<u>TITLE</u>: <u>GRAPHICAL AND ERROR ANALYSIS</u>

<u>AIM</u>: To analyse the motion of a car

APPARATUS: Not Applicable

THEORY

Analytical Representation of Random and Systematic Errors

In measuring any value, the result is not just one number, such as 5.3 cm. It is two numbers, 5.3±0.1 cm. The second number is the experimental uncertainty, or error bar. It usually represents one standard deviation (one sigma) from the first value.

All measurements are subject to some uncertainty (errors). Some of the errors are: personal bias, random errors and systematic errors. As such, it is recommended that repeated measurements are conducted upon which statistical analysis is performed to validate the measurements.

Consider N independent measurements made of the same quantity x. Let the quantities be designated as x_1......x_i......x_N. The mean of these measurements in $\bar{x}$ is given by

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i \qquad (1.2)$$

Where

$$\sum_{i=1}^{N} x_i = x_1 + x_2 \ldots\ldots\ldots. + x_N \qquad (1.3)$$

The difference between every measurement x_i and the mean value $\bar{x}$ is referred to as a *deviation* or residual δx_i and is given by

$$\delta x_i = (x_i - \bar{x}) \qquad (1.4)$$

A better estimate of the uncertainty (experimental error) in the mean is given by the mean deviation which is the mean of the moduli of N deviations, or by the **standard error.**

$$\text{mean deviation} = \overline{\delta x} = \frac{1}{N}\sum |(x_i - \bar{x})| \qquad (1.5)$$

$$\text{standard deviation } \sigma = \sqrt{\frac{1}{(N-1)}\sum(x_i - \bar{x})^2} \qquad (1.6)$$

The error in a measured quantity is conveniently expressed as a percent of the quantity itself. Given the true or known value of the quantity x, the percentage error is given by

$$\%\text{ error} = \frac{x_i - \bar{x}}{x} \times 100\% \qquad (1.7)$$

If the true value is not provided, a percentage deviation of the mean is evaluated as

$$\%\text{ error} = \frac{\overline{\delta x}}{\bar{x}} \times 100\% \qquad (1.8)$$

If a quantity is **raised to the power n**, then the percentage error is **multiplied by n.**

The absolute error is obtained by multiplying the percentage error by the quantity itself and dividing by 100. That is

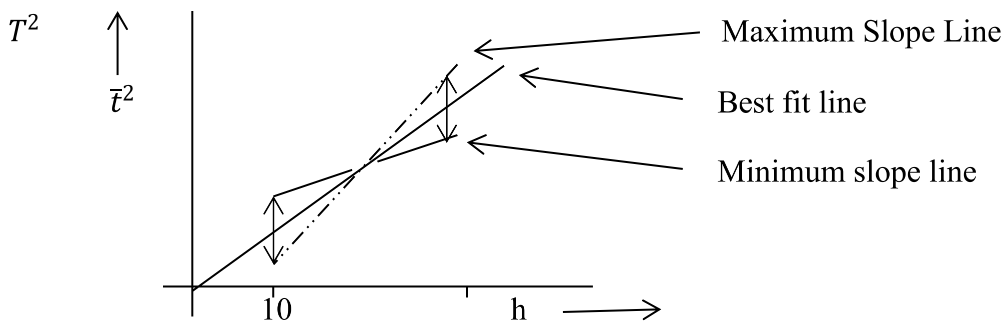
$$\text{Absolute Error} = \frac{(\%\text{ error in x}) \times x}{100} \qquad (1.9)$$

Graphical Presentation of Errors

The measured time of fall T for various lengths of the stone *l* can be used to find *g*. But we cannot average all measurements of T into one mean value plus error and all measurements of *l* into one mean value plus error and use these values to calculate a mean g, since *T* and *l* are different in each measurement. This is because we would not be sampling the same quantity each time and so, statistical error analysis, as dealt with previously for random errors, does not apply.

One approach to this is to calculate a value of g for each *T* and *l,* giving a list of g results, and then determine a mean g with associated error at the end. While this works, it is inefficient, and a graphical approach to this problem is more sensible.

Each data point plotted on the graph has an associated error determined either statistically or from observation of the resolution of the measuring instrument. Thus a value for T^2 of $100s^2$ plotted on the y axis may have an error of$\pm 5s^2$. This error needs to be represented on the graph for each data point *before a proper slope can be drawn.* The error range is represented by drawing a vertical error bar about the data point, i.e. from $T^2 = 95s^2$ to $T^2 = 105s^2$. This is done for every point plotted, on both x and y axes if necessary.



