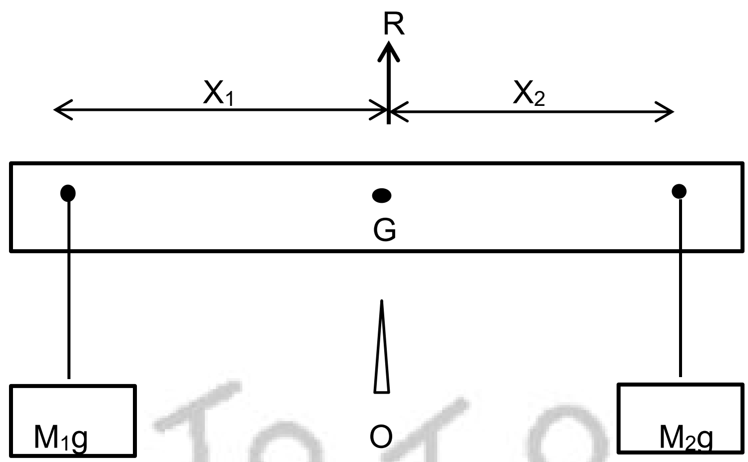

Figure A

4. The rule was pivoted at point O a distance d from the center of gravity G (figure B). The position of M_1 which gave equilibrium was found. Taking torques about O :

$$(M_1\, g)\, x_1 = (M\, g)\, d \qquad (3.3)$$

where M is the mass of the meter rule.

X_1 and d was measured and the value for M was found. It was repeated for 5 other values of d (and hence x_1) so there were six separate values of M determined. the data was recorded in a table.

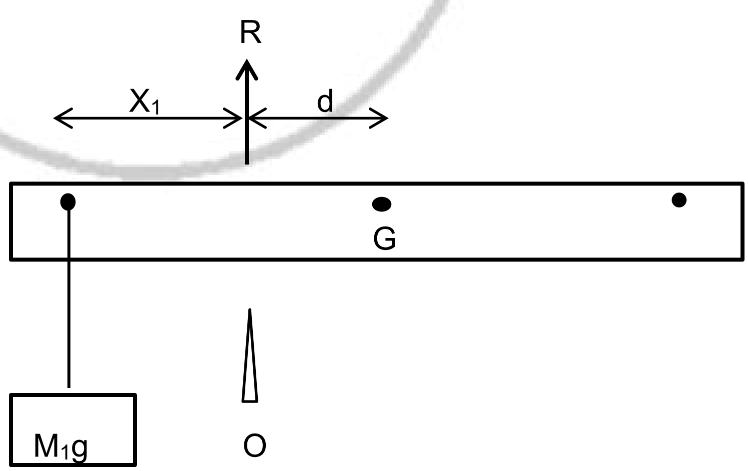

Figure B

5. The meter rule was pivoted again at a distance d from the center of gravity of the meter rule. Using M_1 and M_2 , positions for the two masses which give equilibrium were found (figure c). With the arrangement in equilibrium, the torques were taken about pivot O:

$$(M_1\, g)x_1 = M\, g\, d\ + M_2\, g\, x_2 \tag{3.4}$$

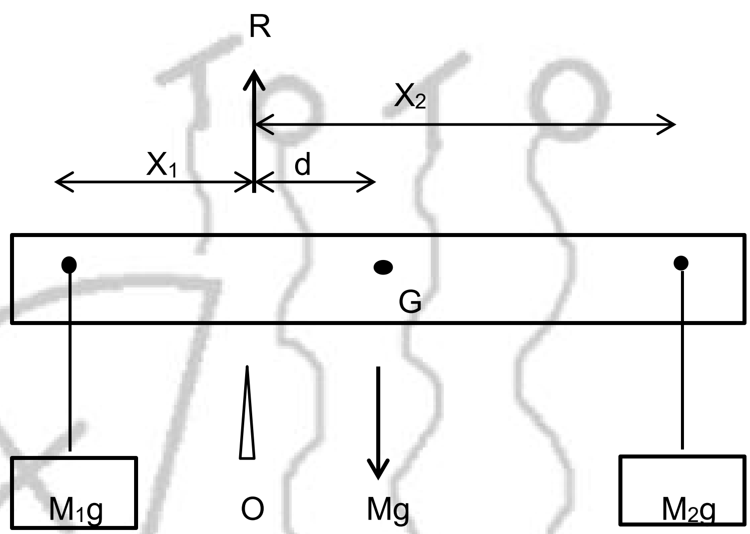

Figure C

O = pivot

G = center of gravity

R = force exerted on the rule by the pivot

M = mass of the meter rule

6. In each arrangement, (figure A, B and C) the pivot exerted an upward force on the metre rule, R. To find the value of this force, the rule was suspended from a spring balance and M_1 and M_2 were positioned on the rule such that the rule and the masses were in equilibrium. The reading on the spring balance was noted. The value of M was determined and compared with the values from 4 and 5.s