The error in the slope is then determined by drawing a line of maximum slope m_{max} through the plotted points and their error ranges and line of minimum slope m_{min}. The average slope M is

$$M = (m_{max} + m_{min})/2 \qquad (1.14)$$

DATA COLLECTION

Table 1: Sampled speed of a car for three trips

No.	Trip 1 (m/s)	Trip 2 (m/s)	Trip 3 (m/s)
1	25	25	25
2	27	27	25
3	27	26	27
4	25	26	27
5	26	26	25
6	25	26	25
7	25	27	27

Table 2: Time of fall of a stone from different heights

Height h (cm)	Time of fall T(s)	$T^2 (s^2)$
100.0	0.4474	0.20016676
120.0	0.4922	0.24226084
130.0	0.5109	0.26101881
140.0	0.5321	0.28313041
150.0	0.5493	0.30173049
160.0	0.5673	0.32182929

DATA ANALYSIS

Table 1

<u>QUESTION 1</u>

<u>For Trip 1</u>

(i) $\sum v$

From equation (1.3)

$$\sum \mathrm{v} = (25 + 27 + 27 + 25 + 26 + 25 + 25)m/s$$

$$\sum \boldsymbol{v} = \underline{\mathbf{180m/s}}$$

(ii) mean v ($\bar{\mathrm{v}}$)

From equation (1.2)

$$\text{mean v} = \bar{\mathrm{v}} = \frac{1}{\mathrm{N}}\sum_{\mathrm{i}=1}^{\mathrm{N}} \mathrm{v_i}$$

$$\text{where N} = 7,\ \sum_{\mathrm{i}=1}^{\mathrm{N}} \mathrm{v_i} = \mathrm{v_1} + \mathrm{v_2} + \ldots\ldots\ldots + \mathrm{v_7} = \sum \mathrm{v} = 180$$

$$\therefore\ \bar{v} = \frac{1}{7}(180)$$

$$\underline{\therefore\ \bar{v} = \mathbf{25.7\ m/s}}$$

(iii) v^2

No.	**V (m/s)**	$\boldsymbol{v^2}$ $\boldsymbol{(m^2/s^2)}$
1	25	625
2	27	729
3	27	729
4	25	625
5	26	676
6	25	625
7	25	625

$$(\bar{v})^2 = (25.7)^2$$
$$\underline{(\bar{v})^2} = \underline{\mathbf{660.5 m/s}}$$

<u>For Trip 2</u>

(i) $\sum v$

From equation (1.3)

$$\sum v = (25 + 27 + 26 + 26 + 26 + 26 + 27) m/s$$

$$\sum \boldsymbol{v} = \underline{\mathbf{183 m/s}}$$

(ii) mean v ($\bar{v}$)

From equation (1.2)

$$\text{mean } v = \bar{v} = \frac{1}{N}\sum_{i=1}^{N} v_i$$

$$\text{where } N = 7, \sum_{i=1}^{N} v_i = v_1 + v_2 + \ldots\ldots\ldots + v_7 = \sum v = 183$$

$$\therefore\ \bar{v} = \frac{1}{7}(183)$$

$$\bar{v} = 26.14285714$$

$$\underline{\therefore}\ \underline{\bar{v}} = \underline{\mathbf{26.1\ m/s}}$$

(iii) v^2

No.	V (m/s)	v^2 (m^2/s^2)
1	25	625
2	27	729
3	26	676
4	26	676
5	26	676
6	26	676
7	27	729

$$(\bar{v})^2 = (26.1)^2$$
$$\overline{(v)^2 = \mathbf{681.2}}(\mathbf{m/s}$$

<u>For Trip 3</u>

(i) $\sum v$

From equation (1.3)

$$\sum \mathrm{v} = (25 + 25 + 27 + 27 + 25 + 25 + 27)\mathrm{m/s}$$

$\sum \boldsymbol{v}$ = **<u>181m/s</u>**

(ii) mean v ($\bar{v}$)