DATA ANALYSIS

1. Centre of Gravity = **<u>50.0cm</u>**
2. M_1= 50g

   X_1=10.5cm

   M_2=40g

   X_2=13.2cm

   For the first torque M_1=f and x_1=d. The units for Torque (T) are Nm and so units for M_1 and x_1 must be converted to S.I units.

$$\frac{50g}{1} \times \frac{1kg}{1000g} \times \frac{9.8N}{1kg} = 0.49N$$

Similarly;

$$\frac{10.5cm}{1} \times \frac{1m}{100cm} = 0.105m$$

From equation (3.1)

$$T = fd$$

$$T = 0.49N \times 0.105m$$

$$\underline{T = \mathbf{0.05145} \approx \mathbf{0.05Nm}}$$

Similarly, for the second torque M_2=f and x_2=d. The units for Torque (T) are Nm. Therefore, units for M_2 and x_2 must be converted to S.I units.

$$\frac{40g}{1} \times \frac{1kg}{1000g} \times \frac{9.8N}{1kg} = 0.392N$$

Similarly;

$$\frac{13.2cm}{1} \times \frac{1m}{100cm} = 0.132m$$

From equation (3.1)

$$T = fd$$

$$T = 0.392N \times 0.132m$$

$$\underline{T = \mathbf{0.051744} \approx \mathbf{0.05Nm}}$$

The torque one the right side of the pivot and the left are equal to each other i.e. **0.05Nm**

3. When X_1 is doubled to 13.2 cm and the masses are still the same, the new value for X_2 is equal to 26.4cm. The new value is approximately two times the one found in part 2. Doubling x_1 means x_2 also has to double for the meter rule to attain equilibrium.

4. **Trial 1**

When x_1=17.5cm=0.175m, M_1=50g=0.05kg and d=10cm=0.1m

$$(M_1 g)x_1 = (Mg)d$$

$$(0.05 \times 9.8)0.175 = (M \times 9.8)0.1$$

$$0.49 \times 0.175 = 9.8M \times 0.1$$

$$0.08575 = 0.98M$$

$$M = \frac{0.08575}{0.98}$$

$$M = 0.0875kg$$

Converting kg to g

$$0.0875kg \times \frac{1000g}{1kg} = 87.5g$$

$$\underline{\therefore M = \mathbf{87.5g}}$$

Trial 2

When x_1=19.2cm=0.192m, M_1=50g=0.05kg and d=11cm =0.11m

$$(M_1 g)x_1 = (Mg)d$$

$$(0.05 \times 9.8)0.192 = (M \times 9.8)0.11$$

$$0.49 \times 0.192 = 1.078M$$

$$0.09408 = 1.078M$$

$$M = \frac{0.09408}{1.078}$$

$$M = 0.0872kg$$

Converting kg to g

$$0.0872kg \times \frac{1000g}{1kg} = 87.2g$$

$$\therefore \underline{M = \mathbf{87.2g}}$$

Trial 3

When x_1=22.8cm=0.228m, M_1=50g=0.05kg and d=13cm =0.13m

$$(M_1 g)x_1 = (\,Mg)d$$

$$(0.05 \times 9.8)0.228 = (M \times 9.8)0.13$$

$$0.49 \times 0.228 = 1.274M$$

$$0.11172 = 1.274M$$

$$M = \frac{0.11172}{1.274}$$

$$M = 0.087692kg$$

Converting kg to g

$$0.087692kg \times \frac{1000g}{1kg} = 87.7g$$

$$\therefore \underline{M = \mathbf{87.7g}}$$

Trial 4

When x_1=26.2cm=0.262m, M_1=50g=0.05kg and d=15cm =0.15m

$$(M_1 g)x_1 = (Mg)d$$

$$(0.05 \times 9.8)0.262 = (M \times 9.8)0.15$$

$$0.49 \times 0.262 = 1.47M$$
$$0.12838 = 1.47M$$
$$M = \frac{0.12838}{1.47}$$
$$M = 0.08733kg$$

Converting kg to g

$$0.08733kg \times \frac{1000g}{1kg} = 87.3g$$
$$\underline{\therefore M = \mathbf{87.3g}}$$