From equation (1.2)

$$\text{mean v} = \bar{v} = \frac{1}{N}\sum_{i=1}^{N} v_i$$

$$\text{where } N = 7, \sum_{i=1}^{N} v_i = v_1 + v_2 + \ldots\ldots\ldots + v_7 = \sum v = 181$$

$$\therefore\ \bar{v} = \frac{1}{7}(181)$$

$$\bar{v} = 25.85714286$$

$$\underline{\therefore\ \bar{v} = \mathbf{25.9\ m/s}}$$

(iii) v^2

No.	V (m/s)	v^2 (m^2/s^2)
1	25	625
2	25	625
3	27	729
4	27	729
5	25	625
6	25	625
7	27	729

$$(\bar{v})^2 = (25.9)^2$$
$$\underline{(\bar{v})^2 = \mathbf{670.8m/s}}$$

QUESTION 2

For Trip 1

1. v

(i) Percentage Error in v

From equation (1.7) Percentage Error $= \frac{\overline{\delta v}}{\bar{v}} \times 100\%$

And from equation (1.5) $\overline{\delta v} = \frac{1}{N}\sum |(v_i - \bar{v})|$

$$\therefore \overline{\delta v} = \frac{1}{7}[|25-25.7| + |27-25.7| + |27-25.7| + |25-25.7| + |26-25.7| + |25-25.7| + |25-25.7|]$$

$$\overline{\delta v} = \frac{1}{7}(0.7 + 1.3 + 1.3 + 0.7 + 0.3 + 0.7 + 0.7)$$

$$\overline{\delta v} = \frac{1}{7}(5.7)$$

$$\underline{\overline{\delta v} = \mathbf{0.81}}$$

$$\text{Percentage Error} = \frac{\overline{\delta v}}{\bar{v}} \times 100\%$$

$$\text{Percentage Error} = \frac{0.81}{25.7} \times 100\%$$

$$\underline{\therefore \textbf{Percentage Error in v} = \mathbf{3.2\%}}$$

(ii) Absolute Error

From equation (1.9) Absolute Error $= \frac{(\%error\ in\ v)\times v}{100}$

$$\text{Absolute Error} = \frac{3.2\% \times 25.7\text{m/s}}{100\%}$$

$$\underline{\textbf{Absolute Error} = \mathbf{0.81m/s}}$$

2. v^2

(i) Percentage Error in v^2

From equation (1.7) Percentage Error $= \frac{\overline{\delta v^2}}{\overline{v^2}} \times 100\%$

And from equation (1.5) $\overline{\delta v^2} = \frac{1}{N}\sum \left|\left(v^2{}_i - \overline{v^2}\right)\right|$

∴ $\overline{\delta v^2} = \frac{1}{7}$[|625-660.5|+|729-660.5|+|729-660.5|+|625-660.5|+|676-660.5|+|625-660.5|+|625-660.5|]

$$\overline{\delta v^2} = \frac{1}{7}(35.5 + 68.5 + 68.5 + 35.5 + 15.5 + 35.5 + 35.5)$$

$$\overline{\delta v^2} = \frac{1}{7}(294.5)$$

$$\underline{\overline{\boldsymbol{\delta v^2}} = \mathbf{42.1}}$$

Percentage Error $= \frac{\overline{\delta v^2}}{\overline{v^2}} \times 100\%$s

$$\text{Percentage Error} = \frac{42.1}{\underline{660.5}} \times 100\%$$

$$\underline{\therefore \textbf{Percentage Error in } \boldsymbol{v^2} = \mathbf{6.4\%}}$$

(ii) Absolute Error

From equation (1.9) Absolute Error $= \frac{(\%error\ in\ v^2)\times v^2}{100}$

$$\text{Absolute Error} = \frac{6.3\% \times 660.5\text{m/s}}{100\%}$$

$$\underline{\textbf{Absolute Error} = \mathbf{42.1m/s}}$$

<u>For Trip 2</u>

1. v

(i) Percentage Error in v

From equation (1.7) Percentage Error $= \frac{\overline{\delta v}}{\bar{v}} \times 100\%$

And from equation (1.5) $\overline{\delta v} = \frac{1}{N}\sum |(v_i - \bar{v})|$

$$\therefore \overline{\delta v} = \frac{1}{7}[|25 - 26.1| + |27 - 26.1| + |26 - 26.1| + |26 - 26.1| + |26 - 26.1| + 26 - 26.1| + |27 - 26.1|]$$

$$\overline{\delta v} = \frac{1}{7}(1.1 + 0.9 + 0.1 + 0.1 + 0.1 + 0.1 + 0.9)$$

$$\overline{\delta v} = \frac{1}{7}(3.3)$$

$$\underline{\overline{\boldsymbol{\delta v}} = \mathbf{0.5}}$$

$$\text{Percentage Error} = \frac{\overline{\delta \mathrm{v}}}{\overline{\mathrm{v}}} \times 100\%$$

$$\text{Percentage Error} = \frac{0.5}{26.1} \times 100\%$$

$$\underline{\therefore \textbf{Percentage Error in v} = \mathbf{1.9\%}}$$

(ii) Absolute Error

From equation (1.9) $\text{Absolute Error} = \frac{(\%error\ in\ v) \times v}{100}$

$$\text{Absolute Error} = \frac{1.9\% \times 26.1\mathrm{m/s}}{100\%}$$

$$\underline{\textbf{Absolute Error} = \mathbf{0.5m/s}}$$

2. v^2

(i) Percentage Error in v^2

From equation (1.7) $\text{Percentage Error} = \frac{\overline{\delta v^2}}{\overline{v^2}} \times 100\%$

And from equation (1.5) $\overline{\delta v^2} = \frac{1}{N} \sum \left| \left(v^2{}_i - \overline{v^2} \right) \right|$

$$\therefore \ \overline{\delta v^2} = \frac{1}{7}[|625 - 681.2| + |729 - 681.2| + |676 - 681.2| + |676 - 681.2| + |676 - 681.2| + |676 - 681.2| + |729 - 681.2|]$$

$$\overline{\delta v^2} = \frac{1}{7}(56.2 + 47.8 + 5.2 + 5.2 + 5.2 + 5.2 + 47.8)$$

$$\overline{\delta v^2} = \frac{1}{7}(172.6)$$

$$\underline{\overline{\boldsymbol{\delta v^2}} = \mathbf{24.7}}$$

$$\text{Percentage Error} = \frac{\overline{\delta v^2}}{\overline{v^2}} \times 100\%$$

$$\text{Percentage Error} = \frac{24.7}{681.2} \times 100\%$$

$$\underline{\therefore \textbf{Percentage Error in } \boldsymbol{v^2} = \mathbf{3.6\ \%}}$$

(ii) Absolute Error

From equation (1.9) $\text{Absolute Error} = \frac{(\%error\ in\ v^2)\times v^2}{100}$

$$\text{Absolute Error} = \frac{3.6\% \times 681.2\text{m/s}}{100\%}$$

$$\underline{\textbf{Absolute Error} = \mathbf{24.7 m/s}}$$

<u>For Trip 3</u>

1. v

(i) Percentage Error in v

From equation (1.7) $\text{Percentage Error} = \frac{\overline{\delta \text{v}}}{\overline{\text{v}}} \times 100\%$

And from equation (1.5) $\overline{\delta v} = \frac{1}{N}\sum |(v_i - \overline{v})|$

$$\therefore\ \overline{\delta v} = \frac{1}{7}[|25 - 25.9| + |25 - 25.9| + |27 - 25.9| + |27 - 25.9| + |25 - 25.9| + |25 - 25.9| + |27 - 25.9|]$$

$$\overline{\delta v} = \frac{1}{11}(0.9 + 0.9 + 1.1 + 1.1 + 0.9 + 0.9 + 1.1)$$

$$\overline{\delta v} = \frac{1}{7}(6.9)$$

$$\overline{\boldsymbol{\delta} \mathbf{v}} = \mathbf{0.99}$$

$$\text{Percentage Error} = \frac{\overline{\delta \text{v}}}{\overline{\text{v}}} \times 100\%$$

$$\text{Percentage Error} = \frac{0.99}{25.9} \times 100\%$$

$$\underline{\therefore \textbf{Percentage Error in v} = \mathbf{3.8\%}}$$

(ii) Absolute Error

From equation (1.9) $\text{Absolute Error} = \frac{(\%error\ in\ v)\times v}{100}$

$$\text{Absolute Error} = \frac{3.8\% \times 25.9\text{m/s}}{100\%}$$

$$\underline{\textbf{Absolute Error} = \textbf{0.99m/s}}$$

2. v^2

(i) Percentage Error in v^2

From equation (1.7) $\text{Percentage Error} = \frac{\overline{\delta v^2}}{\overline{v^2}} \times 100\%$

And from equation (1.5) $\overline{\delta v^2} = \frac{1}{N}\sum \left|\left({v^2}_i - \overline{v^2}\right)\right|$