Trial 5

When x_1=27.9cm=0.279m, M_1=50g=0.05kg and d=16cm =0.16m

$$(M_1 g)x_1 = (Mg)d$$
$$(0.05 \times 9.8)0.279 = (M \times 9.8)0.16$$

$$0.49 \times 0.279 = 1.568M$$
$$0.16268 = 1.568M$$
$$M = \frac{0.13671}{1.568}$$
$$M = 0.0871875kg$$

Converting kg to g

$$0.0871875kg \times \frac{1000g}{1kg} = 87.2g$$
$$\underline{\therefore \mathrm{M} = \mathbf{87.2g}}$$

Trial 6

When x_1=29.6cm=0.296m, M_1=50g=0.05kg and d=17cm =0.17m

$$(M_1 g)x_1 = (Mg)d$$
$$(0.05 \times 9.8)0.296 = (M \times 9.8)0.17$$

$$0.49 \times 0.296 = 1.666M$$

$$0.14504 = 1.666M$$

$$M = \frac{0.14504}{1.666}$$

$$M = 0.08706kg$$

Converting kg to g

$$0.08706kg \times \frac{1000g}{1kg} = 87.1g$$

$$\underline{\therefore M = \mathbf{87.1g}}$$

Position of O (cm)	Position of G (cm)	Position of M_1 (cm)	d (cm)	X_1 (cm)	M (g)
40.0	50.0	22.5	10.0	17.5	87.5
39.0	50.0	19.8	11.0	19.2	87.2
37.0	50.0	14.2	13.0	22.8	87.7
35.0	50.0	8.8	15.0	26.2	87.3
34.0	50.0	6.1	16.0	27.9	87.2
33.0	50.0	3.4	17.0	29.6	87.1

The six values of M are **not** all the same. These variations are due to uncertainties (experimental errors) such as parallax errors and difficulties in positioning of the pivot and the mass and in reading the metre rule. These errors contribute to an experimental error in each value of M.

The best value for the mass of the metre rule is the mean of the six values of M. Hence the mean is;

The mean $\sum m$

From equation (1.3)

$$\sum m = (87.5 + 87.2 + 87.7 + 87.3 + 87.2 + 87.1)\mathbf{g}$$

$$\sum \boldsymbol{m} = \mathbf{524g}$$

(i) mean m ($\overline{m}$)

From equation (1.2)

$$\text{mean m} = \overline{m} = \frac{1}{\mathrm{N}}\sum_{\mathrm{i}=1}^{\mathrm{N}} m_{\mathrm{i}}$$

$$\text{where N} = 6,\ \sum_{\mathrm{i}=1}^{\mathrm{N}} m_{\mathrm{i}} = m_1 + m_2 + \ldots\ldots\ldots + m_6 = \sum m = 180$$

$$\therefore\ \overline{m} = \frac{1}{6}(524)$$

$$\underline{\therefore\ \overline{\boldsymbol{m}} = \mathbf{87.3g}}$$

i. The Mean Deviation

From equation (1.5) $\overline{\delta m} = \frac{1}{N}\sum |(m_i - \overline{m})|$

$$\therefore\ \overline{\delta m} = \frac{1}{6}[|87.5 - 87.3| + |87.2 - 87.3| + |87.7 - 87.3| + |87.3 - 87.3| + |87.2 - 87.3| + |87.1 - 87.3|]$$

$$\overline{\delta v} = \frac{1}{7}(0.2 + 0.1 + 0.4 + 0 + 0.1 + 0.2)$$

$$\overline{\delta m} = \frac{1}{6}(1)$$

$$\overline{\delta m} = \mathbf{0.167g}$$

ii. **The Maximum Deviation**

Maximum deviation is the most deviated figure. This is when│87.7─87.3│ =**0.4g**

iii. **The Standard Deviation**

$$\text{standard deviation } \sigma = \sqrt{\frac{1}{(\mathrm{N}-1)}\sum(\mathrm{M}-\overline{\mathrm{M}})^2}$$

$$= \sqrt{\frac{1}{6-1}\sum[(0.2)^2+(0.1)^2+(0.4)^2+(0)^2+(0.1)^2+(0.2)^2]}$$

$$= \sqrt{\frac{1}{5}\sum(0.04+0.01+0.16+0+0.01+0.04)}$$

$$= \sqrt{\frac{1}{5}(0.26)}$$

$$= \sqrt{0.052}$$

$$= \mathbf{0.23}\mathrm{g}$$

5. M_1=50g=0.05kg, M_2=40g=0.04kg x_1=32.0cm=0.32m, x_2=6.9cm=0.069m, d=15cm=0.15m, g=9.8m/s^2

$$(M_1 g)x_1 = Mgd + M_2 g x_2$$

$$(0.05\times 9.8)0.32 = (M\times 9.8\times 0.15) + (0.04\times 9.8\times 0.69)$$

$$0.1568 = 1.47M + 0.027048$$

$$1.47M = 0.1568 - 0.027048$$

$$M = \frac{0.129752}{1.47}$$

$$M = 0.088266kg$$

Converting kg to g

$$0.088266kg \times \frac{1000g}{1kg} = 88.3g$$

$$\underline{\therefore \mathbf{M = 88.3g}}$$

6. Reading on the spring balance for the arrangement in figure a R= **<u>1.74N</u>**

   Upward forces = Downward forces

   R = Weight of the rule + Weight of masses

   Weight of rule = R — weight of masses

   Weight of rule = 1.74N — [(0.05 + 0.04) × 9.8]

   Weight of rule = 1.74N — 0.882N

   Weight of rule = 0.858N

   By converting weight to mass, from equation (3.2) we have;

$$W = Mg$$

$$M = \frac{W}{g}$$

$$M = \frac{0.858}{9.8}$$

$$M = 0.087551kg$$

   Converting kg to g

$$0.087551kg \times \frac{1000g}{1kg} = 87.6g$$

M=87.6 g

b) Reading on the spring balance for the arrangement in figure b R = **<u>1.33N</u>**

   Upward forces = Downward forces

   R = Weight of the rule + Weight of mass

   Weight of rule = R — weight of mass

   Weight of rule = 1.33N — [(0.05) × 9.8]

   Weight of rule = 1.33N — 0.49N

   Weight of rule = 0.84N

By converting weight to mass, from equation (3.2) we have;

$$W = Mg$$

$$M = \frac{W}{g}$$

$$M = \frac{0.84}{9.8}$$

$$M = 0.085714kg$$

Converting kg to g

$$0.085714kg \times \frac{1000g}{1kg} = 85.7g$$

$$\underline{\therefore M = \mathbf{85.7g}}$$

c) Reading on the spring balance for the arrangement in figure a = **<u>1.73N</u>**

Upward forces = Downward forces

R = Weight of the rule + Weight of masses

Weight of rule = R — weight of masses

Weight of rule = 1.73N — [(0.05 + 0.04) × 9.8]

Weight of rule = 1.73N — 0.882N

Weight of rule = 0.848N

By converting weight to mass, from equation (3.2) we have;

$$W = Mg$$

$$M = \frac{W}{g}$$

$$M = \frac{0.848}{9.8}$$

$$M = 0.086531kg$$

Converting kg to g

$$0.086531kg \times \frac{1000g}{1kg} = 86.5g$$

$$\underline{\therefore M = \mathbf{86.5g}}$$

The values of M found by using the values of R which was determined using the spring balance are close to the values found in 4 and 5**.** The two values for part 4 and 5 are close to the actual value found by taking the measurement from the spring balance. Hence, the error was quite small in the both arrangements**.**

DISCUSSION

In the first part of the experiment the metre was pivoted at its centre of gravity, the mass M_1 was hung at a distance x_1, and M_2 at a distance x_2 from the center of gravity G until it was balanced. The values M_1, x_1, M_2 and x_2 were noted and the torques were calculated. The results showed tat the torque on the left was equal to the torque on the right which was 0.05Nm.It was also found that if the distance is doubled on one side of the system then also the distance on the other side has to double for the meter rule to re-attain equilibrium.

In the second part of the experiment from the principle of moment which states that moments on the right are equal to moments on the left side the value of M for each of the six trials was determined. The six values of M are **not** all the same. These variations are due to uncertainties (experimental errors) such as parallax errors and difficulties in positioning of the pivot and the mass and in reading the metre rule. These errors contribute to an experimental error in each value of M. The best value for the mass of the metre rule is the mean of the six values of M which was found to be 87.3 g with a mean deviation of 0.167 g and a maximum deviation of 0.4 g.The standard deviation was found to 0.23 g.

In the third part of the experiment the mass M was found to be 88.3 g using the principles of moments. The experimental errors arose from things such as difficulties in keeping the system in balance and taking readings. And can be minimized by repeating the experiment and being as accurate as possible when taking the readings.

CONCLUSION

In conclusion, the experiment was a major success, this is because it accurately proved the that when taken from the pivot, the left side moments should be equal to the right moments and that the vector sum of the forces has to be equal to zero for the rule to be attain static equilibrium. The mass of the rule which was found using the spring balance was very close to that found using the spring balance which therefore proved the consistency of the principle of moments.

REFERENCES

P.C. Simpemba, J. Simfukwe and M. Chengo, *PH 110 Laboratory Manual*, (2016), School of Mathematics and Natural Sciences, Department of Physical Sciences, Copperbelt University, Kitwe, Zambia.