$\therefore\ \overline{\delta v^2} = \frac{1}{7}[|625 - 670.8| + |625 - 670.8|+|729-670.8|+|729-670.8|+|625-670.8|+|625-670.8|+|729-670.8|]$

$$\overline{\delta v^2} = \frac{1}{11}(45.8 + 45.8 + 58.2 + 58.2 + 45.8 + 45.8 + 58.2)$$

$$\overline{\delta v^2} = \frac{1}{7}(357.8)$$

$$\overline{\boldsymbol{\delta}\mathbf{v}^2} = \mathbf{51.1m/s}$$

$$\text{Percentage Error} = \frac{\overline{\delta v^2}}{\overline{v^2}} \times 100\%$$

$$\text{Percentage Error} = \frac{51.1}{670.8} \times 100\%$$

$$\underline{\therefore \textbf{Percentage Error in } \boldsymbol{v^2} = \textbf{7.6\%}}$$

(ii) Absolute Error

From equation (1.9) $\text{Absolute Error} = \frac{(\%error\ in\ v^2)\times v^2}{100}$

$$\text{Absolute Error} = \frac{7.6\% \times 670.8\text{m/s}}{100\%}$$

$$\underline{\textbf{Absolute Error} = \textbf{51.1m/s}}—$$

Graph 1

Table 2: Time of fall of a stone from different height

i.T^2

Height h (m)	Time of fall (s)	$T^2(s)^2$
1	0.4474	0.20016676
1.2	0.4922	0.24226084
1.3	0.5109	0.26101884
1.4	0.5321	0.28313041
1.5	0.5493	0.30173049
1.6	0.5673	0.32182929

From points A and B,

$$\text{Average Speed} = \frac{\Delta y}{\Delta x}$$

$$\text{Average Speed} = \frac{1.6 - 1.2}{0.57 - 0.49}$$

$$\text{Average Speed} = \frac{0.4}{0.08}$$

$$\underline{\textbf{Average Speed} = \textbf{5 m/s}}$$

Graph 2

Maximum gradient is at points (0.2,0,825) and (0.32, 0.1775)

$$M_{max} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0.1775 - 0.825}{0.32 - 0.2}$$

$$= \frac{0.95}{0.12}$$

$$\mathbf{M_{max} = 7.9}$$

Minimum gradient is at points (0.2, 1.175) and (0.32,1.425)

$$M_{max} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1.425 - 1.175}{0.32 - 0.2}$$

$$= \frac{0.25}{0.12}$$

$$\mathbf{M_{max} = 2.1}$$

$$\boldsymbol{M} = \frac{\mathrm{M_{max}} + \mathrm{M}_{\boldsymbol{min}}}{\mathbf{2}}$$

$$= \frac{7.9 + 2.1}{\mathbf{2}}$$

$$= \frac{10}{2}$$

$$= \mathbf{5\ m/s^2}$$

$$\%\ \mathrm{error} = \frac{\overline{\alpha} - \alpha}{\alpha} \times 100$$

$$= \frac{9.81 - 5}{9.81} \times 100$$

$$\underline{\%\ \mathbf{Error} = \mathbf{49}\%}$$

DISCUSSION

Part 1: Analyzing motion of a car, several readings were taken so as to come up with the main value because some values are affected by both random and systematic errors in which the observer may have been biased when taking the readings. The average speed for trip 1 was 25.7m/s with a percentage error of 3.2%, hence absolute error 0.81,for trip 2, the average speed was 26.1m/s with percentage error of 1.9%, hence absolute error 0.5,and for trip 3 the average speed was 25.9 with a percentage error of 3.8%,hence absolute error 0.99m/s. These errors can be minimized further by reducing personal bias when taking readings/measurements and reduce on the rounding off of numbers when calculating the measurements.

Part 2: The acceleration of a falling object was taken at different heights so as to obtain the most accurate value but errors could have occurred when timing the falling object.

CONCLUSION

In all experiments of measurements, errors always occur, as shown in the entire experiment, which is why more than one experiment is done so as to obtain several values to come up with a mean value with the smallest percentage error possible.

Sampled speed of a car for three trips: The three trips had different percentage error. The errors were not large and it was seen that the absolute value and the mean deviation had the same value.

The stone falling from different heights: The percentage error was large and this can be attributed to errors in timing by the observer and the accuracy of the instrument used.

REFERENCES

F.Walusa and G.T Baliga, *PH 110 Laboratory Manual*, (2016), School of Mathematics and Natural Sciences, Department of Physical Sciences, Copperbelt University, Kitwe, Zambia.